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INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
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WINTER COLLEGE ON "MULTILEVEL TECHNIQUES IN COMPUTATIONAL PHYSICS"

Physics and Computations with Multiple Scales of Lengths
(21 January - 1 February 1991)

H4.SMR 539/5

Basic Multigrid Ideas

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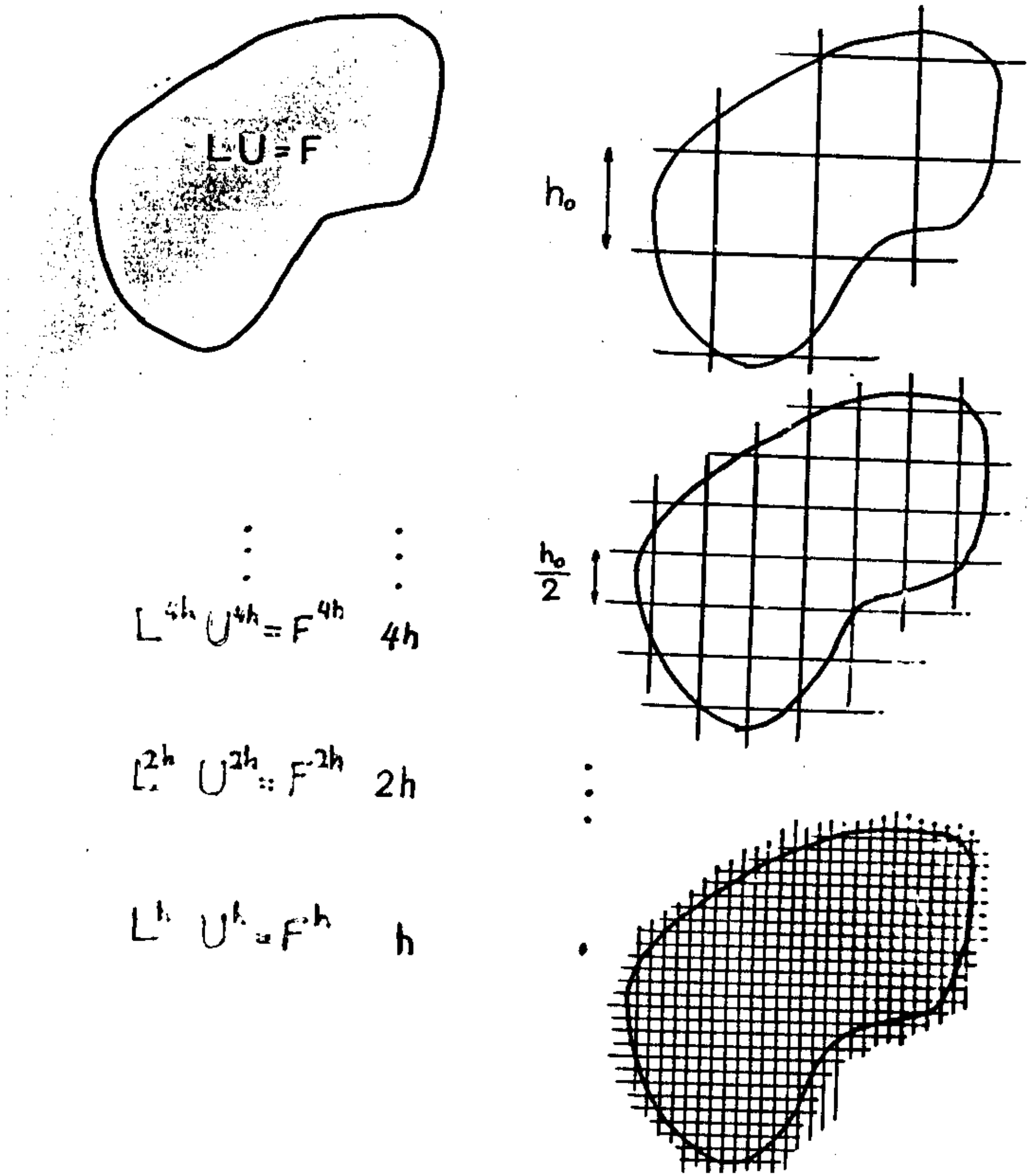
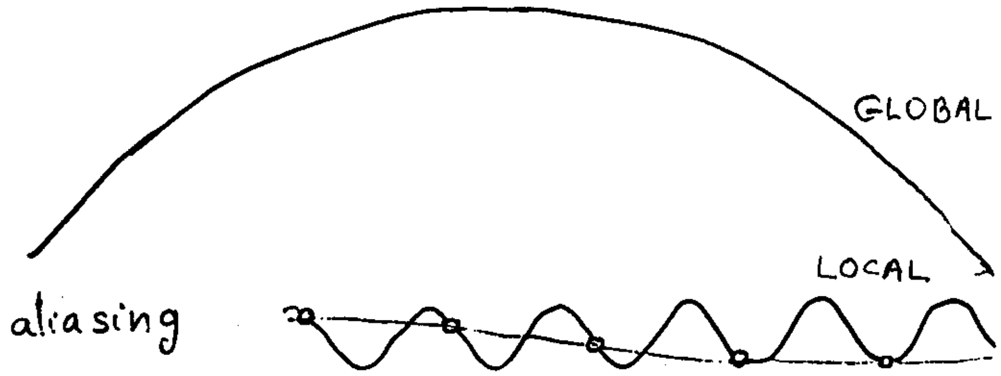
Basic Multigrid ideas





1. First approximation from a coarser grid

2. Relaxation

3. Coarse-grid correction

Elliptic Equations $LU = F$

U is smooth wherever F is smooth.

Non-smooth error created <u>locally</u>, by high-frequency residuals.

⇒ Can be reduced by local relaxation

Efficiency predicted by local mode (Fourier) analysis.

Non-elliptic

Non-smooth error <u>convected</u> along characteristics (streamlines)

⇒ Cannot be reduced locally

Require analysis of "characteristic components"

Relaxation

Basic step:

change u_α to satisfy th
equation at gridpoint α

Relaxation sweep:

repeat for all α

Inefficient for smooth components

component	typical convergence factor per sweep
h-f wave-length $\leq 4h$	.25
smooth wave-length ℓ	$1 - O\left(\frac{h^m}{\ell^m}\right)$

Coarse Grid Correction

$L^h U^h = F^h$

Current approximation: u^h

$L^h u^h = F^h - r^h$ residual

current error: $V^h = U^h - u^h$

$L^h V^h = r^h$

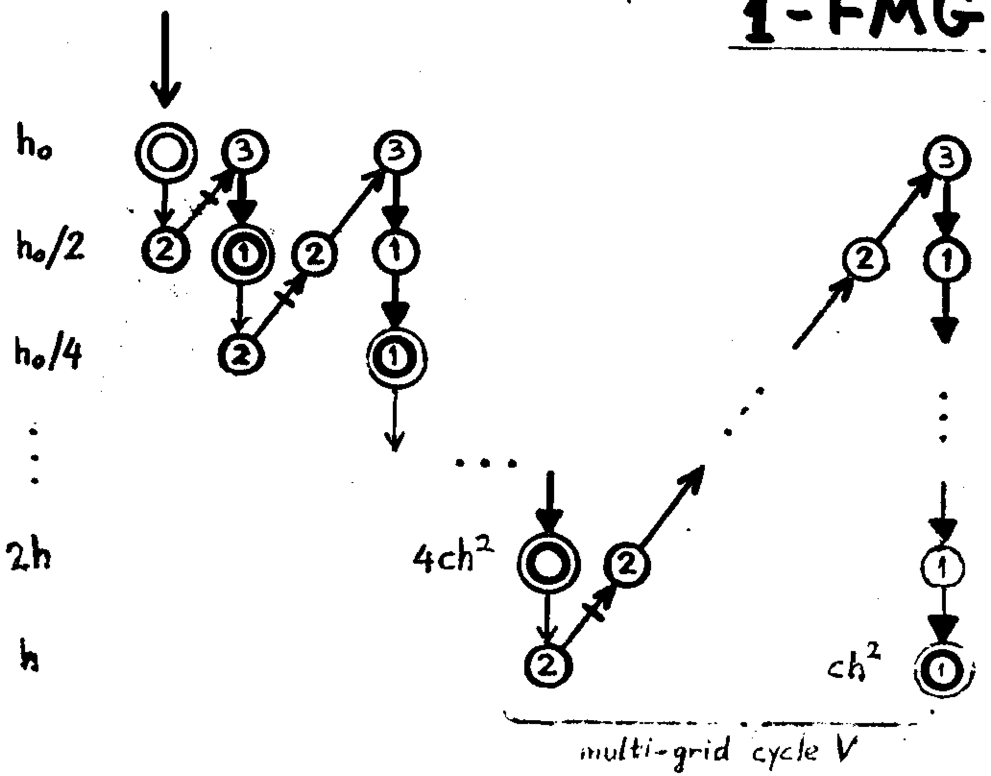
Coarse grid equation: $L^{2h} V^{2h} = I_h^{2h}$

$$u^h_{NEW} = u^h_{OLD} + I_{2h}^h V^{2h}$$

Recursion

FULL MULTIGRID (FMG) ALGORITHM

1-FMG



↓ interpolation (order l+p) to a new grid.

↓ interpolation (order m) of corrections :

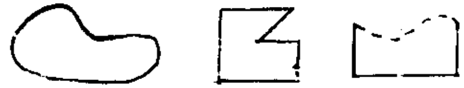
↗ residual transfer

↗ with τ-extrapolation

(r) r relaxation sweeps

◎ algebraic error < truncation error

4-8 Work Units

Any boundary 

Any boundary conditions

Discontinuous coefficients

Non-uniform grids

Non-linear — FAS: $w^{2h} = I_h^{2h} \tilde{u}^h + U^{2h}$

No linearization $\tilde{u} u_x$

Eigenproblems

Non-elliptic BVPs, anisotropy — steady state

singular perturbations — Navier-stokes, Euler

Systems, arbitrary order — transonic, shocks, det L

Non-PDE: Integral equations

Discrete, geometrically-based problems

Compound problems — Optimal control

Rigorously proved (1985+)

for general piecewise continuous elliptic systems on piecewise **uniform** grids ($h \to 0$).

local mode analysis