INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

UNITED NATIONS INDUSTRIAL DEVELOPMENT ORGANIZATION

INTERNATIONAL CENTRE FOR SCIENCE AND HIGH TECHNOLOGY
c/o INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS 34100 TRIESTE (ITALY) VIA GRIGNANO, 9 (ADRIATICO PALACE) P.O. BOX 586 TELEPHONE 040-224572 TELEFAX 040-224575 TELEX 460449 APH I

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System performance determination White Gaussian noise Constant and fading signal

A. D. Spaulding
Institute for Telecommunication Sciences
U.S. Department of Commerce
Boulder, Colorado

SYSTEM PERFORMANCE DETERMINATION

WHITE GAUSSIAN NOISE

CONSTANT AND FADING SIGNAL

we will first consider a simple, but practical example. This will enable us not only to understand the technique, but also to see why the technique will not work for other kinds of fading signals (for example, frequency selective fading, leading to signal distortion).

We will analyze the performance of a binary coherent phase shift keying (CPSK) digital system first, when the signal is constant, and then, from the probability of error characteristic obtained for this constant signal, we will obtain the system performance for slow-flat fading signals. We will do this for all the types of slow-flat fading signals generally considered, starting at the very beginning and analyzing the system's performance using a geometrical approach. This will enable us to picture what is going on in the signal-receiving process.

BI-1.2. CONSTANT SIGNAL PERFORMANCE

To represent a digital system geometrically, we make use of the following fact:

> Any finite set of physical waveforms of duration T, say $S_1(t)$, $S_2(t)$, ..., $S_m(t)$, may be expressed as a linear combination of k orthonormal waveforms $\phi_1(t), \phi_2(t), \ldots \phi_k(t)$, where $k \leq m$.

That is, each signal, $S_i(t)$ can be written as

$$S_i(t) = a_{i1}\phi_1(t)+a_{i2}\phi_2(t)+\ldots+a_{ik}\phi_k(t) \quad , \quad \text{(BI-1)}$$

where the coefficients a_{ij} are given by

$$a_{ij} = \frac{1}{T}\int_0^T S_i(t)\phi_j(t)\,dt.$$

BI-1. SYSTEMS EVALUATION FOR SLOW-FLAT FADING

A. D. Spaulding

BI-1.1. INTRODUCTION

In this section we will develop the simple technique to determine the performance of a telecommunications system with a slow-flat fading signal once a performance characteristic is known for the constant signal. The "slow" in slow-flat fading means the signal amplitude fades slowly enough in time that the signal can be regarded as constant over some time period of interest (such as the time of a signal element in a digital system). The "flat" refers to the spectral behavior of the fading, and implies that the entire signal spectrum fades up and down uniformly so as not to distort the signal.

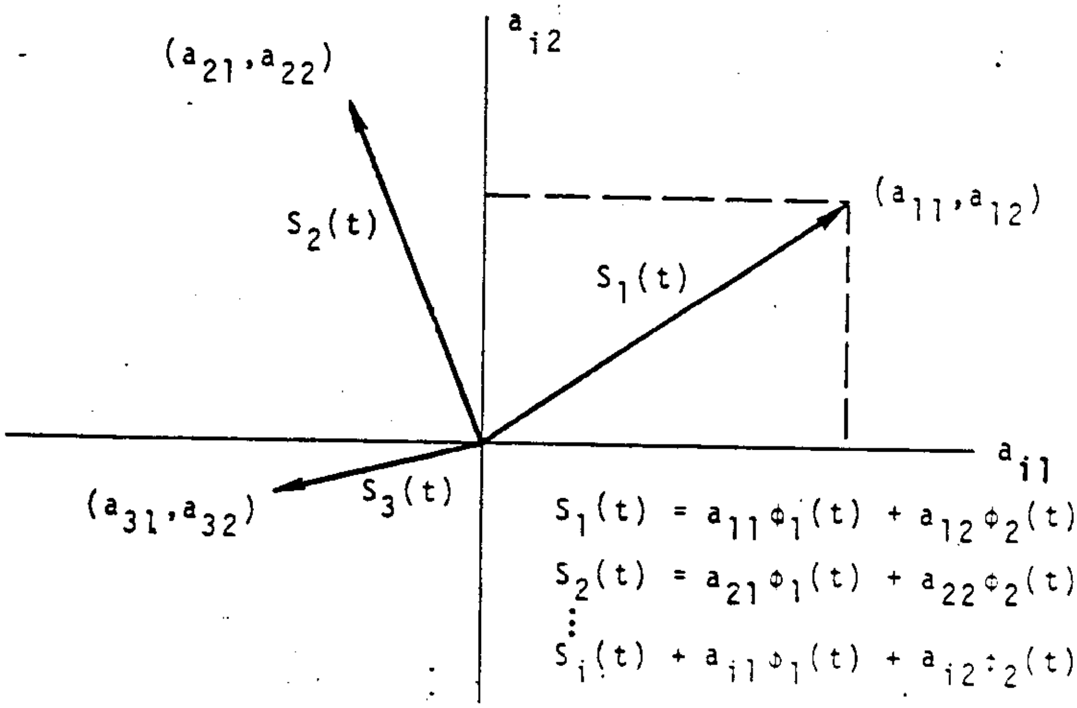
The physical processes that cause fading fall into two broad catagories: (1) absorption and other large volume effects, which result in a random signal normally called scatter; (2) the other category is comprised of numerous specular modes of propagation. The separation of the modes may take place at sharp boundaries of charged particles or reflections from isolated objects, etc. We have an assortment of distinct paths that the wave fronts may take in propagating from the transmitter to the receiver. This phenomenon is commonly called multipath and each path may contain some specular and scatter contributions. In any case, the fading signal received at the receiver becomes random and can be treated only in statistical terms.

In order to understand how a system's performance is degraded by the slow-flat fading signal compared with the performance for a constant signal of the same average power, and how the degree of degradation can be easily calculated, Here the basic waveforms, $\phi_j(t)$, being orthonormal means that

$$\frac{1}{T}\int_0^T \phi_i(t)\phi_j(t)dt = 1 \quad i = j$$
$$= 0 \quad i \neq j .$$

While the above representation looks similar to the familiar Fourier expansion of a waveform, it is different in two important respects. The waveforms $\phi_i(t)$ are not restricted to sine and cosine waveforms, and (BI-1) is exact, even though only k terms are used.

Because of the above, our signaling waveforms, $S_i(t)$, can be represented in the k-dimensional signal space, $\phi_j(t)$, with coordinates given by the a_{ij}. For example, consider a set of signals for which k = 2, then the signals, $S_i(t)$, are given by vectors in the space $\phi_1(t)$, $\phi_2(t)$ as in figure BI-1.



$S_1(t) = a_{11}\phi_1(t) + a_{12}\phi_2(t)$

$S_2(t) = a_{21}\phi_1(t) + a_{22}\phi_2(t)$

$\vdots$

$S_i(t) + a_{i1}\phi_1(t) + a_{i2}\phi_2(t)$

Figure BI-1. Signals represented as vectors in a signal space, k = 2.

As we shall see, the above representation not only allows visualization of what is actually going on in the receiving process, but also allows the variable, t, time to be removed from the problem. Our signals are now represented by simple vectors in ordinary Cartesian coordinates. That is, each signal is now represented by a point in the signal space with coordinates a_{ij}. All the rules of ordinary geometry apply, for example, the "distance" between signals is simply the ordinary distance between the corresponding signal points.

Digital receivers, actually, by various means, compute the coordinates of a received signal and then make a decision based on these coordinates. One obvious receiver implementation is shown in figure BI-2. The actual physical implementations of the digital receiver may be, as in figure BI-2, a matched filter form, etc., but all these forms accomplish precisely the same thing, i.e., to compute the signal coordinates, a_{ij}, and then make a decision as to which signal was sent, based on these a_{ij}.

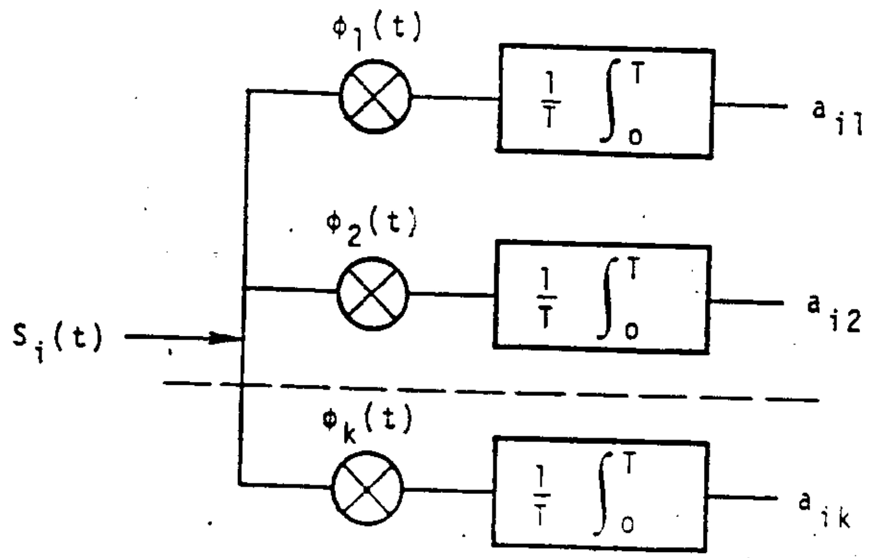


Figure BI-2. Product integrators used to calculate the signal space coordinates of signal $S_i(t)$.

The additive noise, which interferes without signal and causes the receiver to make errors when it tries to decide which one of the m-signalling waveforms was transmitted, also is represented by a point in the receiver's signal space. If n(t) is the received random-noise waveform, then it (like the signal) goes through the product integrators (or whatever), with the result that, as far as the receiver is concerned, the interfering noise is given by

$$n(t) = n_1\phi_1(t)+n_2\phi_2(t)+\ldots+n_k\phi_k(t) \quad . \qquad \text{(BI-2)}$$

Therefore, if the receiver received noise only, the noise would also be represented by a point in the receiver's signal space, the noise coordinates given by n_i, n_2, ..., n_k.

Each of our m signals is represented by a unique point in the signal space. When signal plus noise is received, the result is a point (signal-plus-noise point) that can be anywhere in the signal space, depending on the noise. If each of our m signals is equally apt to have been sent, and are of equal power, the receiver, in order to minimize the average probability of error when it guesses what signal was transmitted, simply guesses the signal whose "point" is closest to the received signal-plus-noise point.

To take a specific example, consider coherent phase-shift-keyed signals. Our m signals are now, say

$$\begin{aligned} S_i(t) &= \sqrt{2W}\cos\left(\omega_0 t+\frac{2\pi i}{m}\right) \quad , \; 0\leq t<T \\ &= 0 \quad \text{elsewhere} \qquad\qquad \text{(BI-3)} \\ i &= 1,2,\ldots,m, \end{aligned}$$

where W is the power in $S_i(t)$ (Watts), and $\omega_0 = 2\pi\ell/T$, for some fixed integer ℓ.

We can choose, then, for our basic waveforms

$$\phi_1(t) = \sqrt{2}\cos \omega_o t$$
$$\phi_2(t) = \sqrt{2}\sin \omega_o t$$

Note that our signal space is two-dimensional (k = 2) no matter what m is.

Consider m = 2, now

$$a_{11} = \frac{1}{T}\int_0^T \sqrt{2W}\cos(\omega_o t+\pi)\sqrt{2}\cos\omega_o t\, dt = -\sqrt{W}$$

$$a_{12} = \frac{1}{T}\int_0^T \sqrt{2W}\cos(\omega_o t+\pi)\sqrt{2}\sin\omega_o t\, dt = 0$$

Likewise, $a_{21} = \sqrt{W}$, $a_{22} = 0$. Therefore, the space and the points representing the two signals are as shown in figure BI-3. The point (n_1, n_2) corresponding to additive noise alone is also shown on figure BI-3.

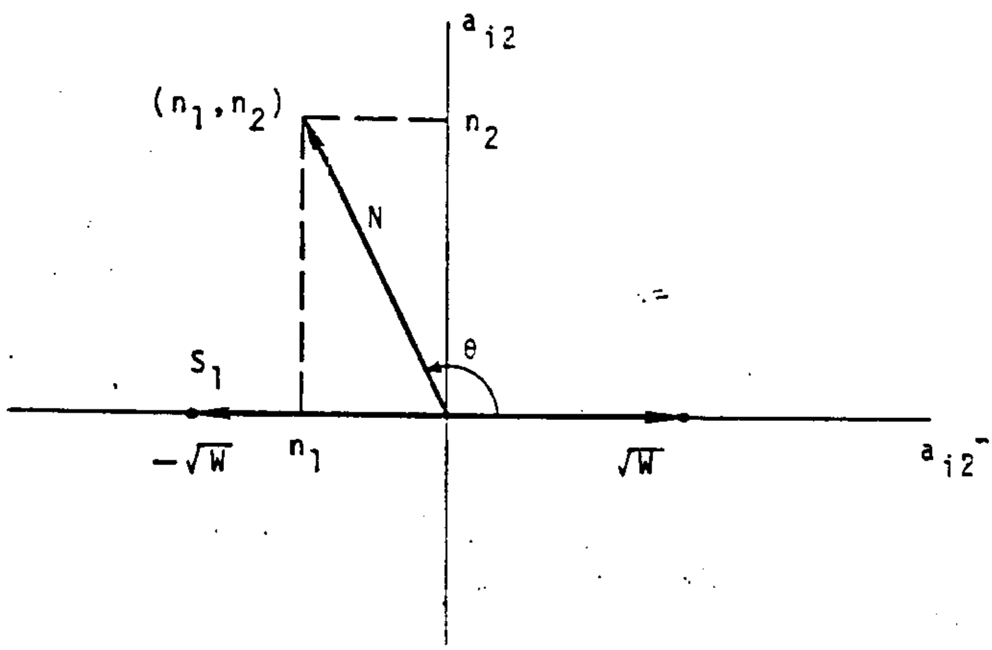


Figure BI-3. The signal space and signal points for binary CPSK, and the noise point (n_1, n_2).

Let the interfering noise be zero mean, white Gaussian, such as it would be if the noise were galactic or receiver front-end noise. N, the noise amplitude after it goes through the receiver and appears on the signal space (fig. BI-3), is Rayleigh distributed. Its probability density function is

$$p_N(y) = \frac{2y}{N_oB} \exp\left[-\frac{y^2}{N_oB}\right], \quad y \geq 0 . \qquad \text{(BI-4)}$$

This says that the probability that the noise amplitude N has a value in the range Y - dy/2 and y + dy/2 is given by $p_N(y)dy$, where N_o is the noise power spectral density (Watts/Hz) and B is the bandwidth (Hz), i.e., N_oB is the noise power. The phase angle θ is uniformly distributed, i.e., its probability density function is

$$p_\theta(x) = \frac{1}{2\pi}, \quad -\pi < x \leq \pi \quad ,$$

i.e., θ has equal probability of being anything between -π and π. The coordinate points are given by $n_1 = N \cos \theta$ and $n_2 = N \sin \theta$. This results in the coordinate points, n_1 and n_2, having zero mean normal distributions,

$$p_{n_1}(x) = \frac{1}{\sqrt{\pi N_oB}} \exp\left[-\frac{x^2}{N_oB}\right], \quad -\infty < x < \infty \qquad \text{(BI-5)}$$

Note that since our development led to the signal being represented on the signal space by a vector of length $\sqrt{W}$, i.e., a rms voltage, the noise appears on the signal space in similar terms. That is, N or the variable y in (BI-4) is the instantaneous rms amplitude of the noise envelope.

Now let us consider the situation where S_2 is sent and we want to compute the probability that the receiver will decide S_1, and thus make an error. The situation is shown in figure BI-4. If the resultant signal-plus-noise point lies in the shaded region (the region whose points are closest to the S_1 point), then the receiver will decide S_1, and make an error. This will happen whenever $\sqrt{W} + n_1$ is less than zero, or p_e = probability of error given that S_2 is transmitted = probability that $\sqrt{W} + n_1 < 0$. The probability, or likelihood, that $\sqrt{W} + n_1 < 0$ depends on the probability distribution of n_1.

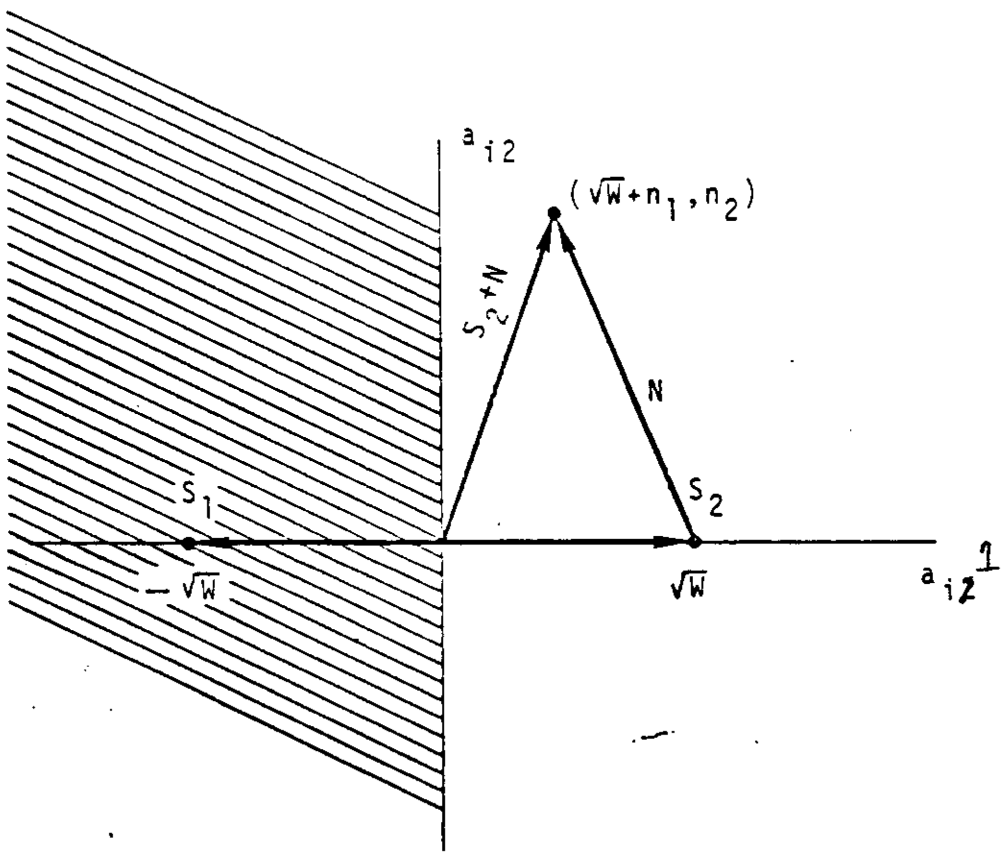


Figure BI-4. The signal-plus-noise point, given that S_2 was transmitted.

In our case

$$p_e = \text{prob}\left[\sqrt{W} + n_1 < 0\right] = \text{prob}\left[n_1 < -\sqrt{W}\right],$$

or, from (BI-5)

$$p_e = \int_{-\infty}^{-\sqrt{W}} \frac{1}{\sqrt{\pi N_o B}} \exp\left(-\frac{x^2}{N_o B}\right) dx \quad ,$$

or

$$p_e = \frac{1}{\sqrt{\pi}} \int_{\sqrt{\frac{W}{N_o B}}}^{\infty} e^{-y^2} dy \quad . \qquad \text{(BI-6)}$$

The performance p_e is a function of the signal-to-noise ratio W/N_oB. It is common to express the signal-to-noise ratio (SNR) as signal energy E (Joules or Watt seconds) to noise power spectral density N_o. For this system, the following are all identical expressions for the SNR:

$$\text{SNR} = \frac{E}{N_o} = \frac{W}{N_o B} = \frac{E}{N_o BT} \quad .$$

The integral (BI-6) can be given in terms of the standard tabulated function called the error function (erf) or

$$p_e = \frac{1}{2}\left[1 - \text{erf}\sqrt{\frac{E}{N_o}}\right] \quad , \qquad \text{(BI-7)}$$

where

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2} dy$$

Let us look more closely at what the above result (BI-6) actually says. If we have in, say the i^{th} bit, the signal level represented by $\sqrt{W}$, and the noise level (in this case represented by n_1), there will or will not be an error in this i^{th} bit, depending on the size of n_1. The integral in (BI-6) says that we are taking an average over an infinity of such i^{th} bits, weighted according to the probability or liklihood that n_1 is of proper size to cause an error. That is, p_e in (Bi-6) represents an average probability of error given that S_2 is sent. If p_e is 10^{-3}, say, then out of m such bits, with m being very, very large, essentially m x 10^{-3} of these bits will be in error. Of course, there is no way of telling which bits will be in error, only the average number. We have considered the above case in which only S_2 was sent. If we repeat for the signal S_1, we obtain the same result. So the probability of error, p_e (BI-6) is the average probability of error for the system.

All digital systems can be put in the above framework and their performance for a constant signal level and for arbitrary additive noise calculated (although, perhaps not so easily as above). Note that for the noise, we required knowledge of the noise as seen by our receiver, how big it was, i.e., its spectral density, N_o, and the probability density of its amplitude. Note also that the performance turned out to be a function of the signal-to-noise ratio E/N_o (or $\frac{W}{N_o B}$).

BI-1.3. FADING SIGNAL PERFORMANCE

We now consider the case where the signal is not constant but fading. Suppose, however, that our signal is not distorted by the fading and that the fading is slow enough that we can consider the signal constant over an appropriate period of time (T seconds in our example). For our example, we still have the same "signal space" representation of the system, but now our two signals are given by (see (BI-3))

$$S_1(t) = -\sqrt{2W_j}\ \cos \omega_o t \ , \ o \leq t \leq T \qquad \text{(BI-8)}$$

$$S_2(t) = \sqrt{2W_j}\ \cos \omega_o t \ , \ o \leq t \leq T$$

where the subscript j denotes the signal level in the j^{th} bit. Note that the only change we have allowed is in the signal amplitude and we require W_j to be constant over the time period occupied by bit j. Having pointed out what, precisely, the "slow-flat" fading rules are, we generally now drop the subscript j, and simply say that the signal amplitude varies according to some fading distribution. This says that now the signal amplitude, just as the noise before, is random and we can only specify the likelihood or probability of it having particular values.

Previously (see fig. BI-3), as we went from bit to bit in our bit stream, the signal points on the a_{i1} axis remained fixed, while the noise point of interest (the coordinate n_1) moved randomly up and down the a_{i1} axis. We computed the average probability of error by averaging over many, many situations (bits) taking into account the probability of n_1 having values which would cause errors.

Now with the fading signal, the signal point also moves randomly up and down the a_{i1} axis as we go from bit to bit. Figure BI-5 shows the situation for three successive bits, considering signal S_2.

As before with the noise, to obtain the average probability of error, we must average over many such bits, taking into account now, the variable signal point (i.e., the probability distribution of the signal amplitude) as well as the variable noise point. This means that our average must now consider both the signal distribution and the noise distribution. Fortunately, this can be accomplished quite easily using the following rule from probability theory:

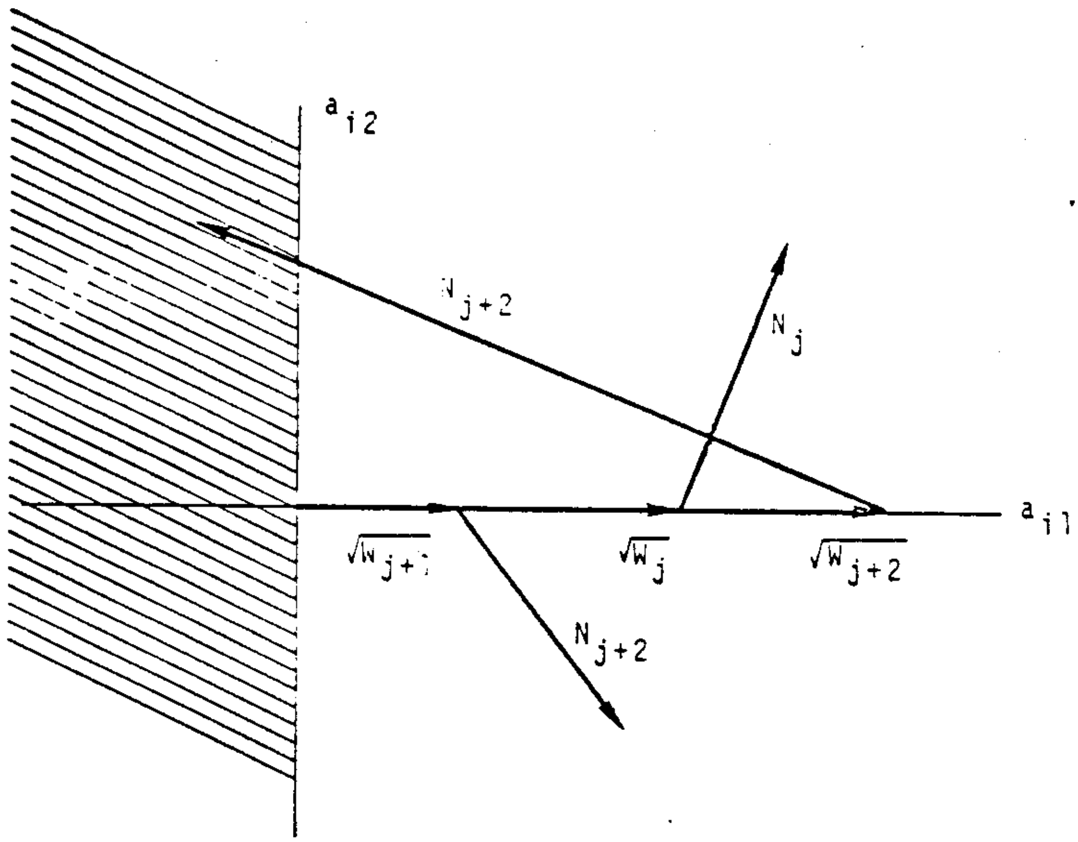


Figure BI-5. Signal plus noise, signal fading.

$$P[A] = \int_{-\infty}^{\infty} P[A|B=x]\; p_B(x)\; dx \;,$$

that is, the probability of event A is given by the probability of event A, given that B has the value x, averaged over all values that B can have.

For our system, we have calculated the performance, given a signal energy E (or power, W) namely, p_e. The above says that for fading signal, we need only multiply the constant signal performance by the probability density function of the fading signal energy and then average (integrate) over all possible values of the signal energy. Therefore, from (BI-7), we have

$$P_e(\text{fading signal}) = \int_0^\infty 1/2 \left[1 - \operatorname{erf}\sqrt{\frac{x}{N_o}}\right] p_E(x)\,dx \, , \qquad \text{(BI-9)}$$

where, in order to keep the variables straight, we have used a "dummy" variable of integration, and $p_E(x)$ denotes the probability density function of E.

Equation (BI-9) gives the solution for the p_e for a fading distribution of signal energy E. Quite often we have given to us, instead, a fading distribution of signal power W or signal amplitude S where $W = S^2/2$. Then, in terms of power, (BI-9) becomes

$$p_e(\text{fading signal}) = \int_C^\infty \frac{1}{2} \left[1 - \operatorname{erf}\sqrt{\frac{x}{N_oB}}\right] p_W(x)\,dx \, , \qquad \text{(BI-10)}$$

where $p_W(x)$ is the fading distribution of signal power W. Note that, of course, (BI-9) and (BI-10) are identical in form. For signal amplitude S,(BI-9) becomes

$$p_e(\text{fading signal}) = \int_0^\infty \frac{1}{2} \left[1 - \operatorname{erf}\frac{x}{\sqrt{2N_oB}}\right] p_S(x)\,dx \, , \qquad \text{(BI-11)}$$

where $p_S(x)$ denotes the distribution of signal amplitude. In (BI-10) the variable of integration x represents signal power W, while in (BI-11), the variable of integration x represents signal amplitude S ($W = S^2/2$).

The question now becomes, what $p_W(x)$ or $p_S(x)$ should we use? Let us first consider the case of a signal whose amplitude fades according to the Rayleigh distribution:

$$p_S(x) = \frac{x}{W_o} e^{-x^2/2W_o} \qquad \text{(BI-12)}$$

where W_o denotes the mean power of the signal; i.e., the mean value of W. Of course, for constant signal, W_o = W. We will see later why the Rayleigh distribution is sometimes a good one to use for multipath signals.

Equation (BI-11) now gives us

$$p_e = \frac{1}{2} \int_0^\infty \left[1 - \text{erf}\,\frac{x}{\sqrt{2N_oB}}\right] \frac{x}{W_o}\, e^{-x^2/2W_o}\, dx \;. \qquad \text{(BI-13)}$$

This integral is easily evaluated (especially with a good table of integrals) to give the known result

$$p_e = \frac{1}{2}\left(1 - \frac{\sqrt{\dfrac{W_o}{N_oB}}}{\sqrt{\dfrac{W_o}{N_oB} + 1}}\right) \;. \qquad \text{(BI-14)}$$

Again, our result came out in terms of the SNR. As discussed previously, the signal power to noise power ratio, W_o/N_oB is equal to the signal energy to noise power spectral density ratio, E_o/N_o, (E_o = mean value of E) for this system.

What we have shown is that the performance of any system with slow flat-fading signal can be calculated using the system performance characteristic in constant signal and the probability distribution of the fading signal. For example, if we had available for an analog system (such as voice) some constant signal performance characteristic (such as articulation index) as a function of signal-to-noise ratio, then we could compute the performance for fading signal as above. We would need to be sure, however, that all the assumptions inherent in "slow" and "flat" were met or were reasonable approximations to the actual physical situation.

In summary, if $g_c(W/N_oB)$ denotes the performance for constant signal, and if $p_W(x)$ denotes the probability density of the signal power W, then the performance of the system in fading signal, $g_f(W_o/N_oB)$, is given by the average over all possible values of W,

$$g_f(W_o/N_oB) = \int_{\text{all } W} g_c(x/N_oB)p_W(x)\ dx\ . \qquad \text{(BI-15)}$$

If $p_S(x)$ is the probability density of the signal amplitude S,

$$gf(W_o/N_oB) = \int_{\text{all } S} g_c(x^2/2N_oB)\ p_S(x)\ dx\ . \qquad \text{(BI-16)}$$

Consider now the cases where either the "slow" assumption, or the "flat" assumption, or both, is not valid. Our receiver will still calculate a signal point no matter what kind of distorted signal the receiver receives. Now, however, the signal points will move randomly and rapidly all over the signal space and the computations of the statistics of such motion will be extremely difficult. Also, the signals are usually spread in time (also frequency), resulting in the received signals occupying more than their allotted (0,T) time slot. The result is that, if we are looking at bit j, for example, there is some signal from bit j-1 still going on, causing interference, i.e., intersymbol interference. This, as well as other problems, indicates why the straightforward approach given in (BI-15,16) cannot be used. For this reason we like to use slow-flat fading approximations whenever possible. The procedures required for system performance calculations in the case of "slow and flat" not being valid are covered in subsequent sections.

BI-1.4 FADING SIGNAL DISTRIBUTIONS

When the signal is propagated from the transmitter to the receiver, it is modified by the propagation media. Quite often the signal travels to the receiver via one, two, or any number of separate paths. If the signal from each of these multipaths is represented by a signal vector, then the receiver sees the vector sum of these signal vectors. The phase angle between any two such vectors is generally on the average, uniformly distributed, i.e., the phase angle has equal chance of being anything between $-\pi$ and π radians. We are interested then in the probability distribution of the amplitude (or power) of the received signal, i.e., the above vector sum.

As mentioned earlier, each path may have some specular and scatter contributions. Scatter comes from large volume effects, and means the signal is scattered into many, many small signal vectors. That is, it is equivalent to multipath with many, many paths such that none of these many, many received signal vectors dominate the others (i.e., sticks out like a "sore thumb"). If we have such a sum of many more or less equal-sized vectors with uniform phase between them, then the amplitude of the vector sum has a Rayleigh distribution. Figure BI-6 (from Neserbergs, 1967) shows the probability-density function of n equal-sized vectors for $n = 1, 2, 3, 4,$ and 6 along with the Rayleigh limit ($n \to \infty$). We see that the "many many" above need only be 5 or 6 before the Rayleigh distribution is a reasonable approximation. In other words, the situation where we have, say 6 or more distinct paths, and the signal components from these paths are essentially equal, then the received signal amplitude is approximately Rayleigh distributed.

Suppose, instead, that we have one specular path (due, for example, to a direct line-of-sight path) and a scatter path, or, equivalently, a number of other paths from which the received signals are more or less equal and small compared to the main signal. An example of one such situation would be "constant groundwave plus Rayleigh fading skywave". There are, of course, many other possibilities. In this case, the received signal amplitude has a Nakagami-Rice distribution,

$$p_S(x) = \frac{x}{\alpha} \exp\left[\frac{-x^2+2\beta}{2\alpha}\right] I_0\left(\frac{\sqrt{2\beta}\ x}{\alpha}\right) \qquad \text{(BI-17)}$$

where α is the power in the Rayleigh vector, β is the power in the constant vector, and I_0 is the zero-order modified Bessel function.

If, as before, N_0 denotes the noise power spectral density, then the signal-to-noise ratio is

$$\frac{W}{N_0 B} = \frac{\alpha+\beta}{N_0 B} \quad . \qquad \text{(BI-18)}$$

The distribution of signal amplitude for the general case of the sum of any number of such Nakagami-Rice vectors and resulting special cases is given by Nesenbergs (1967).

Consider the case where we have a direct ray and a single other path, resulting from a ground reflection. The probability density for the received signal power, W, is then

$$p_W(x) = \frac{1}{\pi} \frac{1}{\sqrt{4k^2\gamma_0^2-(x-(k^2+1)\gamma_0)^2}} \quad ,$$

$$\gamma_0(1-k)^2 \leq x \leq \gamma_0(1+k)^2 \quad . \qquad \text{(BI-19)}$$

where γ_0 is the power of the direct ray and k is the voltage-amplitude ratio of the reflected-to-direct ray (reflection coefficient). The total mean power in the received signal is $\gamma_0(1+k^2)$, or the signal-to-noise ratio is

$$\frac{W}{N_o B} = \frac{Y_o(1+k^2)}{N_o 3} \qquad \text{(BI-20)}$$

Experimental observations of received fading-signal amplitudes over various communication circuits have shown that the signal amplitude, when expressed in decibels, can sometimes be approximated by a normal distribution. That is, the signal amplitude has a log-normal distribution. If, for the signal amplitude, S, we let Y = 20 log S, then

$$p_Y(y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2} \quad , -\infty<y<\infty \ , \qquad \text{(BI-21)}$$

where μ is the mean value of Y(dB) and σ is the standard deviation (dB). The signal distribution for use in (BI-16) is then

$$p_S(x) = \frac{8.686}{x\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{20\log x-\mu}{\sigma}\right)^2} \quad , 0\leq x<\infty \ . \qquad \text{(BI-22)}$$

For log-normal fading signal, the σ is usually given in terms of the "fading range". The fading range is the difference (in dB) between the upper and lower decile values. The upper decile is that value which is exceeded only 10 percent of the time, and the lower decile is that value which is exceeded 90 percent of the time. In terms of the fading range, 2.54 σ = fading range. The average received signal power is

$$W_o = 10^{0.1\mu+0.0115\sigma^2} \qquad \text{(BI-23)}$$

and the signal-to-noise ratio is W_o/N_oB.

The above distributions (BI-12, 17, 19, and 22) pretty well cover all the signal distributions that are generally considered for slow-flat fading. Which one to use depends on the particular kind of propagation path one is interested in. The above distributions of the fading signal say nothing as to how "fast" the signal fades up and down. Therefore, consideration must be given to more than the fading distribution when trying to decide if a slow-flat assumption is valid.

BI-1.5. EXAMPLES AND REFERENCES

In this section we will give the results for our example system (CPSK) for all of the fading distributions considered above. An example for a voice system will also be given.

Figure BI-7 shows the results for the binary CPSK system for constant signal (BI-7), Rayleigh fading signal (BI-14), Nakagami-Rice fading (BI-17) with the power of the constant vector 10 dB above the average power of the Rayleigh vector, Nakagami-Rice fading with the power of the constant vector equal to the average power of the Rayleigh vector, and log-normal fading (BI-22), using a 13.4-dB fading range. Note that Rayleigh fading also has a 13.4-dB fading range.

Figure BI-8 shows the results for the binary CPSK system for the case of constant signal vector plus reflected signal vector. Results are given for constant signal (k=0), and for k = 0.2, 0.6, 0.8, and C.9. We see from figures BI-7 and BI-8 that a very wide range of system performances can be obtained depending on the particular kind of signal fading present.

In order to show the results of using (BI-16) for a voice system, figure BI-9 is included. It shows the performance of a double-sideband AM system in white Gaussian noise and Rayleigh-fading signal. The calculations, via (BI-16), are from the performance in constant signal for a 5.2-kHz IF bandwidth (Cunningham et al., 1947). The performance is given in terms of the phonetically balanced word articulation index.

For the nature of fading signals, extensive bibliographies (Nupen, 1960; Salaman, 1962) are available. A historically significant survey was performed by the National Bureau of Standards (NBS, 1948). A number of good comprehensive texts are also available (Davies, 1965, for example).

The representation of digital systems in geometric terms is covered quite well by Arthers and Dym (Arthers and Dym, 1962). Performance characteristics for systems in fading signal and in nonGaussian impulsive noise (as well as Gaussian noise) are available (Bello, 1965; Corda, 1965; Halton and Spaulding, 1966; Akima et al., 1969; Akima, 1970; etc.

The following list of references includes additional references not cited above. The list is hardly complete, but will provide a great deal of additional information concerning the characterization of the fading channel, and the performance of a wide variety of systems with both constant and fading signal.

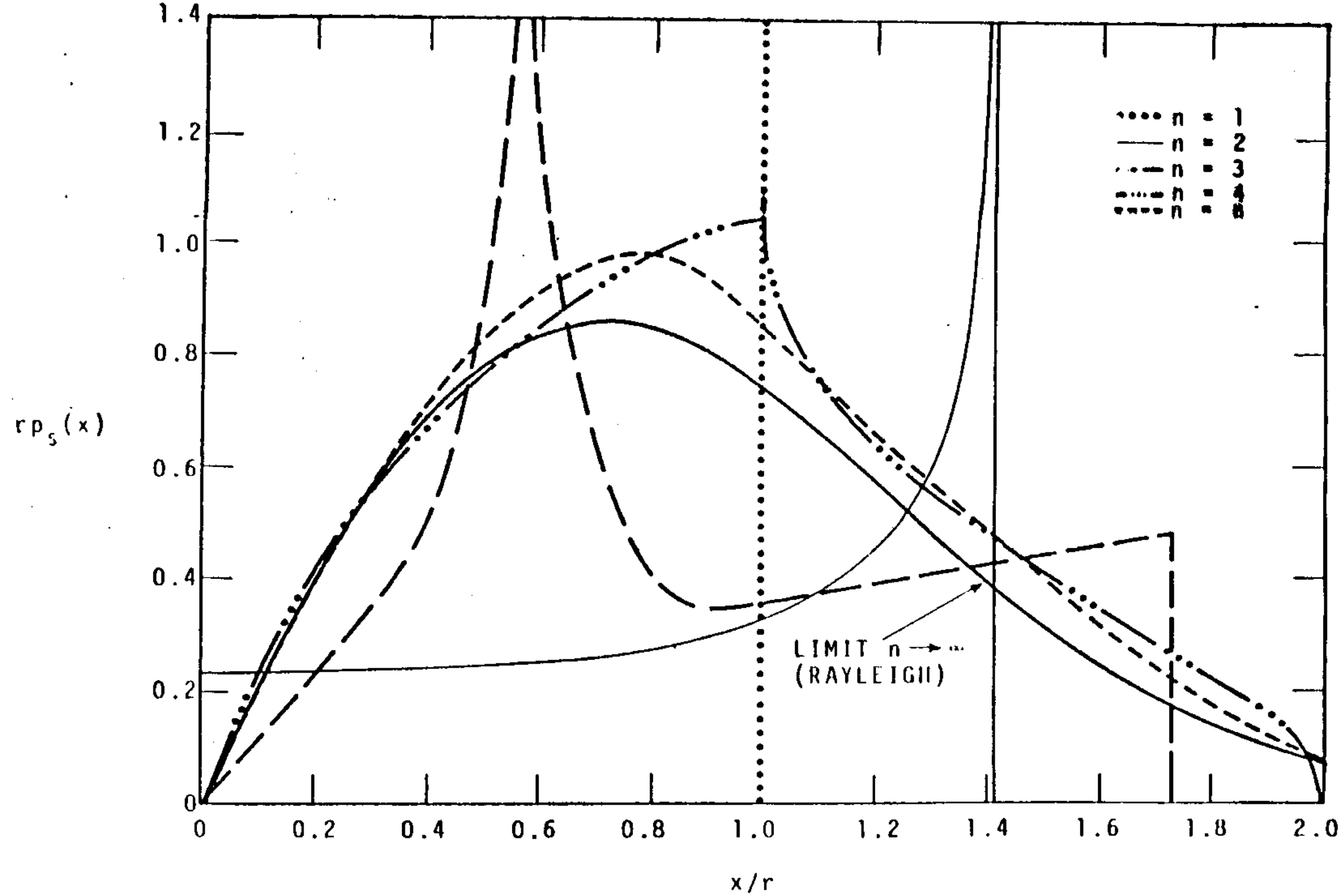


Figure BI-6. Amplitude probability density for sum of n equi-power constant signal vectors. The density functions are given in normalized form where r^2 = total signal power = $n r_1^2$, r_1^2 = power in each of the n-signal vectors.

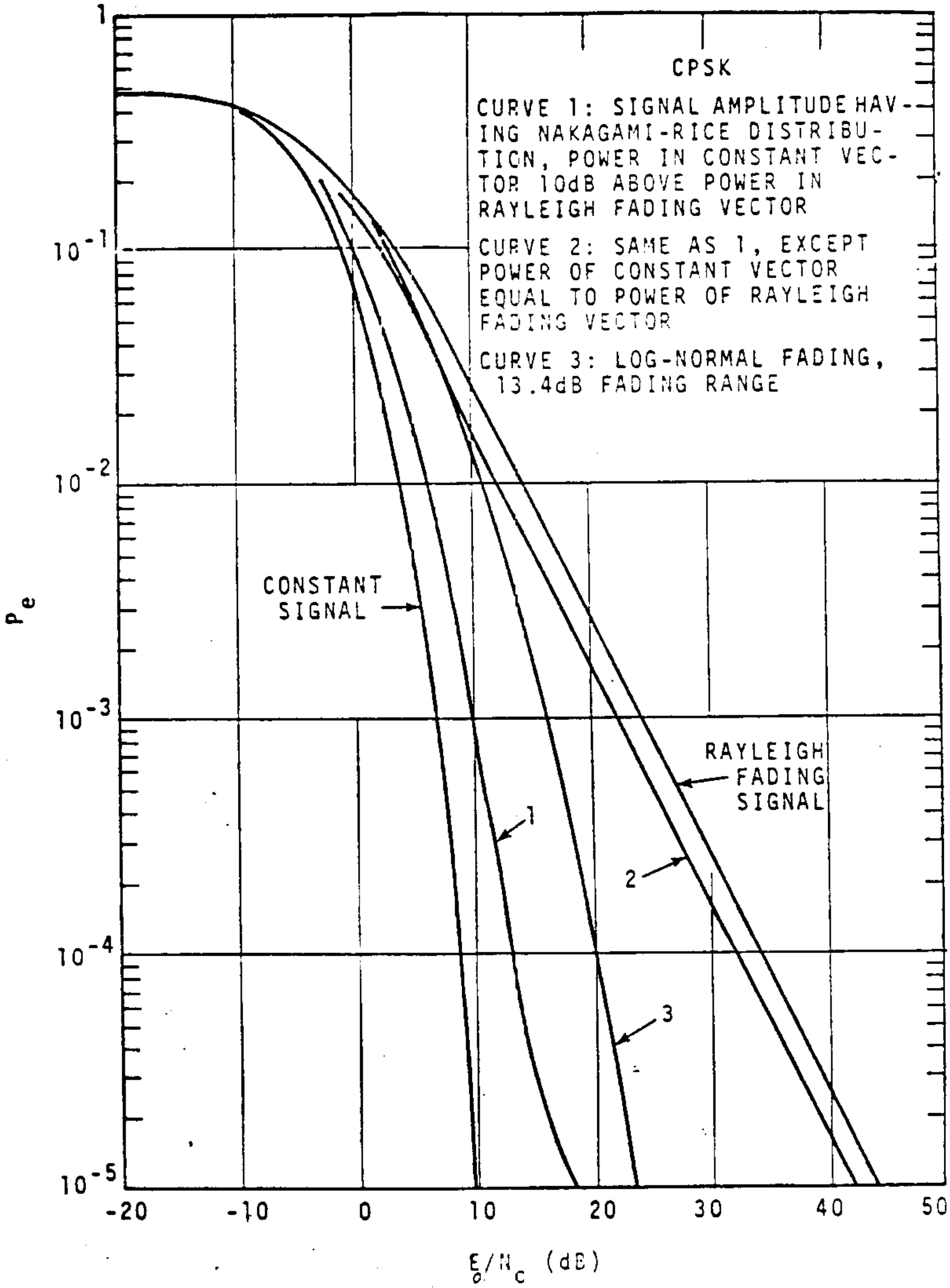


Figure BI-7. Average probability of error vs. signal energy to noise power spectral density ratio for constant signal and various types of fading signals, for a binary coherent phase shift keying system. The noise is Gaussian.

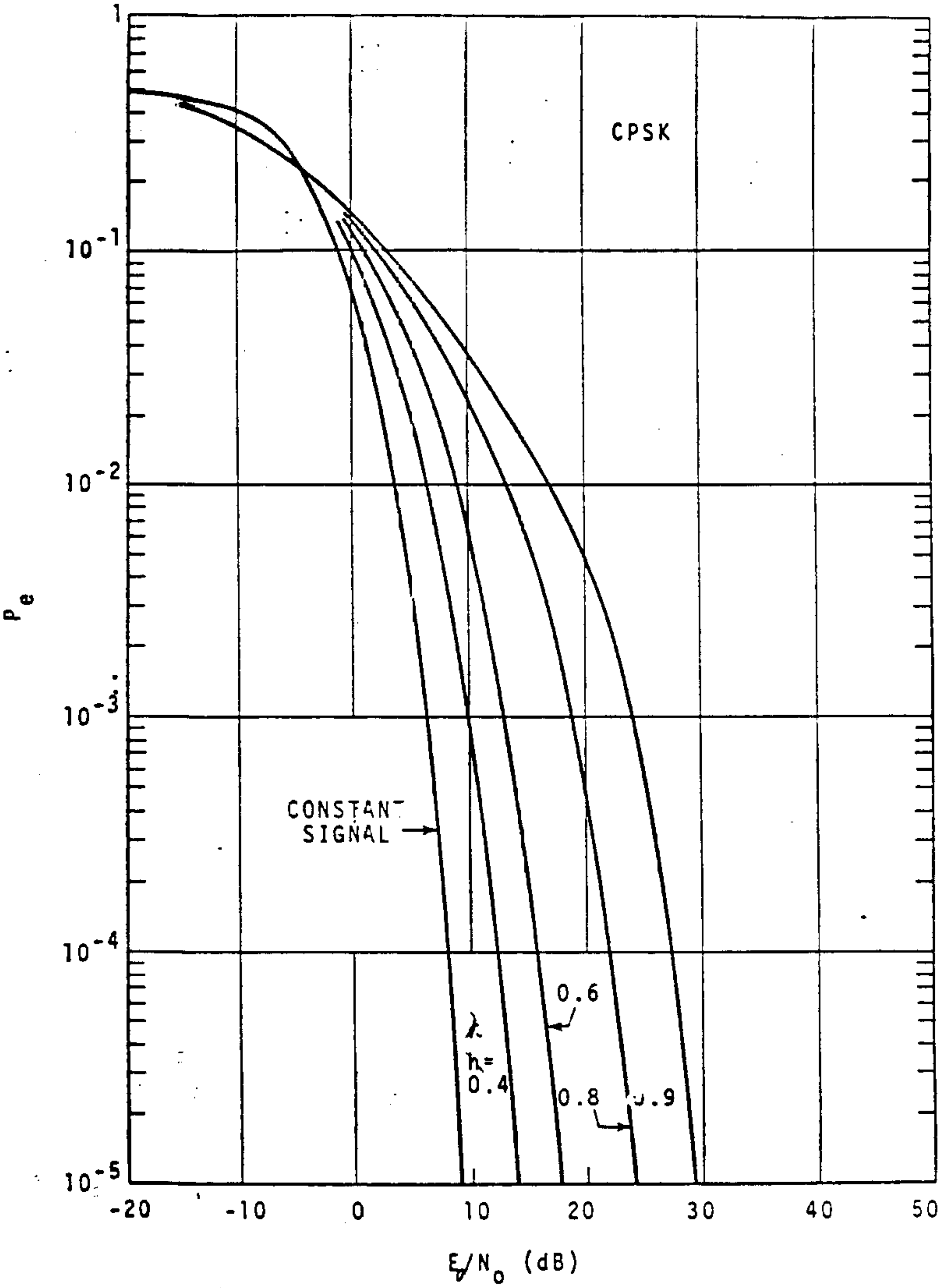


Figure BI-8. Average probability of error vs. signal energy to noise power spectral density ratio for constant signal and fading signal, where the fading signal is composed of a constant signal vector plus a reflected vector with reflection coefficient k. The noise is Gaussian.

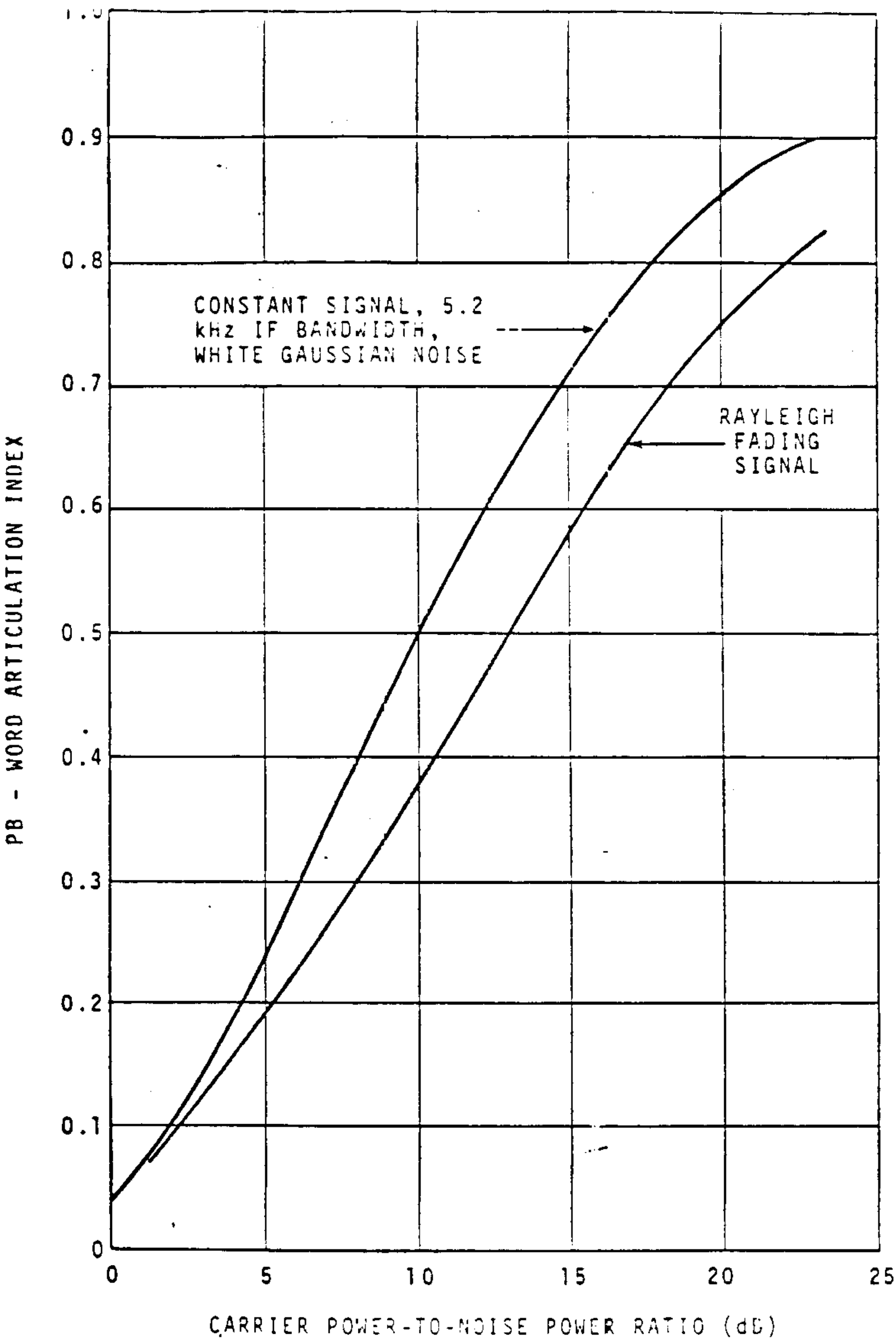


Figure BI-9. Phonetically balanced word articulation index vs. carrier power-to-noise power ratio for DSB-AM constant signal and Gaussian noise (after Cunningham et al., 1947) and for Rayleigh-fading signal.

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