



INTERNATIONAL ATOMIC ENERGY AGENCY  
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION  
**INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS**  
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UNITED NATIONS INDUSTRIAL DEVELOPMENT ORGANIZATION



**INTERNATIONAL CENTRE FOR SCIENCE AND HIGH TECHNOLOGY**

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**Second Training College on Physics and Technology  
of Lasers and Optical Fibres**

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*Why Solitons are the Right Choice*

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## ULTRA LONG DISTANCE HIGH BIT RATE OPTICAL COMMUNICATION SYSTEMS

or

## WHY SOLITONS ARE THE RIGHT CHOICE

# OUTLINE

## I Background; Relevant Optical Fiber Properties

1. Dispersion
2. Loss
3. Nonlinear index  $n_2$
4. Birefringence

## II Impact of Fiber Properties On System Design

### A Compensating for Fiber Properties.

1. Dispersion: Solitons or Transmission at  $\lambda_0$ ?
2. Loss compensation: Regenerators or Optical Amplifiers?  
a A quick look at Erbium doped fiber amplifiers (EDFAs)
3. Nonlinear Index: ASK vs. PSK and FSK

B Summary of Design Considerations. Choice of best candidate systems.

## III Solitons (with frequent comparisons to NRZ)

### A What is a Soliton

- 1 The Nonlinear Schrodinger Equation.

B Ultra Long Distance, High Bit Rate, All Optical Soliton Transmission system.

1. Resistance of solitons to perturbations
2. Effects of Amplifier ASE (the Gordon Haus effect)
3. System performance: Amplifier spacings and Error rates

C Comparison with NRZ

## IV Experiments

### A The Loop

### B Transmission results

- 1 200 Mhz
- 2 1.25 GHz
- 3 2.4 GHz

### C Soliton-Soliton interactions

- 1 Theory
- 2 Experiments

## V Wavelength Division Multiplexing

### A Theory and Results of Computer Simulations

### B Experimental Results (by Andrikson and Olsson)

## VI Conclusions

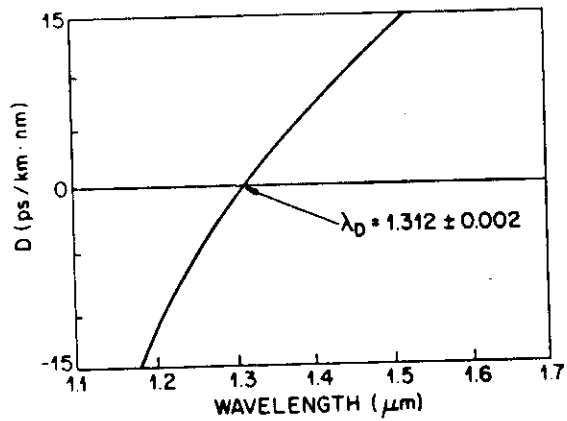
Ultimate System Capacity

## DISPERSION

(Group Velocity Dispersion)

$$D(\lambda) = -\frac{\lambda}{c} \frac{d^2 n(\lambda)}{d\lambda^2}$$

$$L_D = 0.322 \cdot \frac{\pi^2 c}{\lambda^2} \frac{\tau^2}{D}$$



$D < 0$  Normal dispersion regime. Higher frequency (blue shifted,  $\nu > \nu_0$ ) frequency components of a pulse travel slower than the low frequency ( $\nu < \nu_0$ ) components.

$D > 0$  Anomalous dispersion regime. High frequency components of a pulse travel faster than the low frequency components.

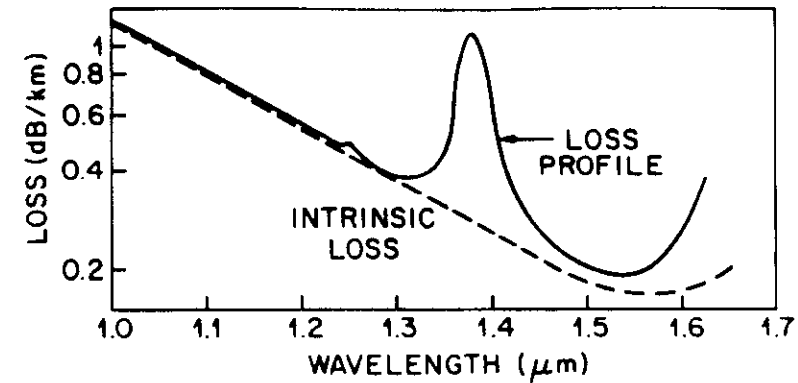
## OPTICAL LOSS

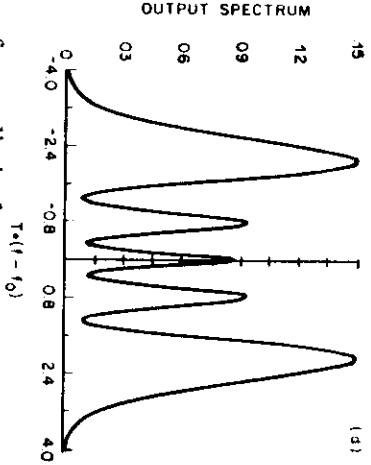
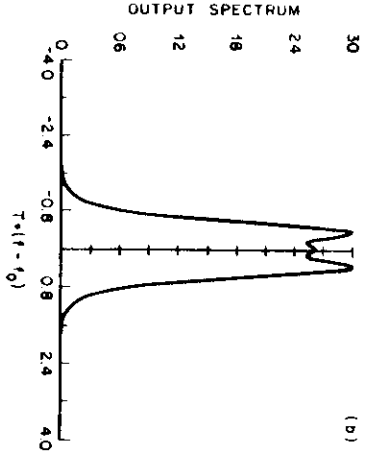
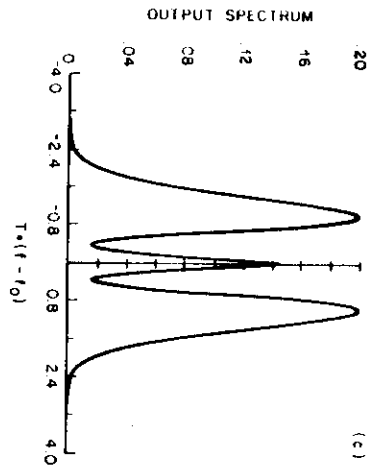
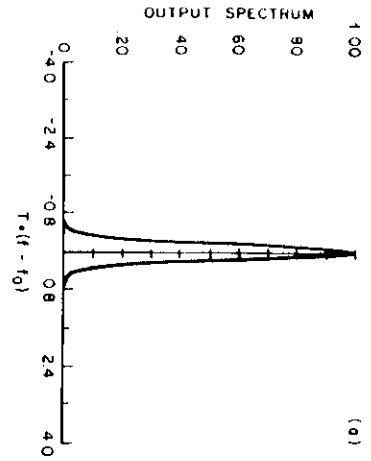
$$I(z) = I_0 e^{-\alpha \cdot z}$$

$$\alpha_{\text{Intrinsic}} \propto \frac{1}{\lambda^4}$$

(Rayleigh scattering from density fluctuations in fiber)

Typical for good fiber  $\alpha = 0.20 \text{ dB/km}$





Spectrum of an initially transform limited Gaussian pulse (a). Spectra of the same pulse with nonlinear phase shifts,  $\Delta\phi$ , of  $\pi$  (b)  $2.5\pi$  (c) and  $4.5\pi$  (d)

## NONLINEAR INDEX

$$n = n_0 + n_2(I)$$

$$n_2 = 3.2 \times 10^{-16} \text{ cm}^2/\text{W}$$

$$\phi_{NL} = k_2 P L$$

Where  $k_2 = \frac{2\pi n_2}{\lambda_0 A_{eff}}$  is the Kerr coefficient.

L is the length of the fiber, and P is the optical power.

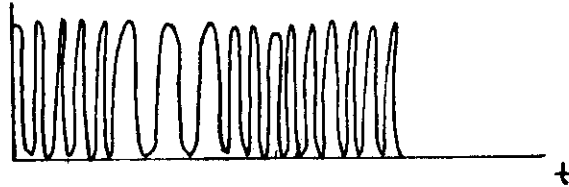
(typical value  $k_2 = 3.7$  radians/W/km)

$$\frac{\partial I}{\partial t} \rightarrow \frac{\partial \phi_{NL}}{\partial t} = \partial \omega$$

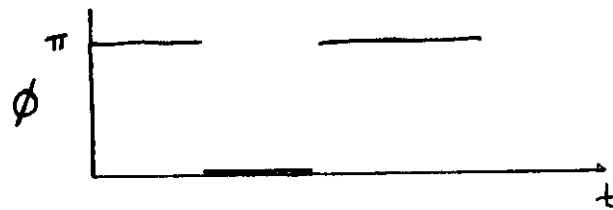
(Self Phase Modulation)

## Signaling Formats

FSK (Frequency Shift Keyed)

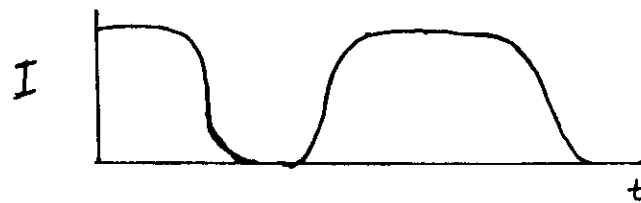


PSK (Phase Shift Keyed)



ASK (Amplitude Shift Keyed)

NRZ (Non Return to Zero)



RZ (Return to Zero)



## Compensating for Dispersion

**Dispersion Shifted Fiber.** Decrease the fiber core size, the waveguide contribution to the dispersion will shift the zero dispersion wavelength. In this way the zero dispersion wavelength can be made to coincide with the wavelength of minimum loss. ( $1.5 \mu\text{m}$ ). This is a fiber optimized for NRZ, PSK and FSK signaling systems.

**Solitons** The soliton has an intensity envelope that produces a SPM that exactly cancels the dispersion. The pulse envelope does not change with time. Allows signal to be at  $1.5 \mu\text{m}$  regardless of D.

## Compensating Fiber Loss

### 1. Electronic Regenerators (present technology)

- Complex
- Expensive (cost increases *rapidly* with bit rate.)
- Not upgradable (bit rate is fixed)
- Not easily compatible with WDM
- Unidirectional

### 2. Optical Amplifiers

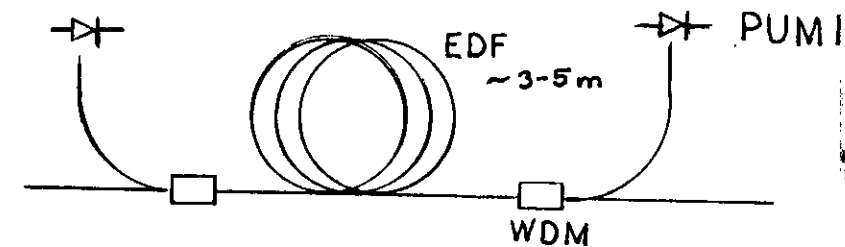
#### • Semiconductor Amplifiers

- Overly sensitive to high frequency noise
- Large excess noise factor
- Large insertion losses (due to mode mismatching)
- Gain is polarization dependent

#### • Erbium Doped Fiber Amplifiers (Technology of choice for future)

## ERBIUM DOPED FIBER AMPLIFIERS

- Low noise  $\beta \approx 1.2 - 1.5$
- Gain independent of signal polarization
- Can be pumped by semiconductor diode lasers at 1480 nm and 980 nm
- Can be spliced right into the signal fiber spans.
- Long lived upper level lifetime  $\sim 100 \mu \text{ sec}$
- Efficient: gain coefficients of  $\sim 6 \text{ db/mW}$
- Powerful:  $P_{sat}^{out}$  of 2.5 dbm at 25 db gain
- Simple and inexpensive!



## Optimum Design Features for an Ultra Long Distance, High Bit Rate Optical Transmission System

- Use ASK format. (Solitons work best!)
- Use Erbium Doped Fiber Amplifiers
- Use Dispersion shifted fiber, at or near  $\lambda_0$
- Use fiber with low polarization dispersion  
 $\leq .3 \text{ ps/km}^{1/2}$

## Effects of Fiber Nonlinear Index

PSK and FSK transmission formats, in an all optical system, experience too much SPM to be useful at high rates for distances more than 2000 km.

ASK formats are the only method that will work for distances of 2000 km or more in an all optical system. NRZ systems are limited to very low power at  $\lambda_0$ . Solitons work the best, since they compensate SPM with dispersion, they are unaffected by the fiber nonlinearity.

## Birefringence

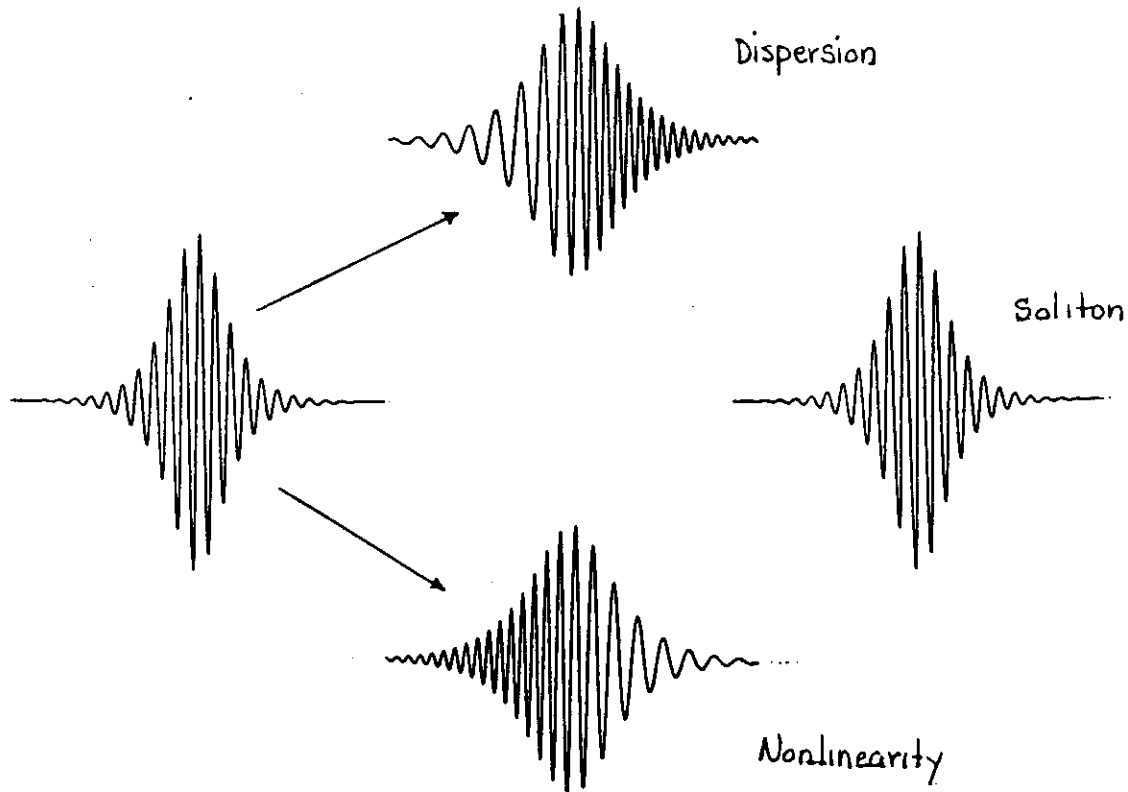
NRZ signals become thoroughly depolarized, and on average, will experience some extra broadening.

Solitons in birefringent fiber are self trapped so long as  $\Delta\beta h^{1/2} \leq 0.3 D^{1/2}$ . If this obtains the soliton will remain in a well defined polarization state and resist the effects of birefringence.

## What is an Optical Soliton

## Begin With The Wave Equation

The exact balance of chromatic dispersion and the self phase modulation due to the small nonlinear index of the glass.



$$\frac{\partial^2 E}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2 (n^2 E)}{\partial t^2}$$

Consider only the nonlinear index (no dispersion)

$$n = n_0 + |E|^2 n_2$$

Make the Slowly Varying Envelope Approximation:

$$E = \phi(z, t) e^{i(\omega_0 t - \beta_0 z)}$$

$$\text{Where } \beta_0 = \frac{\omega_0 n_0}{c}$$

Neglecting second derivatives of  $\phi$  and terms with  $n_2^2$  we get the following propagation equation for the envelope in nonlinear medium.

$$\frac{\partial \phi}{\partial z} + \beta_0 \frac{\partial \phi}{\partial t} = -i \frac{n_2}{n_0} |\phi|^2 \phi$$



Consider a purely dispersive medium.

The propagation constant is approximated by:

$$\beta = \beta_o + \dot{\beta}_o(\omega - \omega_o) + \frac{1}{2} \ddot{\beta}_o(\omega - \omega_o)^2$$

And the pulse envelope is given by:

$$E = \int_{-\infty}^{\infty} A(\omega) e^{i(\omega t - \beta z)} d\omega$$

Again, make the Slowly Varying Envelope Approximation:

$$E = \phi(z, t) e^{i(\omega_o t - \beta_o z)}$$

Substituting into the wave equation we get:

$$\frac{\partial \phi}{\partial z} + \dot{\beta}_o \frac{\partial \phi}{\partial t} = \frac{i}{2} \ddot{\beta}_o \frac{\partial^2 \phi}{\partial t^2}$$

Note that the left hand side of each of these equations is the same.

$$\frac{\partial \phi}{\partial z} + \dot{\beta}_o \frac{\partial \phi}{\partial t} = -i \frac{n_2}{n_o} |\phi|^2 \phi$$

$$\frac{\partial \phi}{\partial z} + \dot{\beta}_o \frac{\partial \phi}{\partial t} = \frac{i}{2} \ddot{\beta}_o \frac{\partial^2 \phi}{\partial t^2}$$

Since the nonlinearity and the dispersion in a typical optical fiber are small the two effects can be considered additive. This gives the proper propagation equation for light in an optical fiber.

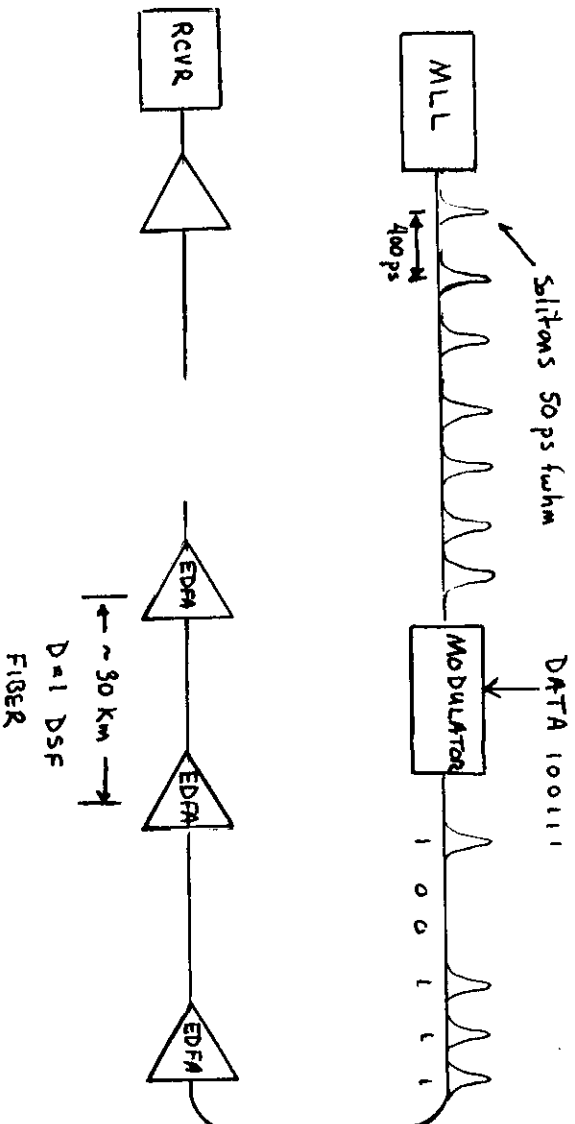
$$\frac{\partial \phi}{\partial z} + \dot{\beta}_o \frac{\partial \phi}{\partial t} = \frac{i}{2} \ddot{\beta}_o \frac{\partial^2 \phi}{\partial t^2} - i \frac{n_2}{n_o} \beta_o |\phi|^2 \phi$$

If we change to units of retarded time and appropriate distance units we rewrite the above equation in a more familiar form:

$$\frac{\partial u}{\partial z'} = \frac{i}{2} \frac{\ddot{\beta}_o}{|\ddot{\beta}_o|} \frac{\partial^2 u}{\partial t'^2} - i |u|^2 u$$

where  $t' = \frac{1}{\tau} (t - \dot{\beta}_o z)$  and  $z' = \frac{|\ddot{\beta}_o|}{\tau^2} z$

## Long Distance Soliton Transmission System



THE NONLINEAR SCHRÖDINGER EQN:

$$-i \frac{\partial u}{\partial z} = \frac{1}{2} \frac{\partial^2 u}{\partial t^2} + |u|^2 u - i\Gamma u$$

ordinary dispersion based on  
 $n = n_0 + n_2 I$

SOLITON:

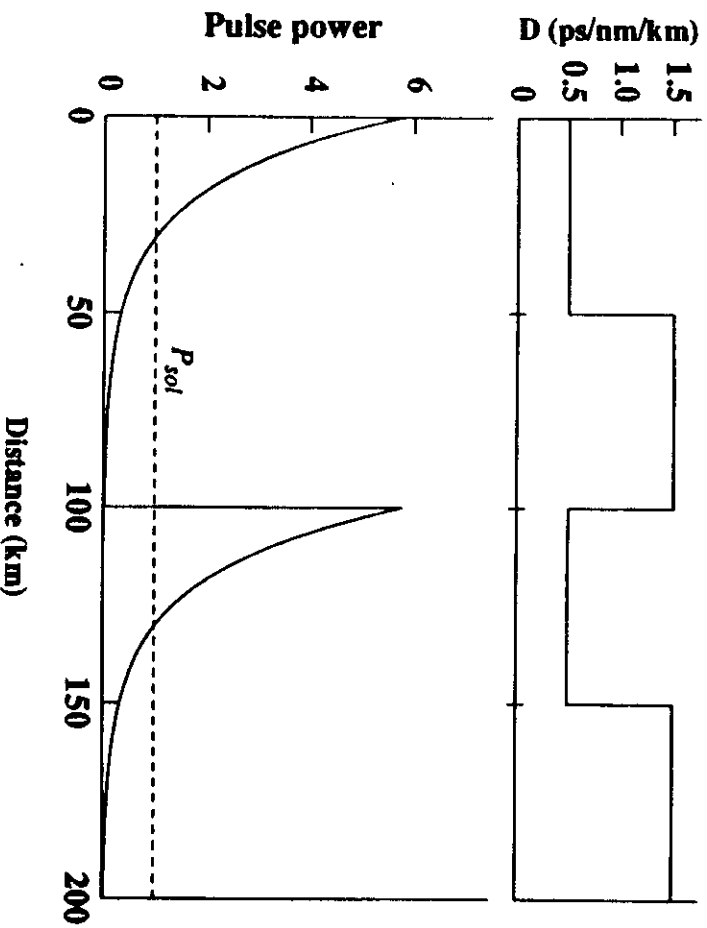
$$u(z, t) = \text{sech}(t) e^{iz/2}$$

SOLITON UNITS:

Length	Time	Power
$\frac{2}{\pi} z_0 = 0.322 \frac{2\pi c}{\lambda^2} \frac{\tau^2}{D}$	$\frac{\tau}{1.763}$	$P_{\text{sol}} = \frac{A_{\text{eff}} \lambda}{4n_2} \frac{\lambda}{z_0} \propto \frac{D}{\tau^2}$

(For  $\tau = 50$  ps,  $D = 1$  ps/nm/km,  
 and  $\lambda = 1550$  nm,  $z_0 \approx 1000$  km.)

## SEGMENT OF MODEL SYSTEM FOR TEST OF SOLITON PROPAGATION THROUGH CHAIN OF LUMPED AMPS AND D.S. FIBER



Note:  $L = 100$  km is several times greater than would be chosen for low accumulated ASE noise.

Thus, the test is an especially rigorous one.

### LONG DISTANCE SOLITON PROPAGATION USING LUMPED AMPLIFIERS AND DISPERSION-SHIFTED FIBER

Let  $L$  be the "amplification period" (span between amplifiers).

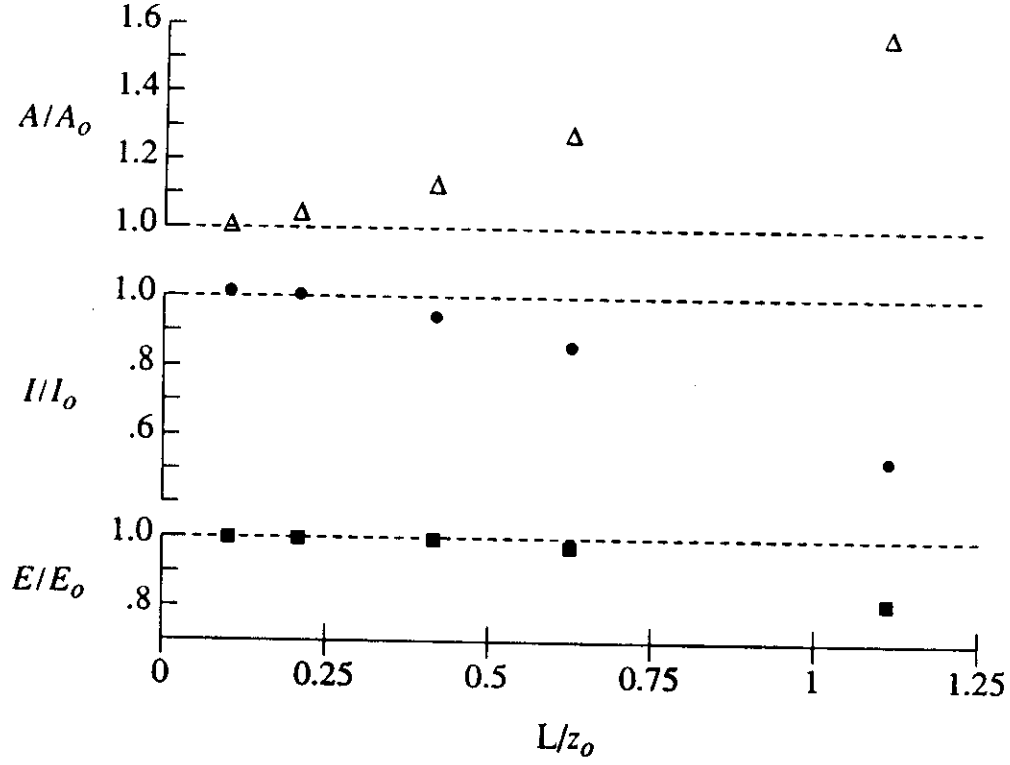
For  $L \ll z_0$ , essentially nothing happens to the pulse shape and width over one period  $L$ .

The nonlinear effect over each  $L$  is determined by the corresponding path-average power.

Thus, if the path-average power is equal to the usual soliton power over each period, one has a perfectly well behaved soliton.

In like manner, by keeping the path-average  $D$  constant from one period to the next, solitons can also tolerate considerable variation in  $D$  over distances  $\ll z_0$ .

### Effects of Periodic Perturbations on Long Distance Soliton Transmission

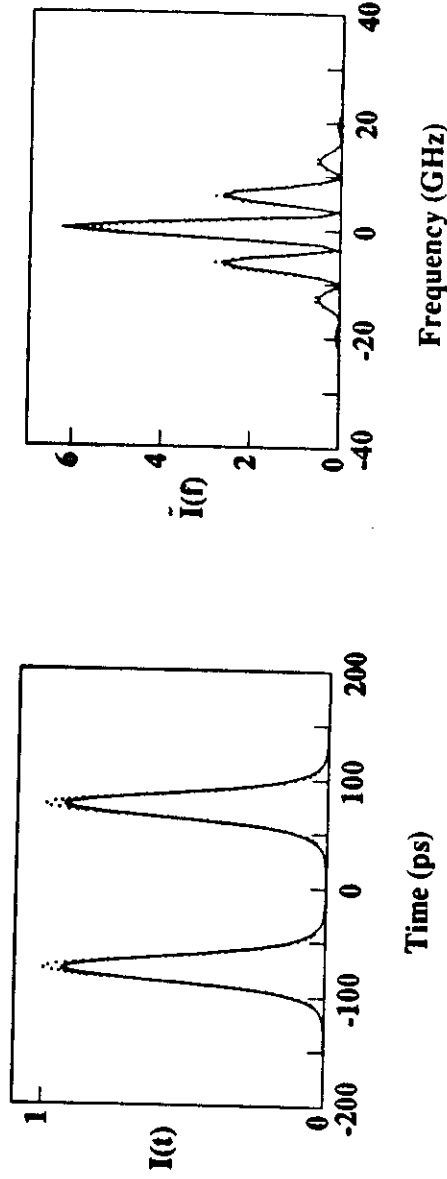


L is the length of one period of the perturbation.

### NUMERICALLY SIMULATED SOLITON TRANSMISSION OVER 9000 KM THROUGH CHAIN OF LUMPED AMPS AND D.S. FIBER AT 1550 nm

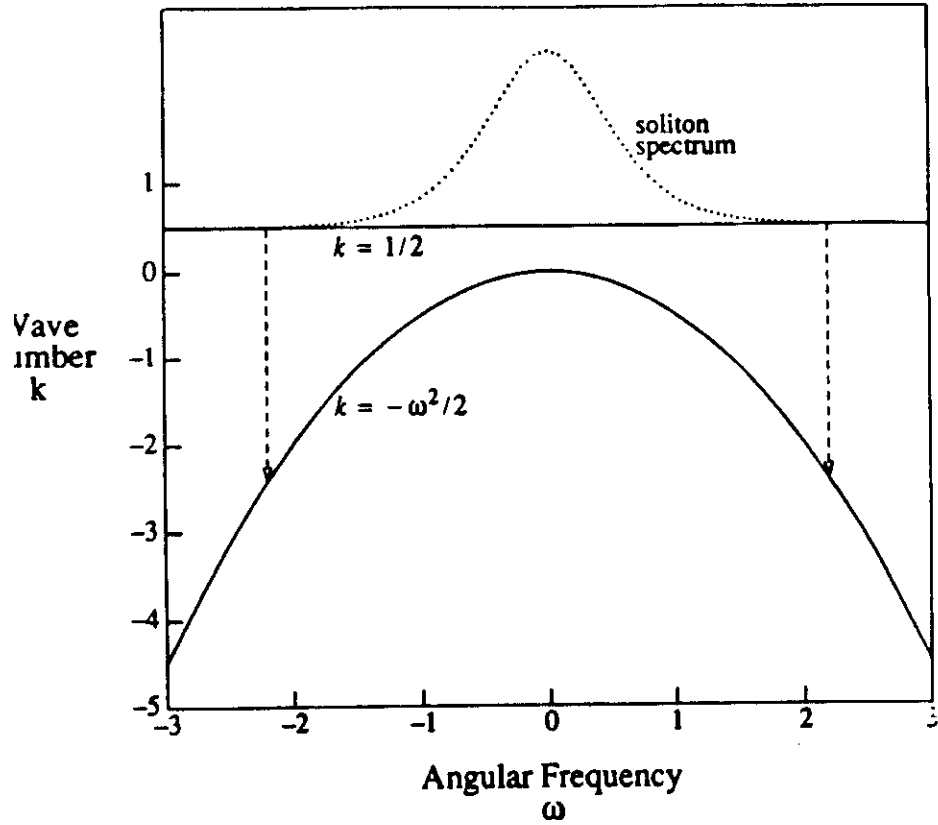
$\tau = 25$  ps and  $\bar{D} = 1$  ps/nm/km  $\Rightarrow z_0 = 250$  km;  $L = 100$  km  
(D steps back and forth every 50 km between 0.5 and 1.5 ps/nm/km.)

Dotted curve: initial pulses; solid curve: after 9000 km



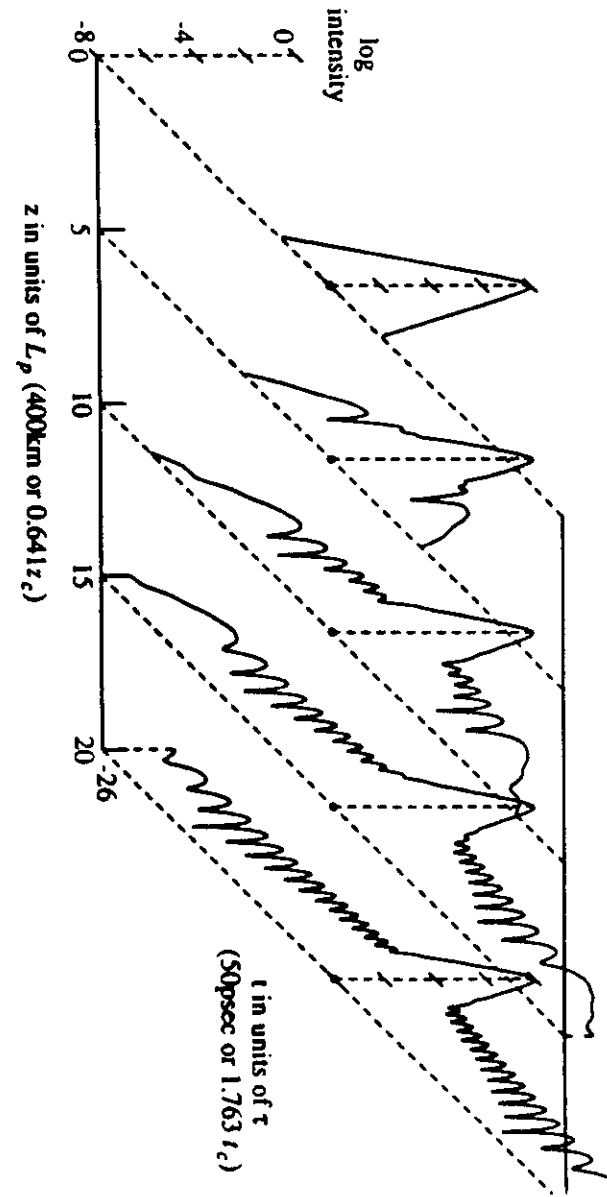
Note: For  $\tau = 35$  and 50 ps ( $z_0 = 480$  and 980 km, respectively), the pulses at 9000 km are virtually indistinguishable from those at input.

## Radiation from a Soliton The Phase Matching Condition



Dispersion curves for the soliton ( $k=1/2$ ) and for purely dispersive radiation ( $k=-\omega^2$ ). Dashed line represents the wave vector of the spatial perturbation needed to phase match the soliton frequency component to the dispersive radiation field.

$$\Gamma_{rad.loss} = \frac{1}{4\Omega} \left[ \pi k A_k \operatorname{sech}\left(\pi \frac{\Omega}{2}\right) \right]^2$$

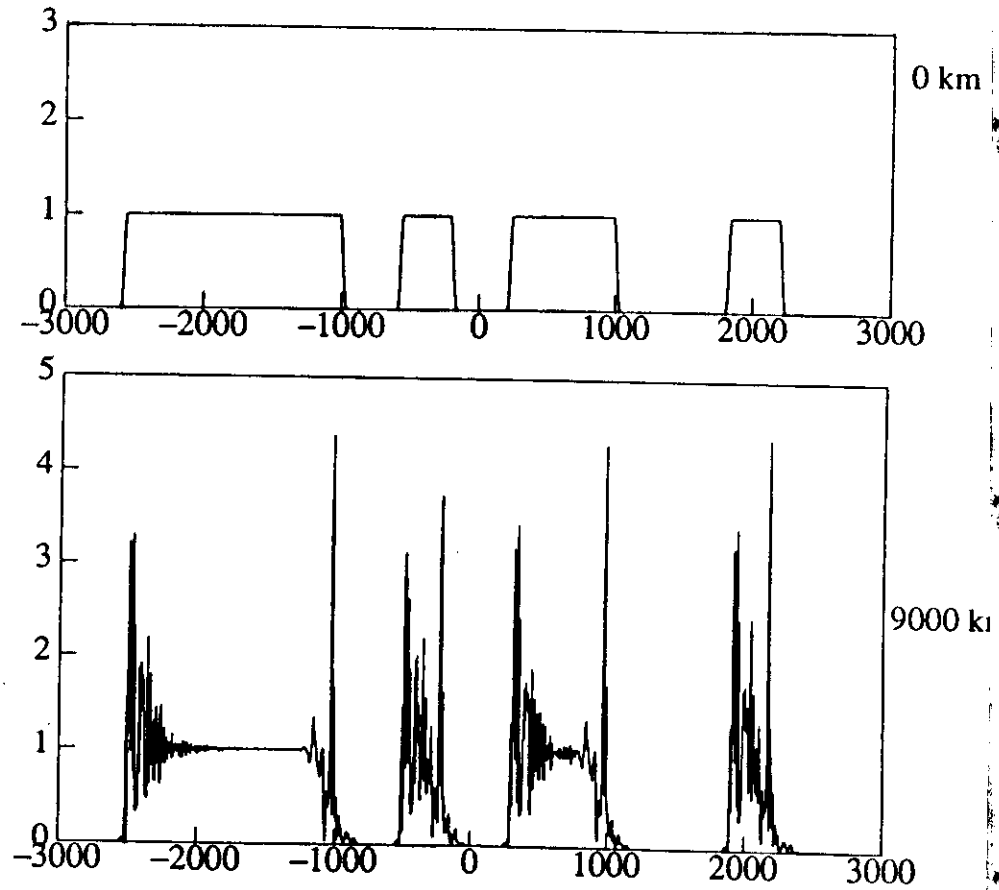


Radiation from a Soliton  
Due to a Periodic Perturbation  
 $A(z) = 0.5 \sin(9.8z)$

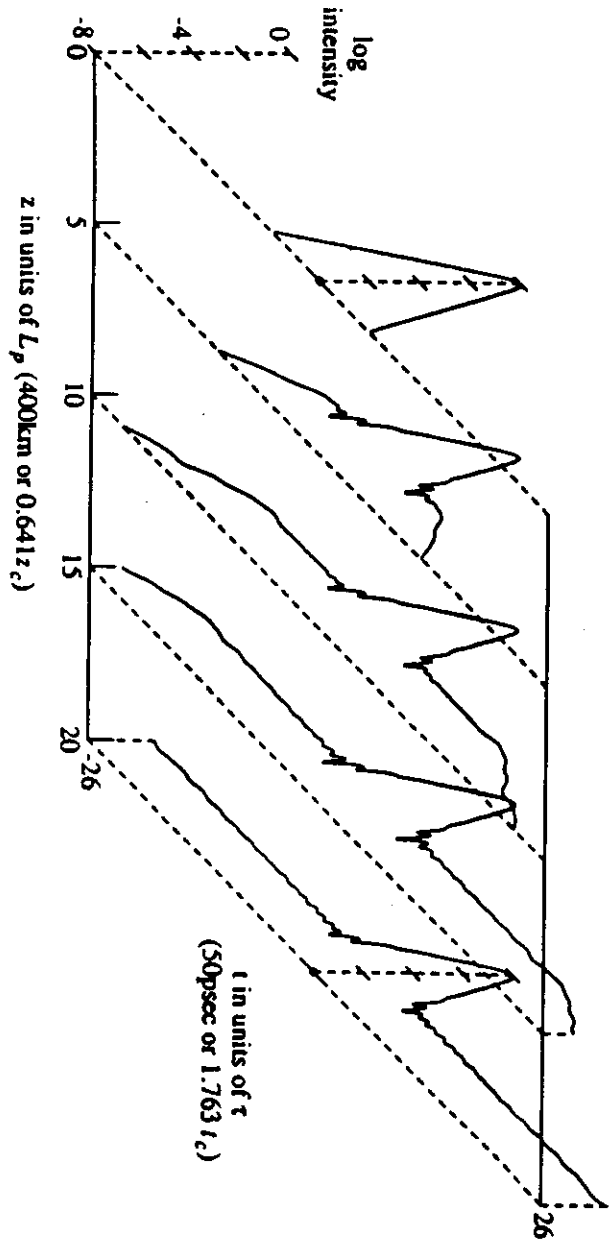
J.P. Gordon

# Computer Simulation of Transmission of NRZ Signals 2.5 GHz

Amplifier Spacing 33 km  
Initial Power 1mW  
Randomly Varying  $D$ , ( $\bar{D}=0$ )

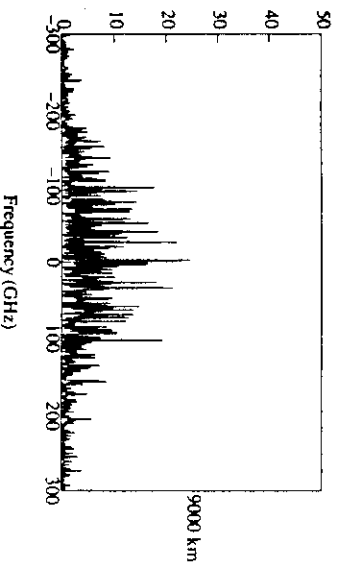
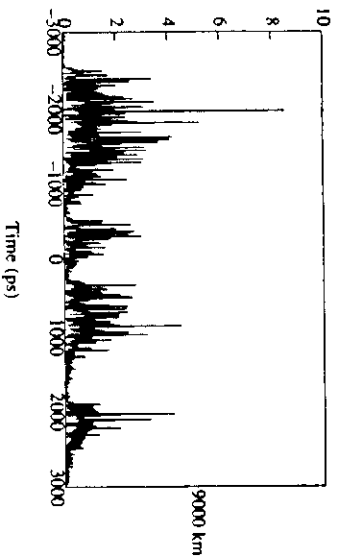
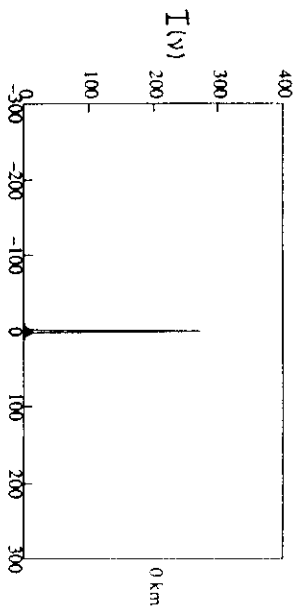
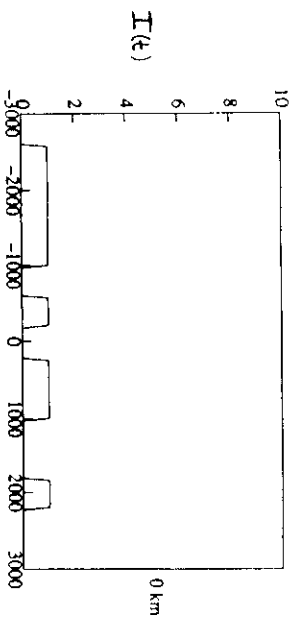


Radiation from a Soliton  
Due to a Periodic Perturbation  
with Corrected Input  
 $A(z) = 0.5 \cos(9.8z)$



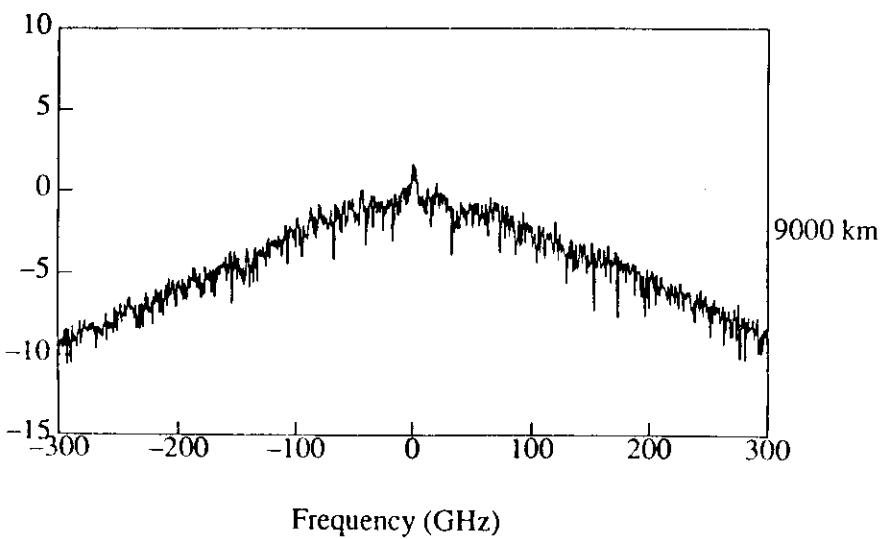
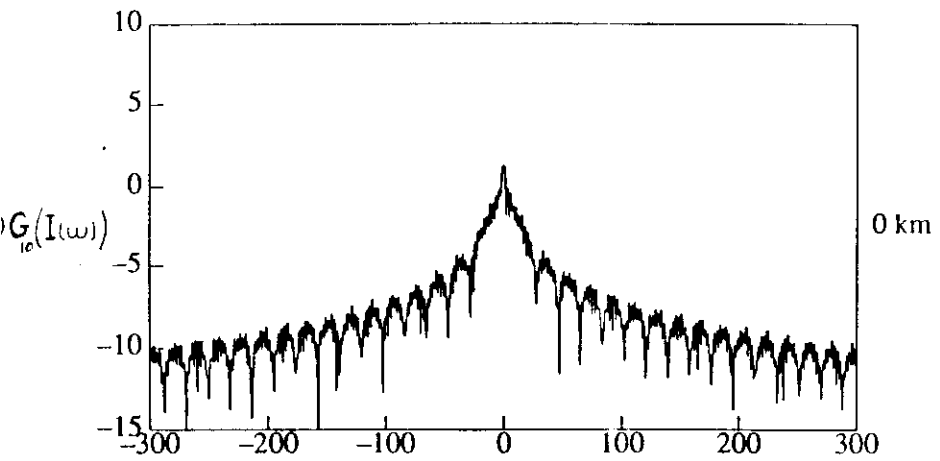
# Computer Simulations of 2.5 GHz NRZ Transmission

$P_0 = 4 \text{ mW}$ ,  $\bar{D} = 0$ , 33km Amplifier spacing



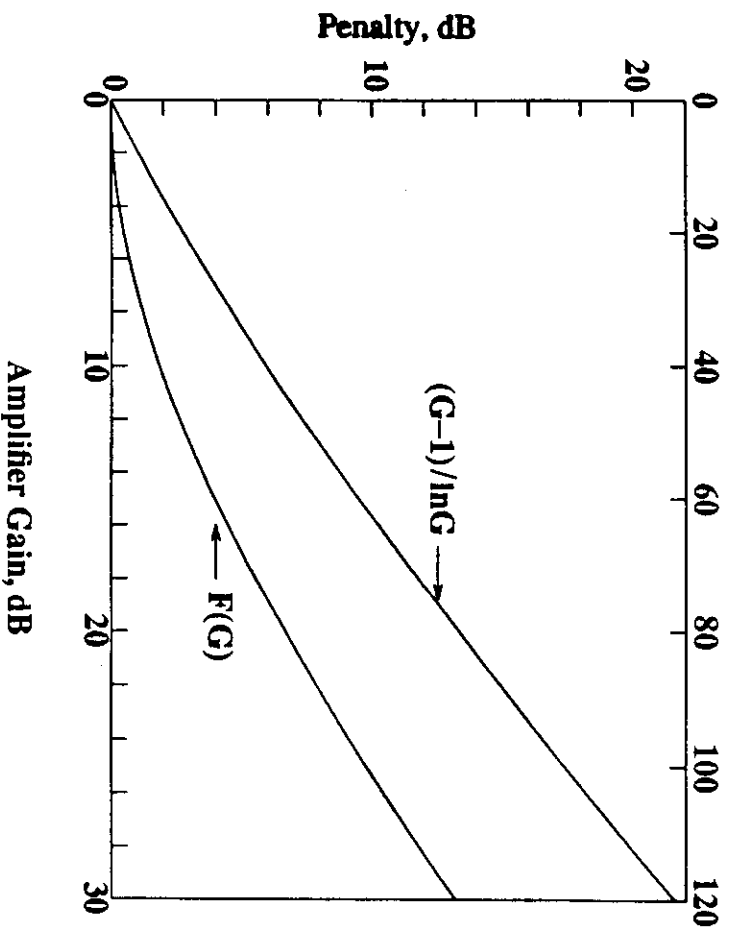
## Spectra of NRZ Pulse Train from Computer Simulations

Amplifier Spacing 33 km  
Initial Power 1mW  
Randomly Varying  $D$ , ( $\bar{D} = 0$ )

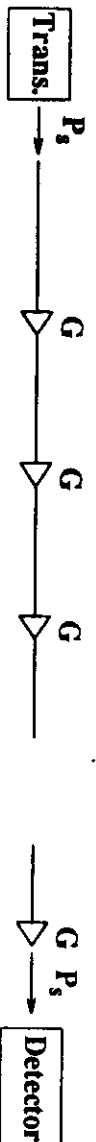


## NOISE PENALTIES VS AMPLIFIER GAIN

Amplifier Spacing, km at .25 dB/km loss



## AMPLIFIED SPONTANEOUS EMISSION NOISE



All optical system with  $N$  amplifiers of gain  $G$  each preceded by  $1/G$  fiber loss factor.

In all-optical transmission, detector noise tends to be insignificant compared to ASE noise.

One can easily show that the ASE spectral density (power per unit bandwidth),  $P(\nu)$ , at the output of the last amplifier in the chain, is

$$P(\nu) = \alpha Z h \nu \beta \frac{(G-1)}{\ln G}$$

where  $\alpha$  is the fiber loss coefficient,  $Z$  the system length,  $h\nu$  the photon energy, and  $\beta$  the amplifier excess spontaneous emission factor.

For solitons, however, where the path-average signal power is fixed, S/N ratio is most conveniently calculated by comparing with the path-averaged  $P(\nu)$ .

Since the ratio of path-average to peak power in each fiber span is  $(G-1)/(G \ln G)$ , the path-average  $P(\nu)$  is

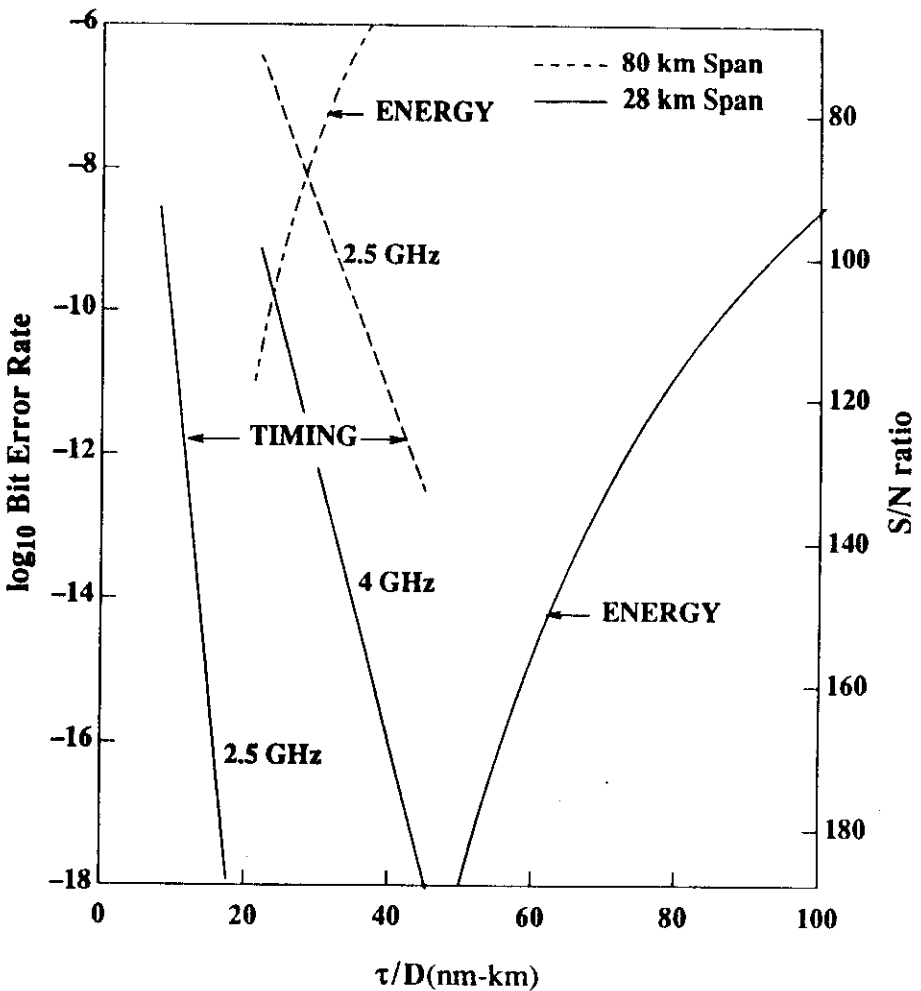
$$\bar{P}(\nu) = \alpha Z h \nu \beta F(G)$$

where the function  $F(G) = \frac{(G-1)^2}{G(\ln G)^2}$  represents an important noise penalty.



## ERROR RATES AT 9000 KM

Assumes 0.25dB/km loss,  $\lambda_s = 1532$  nm, and  $\beta=1.5$   
(erbium amplifiers)



### THE GORDON-HAUS EFFECT: VARIANCE IN PULSE ARRIVAL TIMES FROM AMPLIFIED SPONTANEOUS EMISSION

Amplified spontaneous emission perturbs the soliton velocities such that at the end of a system of length  $Z$ , there is a Gaussian distribution in arrival times with the following variance:

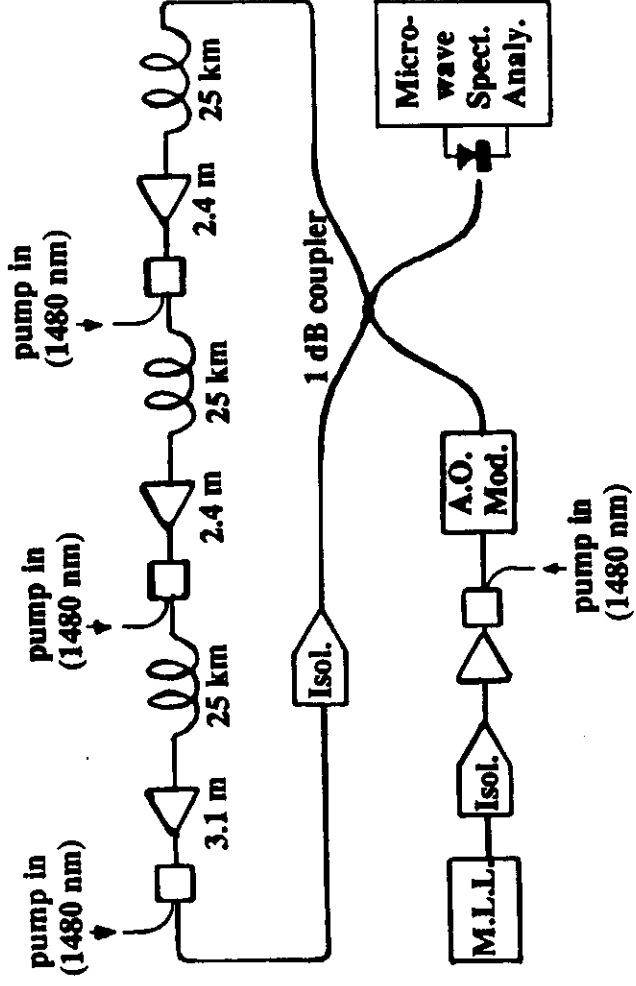
$$\sigma^2 = 4138 \beta F(G) \frac{\alpha_{\text{loss}}}{A_{\text{eff}}} \frac{D}{\tau} Z^3$$

where  $\alpha_{\text{loss}}$  is in  $\text{km}^{-1}$ ,  $D$  in  $\text{ps/nm/km}$ ,  $Z$  in thousands of km,  $\tau$  in ps,  $A_{\text{eff}}$  in  $\mu\text{m}^2$ ,  $G$  and  $\beta$  are the amplifier gain and excess spontaneous emission factor, respectively, and where

$$F(G) = \frac{(G-1)^2}{G(\ln G)^2}$$

Example: Let  $\tau=50$  ps,  $D=1$  ps/nm/km,  $A_{\text{eff}}=35 \mu\text{m}^2$ ,  $\alpha_{\text{loss}}=0.0576/\text{km}$  (0.25 dB/km),  $F=1.24$  (span between amplifiers = 28 km), and  $\beta=1.5$ . Then, for 9000 km, one obtains:

# The Loop Experiment



## SOLITON POWERS

For  $\tau = 50$  ps solitons at  $D = 1.38$  ps/nm/km:

Path-average power:  $P_{sol} \sim 660 \mu\text{W}$

At amplifier output:

$P_{pk} \sim 1.8 \text{ mW}$

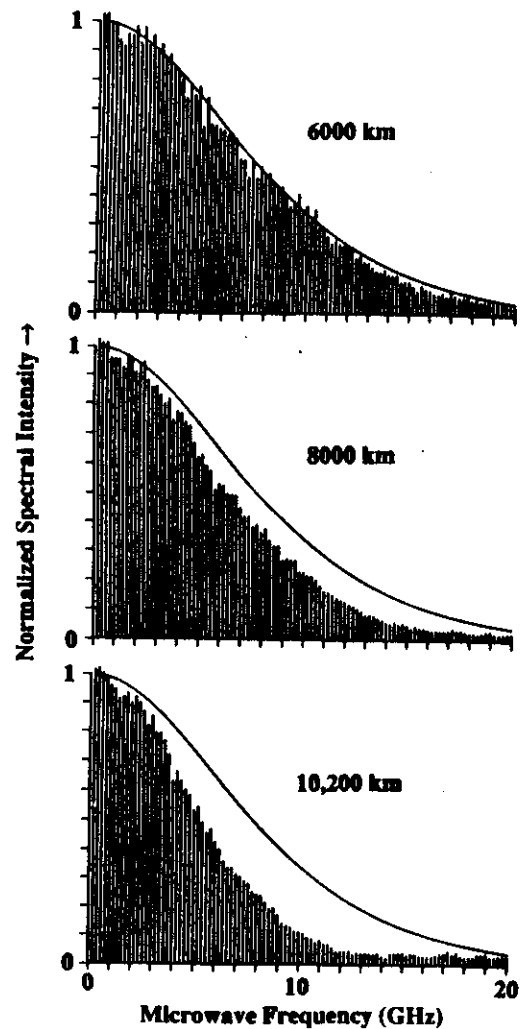
$\bar{P}$  into one bit period at 2.5 Gbit/s:  $\sim 250 \mu\text{W}$

Measured ASE noise power referred to 2.5 GHz:  $\sim 0.8 \mu\text{W}$

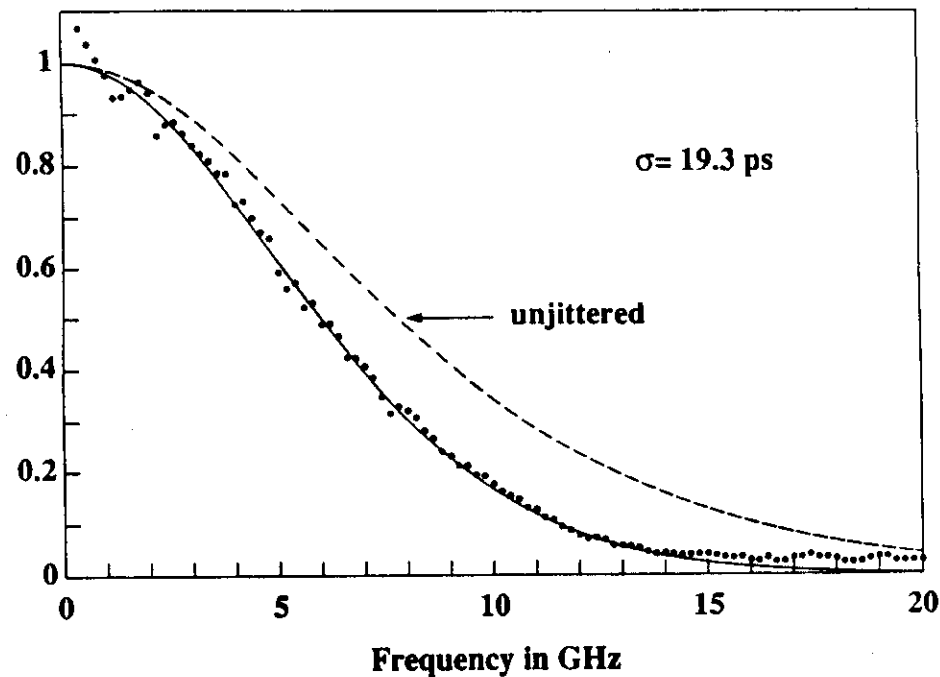
S/N ratio  $> 250$

## Experimental Data Microwave Spectra of the Soliton Pulse Train

Showing the evolution of the soliton spectrum as the pulse train moves down the fiber.



## Soliton Propagation Experiment 9000 km Data

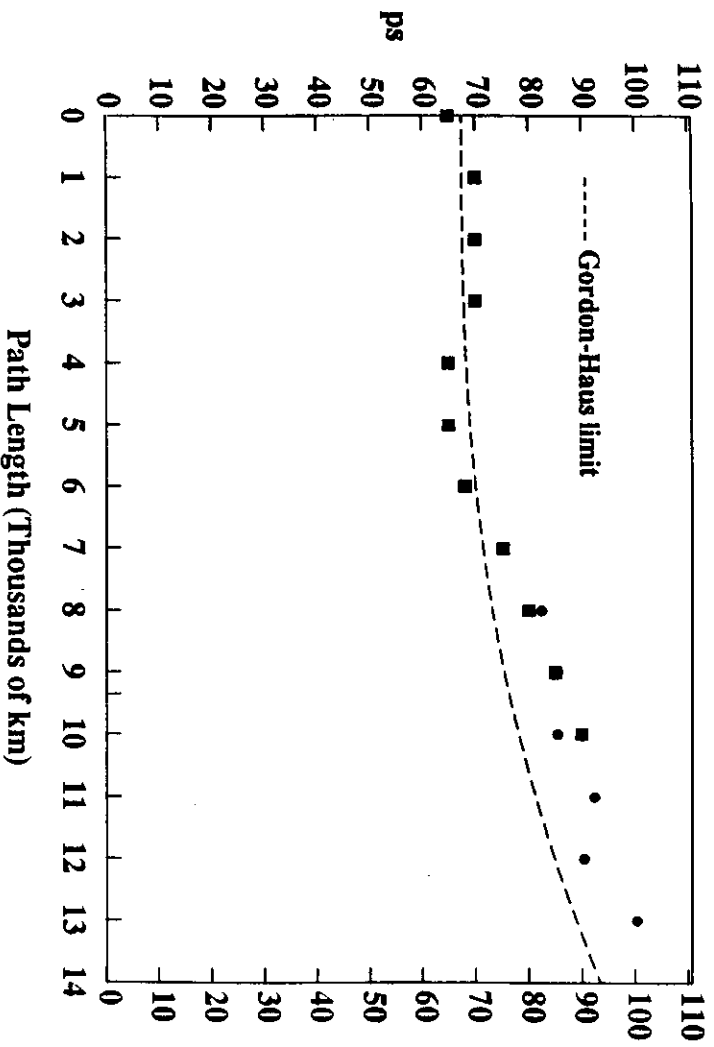


The solid curve is obtained by multiplying the dashed curve by the gaussian:

$$e^{-\frac{1}{2}(2\pi f\sigma)^2}$$

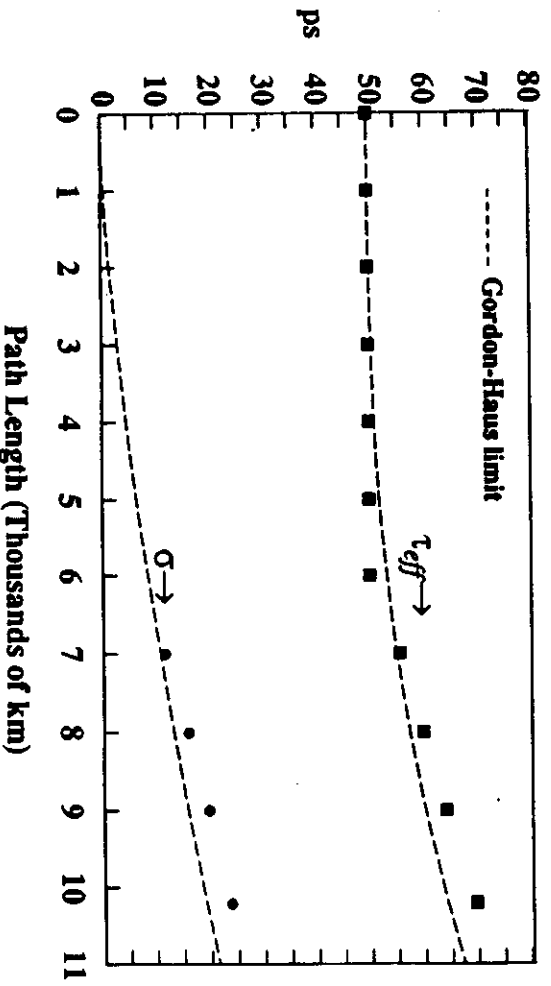
where  $\sigma$  is chosen for best fit to the data.

**SOLITON TRANSMISSION EXPERIMENT USING  
MODE LOCKED LASER DIODE SOURCE  
1.2 GHz pulse repetition rate**

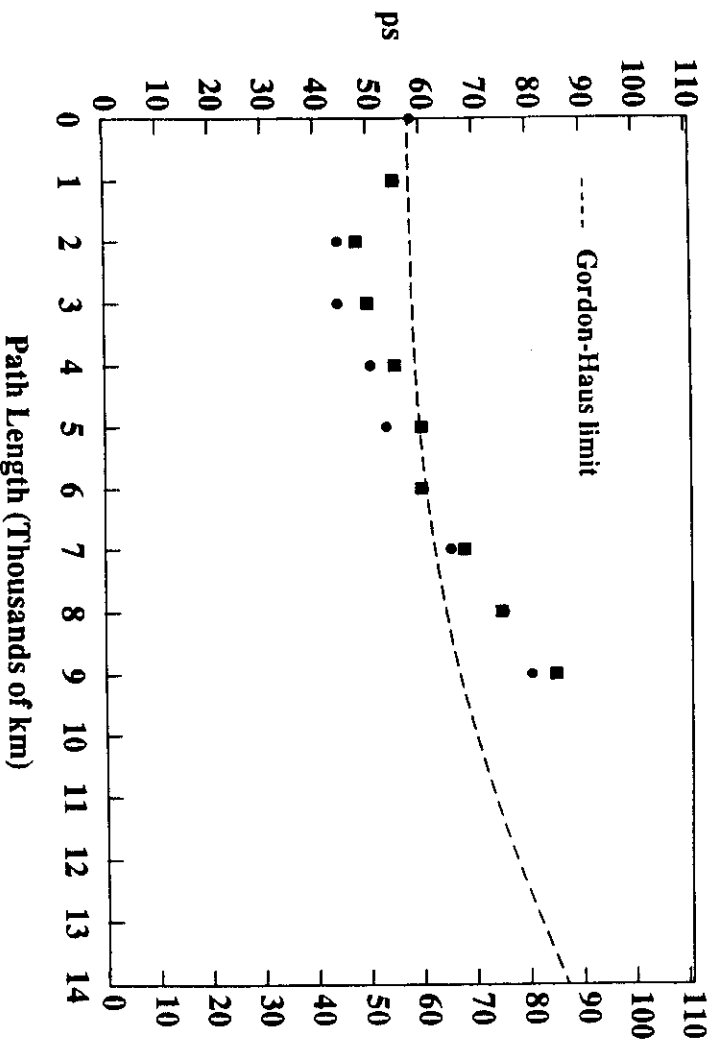


**SOLITON TRANSMISSION EXPERIMENT  
EFFECTIVE PULSE WIDTH AND JITTER VS DISTANCE**

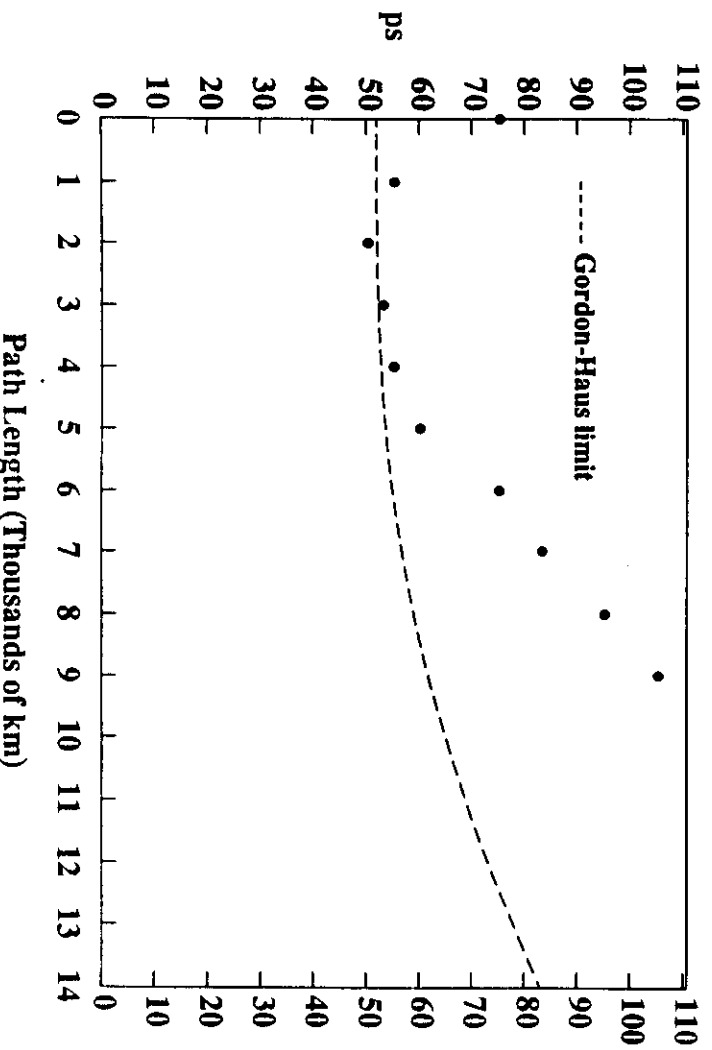
Theoretical curves are for the conditions of experiment,  
viz.,  $\beta$  -1.5, 0.25 dB/km loss, 25 km amp spacing, etc.



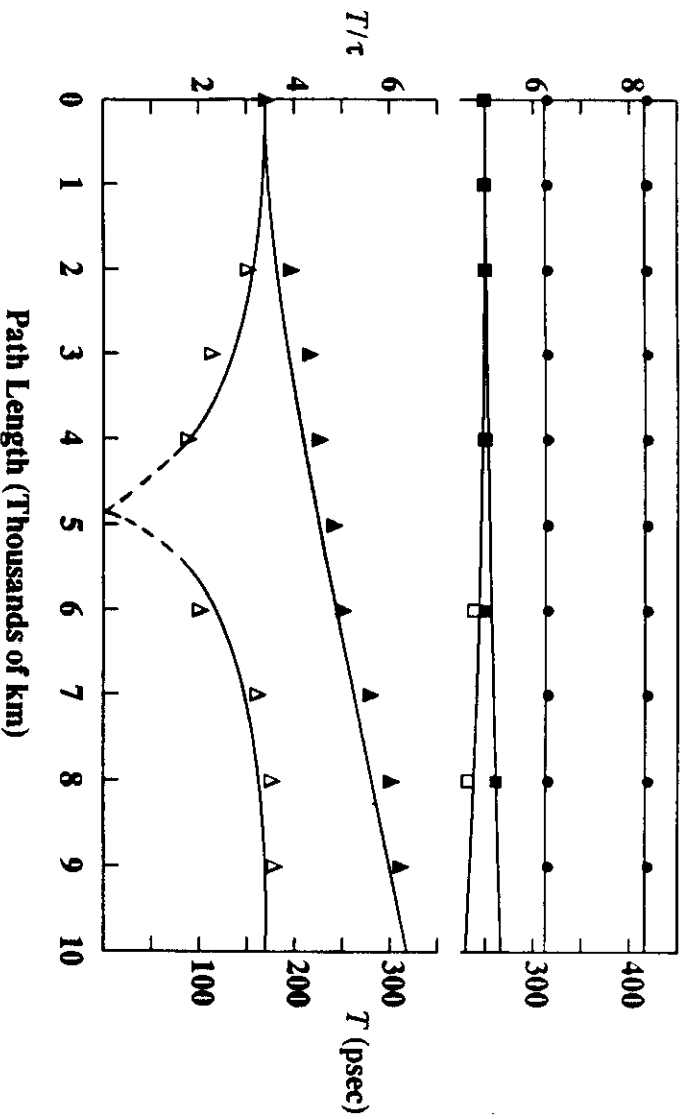
**SOLITON TRANSMISSION EXPERIMENT USING  
 MODE LOCKED LASER DIODE SOURCE  
 2.4 Gbits/s pulse repetition rate**  
 110011001100... pattern imposed by modulator following laser (boxes)  
 110000110000... pattern imposed by modulator following laser (bullets)



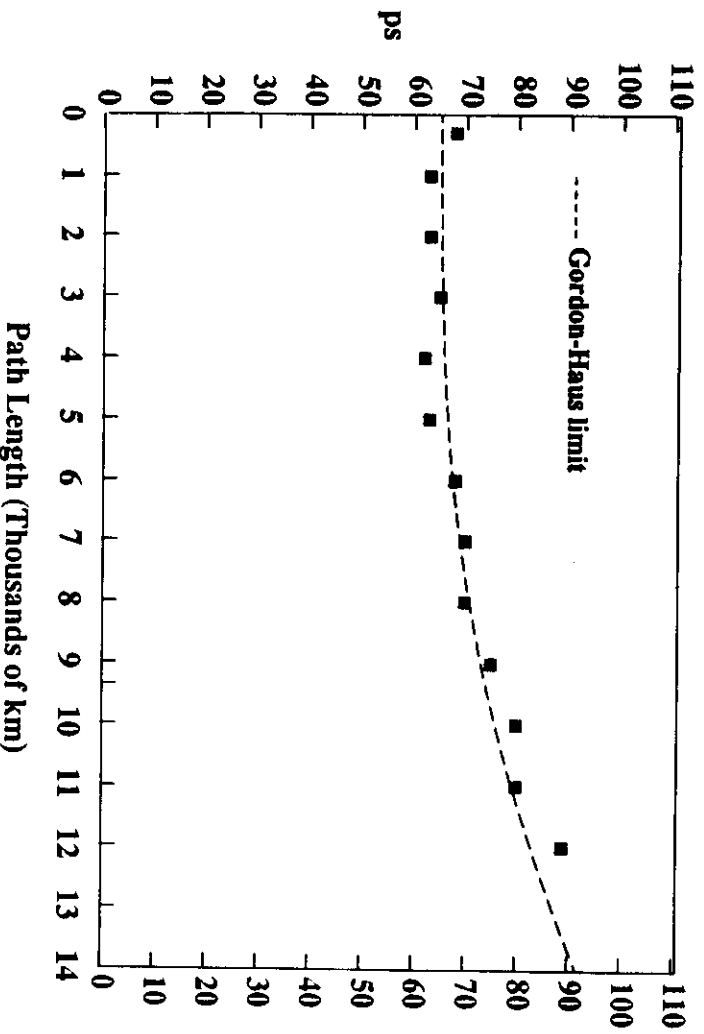
**SOLITON TRANSMISSION EXPERIMENT USING  
 GAIN SWITCHED AND OPTICALLY FILTERED LASER DIODE SOURCE  
 2.4 Gbits/s pulse repetition rate**  
 101010... pattern imposed by modulator following laser



### SOLITON PAIR INTERACTION VS DISTANCE

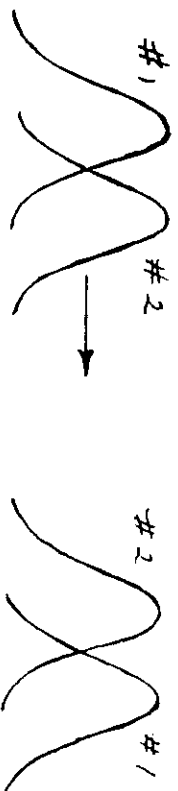


### SOLITON TRANSMISSION EXPERIMENT USING MODE LOCKED LASER DIODE SOURCE 2.4 GHz pulse repetition rate



## SOLITON-SOLITON COLLISIONS IN WDM

Solitons of a higher freq. channel overtake and pass through those of a lower freq. channel:



(#1 at  $f_0 + \Delta f$ ; #2 at  $f_0$ )

Collision length,  $L_{\text{coll}}$ , begins and ends where solitons overlap at their half pwr. points:

$$L_{\text{coll}} = 0.6298 \frac{z_0}{\tau \Delta f} = \frac{2\tau}{D \Delta \lambda}$$

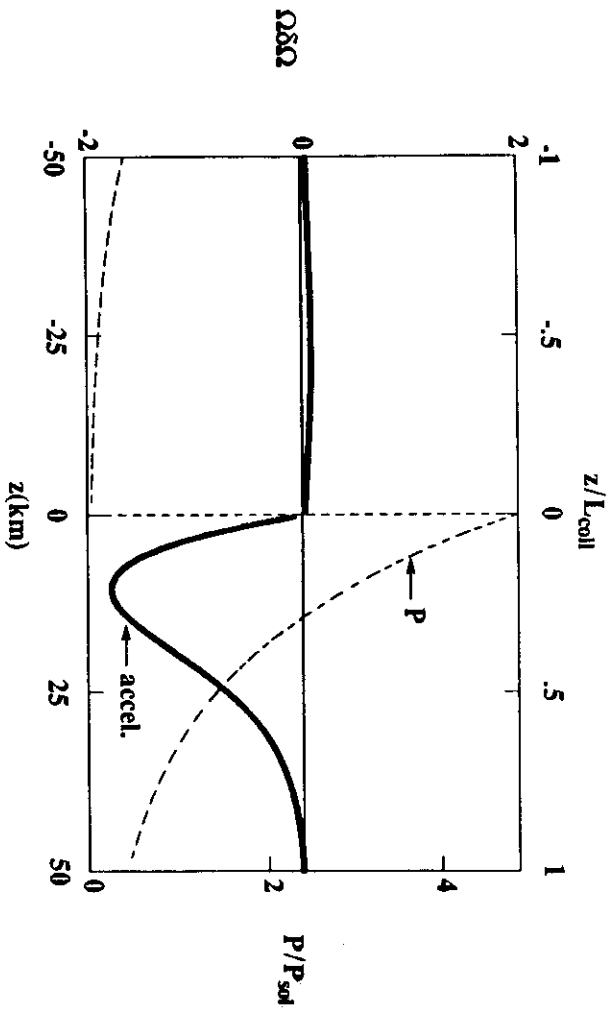
Ex: for  $\tau = 50$  ps,  $D = 1$  ps/nm/km and  $\Delta f = 0.125$  THz ( $\Delta \lambda = 1$  nm at 1550 nm),  $L_{\text{coll}} = 100$  km

[In soliton units, solitons have frequencies  $+\Omega$  and  $-\Omega$ , respectively, where  $\Omega = \text{radians}/\tau c$ . ( $\Delta f/2 = \frac{\Omega}{2\pi\tau c}$ ).  $\Omega$  is also time per unit distance traveled, ie, it is a reciprocal velocity.]

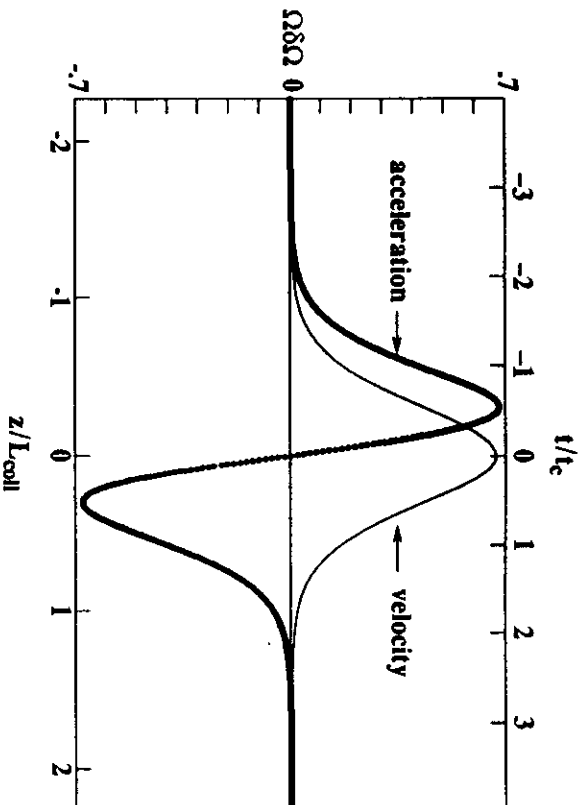
### SUMMARY OF 10000 KM EXPERIMENT:

- Measured timing jitter agrees well with that predicted theoretically.
- Measured S/N ratio ~250
- No measurable polarization dispersion.
- No measurable interaction between pulses separated by  $\geq 5\tau$ .

ACCELERATION OF COLLIDING SOLITONS  
COLLISION CENTERED AT AMPLIFIER (1 AMP/100 km)



ACCELERATION & VELOCITY SHIFT DURING COLLISION OF SOLITONS  
IN LOSSLESS AND OTHERWISE UNPERTURBED FIBER

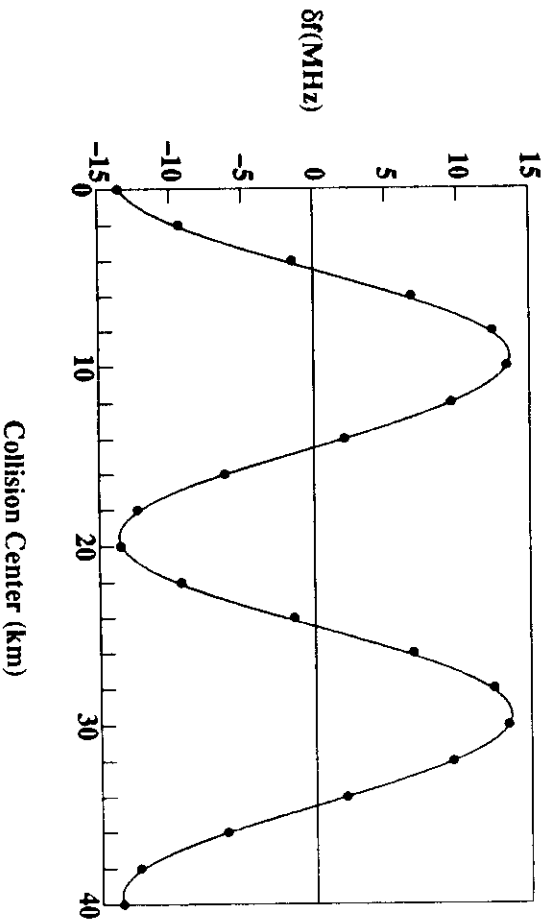


For  $\tau \Delta f > 1$ , pulses are shifted in time by  $\delta t = 0.1786 \frac{1}{\tau(\Delta f)^2}$

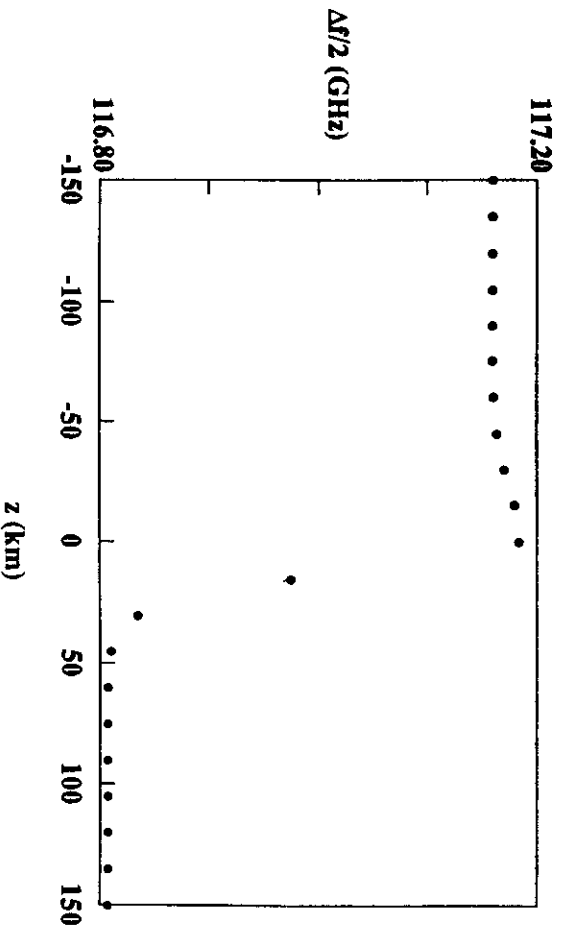
Ex: for  $\tau = 50$  ps and  $\Delta f = 0.03$  THz ( $\Delta \lambda \approx 0.25$  nm at 1550 nm),  $\delta t = 4$  ps.



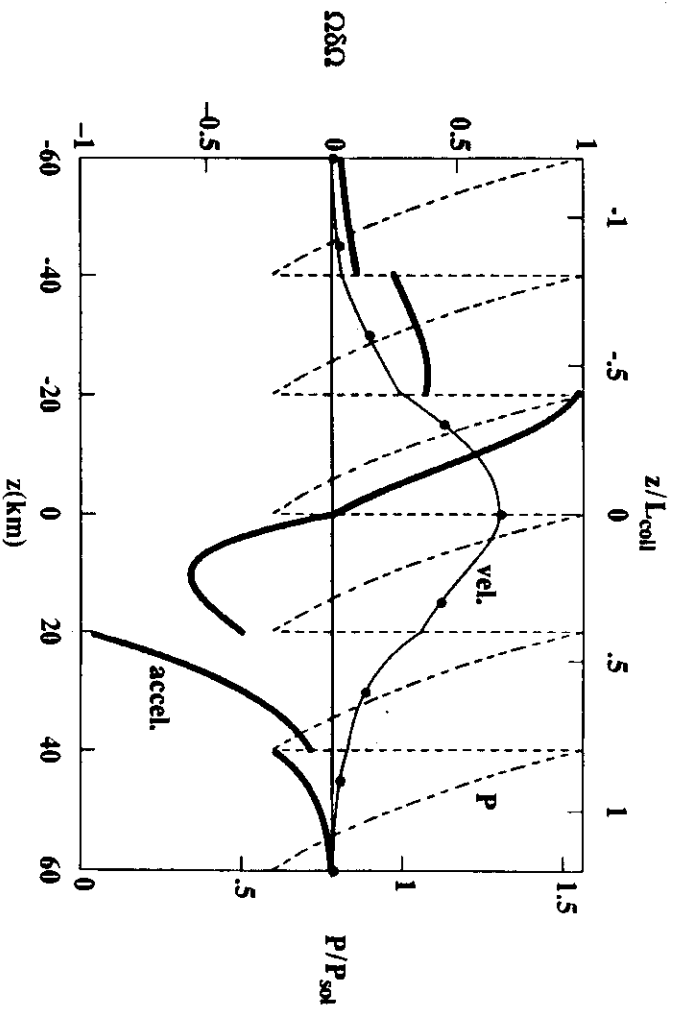
**$\delta f$  OF SOLITONS vs COLLISION CENTER  
LUMPED AMPS AT 0,20,40, etc. km and  $L_{coll} = 20$  km.**



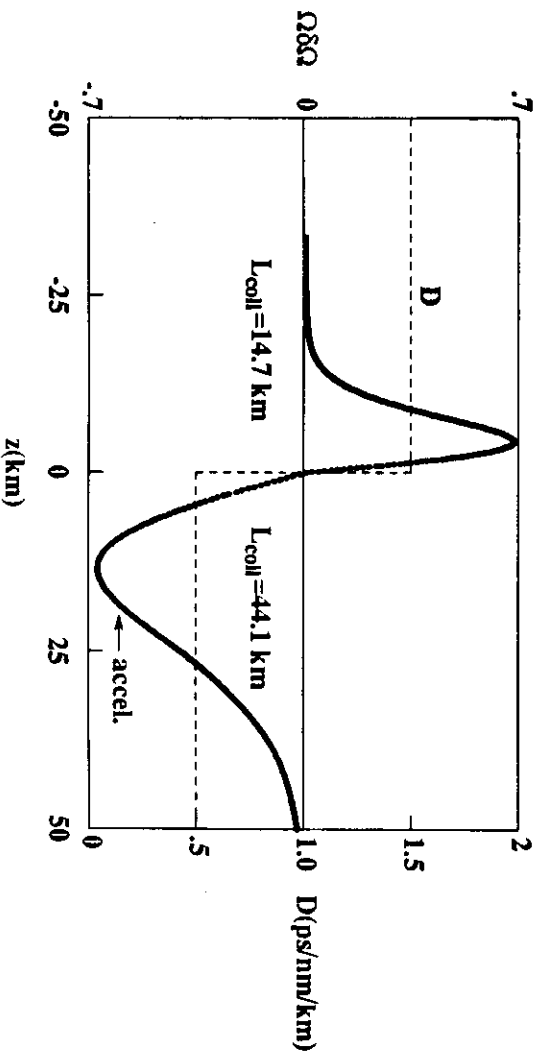
**VELOCITY ( $\Delta f/2$ ) OF COLLIDING SOLITONS  
COLLISION CENTERED AT AMPLIFIER, 1 AMP/100 km**



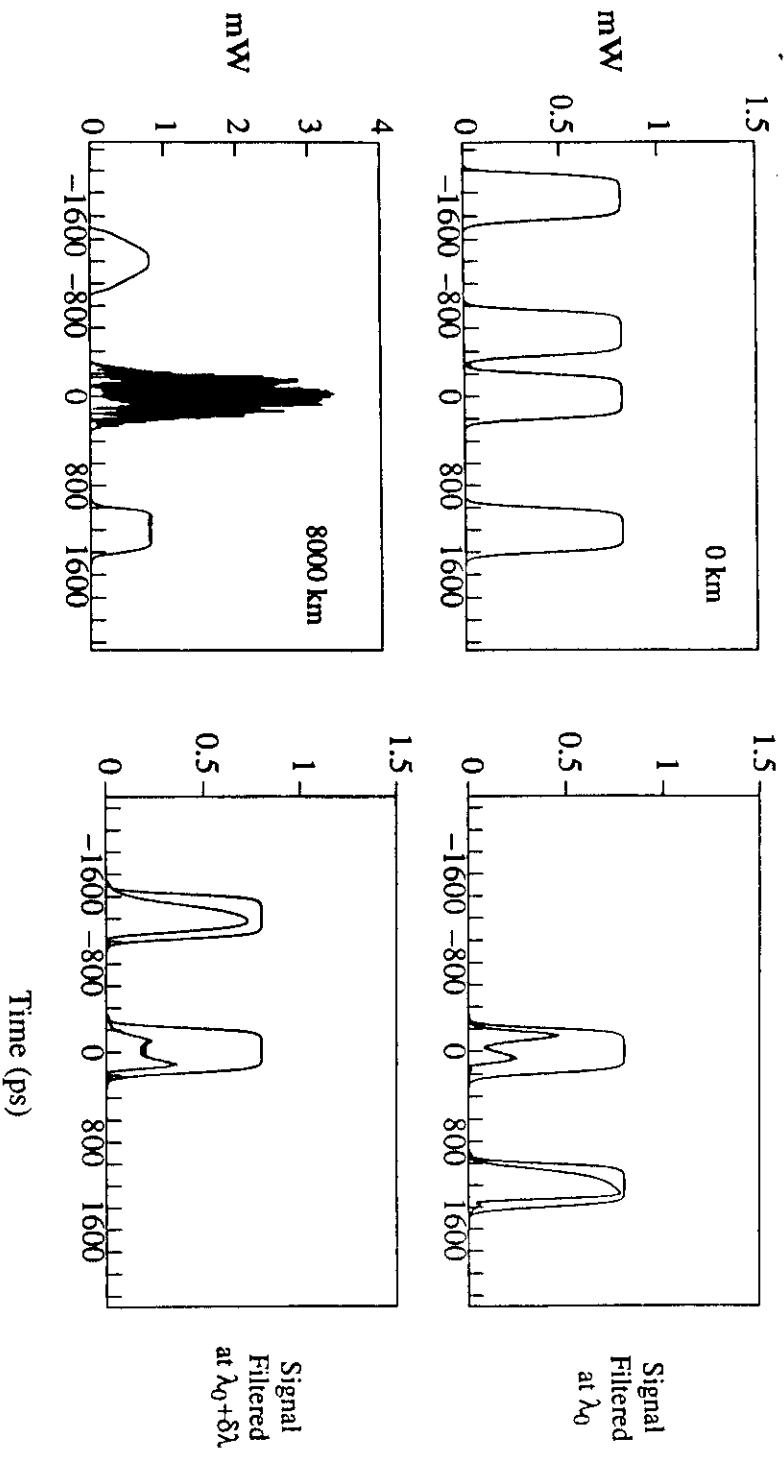
ACCELERATION AND VELOCITY SHIFT OF SOLITONS  
COLLIDING IN SYSTEM WITH LUMPED AMPS EVERY 20 km



ACCELERATION OF SOLITONS COLLIDING IN LOSSLESS FIBER  
COLLISION CENTERED ABOUT A STEP CHANGE IN D

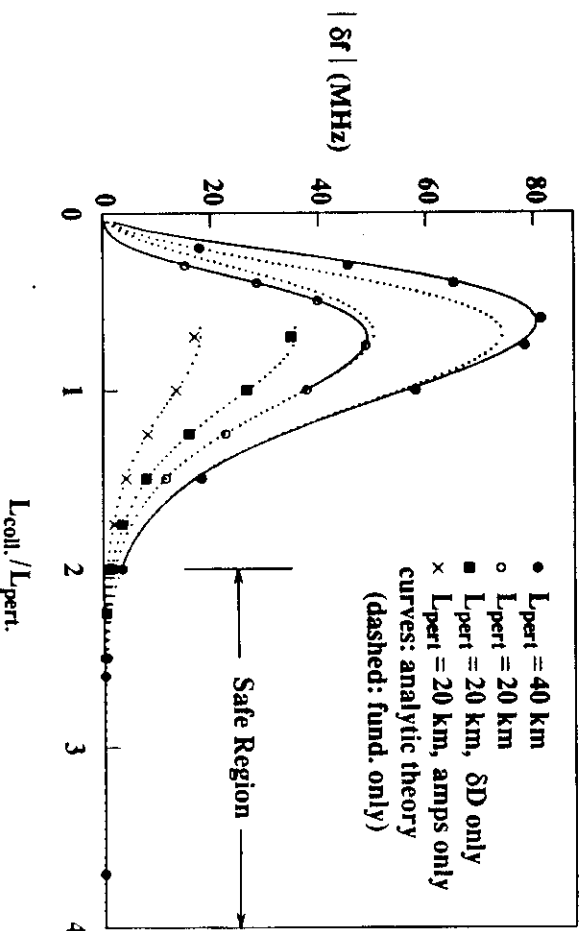


2.5 GBit WDM  
 0.8 mW  
 $\delta\lambda=1.4\text{nm}$   
 34 km amplifier spacing

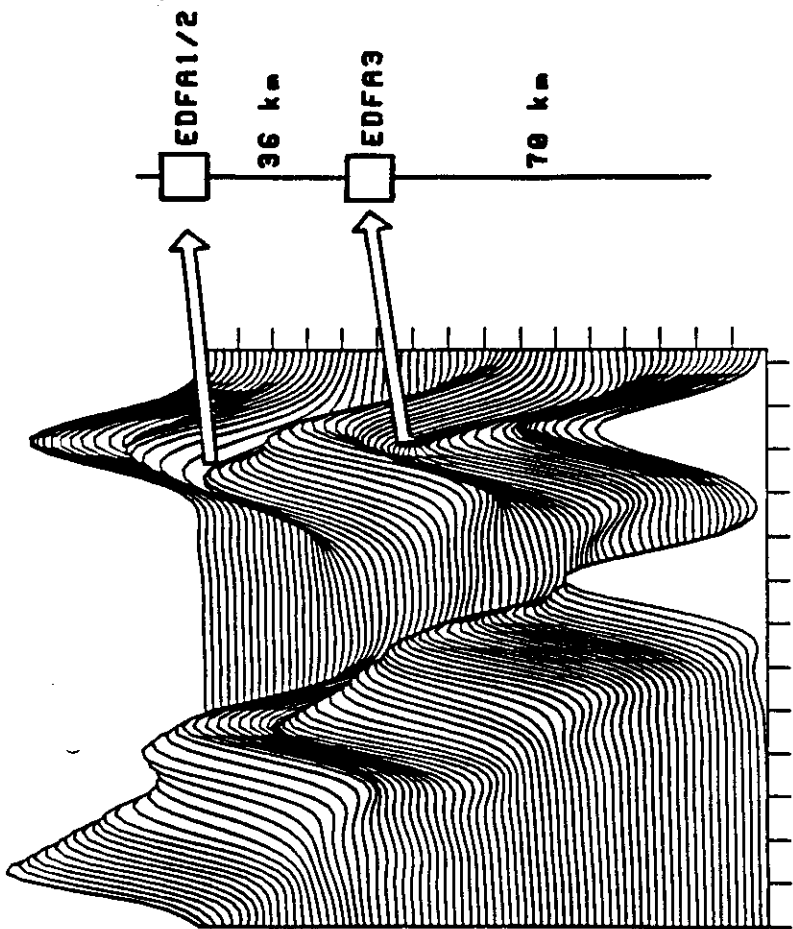


### $\delta f$ OF COLLIDING SOLITONS VS $L_{\text{coll}}/L_{\text{pert}}$

lumped amps every 20 km and  $\delta D = \pm 0.5$  ps/nm/km about  $\bar{D} = 1$

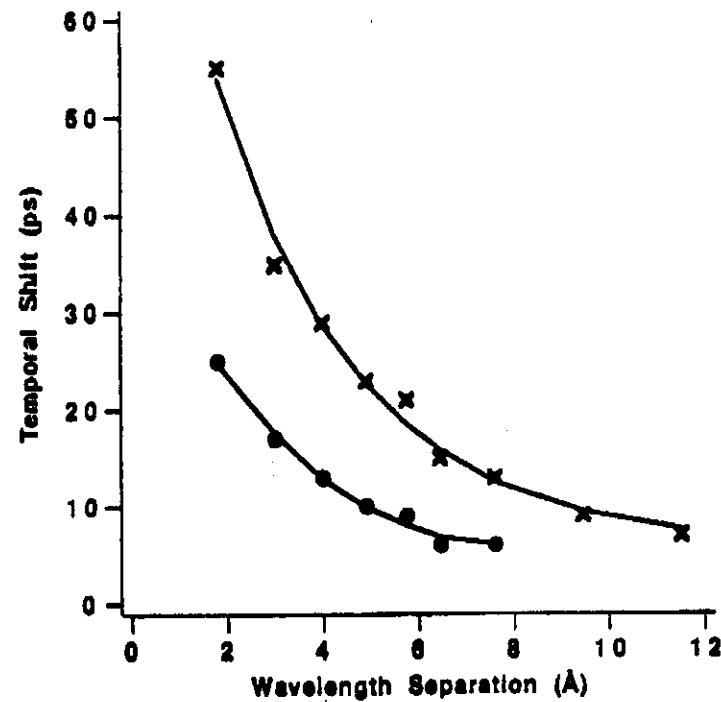


Soliton Collisions  
Effects of Amplifiers on Collisions  
Experimental Data



Data of Andrekson and Olsson (Bell Laboratories)

Temporal Shift due to a Collision  
Experimental Data



$$\delta t = \pm 0.1786 \frac{1}{\tau(\Delta f)^2}$$

Data of Andrekson and Olsson (Bell Laboratories)

## WDM WITH 50 PS SOLITONS

$$Z = 9000 \text{ km}$$

4 Gbits/s per channel

$\bar{D}$ (ps/nm/km)	$\Delta\lambda_{\min}$ (nm)	$\Delta t$ (ps)	$\Delta\lambda_{\max}$ (nm)	No. of channels	$\bar{P}_{\text{sig}}/\text{chan.}$ at amp. outputs ( $\mu\text{W}$ )	Total bidirectional capacity (GBits/s)
1.0	.27	$\pm 19.0$	1.08	5	125	40

### RULES FOR WDM CHANNEL SPACING

The requirement  $L_{\text{coll}} \geq 2L_{\text{peri}}$  determines the maximum allowable channel spacing:

$$\Delta f_{\max} = 0.31 \frac{z_0}{\tau L_{\text{peri}}} \quad \text{or} \quad \Delta\lambda_{\max} = \frac{\tau}{DL_{\text{peri}}}$$

Pulses of the  $i$ th channel will tend to suffer a range of collisions with the  $j$ th channel, from none to a maximum given by  $N_{ij} = Z\tau/(L_{\text{coll}}^i T)$ , where  $Z$  is the total system length and  $T$  is the bit period. There will then be a spread of arrival times about the mean, given by multiplying the time shift per collision by  $N_{ij}/2$ , and summing over all channels  $j \neq i$ , of

$$\Delta t_i = \pm 0.1418 \frac{Z}{z_0} \frac{\tau}{T} \sum_{j \neq i} \frac{1}{(\Delta f)_{ij}}$$

Thus, the max allowable  $\Delta f$  sets a limit on the *minimum* allowable  $\Delta f$ .

[Note: the minimum  $\Delta f$  may instead be determined by the requirement  $\tau \Delta f > 1$ .]

