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ICTP/WMO WORKSHOP ON EXTRA-TROPICAL AND TROPICAL
LIMITED AREA MODELLING
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"Lateral Boundary Conditions for Regional Models"

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LATERAL BOUNDARY CONDITIONS FOR REGIONAL MODELS

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OUTLINE

1) Introduction
2) Well-posedness theory
3) Popular lbc formulations
4) Interactive approach
5) Results

GENERAL STRATEGY

(1) REGIONAL FORECASTS

- early analysis for timeliness
- high res over limited area
- valid for limited time period

(2) HEMISPHERIC/GLOBAL FORECASTS

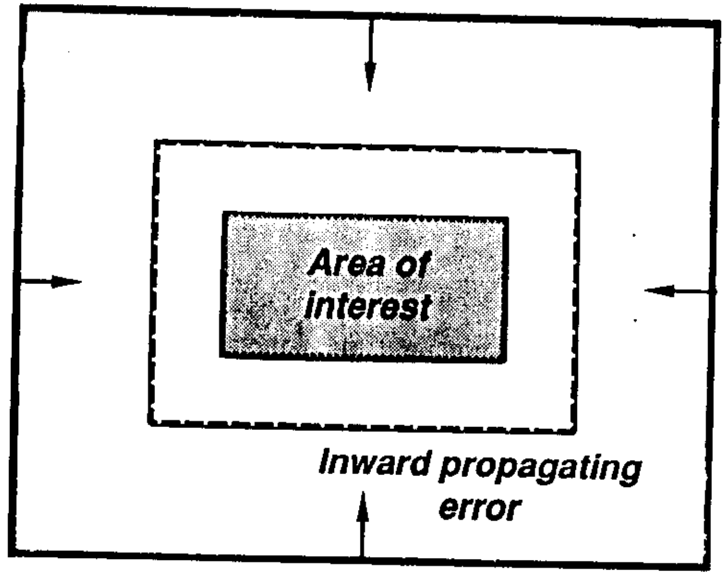
- later analysis with complete data
- lower resolution but larger domain
- valid for longer time periods

CANADIAN CONSTRAINTS

(1) A regional model for Canada is almost hemispheric (for 48 h forecasts)

(2) It is expensive (compared e.g. to European countries) because of size of country and adjacent waters

PROTECTING AN AREA OF INTEREST



■ Area of interest

■ Domain for 24 h forecast

■ Domain for 72 h forecast

REGIONAL MODELS

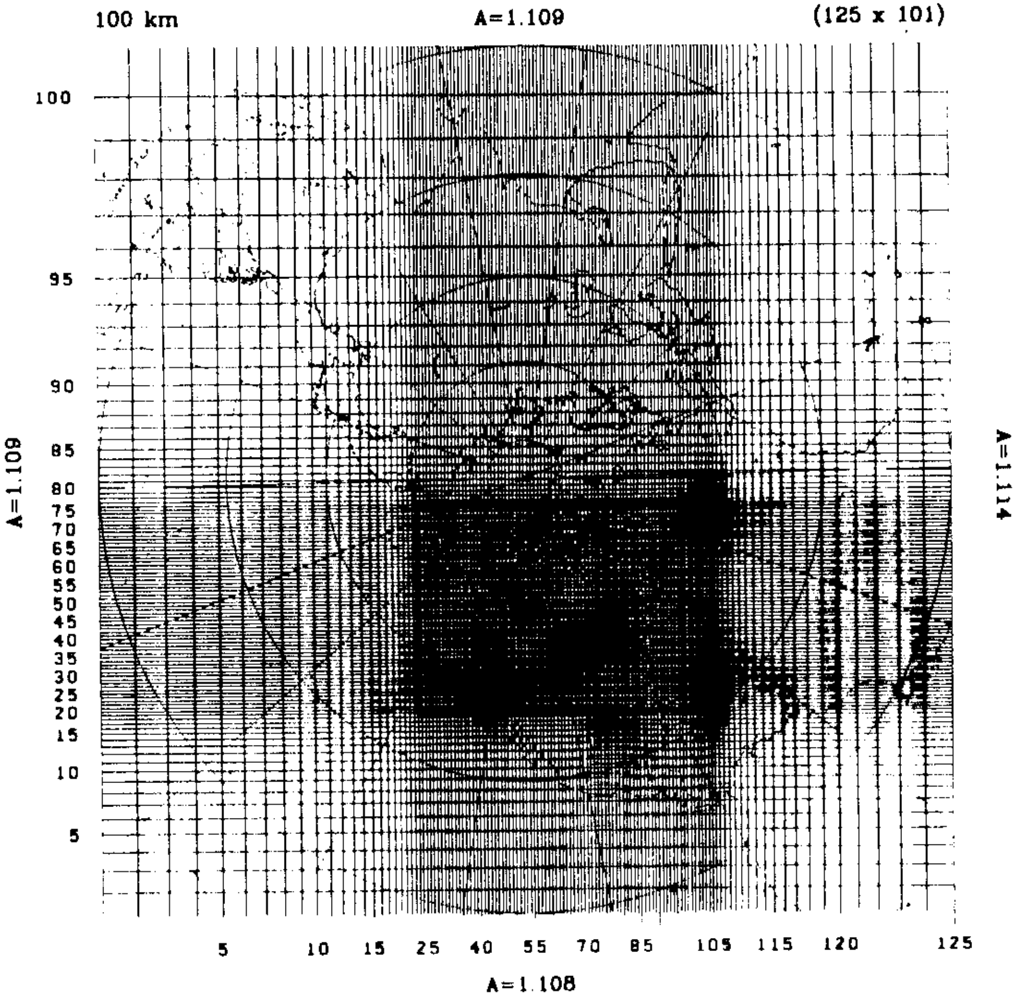
(1) Non-interactive

- needs driving model
- application of open b.c.'s difficult
- boundary-induced error must propagate at speed of Rossby modes, not external gravity modes (1 : 6 speed)

(2) Interactive

- high res / coarse res areas interact during forecast
- smoothly-varying res desirable to reduce problems at resolution interfaces

OPERATIONAL GRID



WELL-POSEDNESS THEORY
(FOR EXISTENCE & UNIQUENESS)

(Oliger & Sundstrom, SIAM J. Appl. Math., 1978)

Have to specify correct # and **type** of lbc's to get **well-posed** problem (whose sol'n over LA should match that of problem solved using periodic cond'ns over sphere)

Under-specification can typically lead to

- instability
- inconsistent approximations (i.e. sol'n of different set of pde's)
- problems of non-uniqueness

Over-specification can typically lead to

- *discontinuous** solution (numerically manifested as noise)
- *inward propagation* of noise from boundary at ***fastest signal speed*** of system (e.g. gravity-wave speed in baro~~clinic~~tropic & hydrostatic primitive equations)

* **Note:** For a *hyperbolic* system of equations, the solution at a point is entirely determined by its *upstream history*. In particular at outflow points. This solution can then contradict value specified at boundary, leading to a *discontinuous* solution..

INVISCID SETS

1) Euler equations of adiabatic gas dynamics

[Quasi-linear set of *hyperbolic* p.d.e.'s for (u,v,w,p,ρ)]

Rigid wall - 1 cond'n everywhere ($\mathbf{V.n} = 0$)

Open domain

Subsonic regions ($|\mathbf{V}|<\phi^{1/2}$):

inflow - 4 cond'ns
outflow - 1 cond'n

(but not obvious how to choose conditions)

Supersonic regions ($|\mathbf{V}|>\phi^{1/2}$):

inflow - 5 cond'ns (specify u,v,w,p,ρ)
outflow - 0

2) Shallow-water equations

Special case of Euler equations, still quasi-linear *hyperbolic* set

Rigid wall - 1 cond'n everywhere ($\mathbf{V.n} = 0$)

Open domain

Subsonic regions ($|\mathbf{V}|<\phi^{1/2}$):

inflow - 2 cond'ns
outflow - 1 cond'n

(there is a family of possible cond'ns)

Supersonic regions ($|\mathbf{V}|>\phi^{1/2}$):

inflow - 3 cond'ns (u,v,ϕ)
outflow - 0

3) **Hydrostatic primitive equations**
(Set is **no longer** *hyperbolic*)

Rigid wall - 1 cond'n everywhere (**V**.n = 0)

Open domain

"Local, pointwise boundary conditions cannot yield a well-posed problem for the open boundary problem for the hydrostatic primitive equ'ns"

VISCOUS SETS

1) **Viscous Euler equations**
(i.e. compressible Navies-Stokes equations)

Quasi-linear hyperbolic set
->*incomplete parabolic* set

Rigid wall - 4 cond'ns everywhere
(**V**.n=0 + family of 3 cond'ns)
(3 more than inviscid case)

Open domain
Have to avoid generating internal viscous boundary layers
=> formulation must reduce to well-posed set for inviscid case.

inflow - 5 cond'ns
(1 more than inviscid case)

outflow - 4 cond'ns
(3 more than inviscid case)

2) Shallow-water equations

Rigid wall - 2 cond'ns everywhere
(e.g. V=0)

Open domain

inflow - 3 cond'ns

outflow - 2 cond'ns

3) Hydrostatic primitive equations

Rigid wall - 3 cond'ns everywhere
(e.g. V=0?)

Open domain Still *ill-posed*

SUMMARY OF THEORY

Following holds for both *inviscid* and *viscous* cases.

Rigid wall

Well-posed: Euler, shallow-water and hydrostatic primitive equation sets.

Open domain

Well-posed: Euler and shallow-water sets.

Ill-posed: Hydrostatic primitive equation sets.

IMPLICATIONS OF WELL-POSEDNESS THEORY FOR NUMERICAL MODELS

1) Over-specification in *non-dissipative* systems

- leads to noise propagation from boundary at *fastest* signal speed of system (gravity-wave speed for barotropic and hydrostatic primitive equations).

2) Over-specification in *dissipative* systems

- also leads to noise propagation from boundary at fastest signal speed, but error is at least damped.

3) Introducing *viscosity*

- raises *order* of equations and # of boundary conditions

- usually introduces *fictitious* internal viscous boundary layers in the fluid. Have to ensure that this error only propagates inwards *slowly*.

4) Best can hope for is that *boundary-induced error propagates at the slowest signal speed* (usually the local wind speed).

5) If a given strategy for an *open* domain works well in a *shallow-water* model, won't necessarily work in a *hydrostatic primitive equations* one.

6) Any set of lbc's for an *open* domain should give a well-posed problem in the special case of a *rigid wall*.

ESTIMATING THE SIZE OF NUMERICAL BUFFER ZONES

Assuming that boundary-induced error propagates at the *local wind speed*, rather than at the much faster speed of the *fastest external gravity wave*,

and **assuming** a maximum-possible wind speed of 75 m/s,

then boundary-induced error can propagate inwards no more than **6500 km/ 24 hours** from an upstream boundary.

This is **considerable**! But still much smaller than if the error propagates as an external gravity wave.

POPULAR LBC STRATEGIES FOR *NON-INTERACTIVE* REGIONAL MODELS

Perkey-Kreitzberg (MWR 1976)

1) Specify time tendencies of prognostic variables in a boundary region of width 4Δx (from driving model).
2) Blend with time tendencies of LA model in boundary region.
3) *Diffuse* results in boundary region

Williamson-Browning (JAM 1974)

1) Specify prognostic variables at inflow only (from driving model).
2) *Diffuse* results in boundary region.

Davies (QJRMS 1976)

1) Relax prognostic variables in a boundary region of 5Δx towards values specified by driving model [i.e. add terms like -K(U-u) to rhs].
2) *Diffuse* results in boundary region.

Robert-Yakimiw (AO 1986)

Variation of Davies' strategy, but variables flattened in a boundary region.

Note: the common ingredient of all approaches

- liberal doses of *diffusion*!

LINEARIZED 1-D SHALLOW-WATER MODEL (Robert & Yakimiw, A-O, 1986)

EQUATIONS

$$\frac{\partial u}{\partial t} + U\frac{\partial u}{\partial x} - fv + \frac{\partial \phi}{\partial x} = 0$$

$$\frac{\partial v}{\partial t} + U\frac{\partial v}{\partial x} + fu = 0$$

$$\frac{\partial \phi}{\partial t} + U\frac{\partial \phi}{\partial x} - fUv + \phi_0\frac{\partial u}{\partial x} = 0$$

INITIAL CONDITION

Slow mode solution such that

ϕ = 2.5 sin kx, wavelength = 2667 km

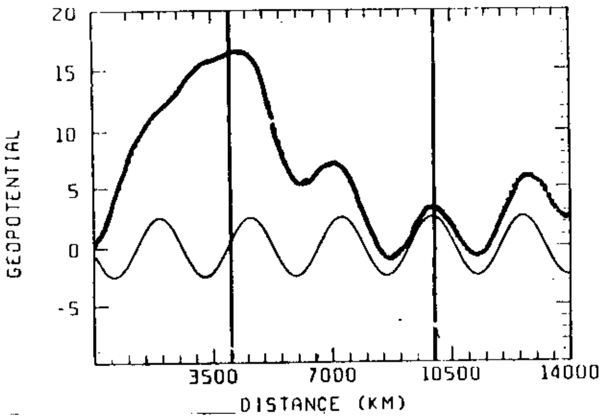
PARAMETERS

U = 46.3 m/s, ϕ_0 = 5600 $dam^2\,s^{-2}$

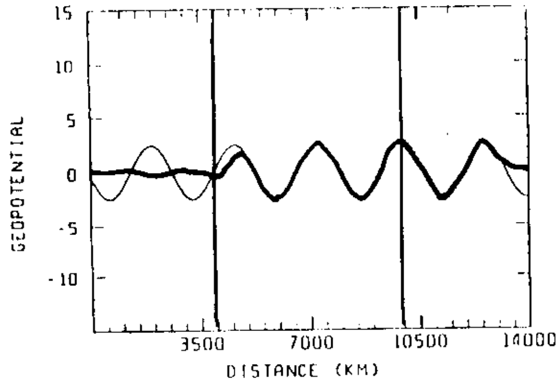
h = 100 km, f = f (45°), L = 14,000 km

FORECAST PERIOD - 24h

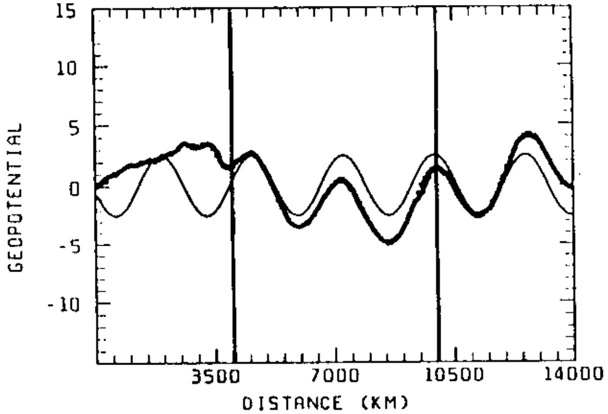
PERKEY - KREITZBERG (MWR 1976)



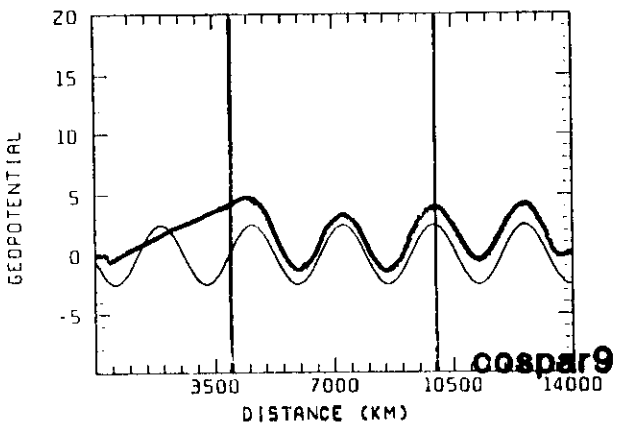
ROBERT - YAKIMIW (A-O, 1986)



WILLIAMSON - BROWNING (JAM 1974)



DAVIES (QJRMS 1976)



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SHALLOW-WATER EXPERIMENTS
(Yakimiw & Robert, AO, 1990)

Control integration

Driving model - T106 spectral

Forecast model - semi-implicit semi-Lagrangian gridpoint

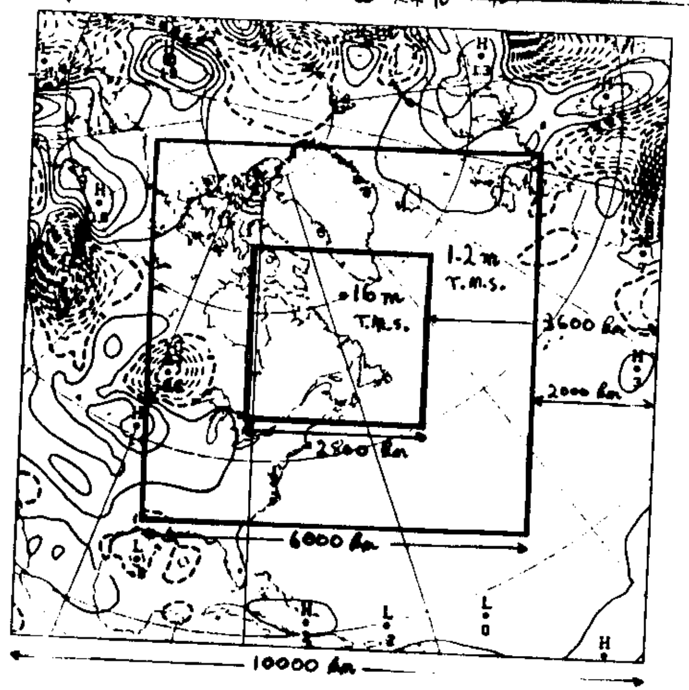
- 127 km res over 235 x 235 quasi-hemispheric domain (~ 30,000 km x 30,000 km)
- $\Delta t = 1h$

Regional integration

Driving model - T31 spectral

Forecast model - semi-implicit semi-Lagrangian gridpoint

- 127 km res over 81 x 81 regional domain (~ 10,000 km x 10,000 km)
- $\Delta t = 1h$



NOTE

These experiments use 500 mb data.

For *baroclinic* models, the maximum wind speed is approximately *twice* as large, and in the worst case the error can propagate inwards *twice* as fast.

Thus any buffer region should be twice as large as for a shallow-water model that uses 500 mb data.

OTHER PROBLEM AREAS

1) Too old a forecast from driving model

(e.g. Gustafsson, Tellus, 1990).

2) Time and space interpolation of forecast of driving model .

3) Unrealistically low growth of perturbations in interior of domain (forecast overly-constrained by boundary constraints), contradicts predictability results over larger domains.

(Errico & Baumhefner, MWR, 1990)

THE INTERACTIVE APPROACH

Here the *coarse-resolution forecast* of the outer domain *interacts* throughout the forecast period with the *fine-resolution* forecast of the area of interest.

Can vary resolution

- abruptly ~~*discretely*~~ across a resolution interface (e.g. NGM)
- *smoothly* away from the high-resolution area of interest (e.g. FER model)

Straightforward to apply a **rigid wall condition** at boundary of outer domain:

- works well if boundary is in tropics
- gives **well-posed** problem.

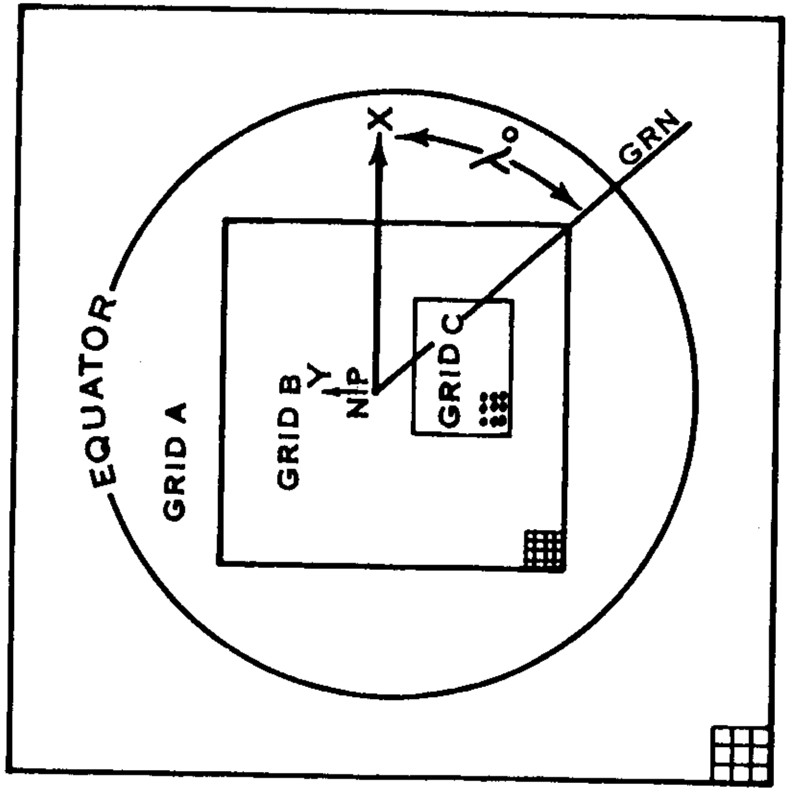
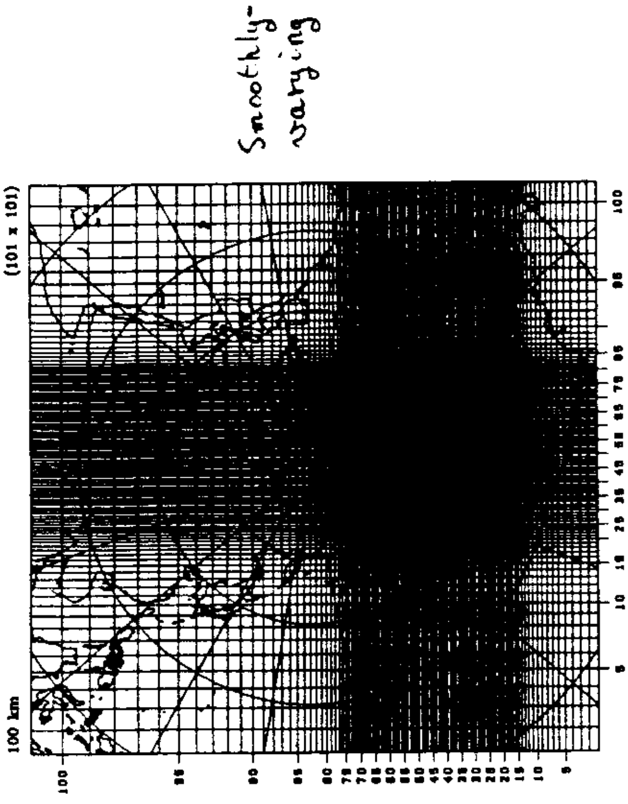
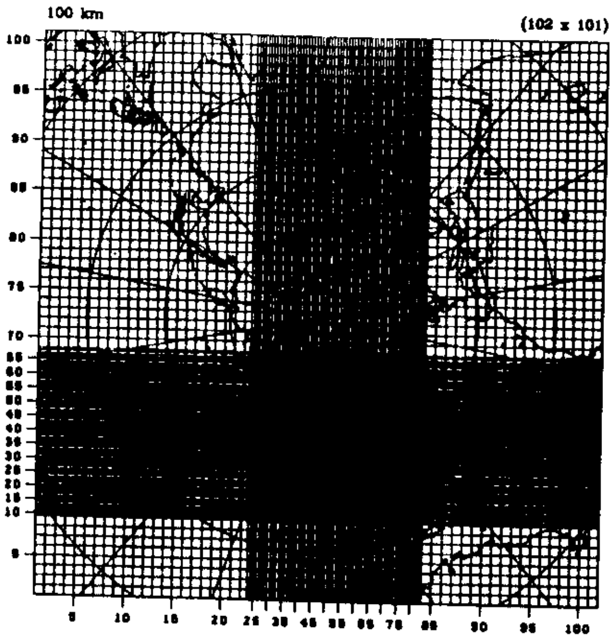


Figure 1.2—Three-grid structure of the NGM, and its orientation with respect to the Greenwich meridian (GRN).

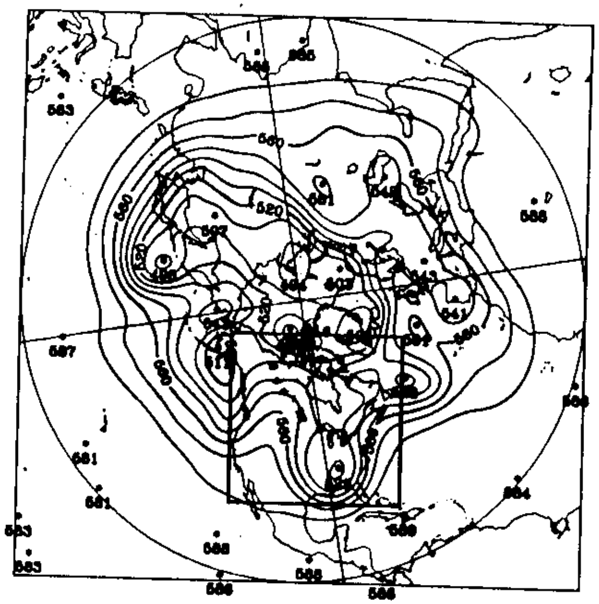


3 FINITE-ELEMENT SHALLOW-WATER REGIONAL MODELS

Eulerian control	Eulerian	Semi-Lagrangian
Staniforth & Mitchell (1978)	Staniforth & Mitchell (1978)	Côté et al (1990)
linear FE's	linear FE's	pseudostaggering
3-time-level	3-time-level	2-time-level
401 x 401 mesh uniform-res	101 x 101 mesh variable-res (61 x 61 uniform)	101 x 101 mesh variable-res (61 x 61 uniform)
dx = 50 km	dx = 100 km	dx = 100 km
dt = 5 mins	dt = 10 mins	dt = 60 mins

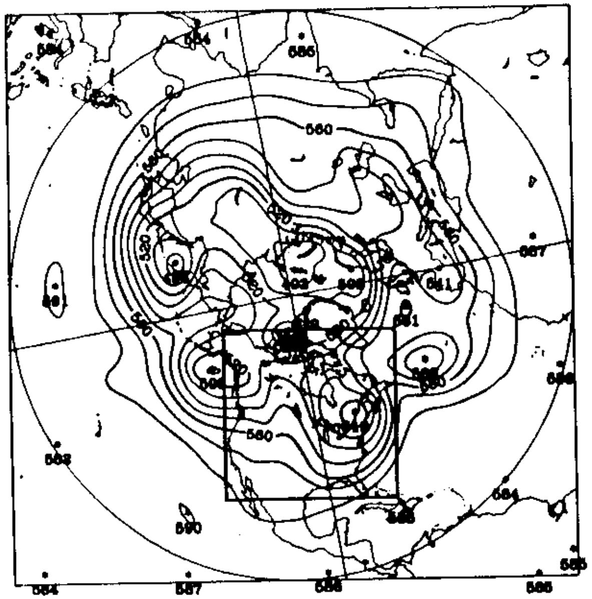


Abruptly-varying



Eulerian control - 0 h

Δx = 50 km

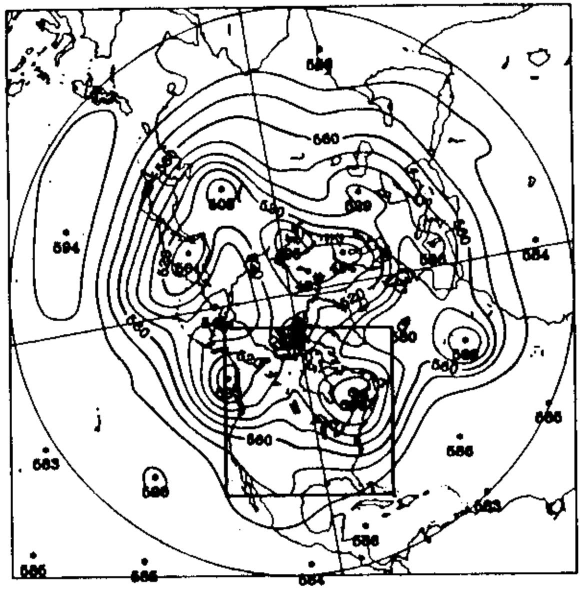


Eulerian

control

- 24 h

Δx = 50 km

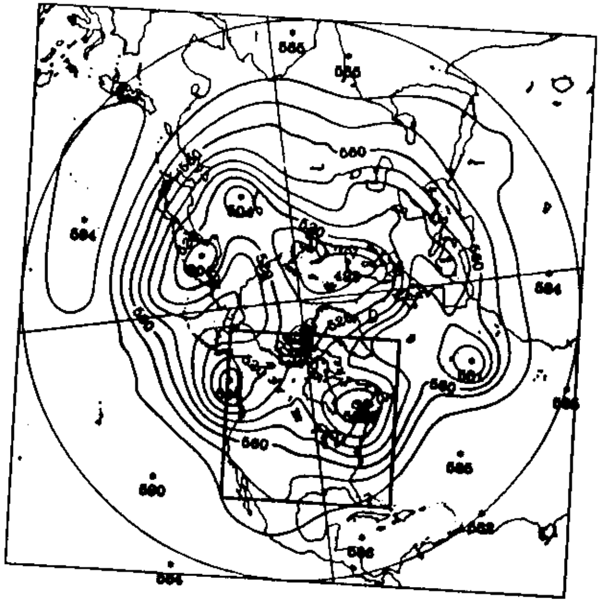


Eulerian

control

- 48 h

Δx = 50 km

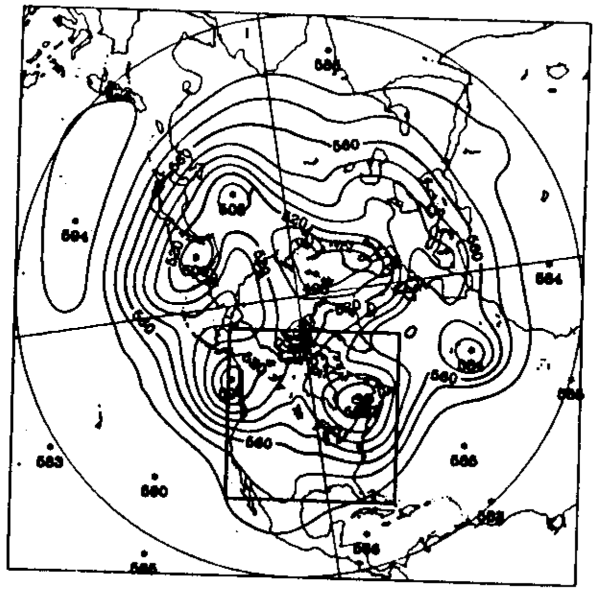


Eulerian

Smoothly-varying mesh

- 48 h

$\Delta x = 100$ km

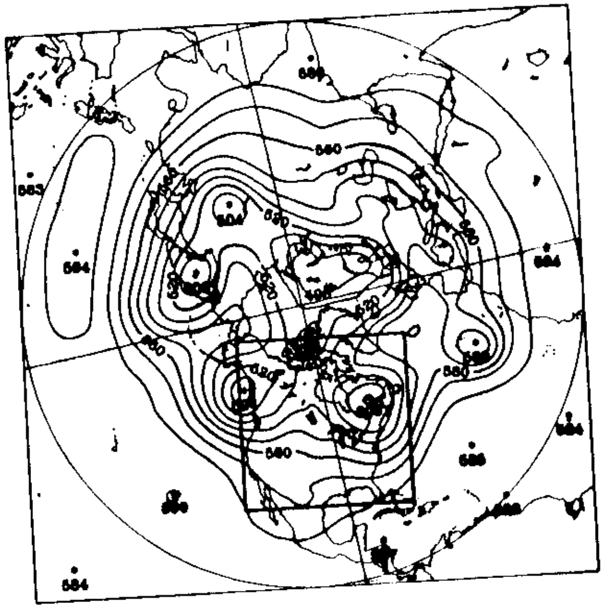


SL

Smoothly-varying mesh

- 48 h

$\Delta x = 100$ km



SL

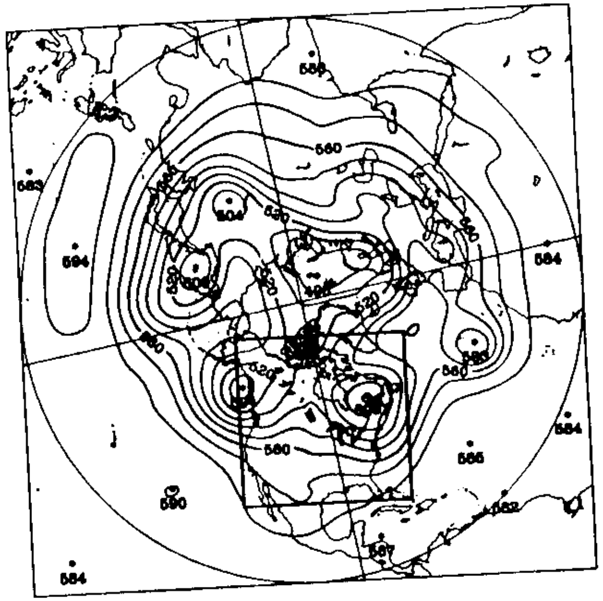
Abruptly-

varying

mesh

- 48 h

Δx = 100 km



SL

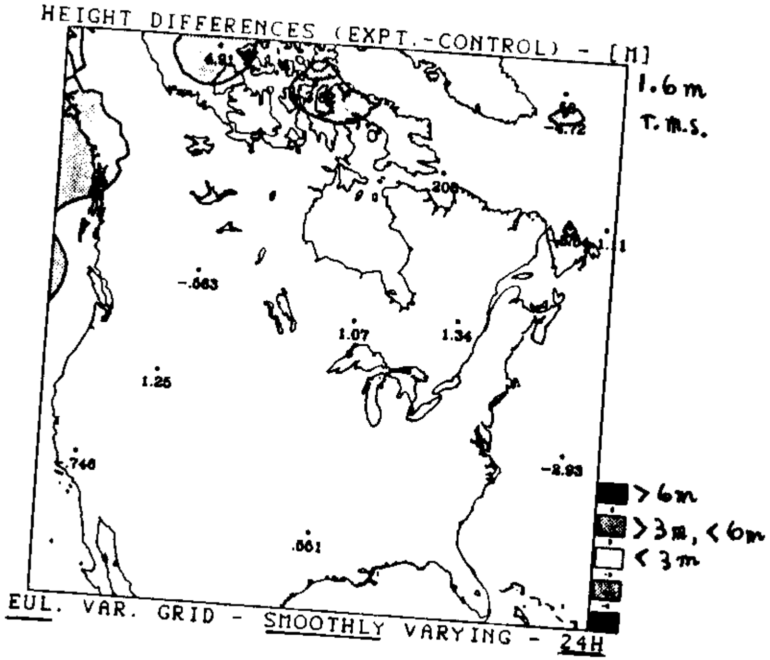
Abruptly-

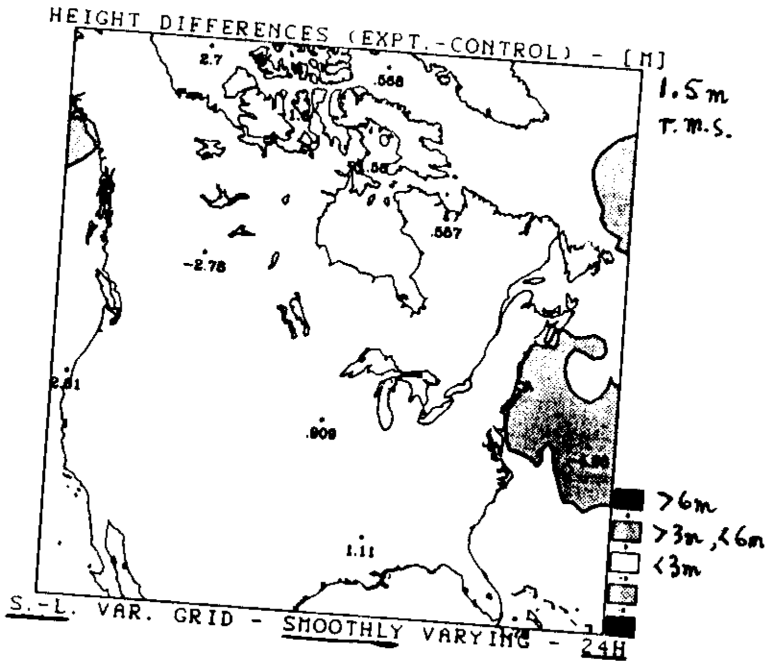
varying

mesh

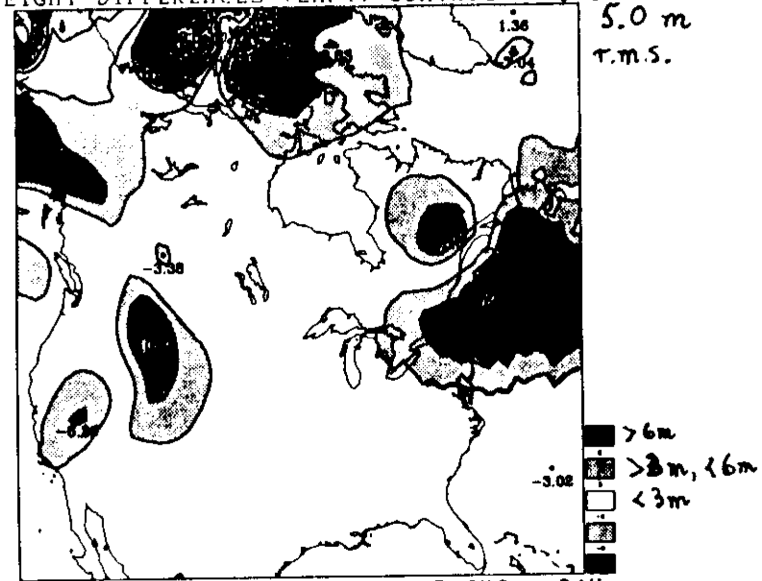
- 48 h

Δx = 100 km


HEIGHT DIFFERENCES (EXPT.-CONTROL) - [M]
1.6m
r.m.s.
-.563
1.07
1.34
1.25
.746
-2.93
.561
>6m
>3m, <6m
<3m
EUL. VAR. GRID - SMOOTHLY VARYING - 24H

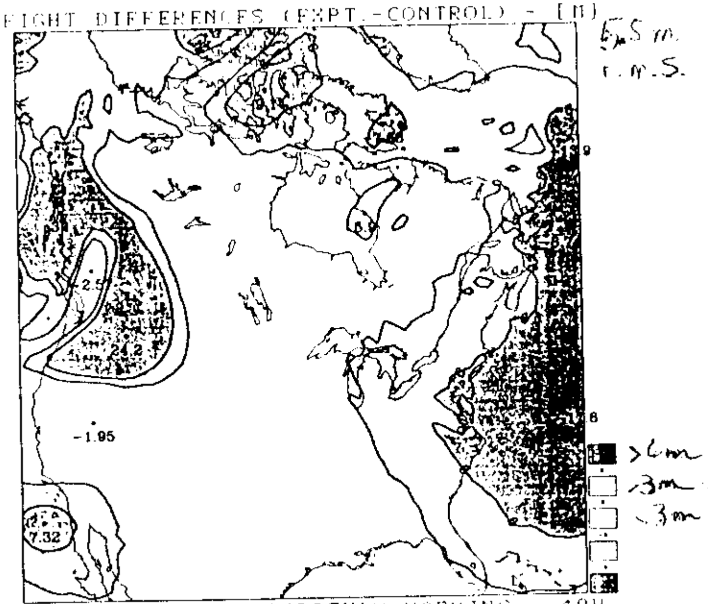

HEIGHT DIFFERENCES (EXPT.-CONTROL) - [M]
1.5m
r.m.s.
2.7
.588
.587
-2.78
.909
1.11
>6m
>3m, <6m
<3m
S.-L. VAR. GRID - SMOOTHLY VARYING - 24H

HEIGHT DIFFERENCES (EXPT.-CONTROL) - [M]



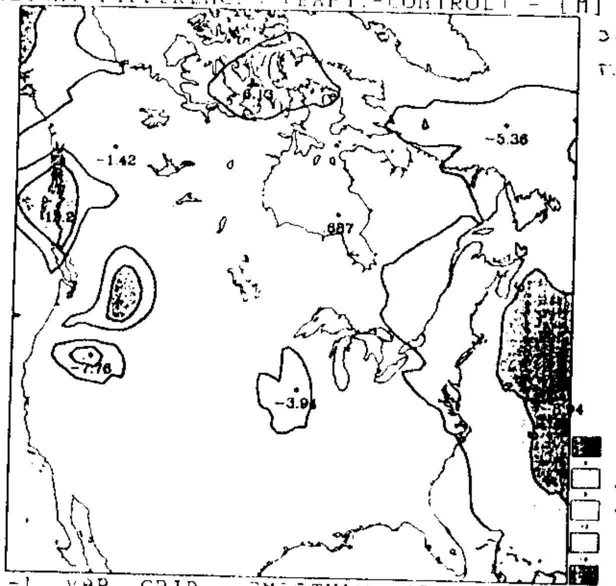
EUL. VAR. GRID - ABRUPTLY VARYING - 24H

HEIGHT DIFFERENCES (EXPT.-CONTROL) - [M]



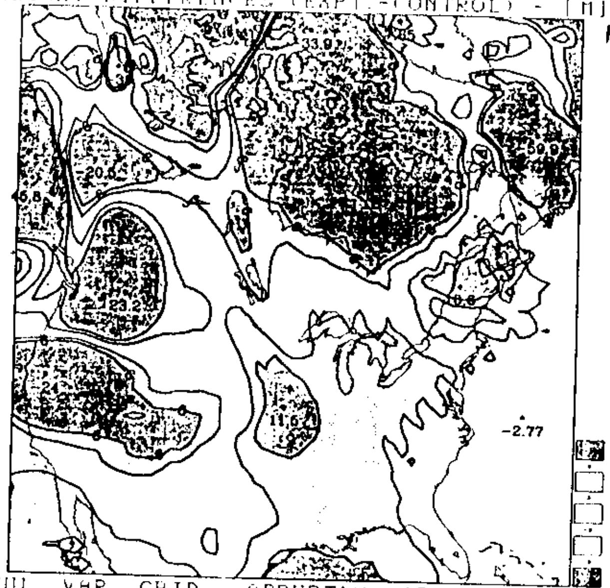
EUL. VAR. GRID - SMOOTHLY VARYING - 48H

HEIGHT DIFFERENCES (EXPT.-CONTROL) - [M]

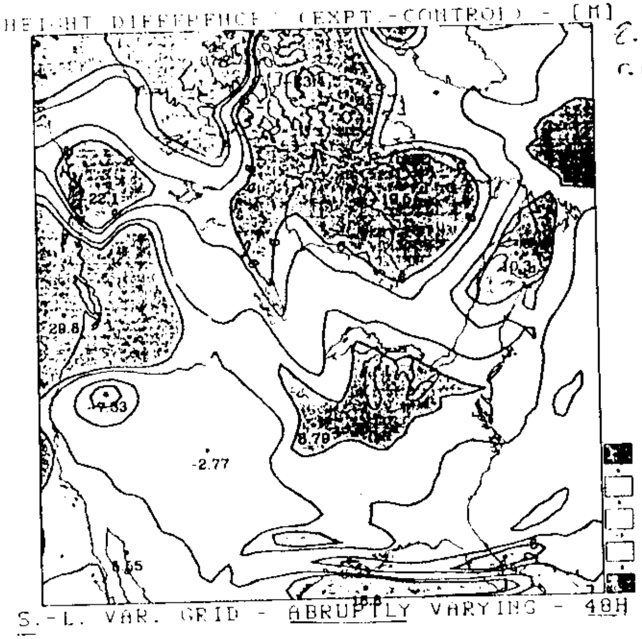


S.-L. VAR. GRID - SMOOTHLY VARYING - 48H

HEIGHT DIFFERENCES (EXPT.-CONTROL) - [M]



EUL. VAR. GRID - ABRUPTLY VARYING - 48H



RMS HEIGHT DIFFERENCE (m)

REGION A

		Eulerian	Semi-Lagrangian
Smoothly varying grid	24H	1.6	1.5
	48H	5.5	3.1
Abruptly varying grid	24H	5.0	4.0
	48H	11.4	8.4

REGION B

		Eulerian	Semi-Lagrangian
Smoothly varying grid	24H	0.6	0.9
	48H	2.1	1.9
Abruptly varying grid	24H	2.4	2.6
	48H	7.0	6.4

<u>CONCLUSIONS</u>

(mine, not necessarily yours!)

1) Applying *open* lateral boundary conditions to *hydrostatic primitive equation* models is fraught with **<u>peril</u>.**

2) It *may* be possible.

3) *Difficulties* of this approach are *greatly underestimated* by regional modelling community.

4) *<u>Variable-res</u>* hemispheric (& global) models an attractive *cost-effective alternative* for regional forecasting to 48 or 72 h.