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I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

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LIMITED AREA MODELLING
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"Higher Order Turbulence Closure Theories & Their Implementation in Numerical Models"

Z.I. JANJIC
University of Belgrade
Department of Meteorology
Belgrade, Yugoslavia

Sources :

Mellor, JAS 1973; Mellor-Yamada, JAS 1974

Mellor, Yamada, RGSP, 1982

Arakawa: Parameterization of the Planetary Boundary Layer, Erice 1984 Lecture Notes

Miyakoda, Sirutis: Manual on the E Physics (Unpublished Manuscript)

Janjić: Physical Package of the HIBU/GFDL/NMC/UCAR Eta Coordinate Limited Area Model (Notes)

—, MWR 1990

—, Numerical Techniques for the Physics in NWP (Unpublished Manuscript)

Rajković: Micrometeorology (Lecture Notes)

Black: The Step-Mountain ... A Documentation (Unpublished Manuscript)

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The basic equations

Notation: $\tilde{u} \equiv U + u$, $\tilde{v} \equiv \ldots$;
Reynolds, ensemble averaging ; properties

$$\overline{AB} = AB + \overline{ab}, \quad \ldots$$

$$\overline{\overline{A}} = \overline{A}, \ldots \qquad \text{(Assumptions)}$$

Total value eqs. $\longrightarrow$

The equations of motion for the mean velocity U_j and mean potential temperature Θ are

$$\frac{\partial U_i}{\partial x_i} = 0, \tag{1}$$

$$\frac{\partial U_j}{\partial t} + \frac{\partial}{\partial x_k}(U_k U_j + \overline{u_k u_j}) + \epsilon_{jkl} f_k U_l$$

Closure?
"K theory" ≡
1st order closure

$$= -\frac{\partial P}{\partial x_j} - g_j \beta \Theta + \nu \nabla^2 U_j, \tag{2}$$

$$\frac{\partial \Theta}{\partial t} + \frac{\partial}{\partial x_k}(U_k \Theta + \overline{u_k \theta}) = \alpha \nabla^2 \Theta, \tag{3}$$

$P \equiv \frac{p}{\rho}$, $g_j \equiv (0,0,g)$, $f_j \equiv (0, f_y, f)$

$\beta \equiv -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p$ the coefficient of thermal expansion, ν the kinematic viscosity, and α the kinematic heat conductivity (or thermal diffusivity). The overbars represent ensemble averages and the lower case terms, u_k and θ, are the fluctuating components of the velocity and temperature

Eqs. for deviations:

Total value eqs. − [Eqs. (1), (2), (3)] $\rightarrow$

$$\frac{\partial u_i}{\partial x_i}=0, \tag{4}$$

$$\frac{\partial u_j}{\partial t}+\frac{\partial}{\partial x_k}(U_k u_j+U_j u_k+u_k u_j-\underline{\overline{u_k u_j}})+\epsilon_{jkl} f_k u_l = -\frac{\partial p}{\partial x_j}-g_j \beta\theta+\nu\nabla^2 u_j, \tag{5}$$

$$\frac{\partial \theta}{\partial t}+\frac{\partial}{\partial k_k}(\Theta u_k+U_k\theta+u_k\theta-\underline{\overline{u_k\theta}})=\alpha\nabla^2\theta. \tag{6}$$

Closure problem. "K-theory" ≡ First order closure
Higher order closure

Eqs. for Reynolds stress and heat conduction moments $\overline{u_i u_j}$, $\overline{u_i \theta}$:

$$\frac{\partial}{\partial t}\overline{u_i u_j} = \overline{u_i \frac{\partial u_j}{\partial t}} + \overline{u_j \frac{\partial u_i}{\partial t}},$$

insert from (5), use (4),

$$\frac{\partial \overline{u_i u_j}}{\partial t} + \frac{\partial}{\partial x_k}\left[U_k \overline{u_i u_j} + \overline{u_k u_i u_j} - \nu \frac{\partial}{\partial x_k}\overline{u_i u_j}\right] + \frac{\partial}{\partial x_j}\overline{p u_i}$$

$$+\frac{\partial}{\partial x_i}\overline{p u_j} + f_k(\epsilon_{jkl}\overline{u_l u_i} + \epsilon_{ikl}\overline{u_l u_j})$$

$$= -\overline{u_k u_i}\frac{\partial U_j}{\partial x_k} - \overline{u_k u_j}\frac{\partial U_i}{\partial x_k} - \beta(g_j \overline{u_i \theta} + g_i \overline{u_j \theta})$$

$$+\overline{p\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)} - 2\nu \overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}}, \quad (7)$$

$$\frac{\partial \overline{u_j \theta}}{\partial t} + \frac{\partial}{\partial x_k}\left[U_k \overline{\theta u_j} + \overline{u_k u_j \theta} - \alpha \overline{u_j \frac{\partial \theta}{\partial x_k}} - \nu \overline{\theta \frac{\partial u_j}{\partial x_k}}\right] + \frac{\partial}{\partial x_j}\overline{p\theta} + \epsilon_{jkl} f_k \overline{u_l \theta}$$

$$= -\overline{u_j u_k}\frac{\partial \Theta}{\partial x_k} - \overline{\theta u_k}\frac{\partial U_j}{\partial x_k} - \beta g_j \overline{\theta^2} + \overline{p \frac{\partial \theta}{\partial x_j}}$$

$$-(\alpha+\nu)\overline{\frac{\partial u_j}{\partial x_k}\frac{\partial \theta}{\partial k_k}}. \quad (8)$$

Eq. (8) involves $\overline{\theta^2}$, an equation for which is obtained from (6) so that

$$\frac{\partial\overline{\theta^2}}{\partial t}+\frac{\partial}{\partial x_k}\left[U_k\overline{\theta^2}+\overline{u_k\theta^2}-\alpha\frac{\partial\overline{\theta^2}}{\partial x_k}\right]$$

$$=-2\overline{u_k\theta}\frac{\partial\Theta}{\partial x_k}-2\alpha\overline{\frac{\partial\theta}{\partial x_k}\frac{\partial\theta}{\partial x_k}}. \quad (9)$$

Continue? Or:

Second order closure:

Modeling assumptions Mellor, 1973 (very similar, Deardorff 1973a,b)

The primary contribution of Rotta (1951) was to suggest an assumption for the term, $\overline{p(\partial u_i/\partial x_j+\partial u_j/\partial x_i)}$, which he called the "energy redistribution term" since one of its functions is to partition energy among the three energy components while not contributing to the total. [Upon contraction, the term drops out of Eq. (7)].

Consider $i=j=1$ with no summation:

$$\frac{\partial}{\partial t}\overline{u_1^2} = \cdots \quad + 2\overline{p\frac{\partial u_1}{\partial x_1}} + \cdots$$

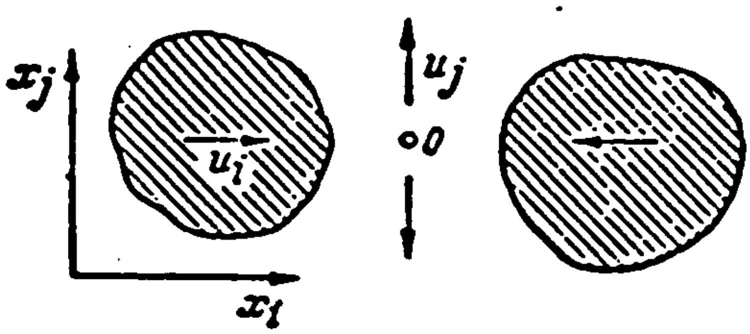

Fig. 1. Zusammentreffen zweier Turbulenzballen.

At point 0:

$p\frac{\partial u_1}{\partial x_1} < 0$,

contributes to a reduction in $\overline{u_1^2}$

The term should be expected to have the maximum absolute value for the component with maximum energy $\overline{u_i^2}$; this energy will be transforming into energy of <u>two</u> remaining components. $\longrightarrow$ equipartition of energy within the three components!

Assume

$$\overline{p\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)}=C_{ijkm}\overline{u_k u_m}+C'_{ijkm}\frac{\partial U_k}{\partial x_m}.$$

isotropic tensors;

$$C_{ijkm} = C_1 \delta_{ij}\delta_{km} + C_2 \delta_{ik}\delta_{jm} + C_3 \delta_{im}\delta_{jk} ;$$

$$C_{ijkm}\overline{u_k u_m} = C_1 \delta_{ij}\overline{u_k u_k} + C_2 \overline{u_i u_j} + C_3 \overline{u_j u_i} .$$

However, for $j=i$, $\overline{p\left(\frac{\partial u_i}{\partial x_i}+\frac{\partial u_i}{\partial x_i}\right)} = 0$, $\longrightarrow$

$$0 = (3C_1 + C_2 + C_3)\overline{u_k u_k} ; \qquad C_1 = -\frac{1}{3}(C_2+C_3)$$

similar reasoning for C_{ijkm},

+ dimensional analysis,

$$\overline{p\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)}$$

$$=-\frac{q}{3l_1}\left(\overline{u_iu_j}-\frac{\delta_{ij}}{3}q^2\right)+Cq^2\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right), \quad (10)$$

$q^2 \equiv \overline{u_k u_k}$, twice turb. kin. energy;

l_1 length scale, C constant to be determ. empirically.

Note that for $j=i=1, 2,$ or 3, the first term on the right hand side desribes reduction in energy for $\overline{u_1^2} > \frac{1}{3}q^2$, .. (increase ... for $\overline{u_1^2} <$ ...) !

Proceeding in similar fashion, we obtain

$$\overline{p\frac{\partial \theta}{\partial x_j}}=-\frac{q}{3l_2}\overline{u_j\theta}. \quad (11)$$

(No term prop. to $\frac{\partial U_k}{\partial x_m}$, since C_{jkm} cannot be isotropic.)

framework is

$$2\nu\overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}}=\frac{2}{3}\frac{q^3}{\Lambda_1}\delta_{ij} \tag{12}$$

and therefore follows Kolmogoroff's (1941) hypothesis of local, small-scale isotropy.

Since there is no isotropic first-order tensor

$$(\alpha+\nu)\overline{\frac{\partial u_j}{\partial x_k}\frac{\partial \theta}{\partial x_k}}=0. \tag{13}$$

However,

$$2\alpha\overline{\frac{\partial \theta}{\partial x_k}\frac{\partial \theta}{\partial x_k}}=2\frac{q}{\Lambda_2}\overline{\theta^2}. \tag{14}$$

"Diffusion" terms, $\overline{u_k u_i u_j}$, ..

($\frac{\partial}{\partial x_k}\overline{u_k u_i u_j}$, .. advection of Reynolds stress moments by turbulent vel. components, ..)

$$\overline{u_k u_i u_j} = C_{ijklmn}\frac{\partial}{\partial x_l}\overline{u_m u_n},$$

$$C_{ijklmn} = C_1\delta_{ij}\delta_{kl}\delta_{mn} + C_2\delta_{ik}\delta_{jl}\delta_{mn} + \cdots,$$

three forms; Mellor chose

$$\overline{u_k u_i u_j} = -q\lambda_1\left(\frac{\partial \overline{u_i u_j}}{\partial x_k} + \frac{\partial \overline{u_i u_k}}{\partial x_j} + \frac{\partial \overline{u_i u_k}}{\partial x_i}\right). \qquad (15)$$

For $\overline{u_k u_j \theta}$, two forms are possible. We choose

$$\overline{u_k u_j \theta} = -q\lambda_2\left(\frac{\partial \overline{u_k \theta}}{\partial x_j} + \frac{\partial \overline{u_j \theta}}{\partial x_k}\right). \qquad (16)$$

Assuming $\overline{u_k \theta^2}$ proportional to $\partial \overline{\theta^2}/\partial x_k$, we obtain

$$\overline{u_k \theta^2} = -q\lambda_3 \frac{\partial \overline{\theta^2}}{\partial x_k}. \qquad (17)$$

It is questionable whether the pressure diffusional terms can be discriminated experimentally. Hanjalic and Launder (1972) assert they are small in the first place. Therefore, for the present we set

$$\overline{p u_i} = \overline{p\theta} = 0 \qquad (18)$$

to complete the required modeling assumptions.

If (10)–(18) are inserted into (7)–(9) and

$$D(\ )/Dt \equiv U_k \partial(\ )/\partial x_k + \partial(\ )/\partial t,$$

we obtain -

$$\frac{D\overline{u_i u_j}}{Dt} + f_k(\epsilon_{jkl}\overline{u_l u_i} + \epsilon_{ikl}\overline{u_l u_j})$$

$$= \frac{\partial}{\partial x_k}\left[q\lambda_1\left(\frac{\partial\overline{u_i u_j}}{\partial x_k} + \frac{\partial\overline{u_i u_k}}{\partial x_j} + \frac{\partial\overline{u_j u_k}}{\partial x_i}\right) + \nu\frac{\partial\overline{u_i u_j}}{\partial x_k}\right]$$

$$-\overline{u_k u_i}\frac{\partial U_j}{\partial x_k} - \overline{u_k u_j}\frac{\partial U_i}{\partial x_k} - \beta(g_j\overline{u_i\theta} + g_i\overline{u_j\theta})$$

$$-\frac{q}{3l_1}\left(\overline{u_i u_j} - \frac{\delta_{ij}}{3}q^2\right) + Cq^2\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) - \frac{2}{3}\frac{q^3}{\Lambda_1}\delta_{ij}, \quad (19)$$

$$\frac{D\overline{u_j\theta}}{Dt} + f_k\epsilon_{jkl}\overline{u_l\theta}$$

$$= \frac{\partial}{\partial x_k}\left[q\lambda_2\left(\frac{\partial\overline{u_j\theta}}{\partial x_k} + \frac{\partial\overline{u_k\theta}}{\partial x_j}\right) + \alpha\overline{u_j\frac{\partial\theta}{\partial x_k}} + \nu\overline{\theta\frac{\partial u_j}{\partial x_k}}\right]$$

$$-\overline{u_j u_k}\frac{\partial\Theta}{\partial x_k} - \overline{\theta u_k}\frac{\partial U_j}{\partial x_k} - \beta g_j\overline{\theta^2} - \frac{q}{3l_2}\overline{u_j\theta}, \quad (20$$

$$\frac{D\overline{\theta^2}}{Dt} = \frac{\partial}{\partial x_k}\left[q\lambda_3\frac{\partial\overline{\theta^2}}{\partial x_k} + \alpha\frac{\partial\overline{\theta^2}}{\partial x_k}\right] - 2\overline{u_k\theta}\frac{\partial\Theta}{\partial x_k} - 2\frac{q}{\Lambda_2}\overline{\theta^2}. \quad (21)$$

Mellor, Yamada 1974: order of magnitude analysis of small deviations of terms from the state of local isotropy.

$$(\ell_1, \ell_2, \Lambda_1, \Lambda_2, \ldots) = (A_1, A_2, B_1, B_2, \ldots)\,\ell$$

ℓ : "master length scale" of turbulence

→ hierarchy of closure models (systems of equations):

"Level 4 Model"

"Level 3 Model" (prognostic eqs. for q^2 and $\overline{\theta^2}$)

* "Level 2 Model" (no prognostic eqs. for turbulence quantities)

"Level 1 Model" (fails to reproduce observed data)

* Later: "Level 2.5 Model": prognostic equation for q^2 only

The Level 2.5 turbulence closure theory

Governing equations (MY82, Janjić 1990):

$$d(q^2/2)/dt - (\partial/\partial z)[l q S_q (\partial/\partial z)(q^2/2)] = P_s + P_b - \varepsilon , \qquad (3.1)$$

$$P_s = -\overline{wu}(\partial U/\partial z) - \overline{wv}(\partial V/\partial z),$$

$$P_b = \beta g \overline{w\theta_v}, \quad \varepsilon = q^3 (B_1 l)^{-1} \qquad (3.2)$$

$$-\overline{wu} = K_M \partial U/\partial z, \quad -\overline{wv} = K_M \partial V/\partial z,$$

$$-\overline{w\theta_v} = K_H \partial \Theta_v/\partial z, \quad -\overline{ws} = K_H \partial S/\partial z, \qquad (3.3)$$

$$K_M = l q S_M, \quad K_H = l q S_H, \qquad (3.4)$$

$$S_M(6A_1A_2G_M) + S_H(1 - 3A_2B_2G_H - 12A_1A_2G_H) = A_2,$$

$$S_M(1 + 6A_1{}^2G_M - 9A_1A_2G_H) -$$

$$S_H(12A_1{}^2G_H + 9A_1A_2G_H) = A_1(1 - 3C_1), \qquad (3.5)$$

$$G_M = l^2 q^{-2}[(\partial U/\partial z)^2 + (\partial V/\partial z)^2],$$

$$G_H = - l^2 q^{-2} \beta g \, \partial \Theta_v/\partial z \qquad (3.6)$$

{ Alternative:

$$P_s + P_b - \varepsilon = [S_M G_M + S_H G_H - B_1{}^{-1}] q^3 l^{-1}$$ }

MY74, Miyakoda and Sirutis 1977:

$$l = l_0 \kappa z (\kappa z + l_0)^{-1}, \quad l_0 = \alpha \left[\int_{pT}^{pS} |z| q \, dp \right] \left[\int_{pT}^{pS} q \, dp \right]^{-1}$$

$$\alpha = \text{const}, \; l_0 \leq 90 \text{ m}$$

For exceptionally large shear, and/or thermal instability, the system (3.5) may degenerate (MY82) in the sense that its determinant may approach zero.

$$G_H \leq .024 , \quad G_M \leq .36 - 15. G_H . \quad (3.9)$$

Note that the limits (3.9) are more restrictive than those that are really necessary in order to prevent the degeneration of the system (3.5). The motivation for such a choice was to avoid unrealistically large exchange coefficients in the cases of unstable stratification.

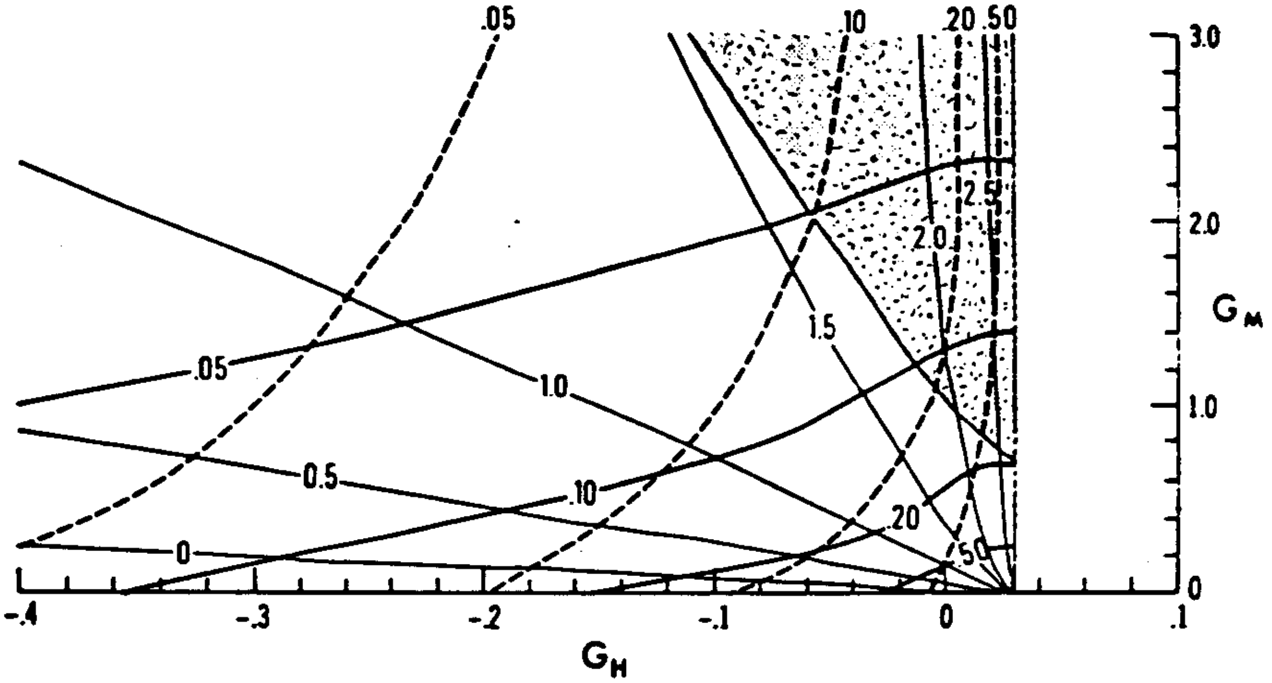

Fig. 3. The stability functions $S_M(G_H, G_M)$ and $S_H(G_H, G_M)$. The heavy solid lines are contours of S_M, whereas th dashed lines are contours of S_H. The lighter solid lines are contours of $(P_s + P_b)/\varepsilon$. One could also draw lines of consta $R_i = G_H/G_M$, which are radial lines on this diagram. The shaded portion is where $\langle w^2 \rangle / q^2 \leq 0.12$.

(Galperin et al., JAS 1 Jan. 88: "Level 2¼ Model" (?)
$S_M, S_H = S_M, S_H(G_H)$, more consistent expansion;
no realizability conditions)

The vertical staggering and the vertical index:

———	z, l, G_M, G_H, S_M, S_H, q^2, $\dot{\eta}$ ———	L−1/2
- - - - -	v, T, S - - - - -	L
———	z, l, G_M, G_H, S_M, S_H, q^2, $\dot{\eta}$ ———	L+1/2
- - - - -	v, T, S - - - - -	L+1

The Level 2 turbulence closure theory

From Level 2.5 for balanced TKE production and dissipation

Governing equations(MY74;MY82;Janjić 1990)

$$R_f = -P_b/P_s, \qquad (4.1)$$

$$R_i = -G_H/G_M = (S_M/S_H)\,R_f, \qquad (4.2)$$

$$S_H = F_1\,(F_2 - F_3 R_f)/(1 - R_f), \qquad (4.3)$$

$$S_M = F_4\,(F_5 - F_6 R_f)\,(F_7 - F_8 R_f)^{-1}\,S_H. \qquad (4.4)$$

F_1, ... F_8 from MY82 A_1, A_2, B_1, B_2, C_1; R_i from (4.2_1), substituting (4.4) into (4.2_2), a quadratic eq. for flux Richardson # with the root

$$R_f=.664[R_i+.1765-(R_i^2-.317\,R_i+.0312)^{1/2}]$$

$$K_M= l^2\{[(\partial U/\partial z)^2+(\partial V/\partial z)^2][B_1(1-R_f)S_M]\}^{1/2}S_M$$

$$K_H= l^2\{[(\partial U/\partial z)^2+(\partial V/\partial z)^2][B_1(1-R_f)S_M]\}^{1/2}S_H$$

[c.f., MY74, Eqs. (66a)-(67b)].

l varies linearly with z reaching the value of the Level 2.5 master length scale at the top of the lowest model layer.

Implementation of the Level 2.5 model

- "Forward-backward": TKE updated first, updated TKE to recalculate exchange coefficients

- Vertical staggering

———— $z, l, G_M, G_H, S_M, S_H, q^2, \dot{\eta}$ ———— $L-1/2$

.... $\mathbf{v}, T, S$ L

———— $z, l, G_M, G_H, S_M, S_H, q^2, \dot{\eta}$ ———— $L+1/2$

.... $\mathbf{v}, T, S$ .... $L+1$

- TKE equation solved in split mode: horizontal and vertical advection, TKE production and dissipation, vertical diffusion treated in sequence.

- Horizontal advection

$$\mathrm{Adv}\,(\mathbf{v},q^2)_{L+1/2} =$$
$$.5\,[\mathrm{Adv}\,(\mathbf{v}_L,q^2_{L+1/2}) + \mathrm{Adv}\,(\mathbf{v}_{L+1},q^2_{L+1/2})]$$

- Vertical advection

$$\overline{\overline{\dot{\eta}}^{\eta}\,\delta_\eta q^2}^{\eta}$$

- Correction of negative values due to advection; production dissipation dominating.

Computational problem of the Level 2.5

TKE production/dissipation in the split mode:

$$\partial(q^2/2)/\partial t = A\, q^3,$$

$$\partial q/\partial t = Aq^2$$

$$A = [S_M G_M + S_H G_H - B_1^{-1}]\, l^{-1}$$

A positive or negative, depends on stability and shear, implicitely through G_M, G_H and l, on TKE.

Possible time differencing scheme

$$q^{\tau+1} = q^{\tau} + A^{\tau}\, \Delta t\, (q^{\tau+1})^2$$

$$q_1^{\tau+1} = [\,1 - (\,1 - 4A^{\tau} q^{\tau} \Delta t)^{1/2}\,]\,(2A^{\tau}\Delta t)^{-1},$$

$$q_2^{\tau+1} = [\,1 + (\,1 - 4A^{\tau} q^{\tau} \Delta t)^{1/2}\,]\,(2A^{\tau}\Delta t)^{-1}.$$

- $\Delta t \to 0$, $q_1^{\tau+1} \to q^{\tau}$, q_1 physical solution; q_2 computational, should be damped or removed.
- Another problem, unless $\Delta t \approx 1$ min, expression under square root negative, rather disappointing, prohibitively expensive

- If the time step limitation locally exceeded, *TKE growth rate* restricted

$$(q^{\tau+1})^2 = [1-(1-4A^{\tau}q^{\tau}\Delta t)^{1/2}]^2(2A^{\tau}q^{\tau}\Delta t)^{-2}(q^{\tau})^2, \quad \textit{if}\ 1-4A^{\tau}\Delta t q^{\tau} \geq 0;$$

$$(q^{\tau+1})^2 = 4\,(q^{\tau})^2, \qquad \textit{if}\ 1-4A^{\tau}\Delta t q^{\tau} < 0.$$

If the time step limitation exceeded, TKE still quadruples in a single time step

- Slowing down analogous to polar filtering and implied deceleration of gravity waves associated with the application of the semi-implicit scheme.

- Essential in order to get the Level 2.5 scheme working in the eta model.

ıes, and the shaded
by the mountains.
36 hour (lower left
o late afternoons,
ınd 48 hour (lower
ınd to early morn-
, the turbulent en-
cle, reaching max-
^{2} in the afternoons,
lower levels in the
large water surfaces
imes.
section of $\log_{10}q^2$.
N, 96°W to 55°N,
ond to the 36 hour
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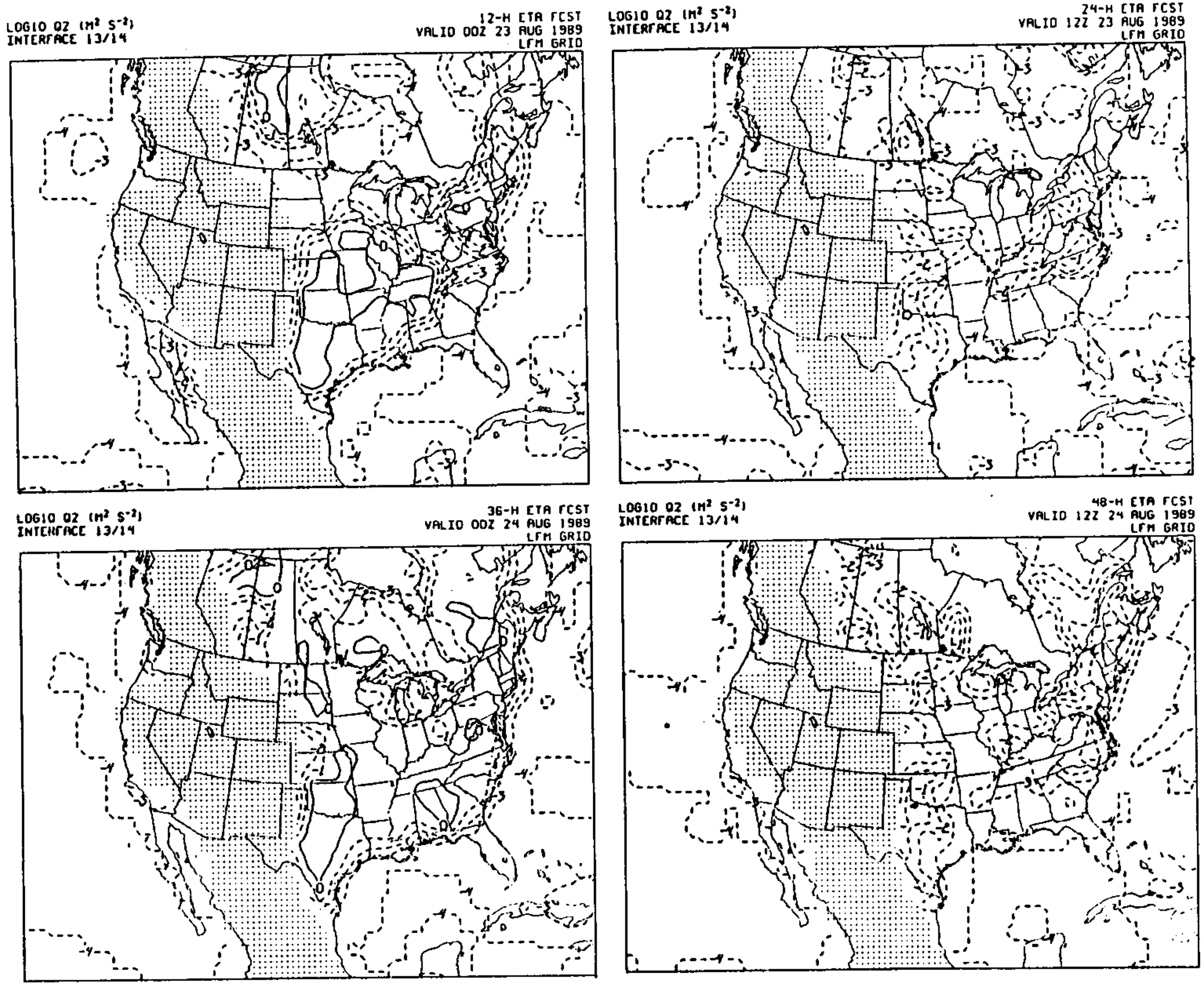

FIG. 6. Contour of $\log_{10}q^2$ (q^2 in $m^2 s^{-2}$) at a constant eta surface. The fields shown correspond to: 12 hour (upper left panel), 24 hour (upper right panel), 36 hour (lower left panel) and 48 hour (lower right panel) forecast times. The dashed contours correspond to negative values of $\log_{10}q^2$. The shaded area represent the volume blocked by the mountains.

more. The improvements reached about a factor of 2 for the 1.25 and 1.50 inch categories. These improve-

eta model development (M+), and in the runs with the full physics (Black and Janjić 1988), is that of Ap-

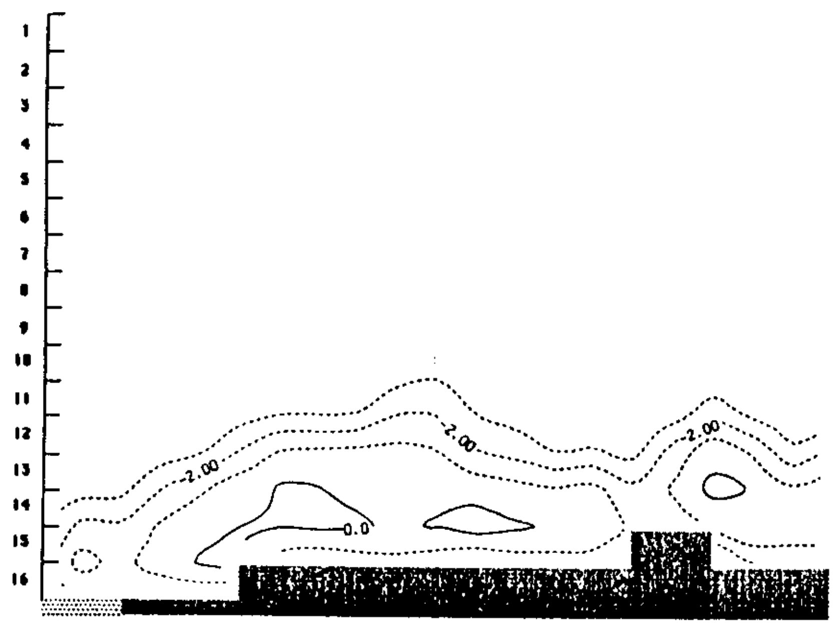

FIG. 7. Vertical cross section of $\log_{10}q^2$ (q^2 in $m^2 s^{-2}$) at 36 hours extending from 26°N 96°W to 55°N 87°W. The shaded steps represent the model topography. Model layers are indicated along the vertical axis. Lighter shading on the left indicates the sea surface. The dashed contours correspond to negative values of $\log_{10}q^2$.

As an additional refinement of the physical package, a viscous sublayer (e.g., Zilitinkevitch 1970; Pielke 1984) has been recently introduced next to the surface (Black and Mesinger 1989). Due to potentially reduced

zontal resolution, this result cannot be co relevant for intercomparison of the two m dicates, however, the benefits that one m from the application of regional models refine the global forecasts.

-20

7. Conclusions

In an atmospheric model using the ste eta coordinate, three major problems ca pated: (1) the internal boundaries at the v of the mountain walls, (2) vectorization physical package. The first two were add successfully solved, by Mesinger et al. (1 third one, that of the physical package, is this paper.

In the case of the eta coordinate, desig prehensive physical package is complicated of experience with the step-like mountain tion, particularly concerning the representa The Level 2.5 turbulence closure model in Yamada hierarchy (Mellor and Yamada cf. Vager and Zilitinkevitch 1968; Zilitinke was chosen to represent the turbulence ab face layer. The model was implemented splitting. In early experiments, however, a s

...er 1989). Due to potentially reduced
...ure transfer resulting from the presence
...r, its incorporation into the model was
by a return to the conventional use of
...vection scheme (Betts 1986; Betts and
in the sense that the shallow convection
...n the situations when insufficient mois-
...in vertical columns prevents the deep
produce positive precipitation. A con-
...rovement of an east coast storm forecast
with these modifications (Black and
...)).

...pointed out, the eta model has also been
...in the tropics (Lazić and Telenta 1988).
...ed to be important for testing the con-
...e and convectively driven circulations.
...as tested in 48 hour simulations of the
...nes Connie, Irma, Damien and Jason
from the Australian Monsoon Experi-
...) period. The initialized (then) opera-
...F analyses were used to specify the initial
...d the boundary conditions were derived
...ved ECMWF forecasts. For the analyses
...ime GTS data were available, excluding
...node AMEX data. From the synoptic
the results were considered as remarkably
...e circumstances, both in absolute terms,
to the results obtained with other models
...elenta 1988; Lazić 1990). In particular,
...owed the ability to predict the genesis of
...ecasts also compared favorably with the
...ational products (communicated by La-
...e, since the eta model had higher hori-

lem was encountered: TKE was taking on too large or negative values in large parts of the integration domain, leading eventually to numerical instability. The problem was found to be of numerical origin, and related to the treatment of the TKE production/dissipation term. A suitably designed time-differencing scheme eliminated this problem. With this scheme, TKE adjusts quickly to the forcing irrespectively of the initial conditions, and behaves well, staying within the bounds expected from physical considerations and high resolution PBL simulations. Except for rather insignificant errors produced in the advection step, no artificial con-

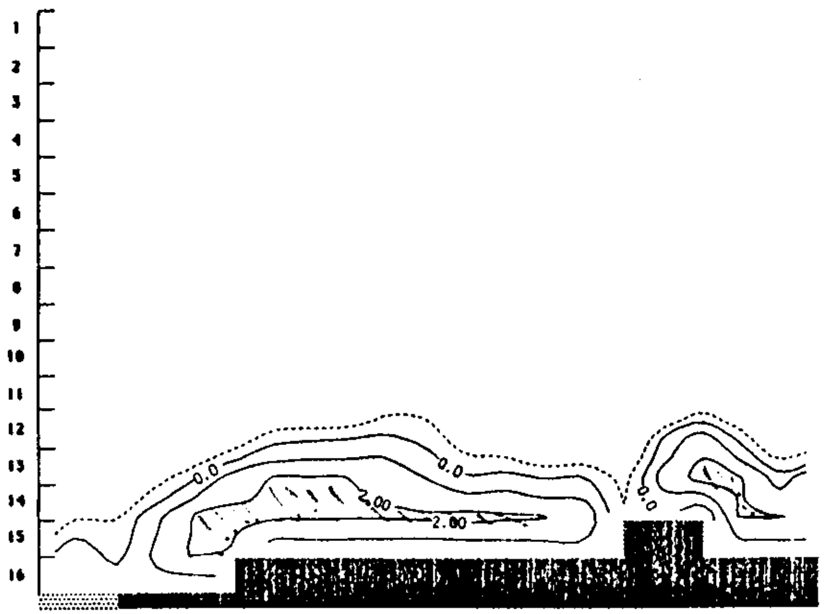

FIG. 8. Vertical cross section as in Fig. 7, but for $\log_{10}$ of matched Level 2 and Level 2.5 heat exchange coefficients (in $m^2 s^{-1}$).

(cf. e.g., Miyakoda and...
termittent turbulent tra...
der unstable conditions...
depth of the surface lay...
heat capacity become c...
cent layer of the air.

Following many oth...
order nonlinear lateral...
the eta coordinate mod...
pends on deformation...
koda and Sirutis 1983...
1988). The ratio of the...
mentum and heat exch...
with the Mellor–Yama...
order diffusion coeffici...
ceed values that are al...
smaller than those typi...
the major centers; an ap...
nitude smaller coefficier...
of the smaller grid dis...
divergence damping a...
maintaining the smoot...
or accelerating the geos...

With several minor...
Miller approach has bee...
cumulus convection (...
1986). The formulation...
is rather conventional,...
of precipitating water i...
the condensation level.

The radiation schem...
dependently. Instead, ...
radiation scheme wit...