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SECOND COLLEGE ON THEORETICAL AND EXPERIMENTAL RADIOPROPAGATION PHYSICS
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IONOSPHERIC TIME DELAY EFFECT ON EARTH-SPACE LINKS

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1- Fundamentals of signal delay

Let us consider a signal transmitted by a satellite, crossing the free space, the ionosphere, the stratosphere, and finally received at ground level. For simplicity assume that the satellite transmitter -Tx- and the ground receiver -Rx- are both stationary. If the wavelength λ and the total phase Φ between Tx and Rx are known, the range ρ can be calculate (see fig.1):

$$\rho = \lambda \phi$$

To measure the range practically it must be known:
- the fractional transmitted phase of the carrier at time t
- the expected range sufficiently accurate to assume the correct number of cycles -accurate means better than one half cycle-.

The Tx and Rx phase can be written:

$$\phi_{Tx} = \omega\, t \quad \phi_{Rx} = \omega(t - \frac{\rho}{c})$$

$$\phi = \phi_{Tx} - \phi_{Rx} = \omega\, \frac{\rho}{c}$$

this is the range measurement called "phase delay measure". Of course, to know the transmitted phase at time t becomes to know the Tx time with respect to the receiver time. This means the knowledge of the time offset τ_c between Tx and Rx; any error on this offset $\Delta\tau_c$ becomes an error $\omega\Delta\tau_c$ in the phase and $c\Delta\tau_c$ in the range.

In the case the offset is unknown -the most probable case-, the station time can be used, the Tx phase is affected by the error of the full offset $\omega\tau_c$ and the measured phase becomes:

$$\phi = \omega(\frac{\rho}{c} + \tau_c)$$

The quantity in parentheses is called pseudo-range and can be useful in many applications, when the offset can be later known or lost during differentiation. In any case the cycles ambiguity must still be solved.

If we consider not only a carrier, but a signal -i.e. any kind of modulation-, there are two basic types of delay to examine:
- a non dispersive delay in which all the frequencies are delayed of the same amount and therefore the signal results unaltered;

- a dispersive delay, when different components are delayed of different amounts, depending on the frequency.

In the first case phase and frequency are linearly related: $\phi = \tau\omega$; the derivative $\frac{d\phi}{d\omega}$ is the constant τ (fig.2). In the second case τ is a function of ω. This derivative, evaluated at a frequency ω_0, is called the group delay at ω_0 and give the actual time delay of a narrow wave packet centered at that frequency. A measurement of signal delay that is an estimation of that derivative is a "group delay measurement". We must note that in a non dispersive medium, the phase delay and the group delay give the same correct result:

$$\frac{d\phi}{d\omega} = \frac{\phi}{\omega} = \tau$$

for dispersive media this is not true: only the group delay $\frac{d\phi}{d\omega}$ is the actual time delay at every frequency.

In the real case the signal is affected by many different delays between Rx and Tx, which can be summarized:

non dispersive delays

- τ_{Tx} trough Rx electronics
- τ_{Rx} trough Tx electronics
- τ_{trop} trough troposphere
- τ_c clock offset -this is an apparent delay-

dispersive delays

- $\phi_{Rx}(\omega)$ dispersion by Rx electronics
- $\phi_{Tx}(\omega)$ dispersion by Tx electronics
- ϕ_h phase shift (constant for all frequencies) due to heterodyning
- $\phi_{ion}(\omega)$ dispersion by ionosphere

In the following the group delay and phase delay due to the ionosphere will be pointed out.

2-The ionospheric delay

The neutral part and the heavy ions of the ionosphere is characterized by a very low molecular density and no charge so that it has no effect on the propagation of the electromagnetic waves at radio frequencies. The electrons, on the contrary, affect heavily the propagation, due to their very little mass and consequent very high mobility. So that a refraction index depending on the electron density must be defined (see the magnetoionic theory in Davies, 1969). A simple but important formula can be written as example (no collision, no magnetic field):

$$n^2 = 1 - \frac{e^2}{\epsilon_o m \omega^2} N$$

$$n^2 = 1 - \frac{k}{f^2} N$$

and, at the first order:

$$n = 1 - \frac{k}{2f^2} N$$

where:

e charge of electron	1.602 10-19 C
m rest mass of electron	9.107 10-31 kg
ϵ_0 permittivity of the free space	8.854 10-12 F/m

N is the electron density in el/m^3

f is the frequency of the signal in Hz

k is the constant 80.6

$\omega = 2\pi f$

If a vertical profile of the electron density is available, a precise path length and the related delay can be easily computed for each signal. But the ionosphere depends on many physical processes affected by many parameters as:

- time of the day
- season
- latitude
- solar activity
- magnetic activity.

So it is impossible to model a "mean ionosphere" to compute a "mean delay". Different methods must be applied to different measurement systems in order to compute, or to correct, or finally in order to don't take into account the ionospheric effects.

In general these effects are due to the whole ray path, and the total effect depends on the integral of the refractive index along the path and consequently on the integral of the electron density N i.e. of the quantity:

$$TEC=\int_{Rx}^{Tx} N(s)ds$$

where:

N(s) is the electron density in el/m_3

ds is the ray path element in m

TEC Total Electron Content results in el/m^2

TEC is therefore the total number of free charges (electrons) contained in one square meter section tube along the ray path between transmitter and receiver.

Most of the effects of the ionosphere on the crossing signals can be modeled by using only this ionospheric parameter.

If we consider the optical path between satellite and ground station as a straight line -in the case of the ionosphere the error is of an order greater then the first one (7)-, we can write:

$$\rho=\int_S nds=\int_S (1-\frac{kN}{f^2})^{\frac{1}{2}}ds$$

At a 1° order approximation, we have

$$\rho=S-\frac{1}{2f^2}\int_S kNds$$

the ion group delay is:

$$\Delta t = \frac{\rho - S}{c} = \frac{k}{2}\frac{1}{cf^2}\int_s N ds$$

that is:

$$\Delta t = \frac{40.3}{cf^2} TEC \quad = \frac{1.34\ 10^{-7}}{f^2} TEC$$

where:
t is the time in sec
f is the signal frequency in Hz

The ionospheric time delay is therefore directly proportional to the TEC and inverse proportional to the square of the frequency.

As said above, the TEC is a function of many variables as local time, season, geographic position, solar and magnetic activity. In Fig 4 a typical TEC is shown: it is highest within a few hours of local noon and it is greater in a region situated between plus and minus 20 degrees away from the geomagnetic equator -equatorial anomaly region-. Accurate determination of monthly median at any station can reduce the ionospheric error (rms) in the delay determination of approximately 70-80 %. TEC models are usually insufficient to compute the expected path delay at any time, but a good representation of the monthly mean behavior of the TEC is a good tool to reduce the errors when direct measurement are impossible. So a particular effort must be done in developing models of TEC using as less input parameters as possible and the minimum computation time.

In many applications as, for example, in geodesy, the path length must be computed, measuring the path delay or the phase delay. If two coherent near frequencies are available on the Tx, the dispersivity of the ionosphere becomes a great tools both to correct for the delays and to measure the TEC. In fact it can be written:

$$\Delta t_1 = \frac{40.3}{cf_1^2} TEC \quad \Delta t_2 = \frac{40.3}{cf_2^2} TEC$$

the time delays difference $\Delta t_1 - \Delta t_2$ can be measured, the frequencies are known, so the TEC can be computed and used to get absolute values of path i.e. Earth- satellite distance. The ionospheric structure doesn't matter, the only limit is that the ionosphere must not make different the propagation path of waves of different frequencies; this assumption is allowed if the frequencies are very close and possibly high.

Some useful equations for numeric calculation follow.

From:

$$\Delta t - \tau - \frac{d\phi}{d\omega}$$

derives:

$$\phi - - \frac{k'}{\omega}$$

So the phase advance can be written:

$$\phi - - \frac{1.34\ 10^{-7}}{f} TEC$$

and the ion phase delay $= \frac{\phi}{\omega} - - \frac{k'}{\omega^2} - \quad - \frac{1.34\ 10{-}7}{\omega^2} TEC$

Thus the ionosphere alters group and phase delay measurements by equal amounts but with opposite sign. Any modulation on the signal will be delayed of the positive group delay.

Finally, it can be written:

$$\Delta f - \frac{d\phi}{dt} - \frac{1.34\ 10^{-7}}{f} \frac{d(TEC)}{dt}$$

where: $\Delta\phi$ is in cycles.

3- The GPS

The GPS -Global Positioning System- is an advanced navigation system, in which a user would determine his position or two users would determine their relative position by measuring pseudo range or phase delay of the signals transmitted from four satellites. The user needs measurements from four satellites to determine his position in tree dimensions and to correct for the time reference of his clock against the time information received. The System will consist of twenty four (reduced in the new program to eighteen) satellites with eight (six) satellites in each of tree orbital planes. Each orbit will be highly circular with an inclination of 63° and the orbital planes will be displaced from one another by 120°. The altitude of the satellites above the surface of the Earth is about 20.000 Km. The orbital period is to be precisely twelve sidereal hours. So each satellite repeats the some ground track day after day.

The GPS signal structure is quite complicated. Briefly, there are two carriers at frequencies L_1=1.57542 GHz (154*10.23 Mhz) and L_2=1.2276 GHz (120*10.23 MHz), both biphase modulated by identical pseudo-random square wave codes having a chip rate of 10.23 Mbs. These are the precise codes or P-codes. The square wave phase modulation switches between ±90°, which is equivalent to a square wave amplitude modulation switching between ±1. Thus it can be written:

$$S_1(t) - P(t)\ A\ \cos\omega_{L_1} t$$
$$S_2(t) - P(t)\ B\ \cos\omega_{L_2} t$$

where P(t) is the random square wave taking the values ±1. In addition the L_1 carrier is phase modulated in quadrature (i.e. with a 90° phase shift) by a pseudo random code with a chip rate of 1.023 Mbs. This is the C/A code (coarse/acquisition). Thus the $S_1(t)$ must be written:

$$S_1(t) - P(t)\ A\ \cos\omega_{L_1} t + C(t)\ A'\ \sin\omega_{L_1} t$$

where the C(t) is the code switching between ±1. Furthermore a signal of 50 Hz, BPSK modulated containing time, orbital etc data, modulates both L_1 and L_2, resulting in a multiplying factor $D(t)\cos\omega_{50Hz}t$. The carrier is suppressed in the signal spectrum by this modulation; however it is recoverable in the receiver for phase delay measurement whether if the codes are known -by multiplying the received signal with the code-, or if it is possible in some way to reduce the band of the signal to the band of the information (some hundreds of Hz as the modulation of the information is 50 Hz) by squaring the signal. Pseudo range is measured both from the recovered carrier phase delay and by correlation between the same code (also unknown) at the two frequencies.

Remembering the previous section, it can be written

$$\phi(\omega) - \omega\left(\frac{\rho}{c} + \tau_c + \tau_{Tx} + \tau_{Rx} + \tau_{trop}\right) + \phi_{Tx} + \phi_{Rx} + \phi_h + \phi_{ion}$$

We forget in this lecture all the delays but the ionospheric one. The time delay $\Delta t(L_1)$ for the L_1 signal and the difference in time delay between the two signals $\delta(\Delta t)$ can be computed:

$$\Delta t(L_1) = 5.415 \times 10^{-26}\ TEC$$
$$and$$
$$\delta(\Delta t) = \frac{\Delta t(L_1)}{1.5457}$$
$$where$$
$$\Delta t(L_1)\ in\ \mathrm{sec}$$
$$TEC\ in\ el/m^2$$

from these, it is possible to write the difference between the delays of the two signals L_1 and L_2:

$$\delta(\Delta t) = .35033 \times TEC$$
$$where:$$
$$\delta(\Delta t)\ in\ nsec$$
$$TEC\ in\ 10^{16}\frac{el}{m^2}$$

This means that a differential delay between the two frequencies of 1 nanosec is equivalent to 2.8 10^{16} el/m^2.

In conclusion two kinds of measurements can be carried out: pseudo range measurements, by detecting the time of correlation between the transmitted code and the station generated code to measure absolute position. The ionosphere can delay the arrival time of the signal, at these frequencies, over 100 meters in range in the worst case (high TEC, very slant ray path); the expected accuracy in position determination is 16 meters. Using the two frequencies method this delay can be automatically corrected: the residual ionospheric error -due to phase detecting error- can be estimated roughly 2 meters r.m.s. A global error of 3 meters using the P code and of 30 m with the C/A code is expected. The second method consists in reconstructing the carrier (knowing the code or by squaring as said above in case of high S/N ratio) and consequently in measuring differences in phase. In the case of absolute position measurement, the accuracy is of the same order of the range measurements (it could be better in principle, but the imprecise knowledge of the orbit reduce the accuracy); if the relative position between two different stations is measured an accuracy of few centimeters can be expected proportionally to the distance.

3- Polarization rotation

Another effect of the ionospheric delay on a signal is the so called Faraday rotation of the polarization, caused by the different phase velocity of both ordinary and extraordinary component of the signal. The angle of rotation can be derived from the magnetoionic theory (Davies K.,69):

$$\Omega = \frac{k}{f^2}\int_{Rx}^{Tx} B \cos\theta \, N \, ds$$

where:

Ω is the amount of Faraday rotation in radian and it can be written: $\Omega = \Omega_m + \Omega_i + \Omega_0$, where Ω_m is the measured rotation, Ω_i is the initial polarization at the satellite, Ω_0 is the ambiguity.

k is a constant that includes the electron's charge and mass, the velocity of light, and other physical constants. Its value is, for our purposes, is $2.36 \cdot 10^{-5}$.

f is the frequency in Hertz

B is the magnetic field intensity in gammas (1 gamma=10^{-5} gauss).

θ is the angle between the direction of propagation and the magnetic field direction

$\int N ds$ is the TEC in electrons per square m.

This rotation of the polarization can induce other error in pseudo range measurements if linear polarization is used. In the case of the GPS the two components are both right circular polarized - i.e. clockwise looking the Earth from the satellite.

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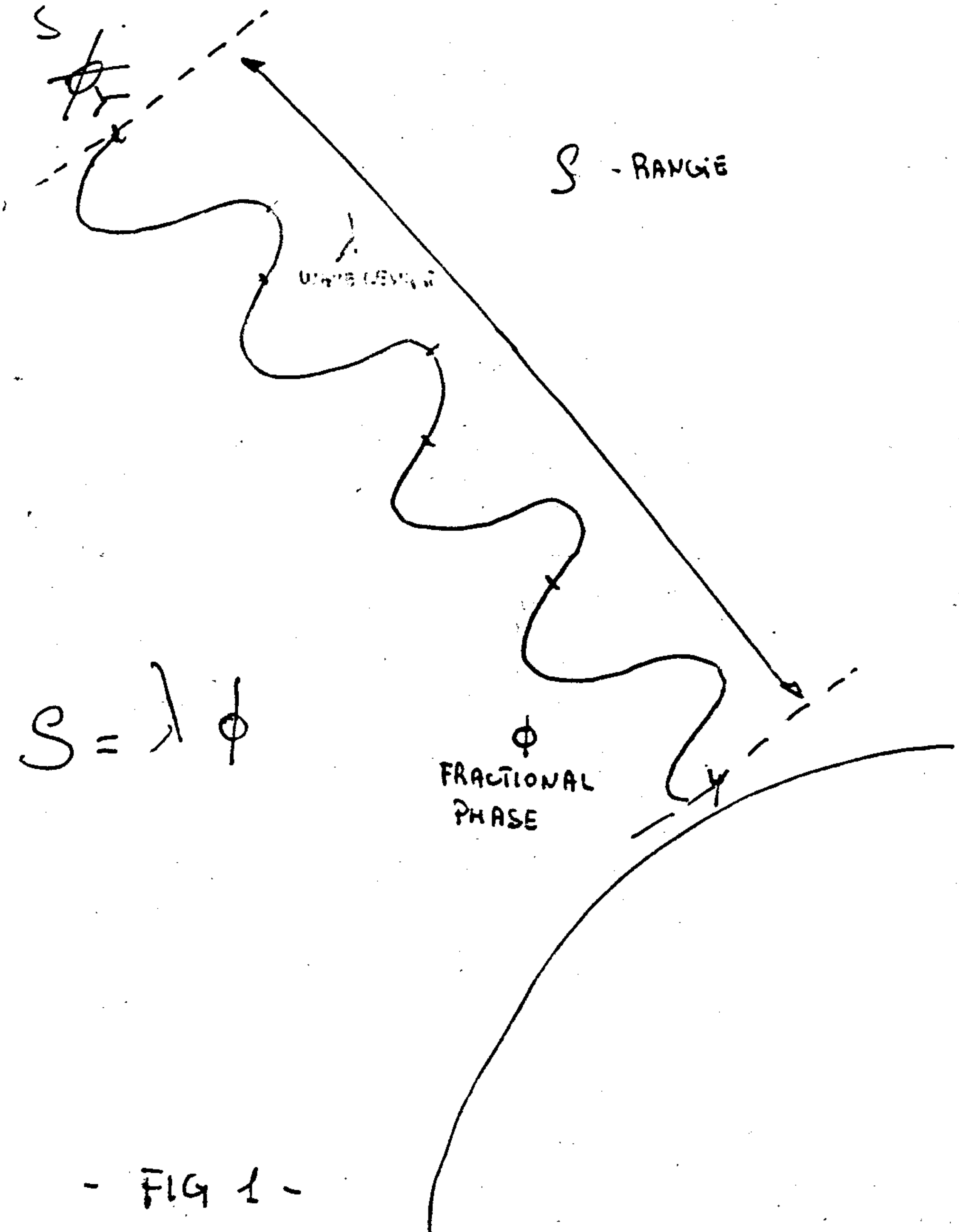


$$S = \lambda \phi$$

- FIG 1 -

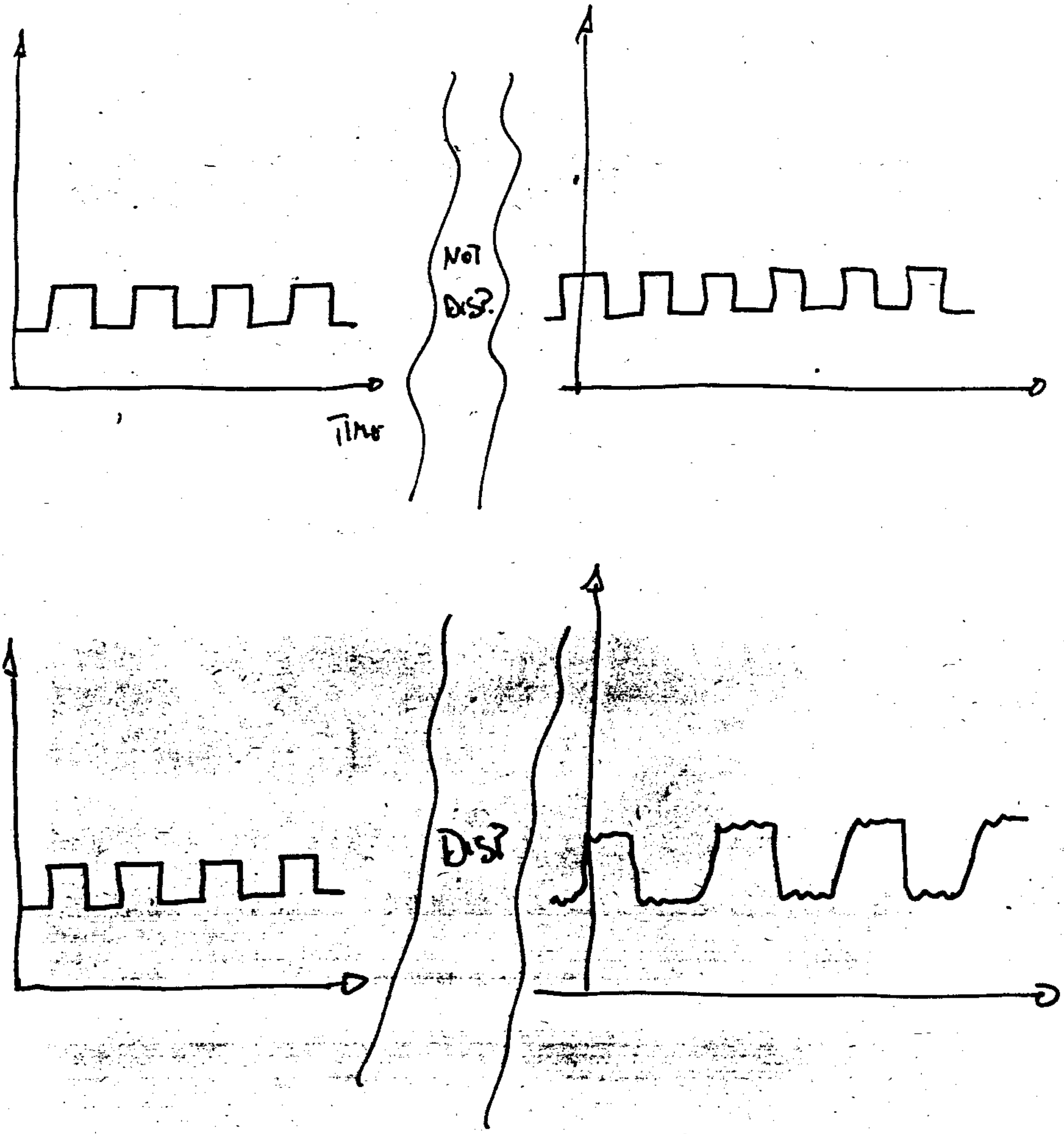


FIG. 2

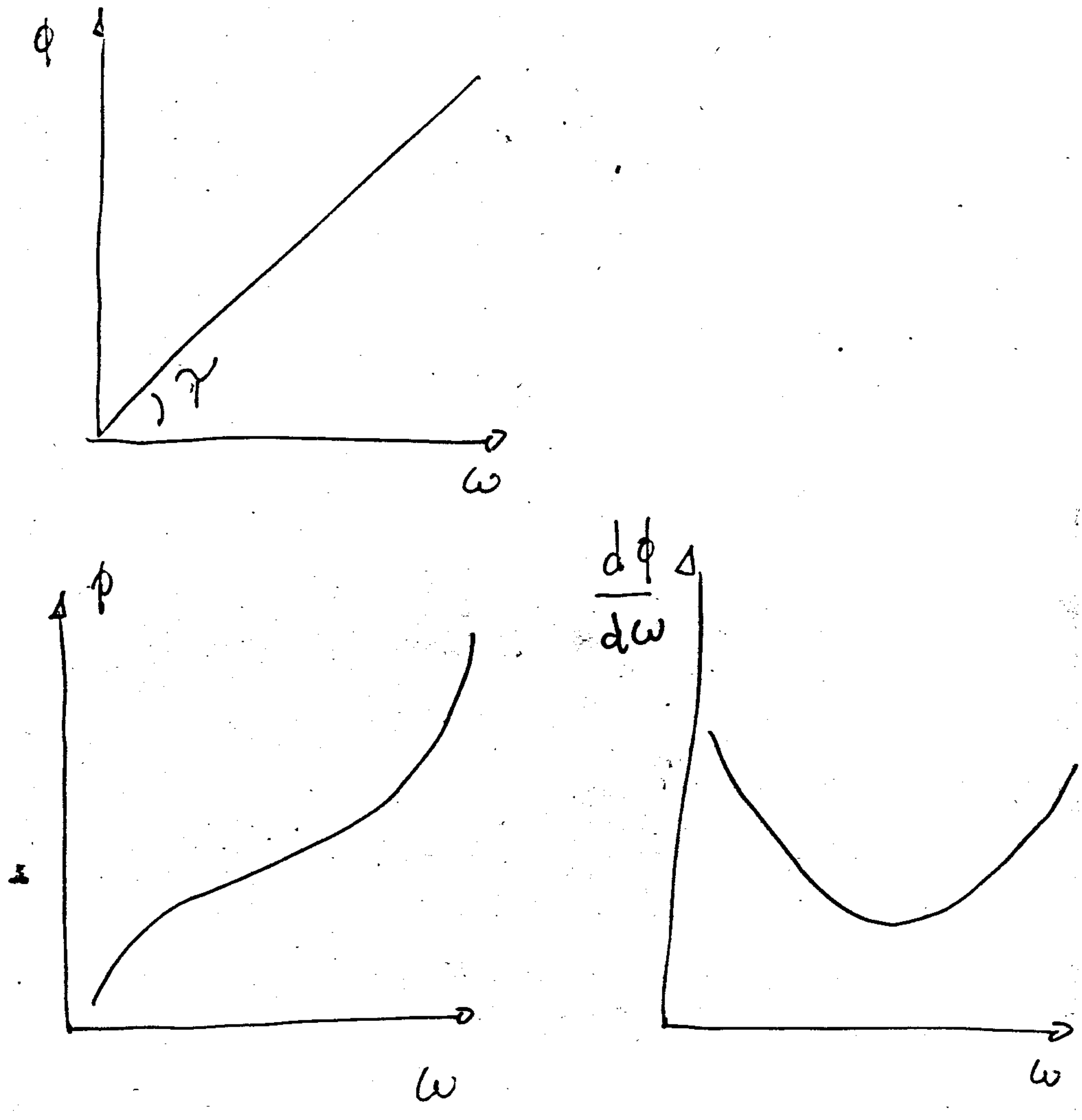


FIG. 3

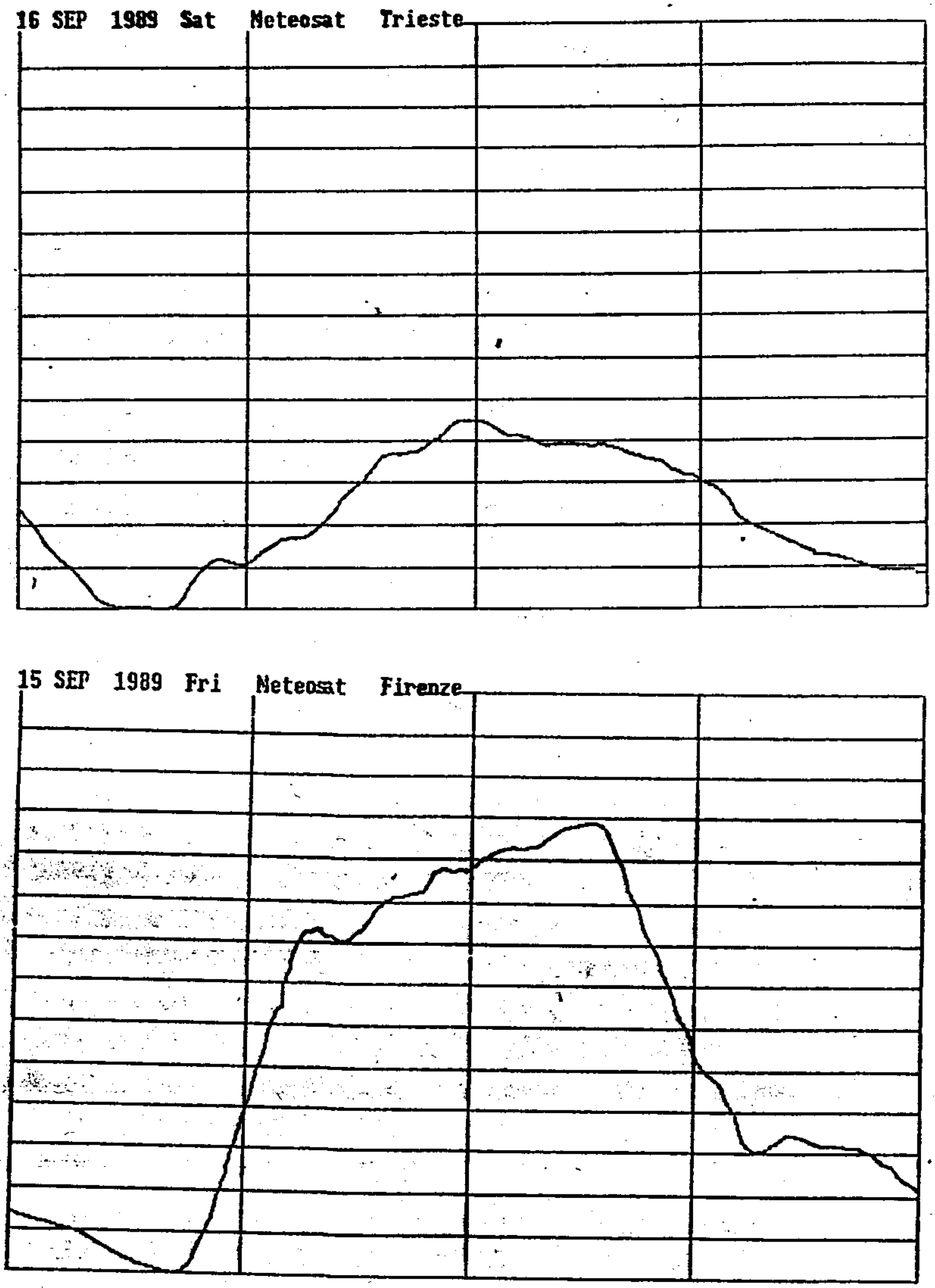
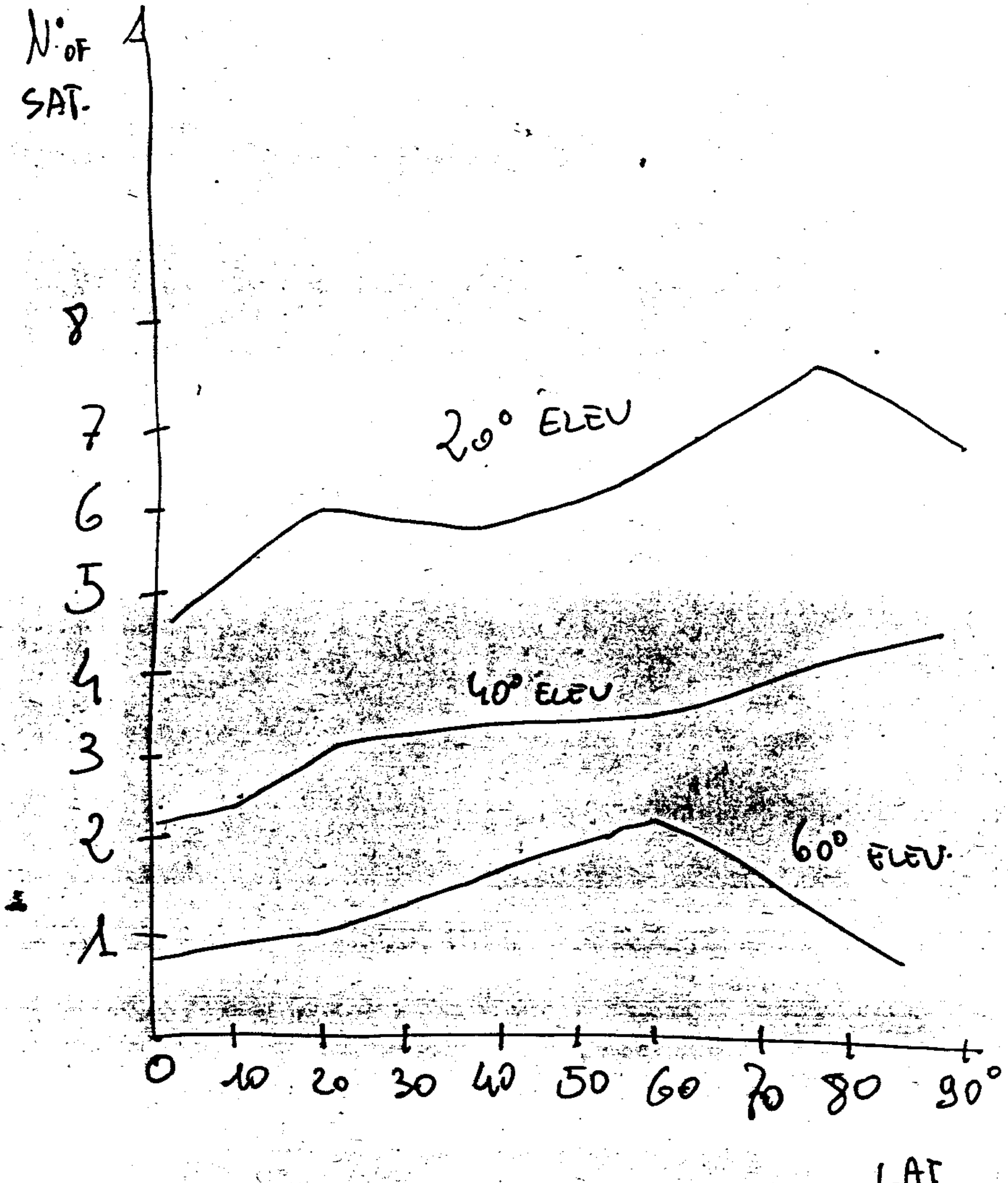

16 SEP 1989 Sat Meteosat Trieste
15 SEP 1989 Fri Meteosat Firenze

FIG. 4



N° OF SAT.
8
7
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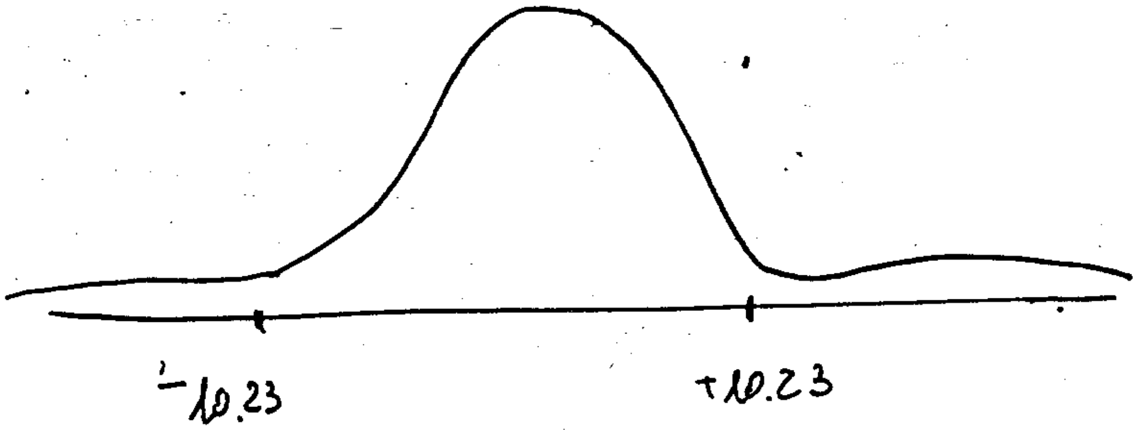
80
90°
LAT



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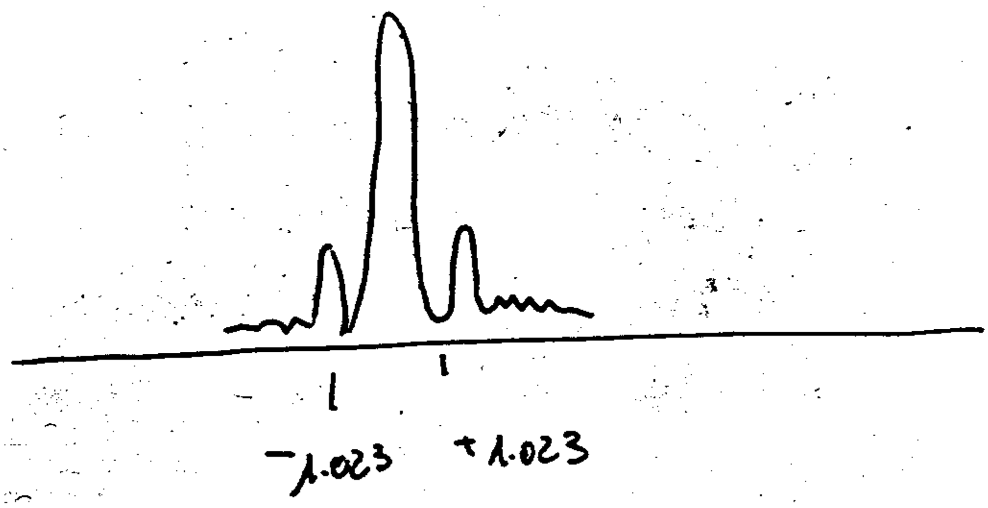

L_1 = SUM OF THE TWO