



INTERNATIONAL ATOMIC ENERGY AGENCY  
 UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION  
**INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS**  
 I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY. CABLE: CENTRATOM TRIESTE



UNITED NATIONS INDUSTRIAL DEVELOPMENT ORGANIZATION



**INTERNATIONAL CENTRE FOR SCIENCE AND HIGH TECHNOLOGY**

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS - P.O. BOX 586 TRIESTE ITALY - P.O. BOX 586 TRIESTE ITALY - P.O. BOX 586 TRIESTE ITALY

H4.SMR/540-28

**Second Training College on Physics and Technology  
 of Lasers and Optical Fibres**

21 January - 15 February 1991

*Integrated Optoelectronics*

R. Baets  
 University of Gent  
 Gent, Belgium

Trieste, Feb 11-12, 1991

# INTEGRATED OPTOELECTRONICS

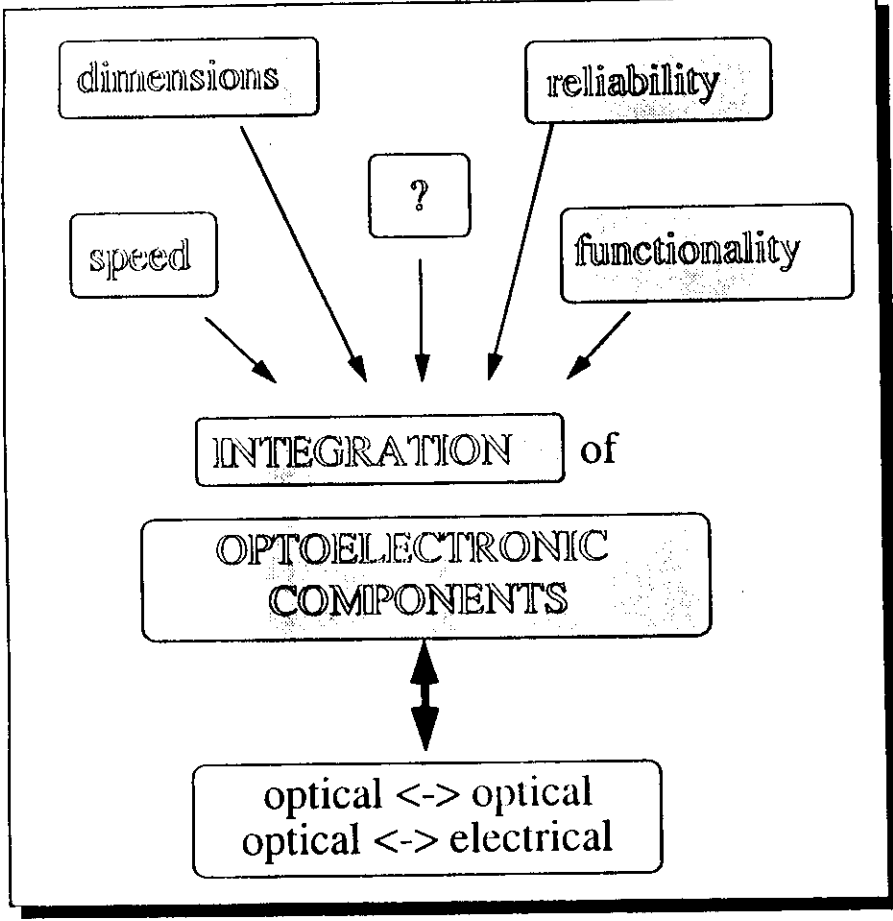
R. BAETS

University of Gent  
 Belgium

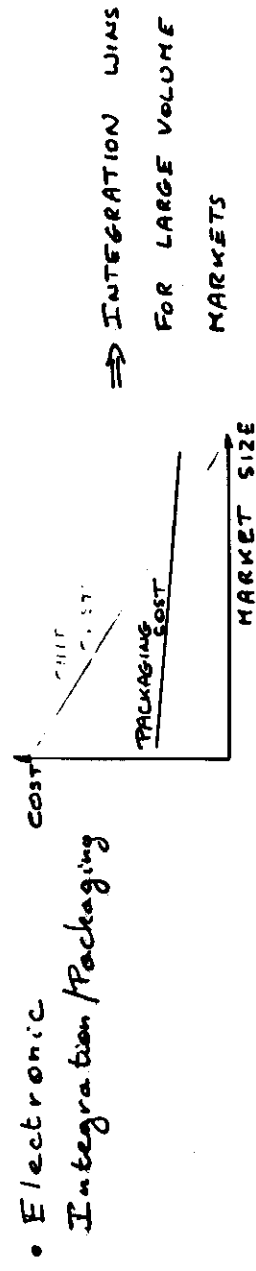
## TOPICS

- INTEGRATION : WHY?
- BUILDING BLOCKS : DEVICES
- INTEGRATION TECHNOLOGY
  - EXAMPLES
  - MODELLING

# Introduction



## INTEGRATION VERSUS PACKAGING



- Electronic Integration/Packaging
  - ⇒ Integration Wins FOR LARGE VOLUME MARKETS
- Optoelectronic Integration/Packaging
  - Packaging at optical interface is difficult
  - Integration of dissimilar devices is difficult
  - ⇒ Advanced Integration Technology needed
  - ⇒ very expensive technology
  - ⇒ high volume markets needed

# Introduction



communications and datastorage

## ever increasing demands on:

- \* data quantity
- \* speed
- \* reliability

## we already have:

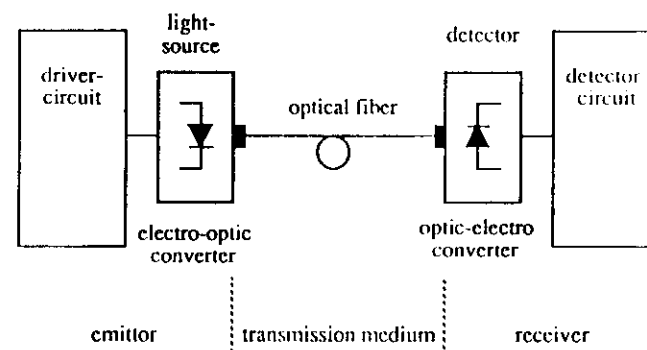
- \* high performance transmission media (fibers)
- \* high performance sources (laserdiodes)
- \* high performance receivers (detectors)
- \* high performance electronics

# Introduction



## Important field of application:

### Optical transmission



optical communications (fibers)  
optical networks (LAN)  
optical interconnect  
optical computing

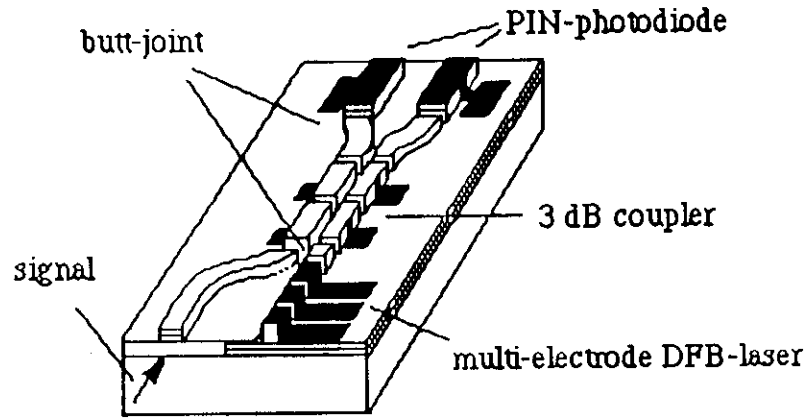
.....

# Introduction



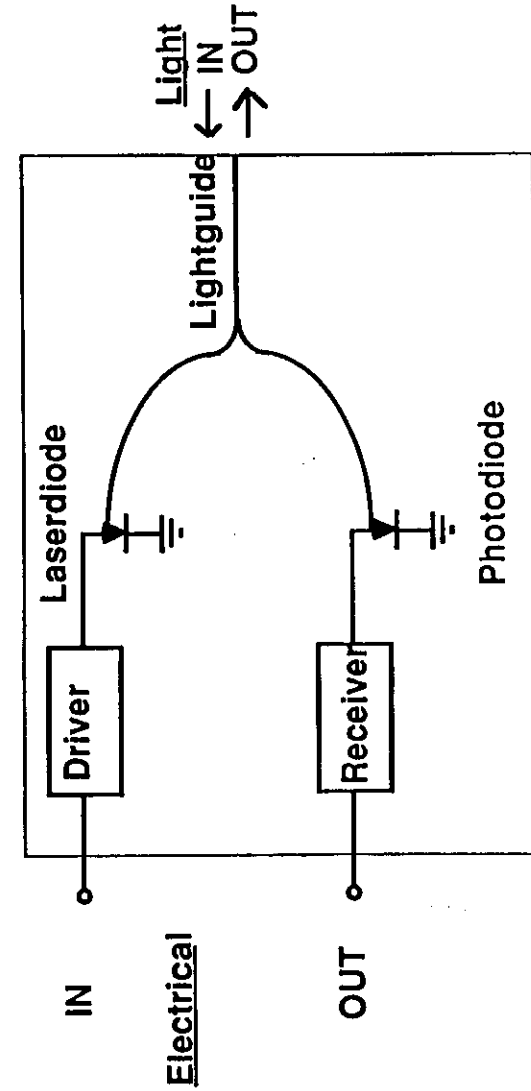
integration soon leads to  
very complex circuits

integrated receiver for  
coherent communication



(NTT-Japan, AT&T-USA)

## OEIC FUNCTION

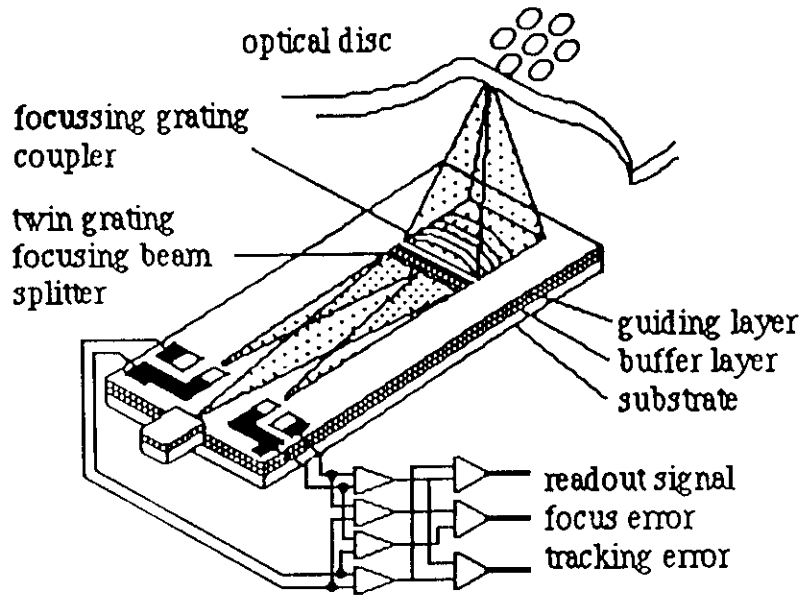


# Introduction



optics is more  
than optical communications

optical head for optical discs



## TERMINOLOGY

OPTOELECTRONICS - AIC STANDARDS

OEIC : Optoelectronic IC

= Integration of electronic circuits with optoelectronic devices  
(few optical interfaces)

PIC : Photonic IC

= Integration of active and passive optoelectronic devices  
(mainly of waveguide type)

ATT - Bell Labs terminology

IOC : Integrated Optic Circuit

= Integration of planar waveguide devices (mainly passive)

## Building blocks



### Electronic components

- \* transistors

Si - GaAs - ...

### Optoelectronic components

- \* light sources
- \* detectors

III-V - Si - ...

### Optical components

- \* waveguides
- \* optical switches

LiNbO<sub>3</sub> - III-V - ....

## Building blocks



### Electronic components

bipolar - field effect (FET)

JFET

Si ...  
GaAs ...

MOSFET

Si (N) (P) (C) MOS, ...  
GaAs

MESFET

Si ...  
GaAs ...

Si, III-V

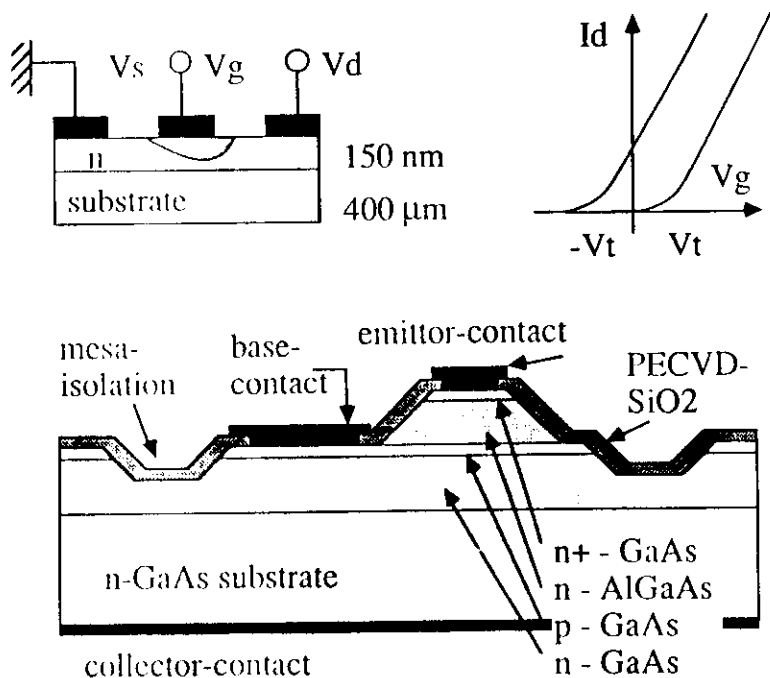
Si: integration = materials problem

GaAs: integration = structural problem

## 2 - Building blocks



### GaAs MESFET $\leftrightarrow$ HBT



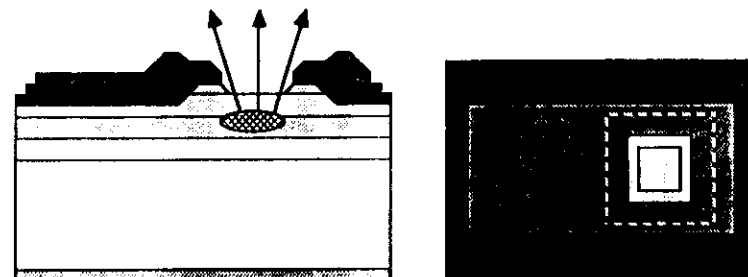
## 2 - Building blocks



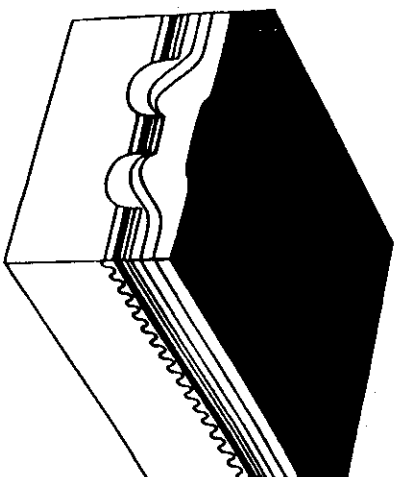
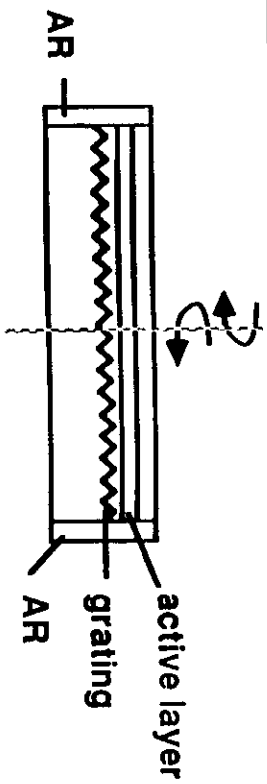
### Optoelectronic components

Lasers, LED's, Detectors

III-V

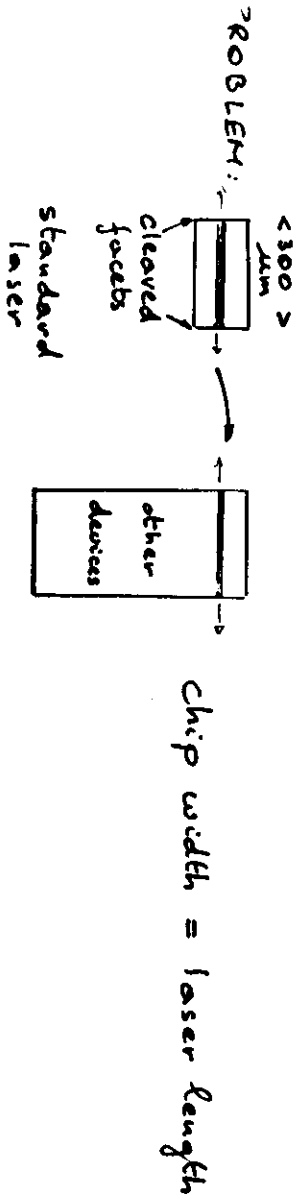


# SLM LASERS DISTRIBUTED FEEDBACK (DFB) LASER

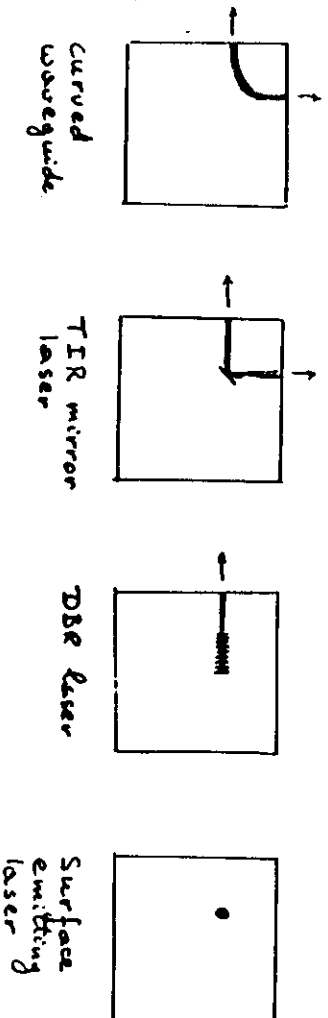


DCPBH - LASER

## INTEGRATION OF LASER DIODES THE CAVITY LENGTH PROBLEM



SOLUTIONS:





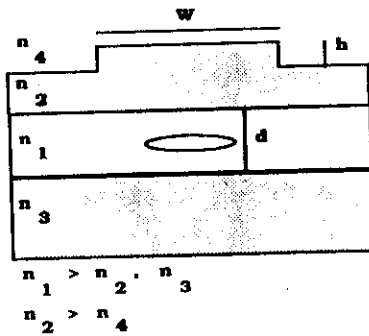
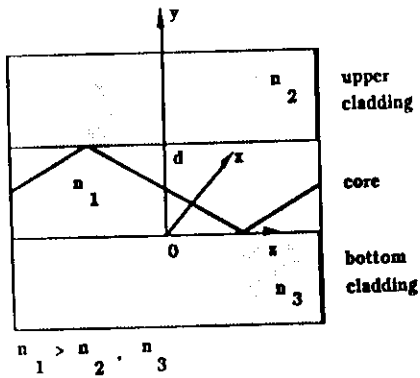
# Building blocks



## Optical components

Passive elements (waveguides, splitters,...)  
III-V, Si, LiNbO<sub>3</sub>, glass, polymers,...

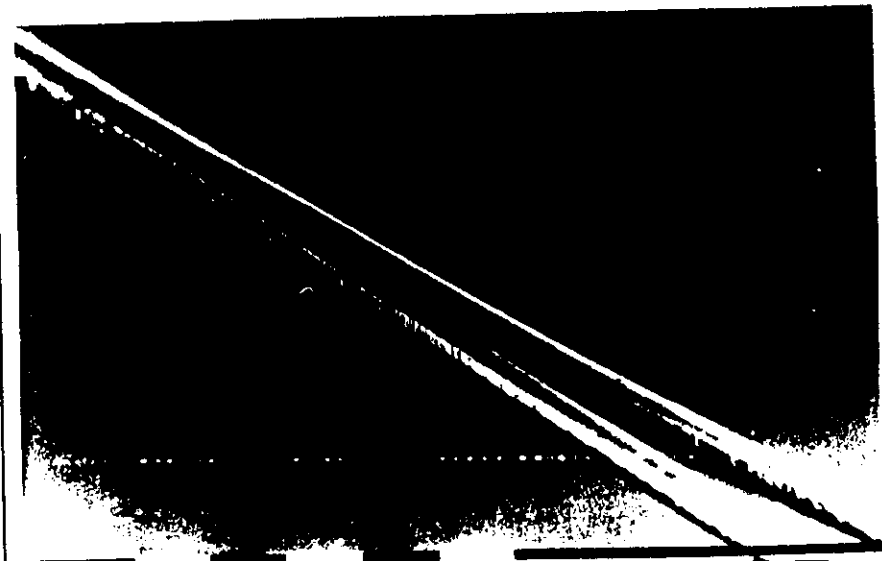
Active elements (switches, modulators,...)  
III-V, LiNbO<sub>3</sub>, polymers,...



# 2 - Building blocks



## RIE-etched waveguide

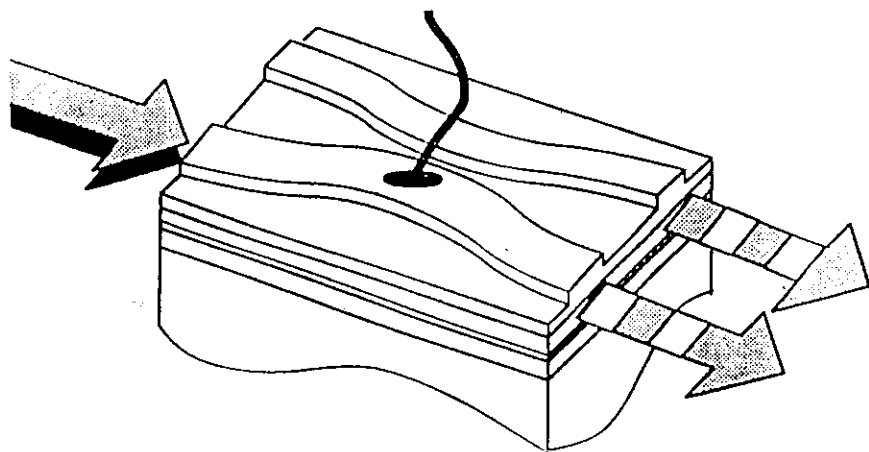


10  $\mu$ m 301kU 110E3 2213/87 RUG-LEM

## 2 - Building blocks



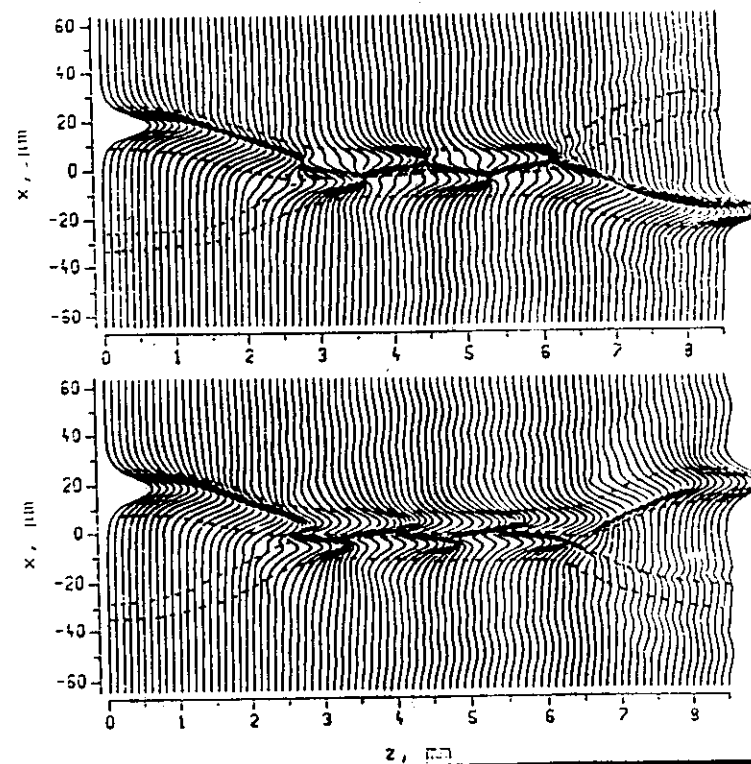
Waveguides for integrated optics:  
directional coupler



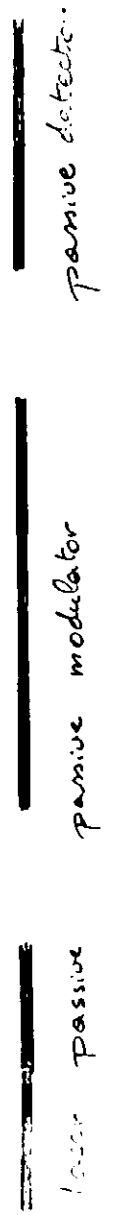
## 2 - Building blocks



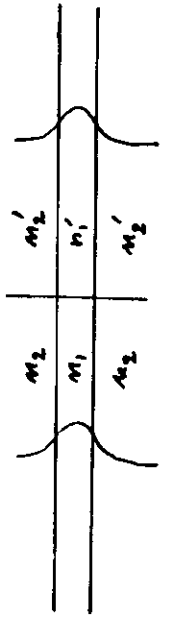
Waveguides for integrated optics:  
directional coupler



# COUPLING BETWEEN ACTIVE & PASSIVE WAVEGUIDES



Ideal coupling:



very difficult to make

$m_1 - n_2 \approx m'_1 - n'_2 \rightarrow$  mode matching

$m_1 - m'_1$  and  $n_2 - n'_2$  small  $\rightarrow$  little reflection (except if laser facet is needed)

## COUPLING BETWEEN ACTIVE AND PASSIVE WAVEGUIDES



Butt coupling



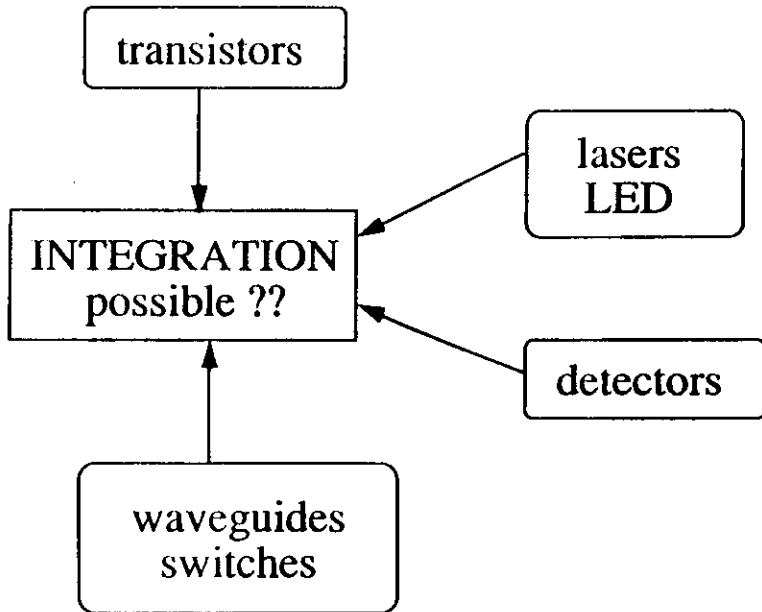
Evanescent field coupling (works very well for detectors)



Quantum well active device

- two growth steps
- growth interruption near active layer
- incompatibility between doping in active and passive region

# Techniques



# Techniques



Different techniques can be used:

- \* **hybrid integration**
- \* **monolithic integration**
  - homo-epitaxial growth
  - hetero-epitaxial growth

Alternatives:

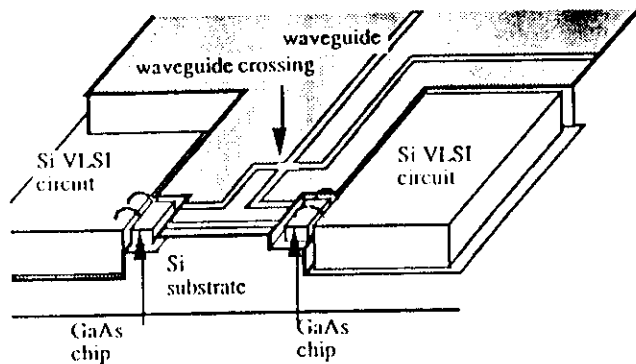
- \* **flip-chip integration**
- \* **epi-lift-off**

# Techniques



## hybrid integration

Integration of different chips:  
by means of epoxy / solder  
on the same carrier  
interconnections via wires



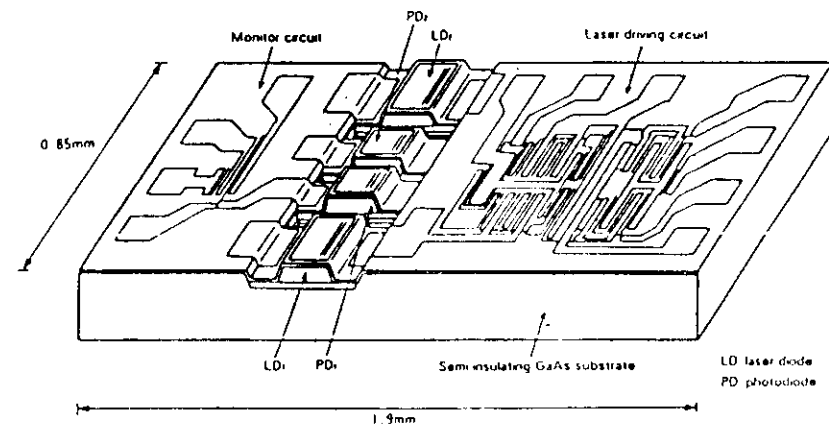
# 3 - Techniques



## monolithic integration

Integration of different chips:  
via "crystalline" growth  
on the same carrier  
interconnections on chip-level

Structure of high speed modulation laser diode



# Techniques



## monolithic integration

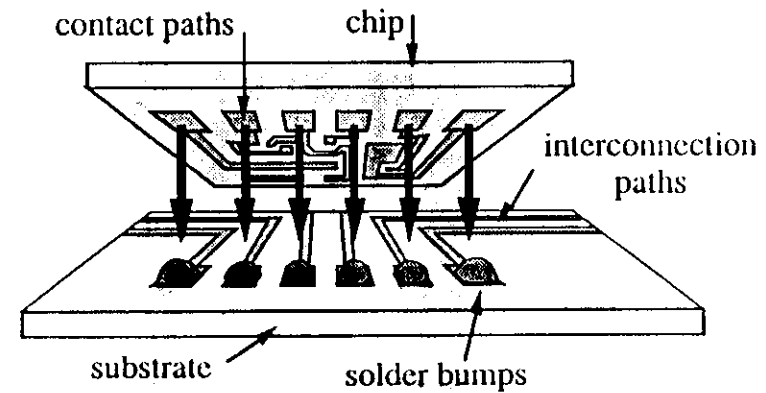
problems:

- \* materials incompatibility  
(GaAs on Si, GaAs on InP,...)
- \* different layerstructures  
(Laser + waveguide,...)
- \* chip processing difficulty
- \* lasers are power hungry devices

# Techniques

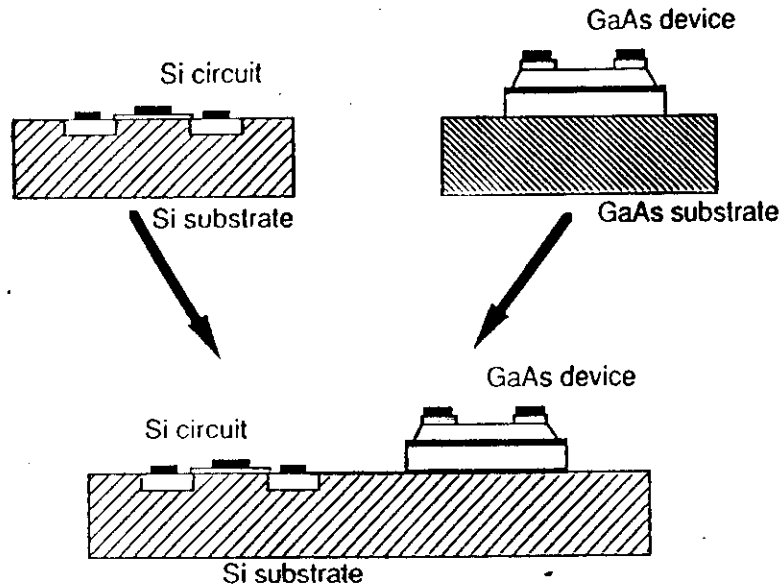


## flip-chip integration



## Example

### GaAs on Si integration



## Advantages

- GaAs and Si devices optimized separately
- final integration similar to monolithic integration (using heteroepitaxy)

## Problems

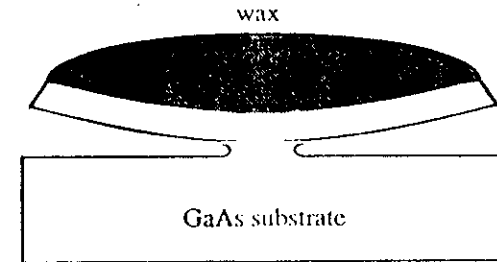
- very fragile ELO film (to be supported)
- alignment
- degradation

## 3. LED epi-lift-off

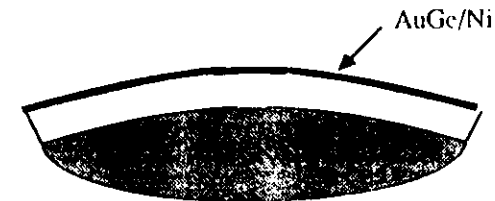
### ELO strategy 2

using a wax carrier

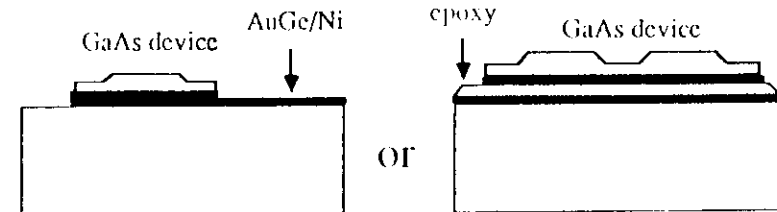
- ELO etch in HF:DI (1:5)



- Deposition of AuGe/Ni for backside contact



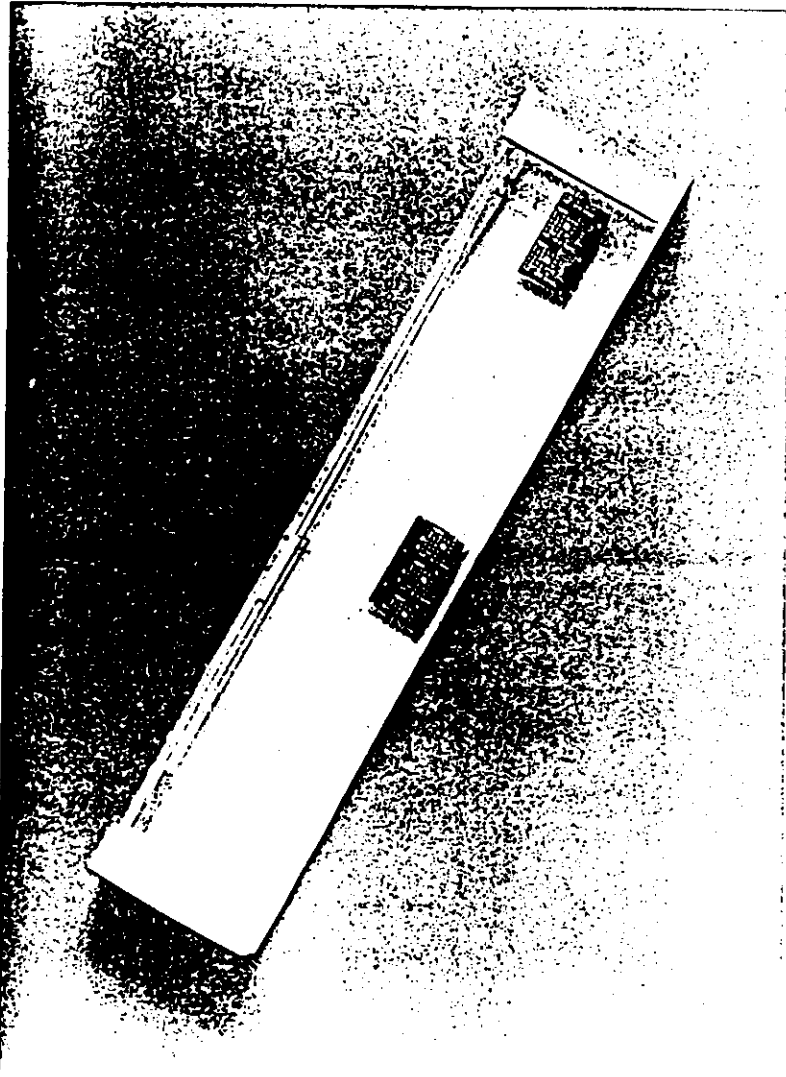
- Adhesion to Si



fast alloying at 450 °C

with silver epoxy

1mm301KU 486E1 0560/90 RUG-LEM



Photograph of 1 x 2 lithium niobate optical switch  
with epitaxial lift-off GaAs MESFET circuits

**GEC-Marconi**



# Processing



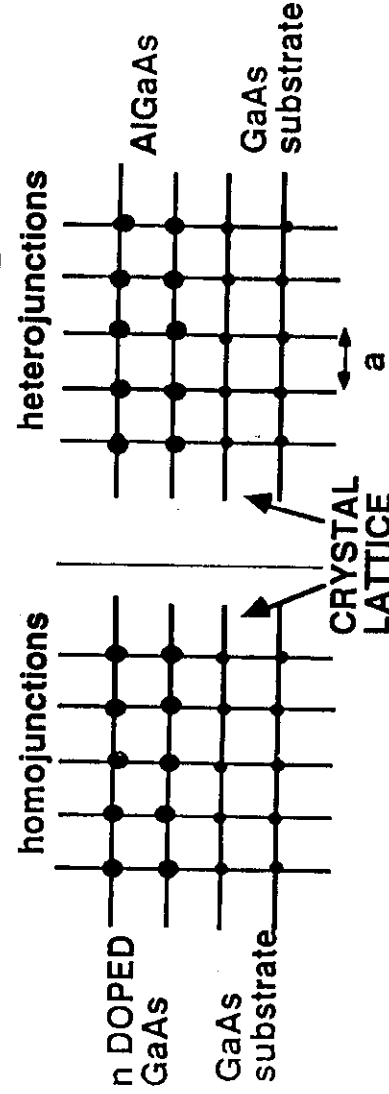
Integration (monolithic, ELO,...):

- \* change design
- \* change processes
  - epitaxial growth
  - lithography
  - metallisations
  - etching
  - .....

Problems can differ very much  
**creativity and flexibility required**

## EPITAXIAL GROWTH

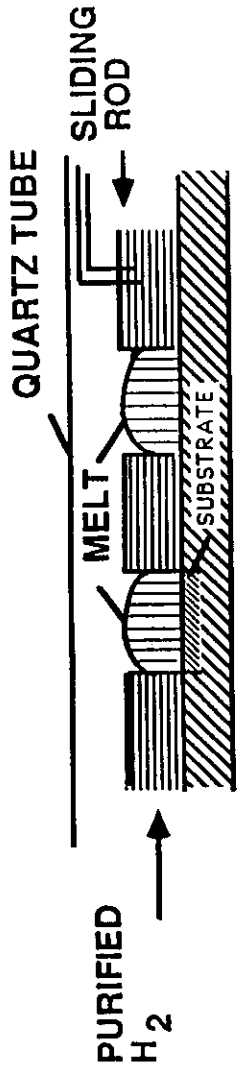
GROWTH OF MONOCRYSTALLINE MATERIAL  
ON SAME OR DIFFERENT CRYSTAL



REQUIREMENT : IDENTICAL LATTICE CONSTANT (a)

TECHNIQUES : LPE  
(MO)CVD  
MBE

## LIQUID PHASE EPITAXY (LPE)

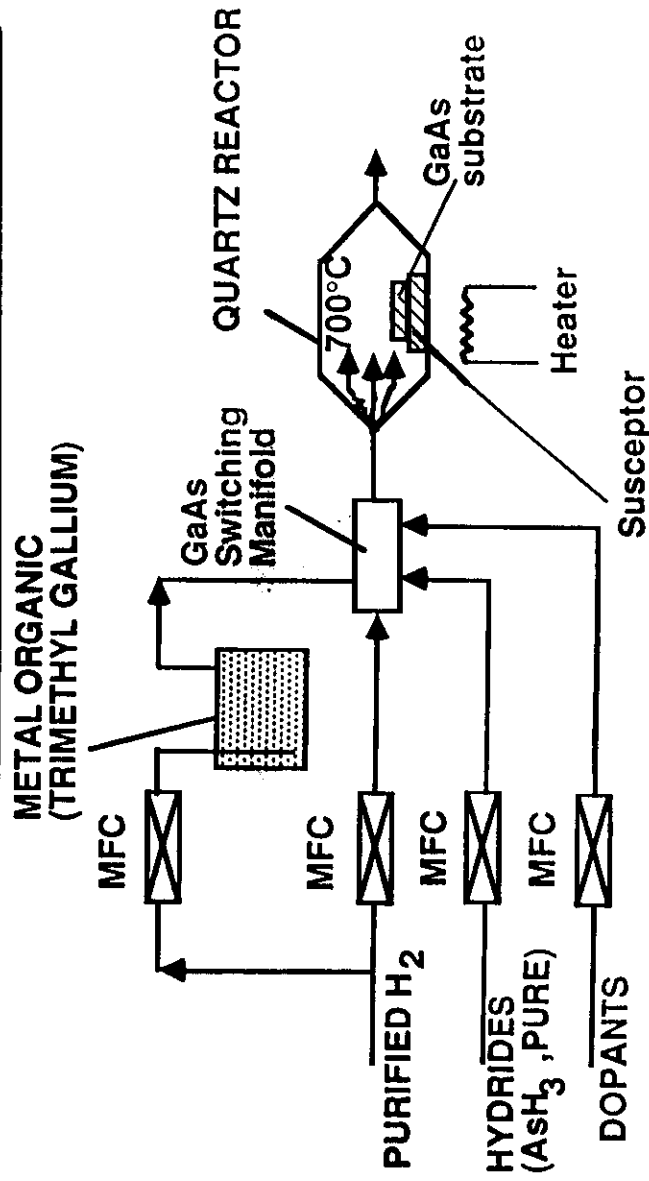


800°C, COOLING AT 1°C/MIN

MELT : In + Ga + As + P + (dopants)  
SUBSTRATE : InP

- THERMODYNAMICAL EQUILIBRIUM PROCESS
- FABRICATION METHOD OF LASER DIODES
- THINNEST LAYER : 0.1 μm ; SMALL AREA

## PRINCIPLE OF MOCVD REACTOR FOR EPITAXIAL GROWTH



## (METAL ORGANIC) CHEMICAL VAPOR DEPOSITION (MO)CVD

EPITAXIAL GROWTH FROM THE VAPOR PHASE

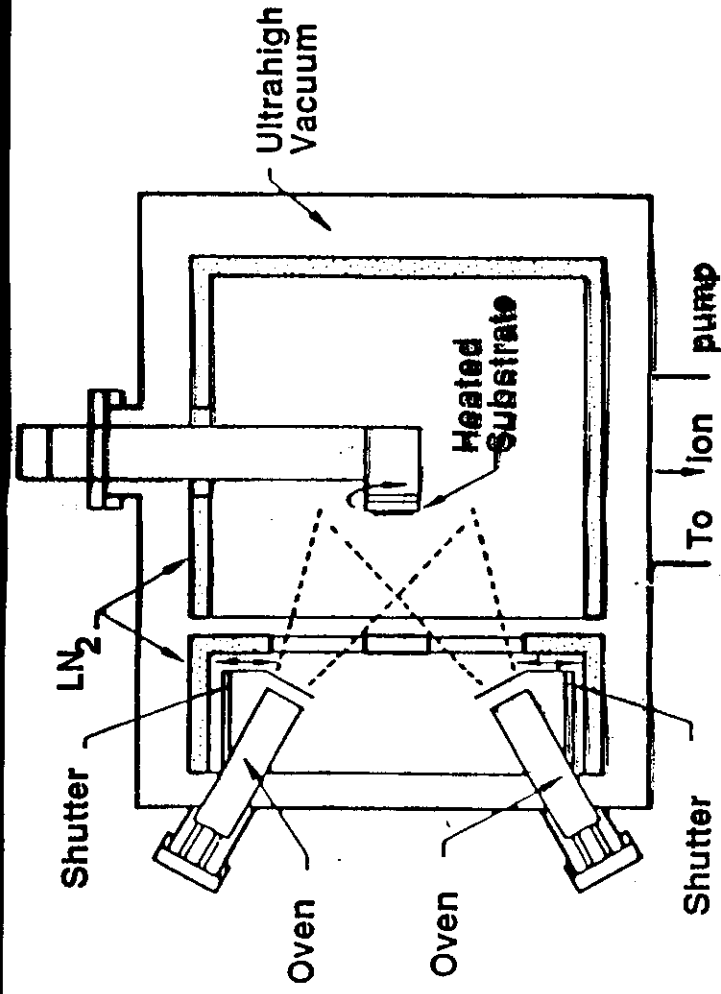
ADVANTAGES

- GROWTH OVER LARGE AREA
- VERY THIN LAYERS ( 10 - 100 Å) POSSIBLE
- SHARP INTERFACES

PROBLEM: SAFETY

- HIGHLY TOXIC ARSINE
- (PYROFORIC METALORGANICS)
- (EXPLOSIVE  $H_2$ )

## MOLECULAR BEAM EPITAXY (MBE)



# Processing

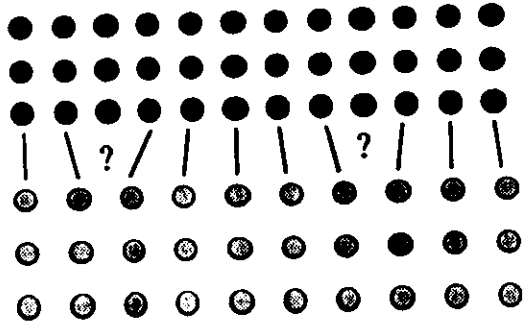


## epitaxial growth

hetero-epitaxy:

≠ crystallographic structure

≠ thermal expansion



## HETEROEPITAXY : GaAs on Si

### WHAT:

**EPITAXIAL GROWTH OF GaAs LAYERS ON SILICON SUBSTRATES**

### POTENTIAL APPLICATIONS:

- Monolithic integration of GaAs and Silicon devices with the goal of combining the sophistication of Si VLSI technology with the superior speed and/or optoelectronic capabilities of GaAs-based microelectronics
- The use of Si as a substrate for GaAs epitaxial deposition (e.g. solar cells)

FIRST PUBLISHED REPORT IN 1983

## **GaAs on Si PERFORMANCE / MATERIAL PROBLEMS**

### GENERIC PROBLEMS OF HETEROEPI TAXY

LATTICE MISMATCH (4%)

SYMMETRY MISMATCH

THERMAL EXPANSION MISMATCH

COMPLETE REMOVAL OF NATIVE OXIDE FROM SI SURFACE

## **GaAs on Si PERFORMANCE PROBLEMS**

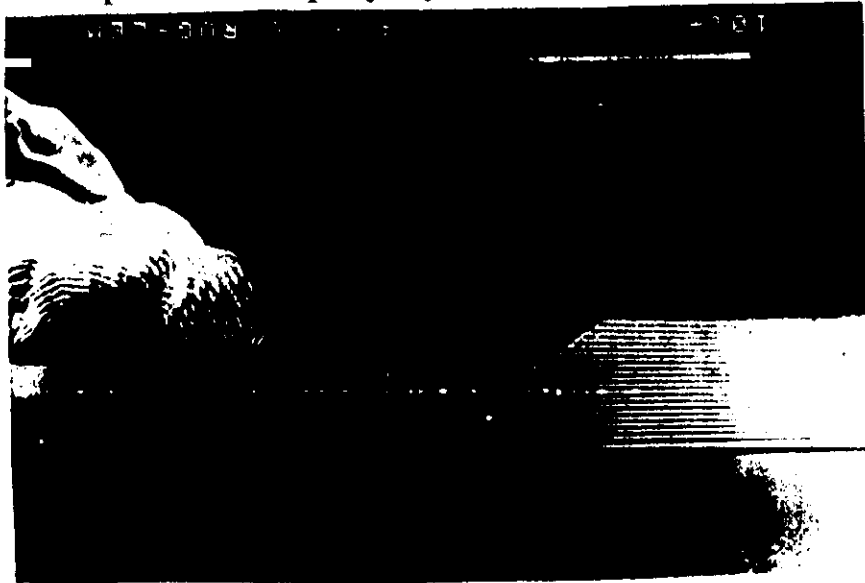
- MAJORITY-CARRIER DEVICES (MESFET, HEMT) :  
PERFORMANCES COMPARABLE TO DEVICES  
FABRICATED IN GaAs
- MINORITY-CARRIER DEVICES (LED, LASER, ...) :  
CURRENTLY INFERIOR PERFORMANCE

## 4 - Processing



### epitaxial growth

selective and/or non planar growth:  
formation of different crystallographic  
planes and polycrystalline material

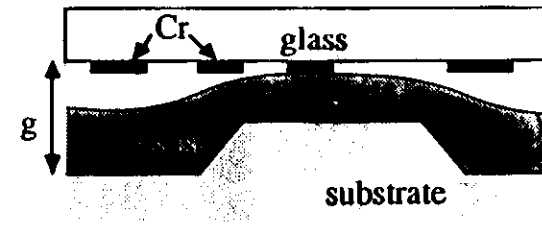


## Processing



### lithografie

non-planarity results in problems for:  
minimum feature size



$$W = \sqrt{\lambda \cdot g}$$

# 4 - Processing

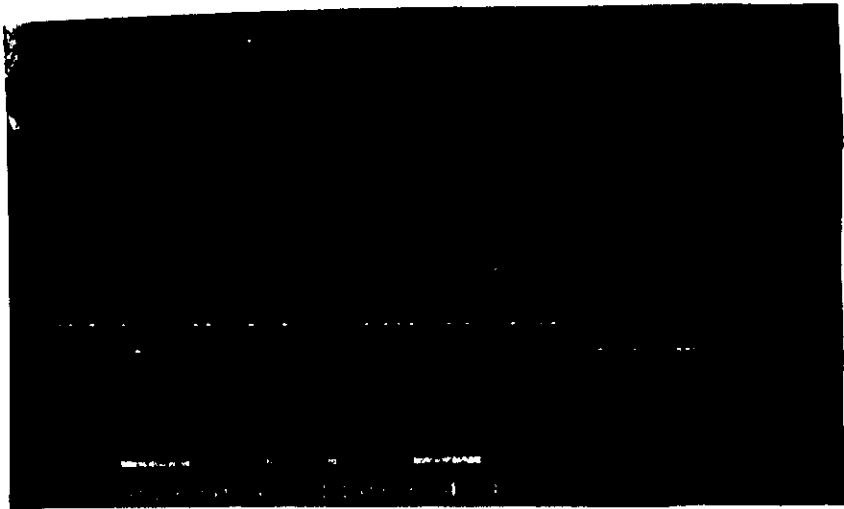


## resist profile for lift off

after deposition of metal



after lift-off

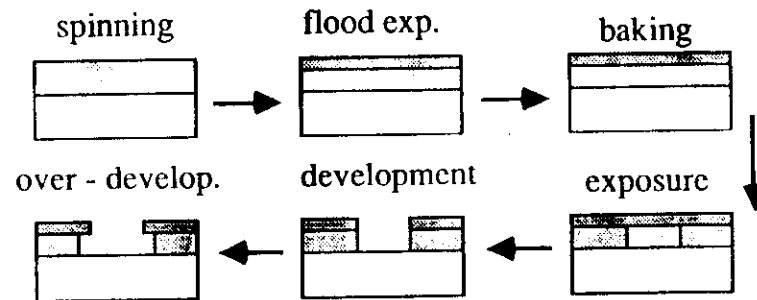


# 4 - Processing



## lithografie

advanced resists and lithography: NEPPOS



## Processing



### metallisations

Different materials required:

ohmic contacts (Au, Ge, Ni, Zn, Ti,  
Pt, Pd, Ge, ...)

Schottky contacts (Ti, W, ...)

interconnection (Al, Au, ...)

extra (Cu, Ag, In,...)

all must be compatible:

with each other

with used materials

## Processing



### etching

Different materials require:

- \* variety on etchants  
(selective and non-selective)
- \* perfect control on etched profile  
(crystallographic orientation,  
layerstructure)

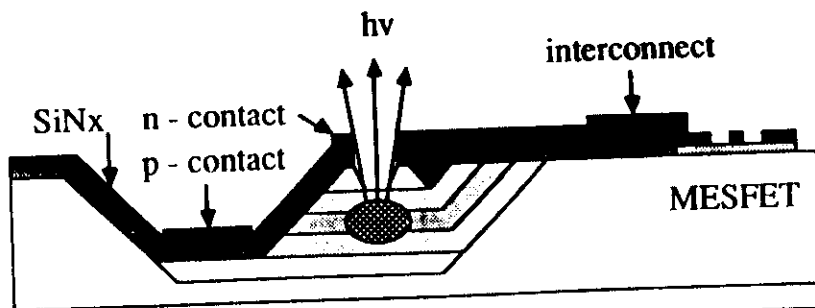
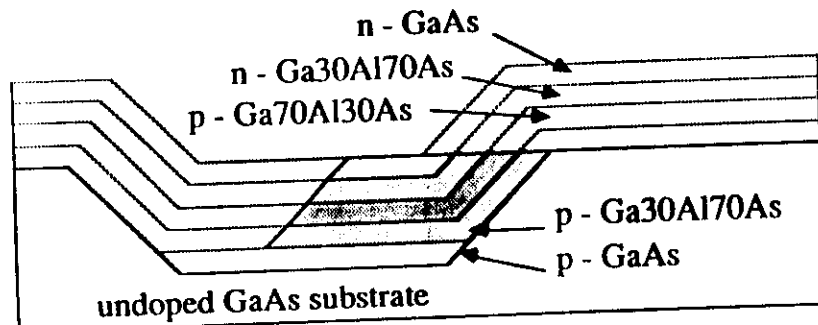


# Processing



## etching

influence of layerstructure  
on etched profile



# Processing



## etching

Dry etching processes

Several forms:

- Reactive Ion Etching (RIE)
- Reactive Ion Beam Etching (RIBE)
- Ion Beam Milling
- .....

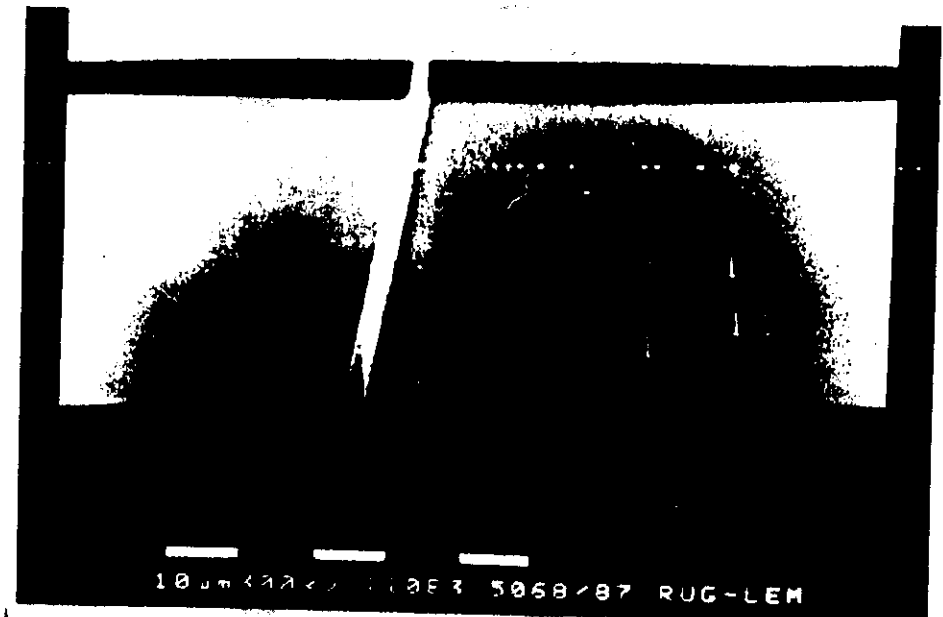
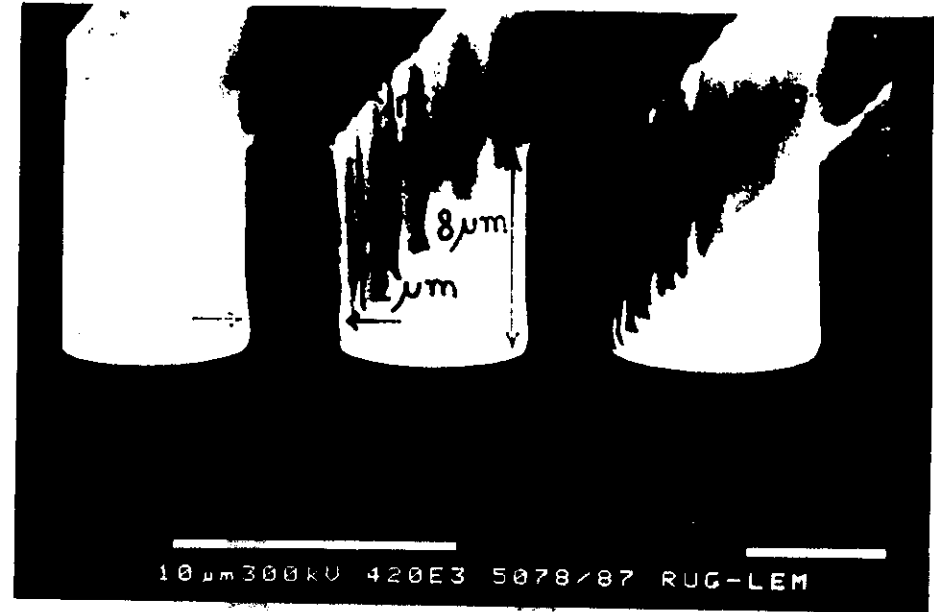
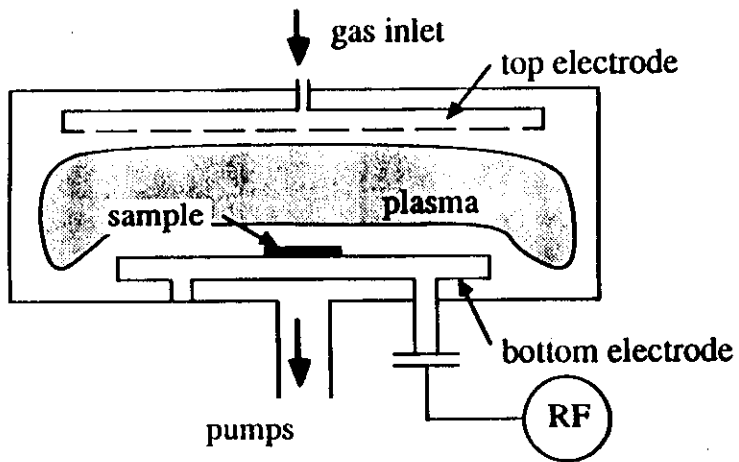
Ions hit surface with certain velocity  
Combination of impact and chemical effect

- + strongly anisotropic etching possible
- + independent etching of different materials
- damage effects
- low selectivity

# Processing



## Reactive Ion Etching



# 4 - Processing



Reactive Ion Etching of quantum dots



(Glasgow University)

## MATERIAL COMBINATIONS OEIC

E device(s)	Electronics	# reports	techniques	examples
AlGaAs	(Al)GaAs	+	monolithic	receivers, laser drivers
InP	InP	+	monolithic	" "
Si - detector	Si	++	monolithic	CCD
- modulator	Si	-	monolithic	
- emitter	Si	-	monolithic	
InP	GaAs	+/-	het. epit. / ELO	receivers, laser drivers
AlGaAs	Si	+/-	het. epit. / ELO	laser or modulator driver
InP	Si	-	het. epit. / ELO	" "
Si, NbO <sub>3</sub> , ...	GaAs	-	ELO	+ receiver switch driver
lasers	amorphous Si II-VI	++	CVD	active matrix LCD

MATERIAL COMBINATIONS  
PIC

Active OE device	Passive waveguide	# reports	techniques
InP	InP	+	monolithic
GaAs	GaAs	+	monolithic
GaAs or InP	polymeric waveguide	-	spin-on film
GaAs or InP	glass	-	hybrid / ELO

# Examples



receiver:

- \* general performances
- \* photoreceiver via flip-chip

source:

- \* Laser - waveguide
- \* GaAs on InP transmitter

systems:

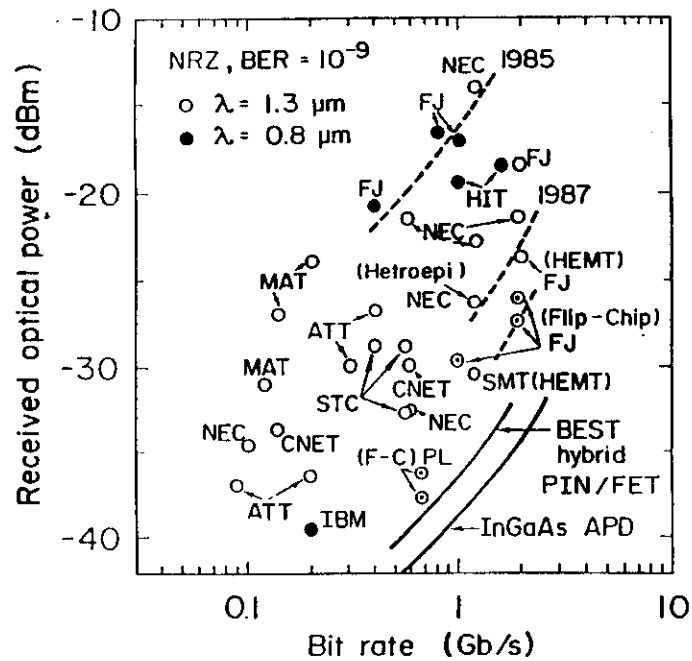
- \* IBM optoelectronic link
- \* optically coupled 3D memory

# 5 - Examples



receiver:

\* general performances



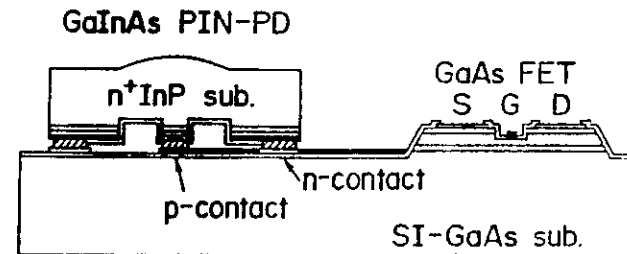
# 5 - Examples



receiver:

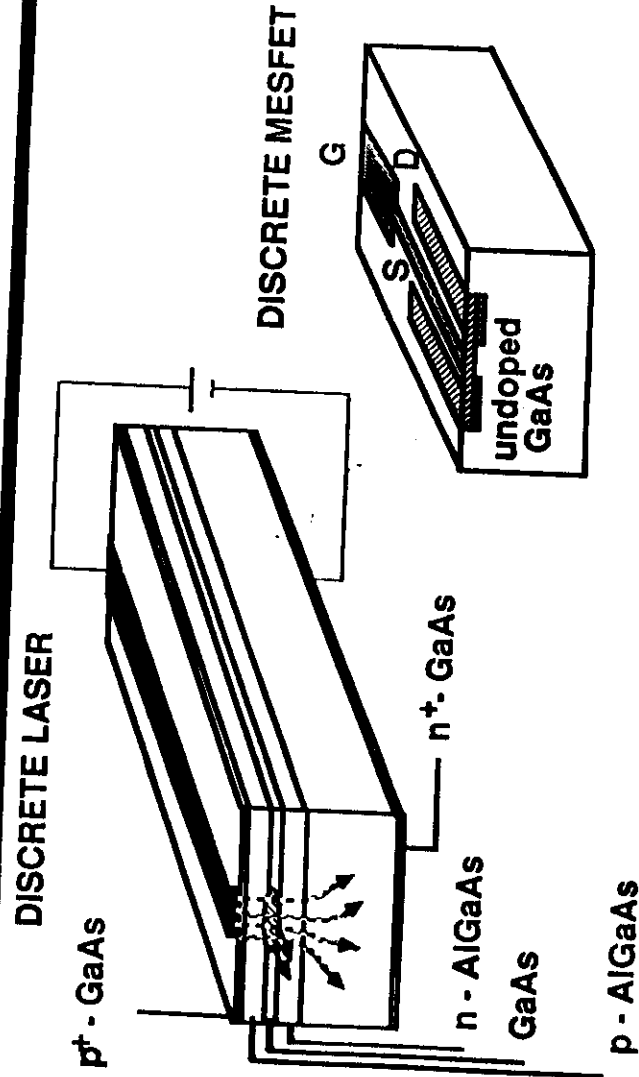
\* photoreceiver via flip-chip

GaInAs PIN/GaAs amplifier receiver



Makiuchi et al. (Fujitsu), Electr. Let. vol 24 No 16

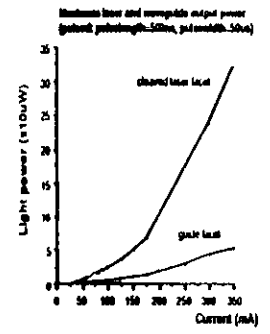
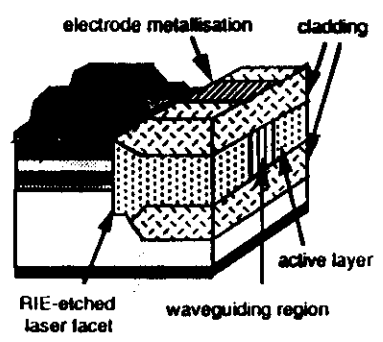
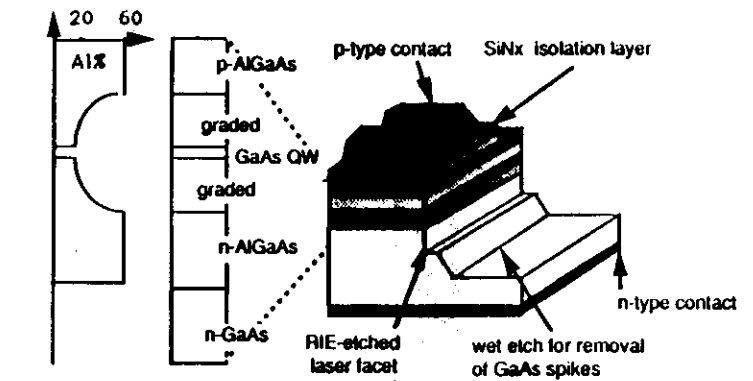
# OEIC EXAMPLE LASER + MESFET



# Examples



source:  
\* Laser (GaAs) - waveguide (polymers)

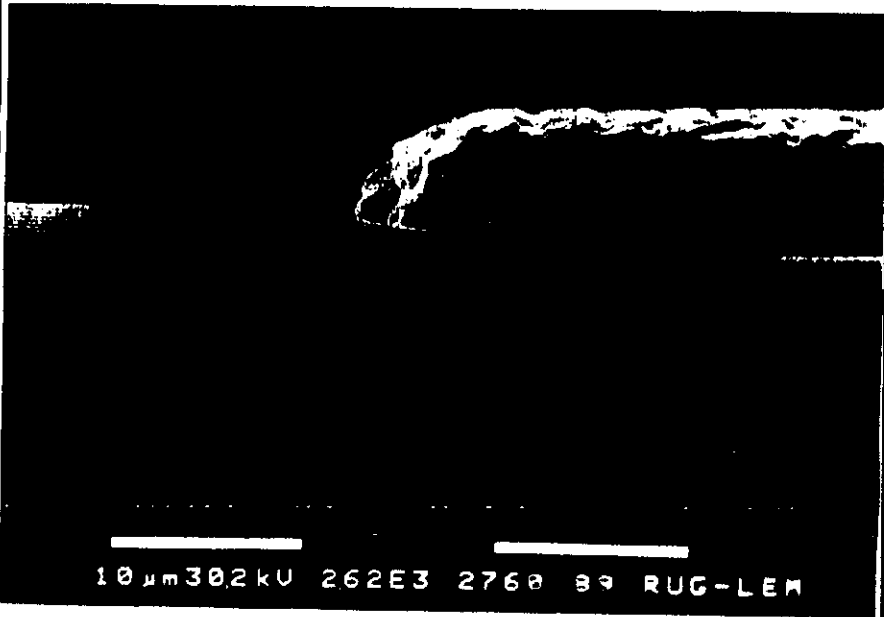


# 5 - Examples



source:

\* Laser - waveguide (III-V)

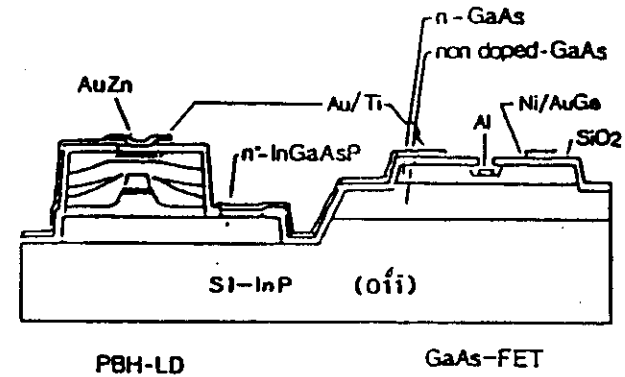


# 5 - Examples

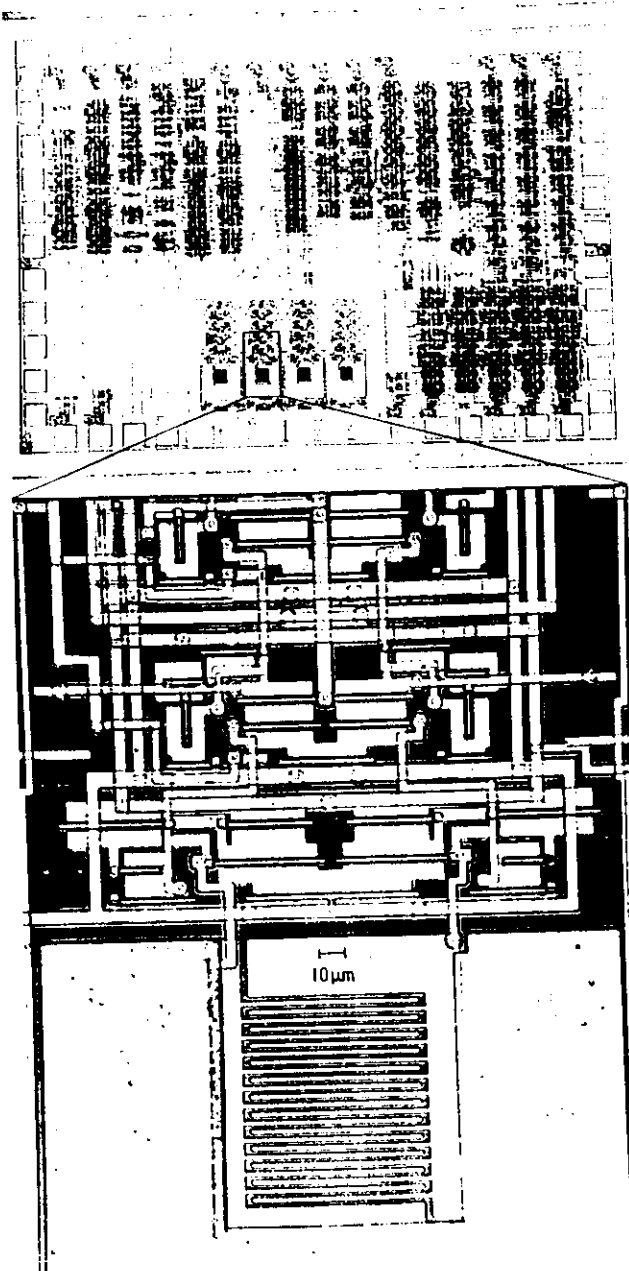


source:

\* GaAs on InP transmitter



Suzuki et al., Techn Dig. ECOC'88



## Conclusions



### INTEGRATION OF OPTOELECTRONIC COMPONENTS

**necessary for:**

- keeping high performances of individual components in complete systems
- reduce power consumption
- eliminate sensitivity to environment
- increased functionality

**slowed down by:**

- huge problems during fabrication

**but:**

- feasability is proven
- high expectations (performance, cost,..)



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MODELLING AND DESIGN  
OF  
INTEGRATED OPTIC CIRCUITS

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NG AND DESIGN OF  
TED OPTIC CIRCUITS

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**Outline**

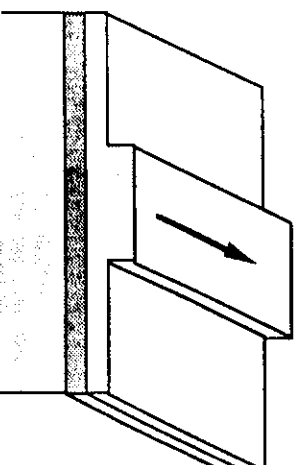
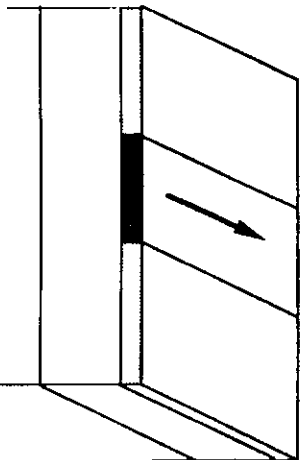
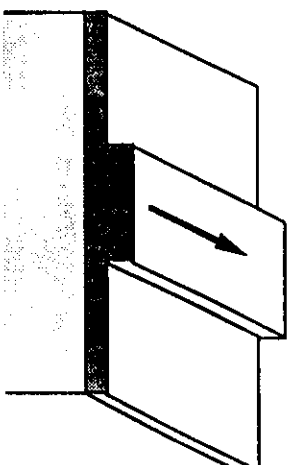
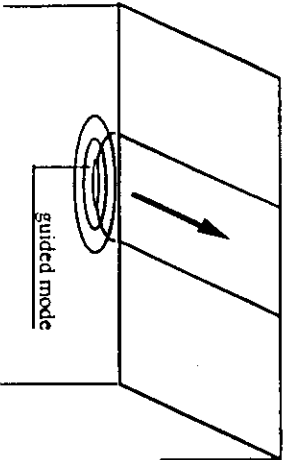
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- INTRODUCTION
  - 2D MODE SOLVERS
    - OPTICAL WAVEGUIDE MODES : DEFINITIONS
    - EFFECTIVE INDEX METHOD
    - SCALAR FINITE DIFFERENCE METHOD
    - VECTORIAL FINITE DIFFERENCE METHOD
  - BEAM PROPAGATION METHOD
  - TRANSFER MATRIX METHOD
  - COUPLED MODE THEORY
-

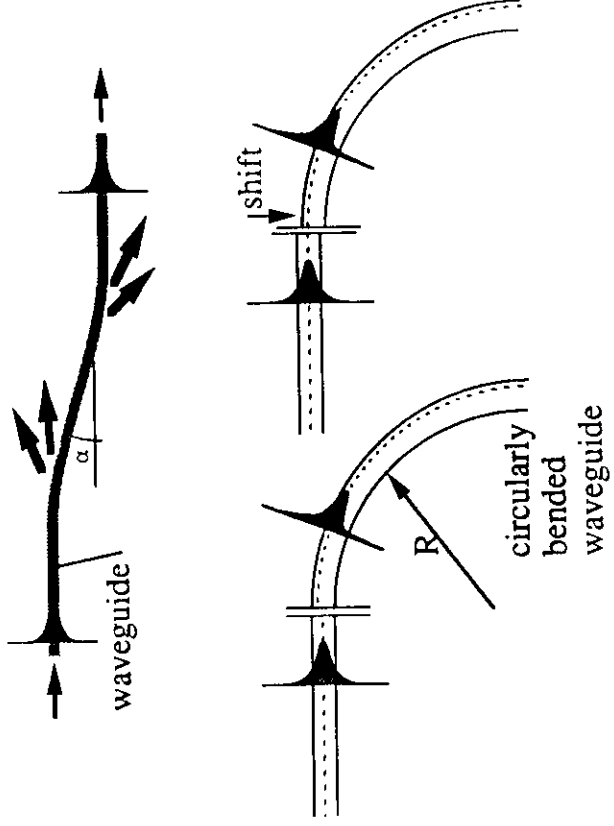
## Basic IOIC's Components :

- Passive
  - waveguides
  - bends
  - Y-junctions
  - crossings
  - directional couplers
  - structures with gratings
- Electrooptic
  - phase and intensity modulators
  - space switches

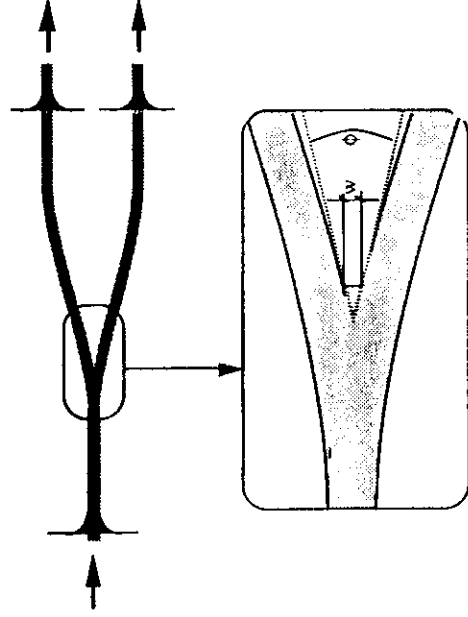
## Basic IOIC's Components : waveguides

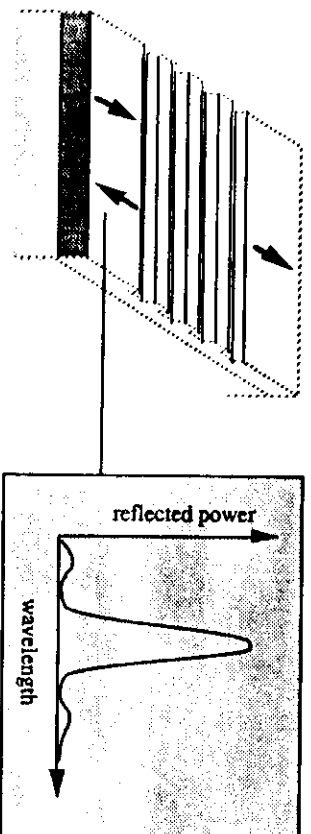
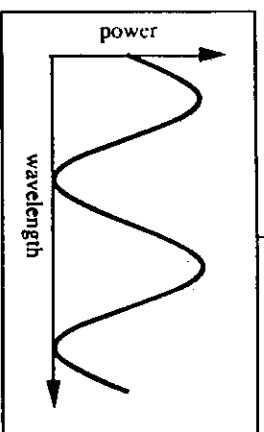
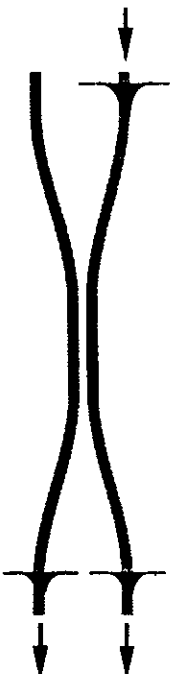


## Basic IOC's Components : bends

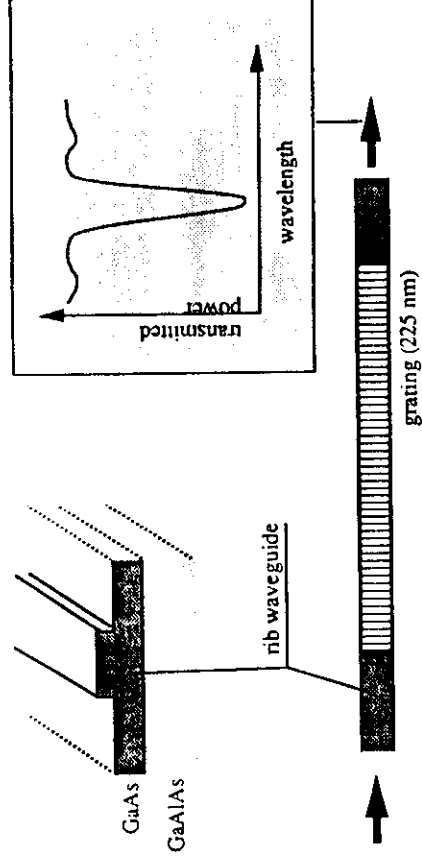


## Basic IOC's Components : Y-junctions

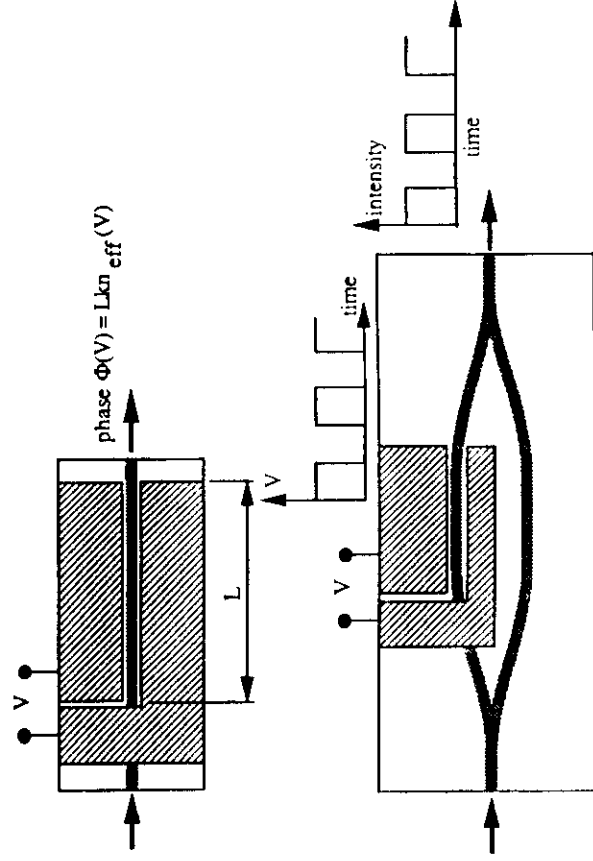




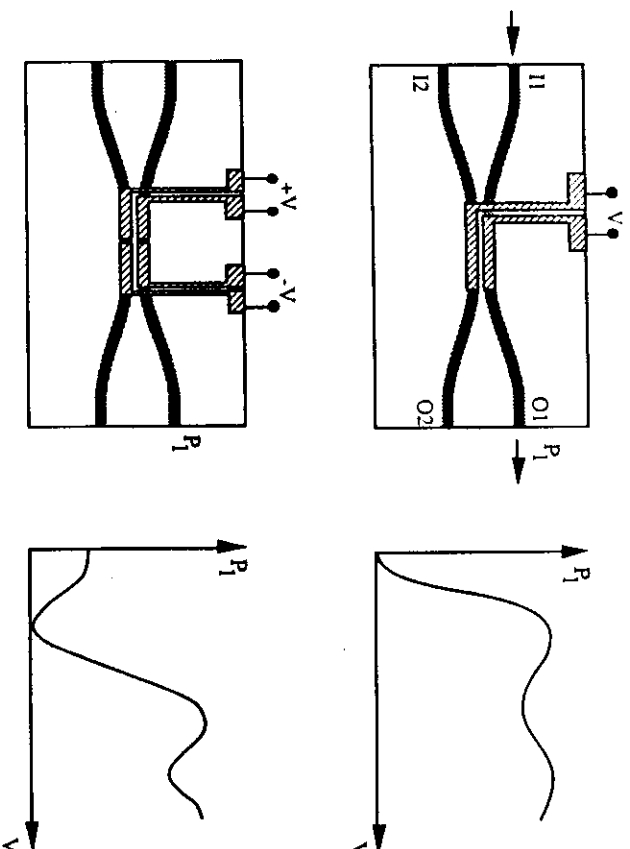
## Basic IOC's Components : Gratings (2)



## Basic IOC's Components Electrooptic phase and intensity modulators



## Basic IOC's Components Electrooptic space switches



MODELING AND DESIGN OF  
INTEGRATED OPTIC CIRCUITS  
2d mode solvers

## Optical Waveguide Modes : definitions

### Maxwell's equations

[time-dependence :  $\exp(j\omega t)$ ]

$$\vec{\nabla} \times \vec{E} = -j\omega\mu_0\vec{H}$$

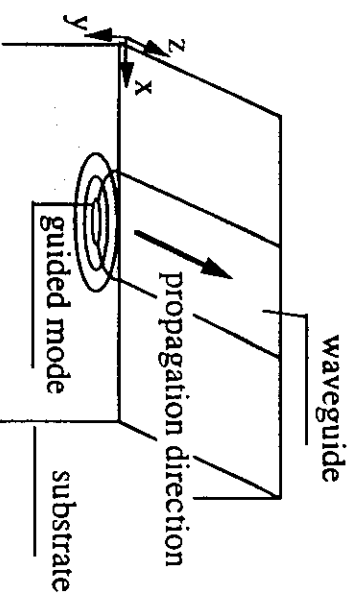
$$\vec{\nabla} \times \vec{H} = j\omega\epsilon_0\epsilon_r\vec{E}$$

$$\vec{\nabla} \cdot (\epsilon_r \vec{E}) = 0$$

$$\vec{\nabla} \cdot \vec{H} = 0$$

+ continuity conditions for  $\vec{E}$ ,  $\vec{H}$

+ radiation condition at  $\infty$



## Optical Waveguide Modes : definitions

$$\nabla^2 \vec{H} + \frac{\nabla \epsilon_r}{\epsilon_r} \times (\nabla \times \vec{H}) + k_0^2 \epsilon_r \vec{H} = 0$$

$$\nabla^2 \vec{E} + \nabla \left( \frac{\nabla \epsilon_r}{\epsilon_r} \cdot \vec{E} \right) + k_0^2 \epsilon_r \vec{E} = 0$$

$$k_0^2 = \omega^2 \epsilon_0 \mu_0$$

→ modal field representation

$$\vec{H}(x, y, z) = \vec{h}(x, y) \exp(-j\beta z) = (\vec{h}_t + \vec{h}_z) \exp(-j\beta z)$$

$$\vec{E}(x, y, z) = \vec{e}(x, y) \exp(-j\beta z) = (\vec{e}_t + \vec{e}_z) \exp(-j\beta z)$$

→  $\vec{e}_t, \vec{h}_t$  – transversal components

→  $\vec{e}_z, \vec{h}_z$  – longitudinal components

→  $\beta$  – real propagation constant

## Optical Waveguide Modes : definitions

Transversal components → complete vectorial field

either :

$$\vec{e}_t : \nabla_t^2 \vec{e}_t + (k_0^2 \epsilon_r - \beta^2) \vec{e}_t = -\nabla_t \left( \frac{\nabla_t \epsilon_r}{\epsilon_r} \cdot \vec{e}_t \right)$$

→  $\vec{e}_z, \vec{h}_z$  and  $\vec{h}_t$  can be computed from  $\vec{e}_t$

or :

$$\vec{h}_t : \nabla_t^2 \vec{h}_t + (k_0^2 \epsilon_r - \beta^2) \vec{h}_t = -\frac{\nabla_t \epsilon_r}{\epsilon_r} \times (\nabla_t \times \vec{h}_t)$$

→  $\vec{e}_z, \vec{h}_z$  and  $\vec{e}_t$  can be computed from  $\vec{h}_t$

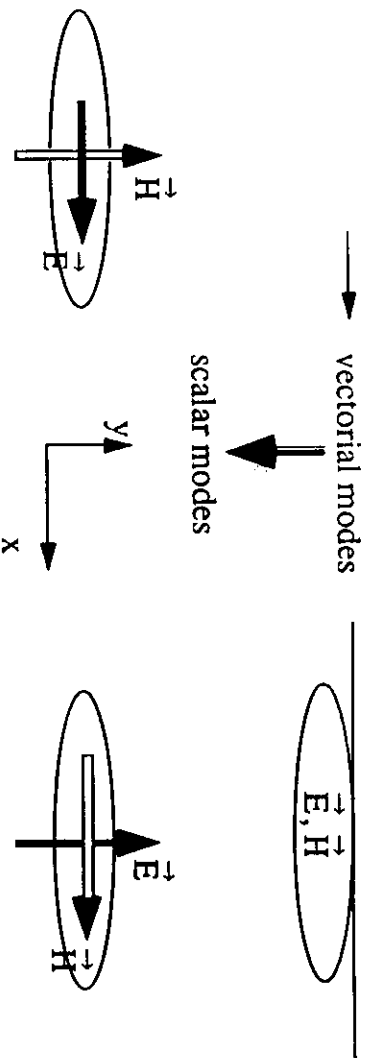
+ boundary conditions for  $\vec{E}, \vec{H}$

+ conditions at  $\infty$

## Optical Waveguide Modes : definitions

Simplification of vectorial modes  $\longrightarrow$  scalar approximation

- Typical planar waveguide :
  - width  $W >$  height  $H$
  - lateral index change small



quasi - TE  
 $(|\vec{E}_x|, |\vec{H}_y|) \gg (|\vec{E}_y|, |\vec{E}_z|, |\vec{H}_x|)$

quasi - TM  
 $(|\vec{H}_x|, |\vec{E}_y|) \gg (|\vec{H}_y|, |\vec{H}_z|, |\vec{E}_x|)$

## Optical Waveguide Modes : definitions

Example : equations for quasi - TE mode

$$\vec{E}_y = 0$$

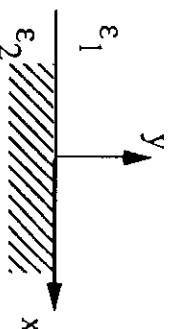
$$\rightarrow \nabla_t^2 e_x + (k_0^2 \epsilon_r - \beta^2) e_x = 0$$

- \*  $e_x = \phi$  (a scalar function)
- \*  $(e_z, h_x, h_y, h_z)$  can be computed
- \* small lateral index variations :

$$\rightarrow e_z \approx 0, h_x \approx 0, h_y \approx \frac{\beta}{\omega \mu_0} e_x$$

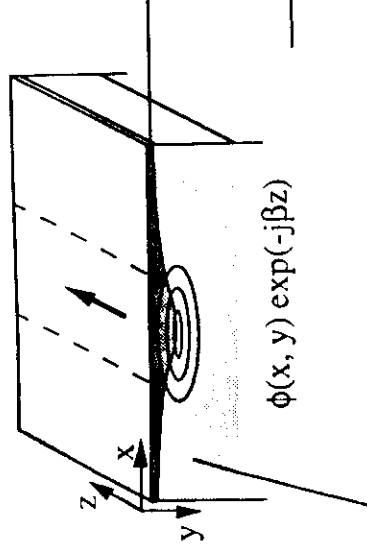
$\rightarrow$  + boundary conditions

TE :  $e_x$  and  $\frac{\partial e_x}{\partial y}$  continuous





## Effective Index Method : theory



$$\phi(x, y) \exp(-j\beta z)$$

modal field - scalar approximation

$$\nabla_T^2 \phi + [k^2 n^2(x, y) - \beta^2] \phi = 0$$

$$n(x, y) = n_0(y) + \Delta n(x, y)$$

$$\phi(x, y) = ? \quad \beta = ?$$

assumption

$$\phi(x, y) = F(x, y) \cdot G(x)$$

where fast variations of  $\phi$  along  $x$  are taken up in  $G(x)$

$$\rightarrow \frac{\partial F(x, y)}{\partial x} \approx 0$$

## Effective Index Method : Theory

$$\text{Eq(*)} \rightarrow \frac{1}{G} \frac{d^2 G}{dx^2} + \frac{1}{F} \frac{\partial^2 F}{\partial y^2} + (k^2 n^2 - \beta^2) = 0$$

function of  $x$       function of  $x, y$

$$\frac{1}{F} \frac{\partial^2 F}{\partial y^2} + n^2 k^2 = \gamma^2(x)$$

$$\frac{1}{G} \frac{d^2 G}{dx^2} - \beta^2 = -\gamma^2(x)$$

$$\gamma = kn_{\text{eff}}$$

$$\frac{\partial^2 F}{\partial y^2} + [k^2 n^2 - k^2 n_{\text{eff}}^2(x)] F = 0 \rightarrow F(x, y), n_{\text{eff}}(x)$$

1d wave equation (to be solved for each  $x$ )

$$\frac{d^2 G}{dx^2} + [k^2 n_{\text{eff}}^2(x) - \beta^2] G = 0 \rightarrow G(x), \beta$$

1d wave equation to be solved once

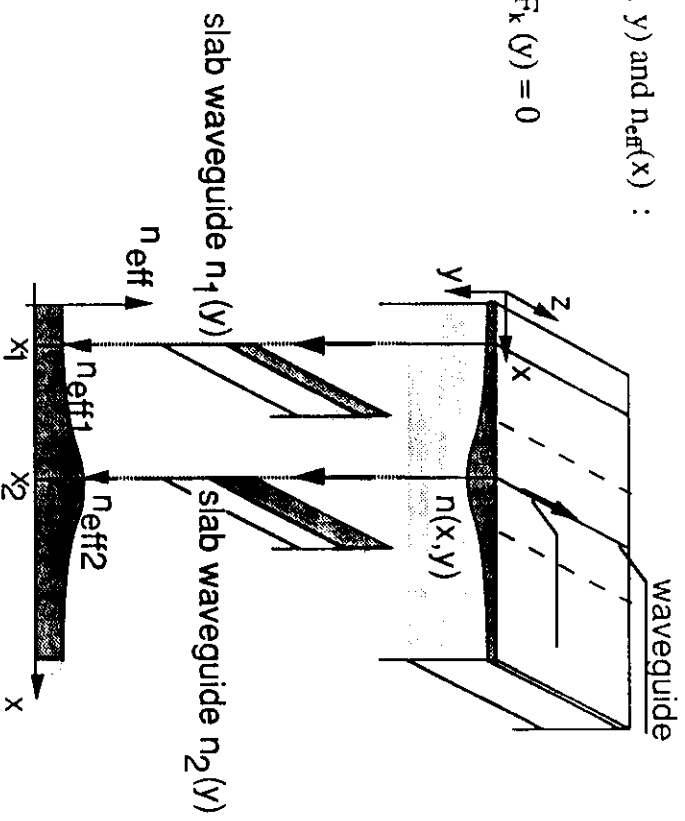
- This finally yields  $\phi(x, y) = F, G$ , and  $\beta$ .

## Effective Index Method : Theory

- discretized solution for  $F(x, y)$  and  $n_{eff}(x)$  :

$$\frac{d^2 F_k(y)}{dy^2} + k^2 (n_k^2(y) - n_{eff}^2(x)) F_k(y) = 0$$

$$\phi(x_k, y) = F_k(y) \cdot G(x_k)$$



## Effective Index Method : Example

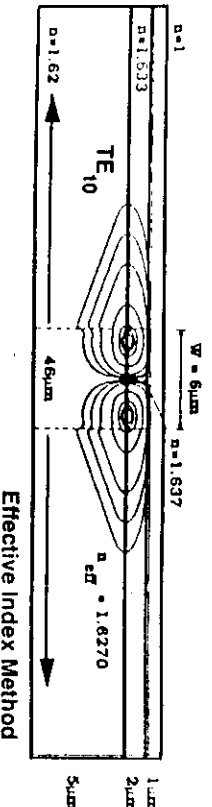
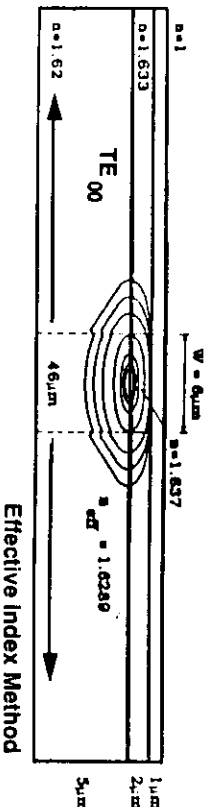
- channel waveguide in polymers

$$\lambda = 845 \text{ nm}$$

contour plots of  $|\phi(x, y)|$

(0.05, 0.1, 0.2, 0.5, 0.8, 0.90, 0.95)

→ note rapid transitions of  $|\phi(x, y)|$



Effective Index Method

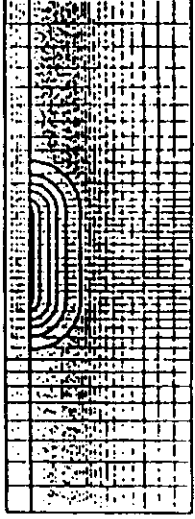
## Scalar Finite Difference Method

BASIC EQUATION :

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + (k_0^2 \epsilon_r - \beta^2) \phi = 0 \quad (1)$$

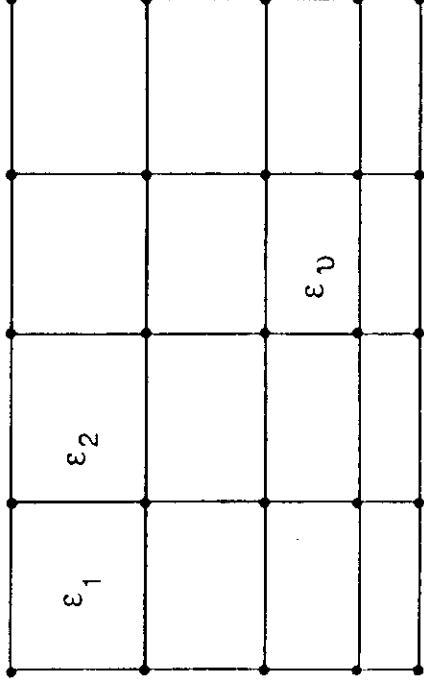
\* A finite cross section is defined by enclosing the guide in a rectangular box, where  $\phi = 0$  on the side walls

\* In this box a graded mesh is defined



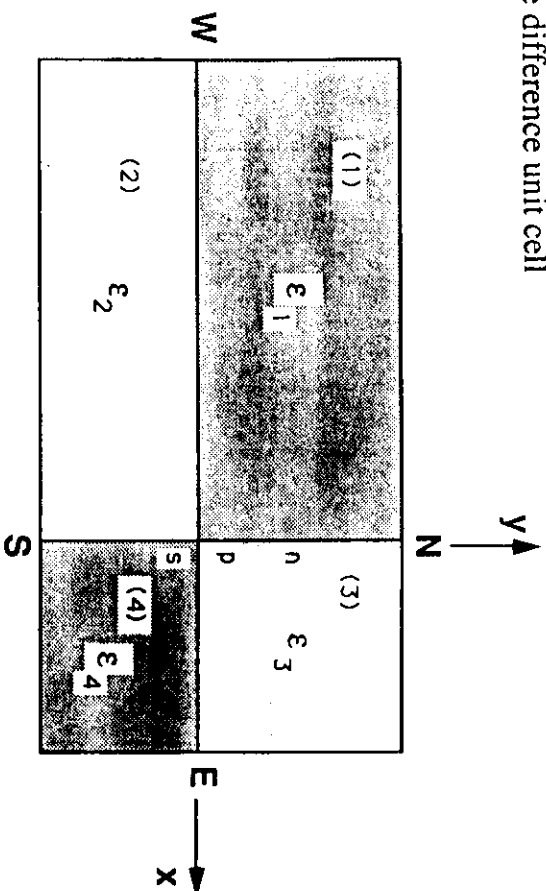
## Scalar Finite Difference Method

\* In each cell of the mesh the relative permittivity is assumed to be constant.  
Equation (1) applies in each subregion  $S_{ij}$ , with  $\epsilon_r = \epsilon_{ij}$ .



## Scalar Finite Difference Method

\* Finite difference unit cell



## Scalar Finite Difference Method

Continuity conditions :

$\left. \begin{matrix} \phi \\ \frac{\partial \phi}{\partial n} \end{matrix} \right\}$  continuous at boundary between S and N

Substitution of the discretized form of  $\nabla^2 \phi$  and the continuity conditions into (1) leads to :

$$\begin{aligned}
 & \frac{2}{w(e+w)} \phi_w + \frac{2}{s(n+s)} \phi_s - \left( \frac{2}{w(e+w)} + \frac{2}{e(e+w)} + \frac{2}{s(n+s)} \right) \phi_p \\
 & + \frac{2}{n(n+s)} \phi_n + \frac{2}{e(e+w)} \phi_e + \frac{2}{n(n+s)} \phi_N \\
 & + k_0^2 \frac{wne_1 + wse_2 + ese_3 + ene_4}{(e+w)(n+s)} \phi_p - \beta^2 \phi_p = 0
 \end{aligned} \quad (2)$$

## Scalar Finite Difference Method

Equation (2) holds for each node point P. The resultant eigenvalue equation is of the form

$$[[A] - \beta^2 [U]] [X] = 0$$

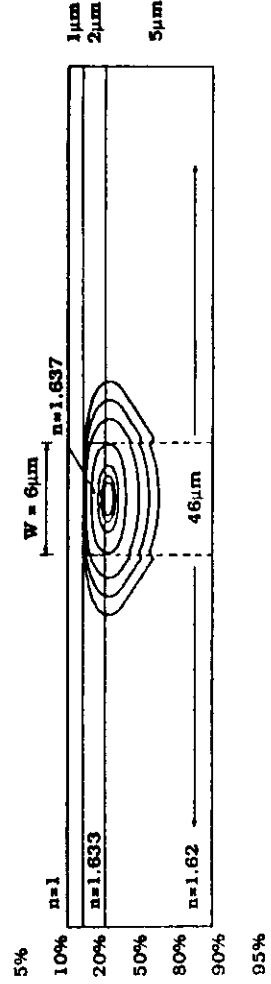
with

$$[X] = [\phi_1, \phi_2, \dots, \phi_{NTOT}]^T$$

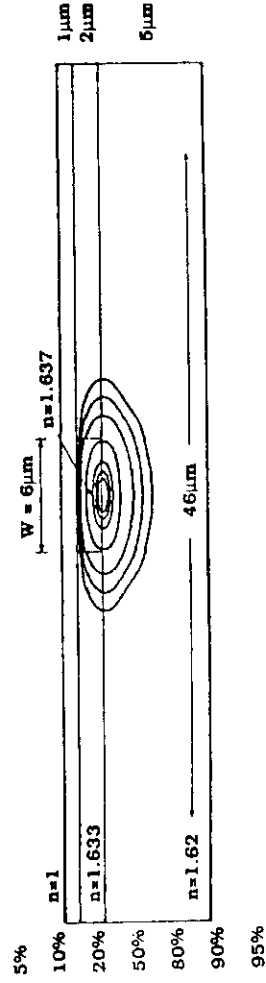
[U] is the unit matrix, and NTOT is the total number of mesh points. The matrix [A] is a real, but generally not symmetric, sparse matrix. Eigenvalues and corresponding eigenvectors of [A] are found by a simultaneous iteration algorithm.

## Scalar Finite Difference Method

### EFFECTIVE INDEX METHOD

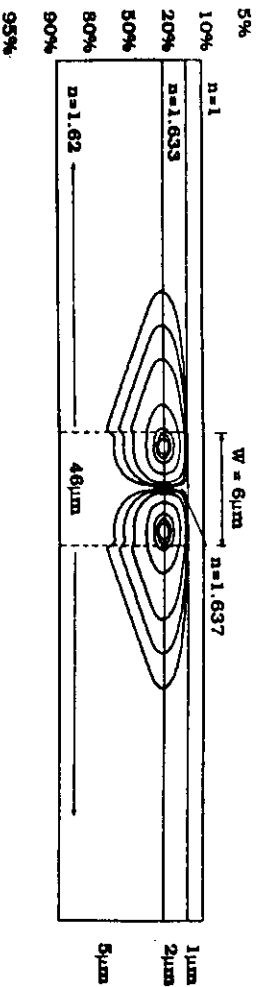


### FINITE DIFFERENCE METHOD

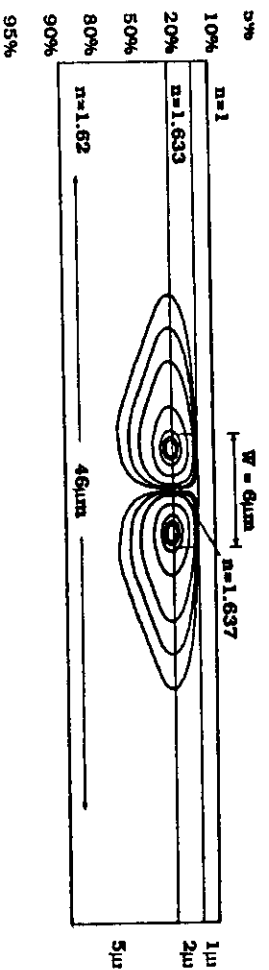


## Scalar Finite Difference Method

### EFFECTIVE INDEX METHOD



### FINITE DIFFERENCE METHOD

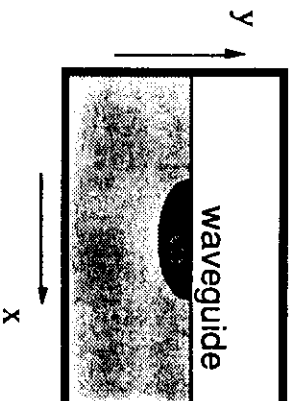


## Vectorial Finite Difference Method : Theory

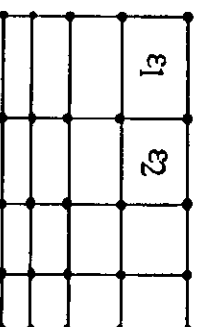
Basic equation :

$$(*) \quad \nabla_t^2 \vec{h}_t + (k_0^2 \epsilon_r - \beta^2) \vec{h}_t = -\frac{\nabla_t \epsilon_r}{\epsilon_r} \times (\nabla_t \times \vec{h}_t)$$

- finite cross section



- graded mesh
- each cell constant permittivity



## Vectorial Finite Difference Method : Theory

- 4 elementary cells :

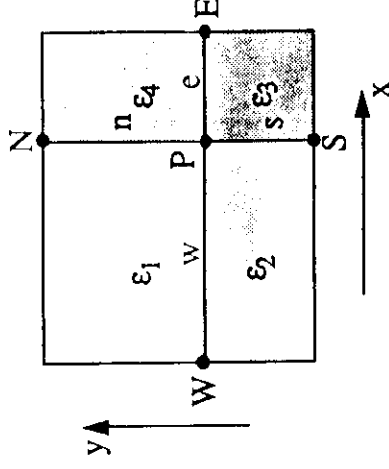
$$\text{eq}(\ast) \longrightarrow \left\{ \begin{array}{l} \nabla_x^2 h_x^{(v)} + k_0^2 h_x^{(v)} = 0 \\ \nabla_y^2 h_y^{(v)} + k_0^2 h_y^{(v)} = 0 \\ k_0^2 = k_0^2 \epsilon_0 - \beta^2 \end{array} \right.$$

- finite difference representation  
+ implementation of  
boundary conditions give :

$$h_{xP} = f_1 (h_{x(W, N, E, S)}, h_{y(W, N, E, S)})$$

$$-h_{yP} = f_2 (h_{x(W, N, E, S)}, h_{y(W, N, E, S)})$$

$$\text{(scalar case : } \Phi_P = f(\Phi_{W, N, E, S}) \text{)}$$



## Vectorial Finite Difference Method : Theory - eigenvalue equation

- eigenvalue equation for NTOT node points

$$\{[A] - \lambda[U]\} [X] = 0$$

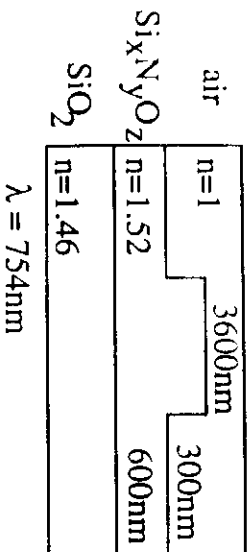
$$\lambda = \beta^2$$

$$[X] = [h_{x1}, h_{y1}, h_{x2}, h_{y2}, \dots, h_{xNTOT}, h_{yNTOT}]^T$$

- solution : a standard numerical procedure

## Vectorial Finite Difference Method : Example : rib waveguide 1

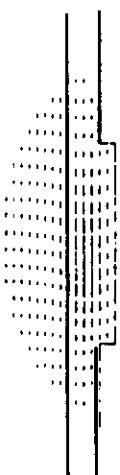
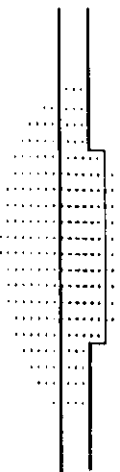
Waveguide geometry



Scalar FDM :



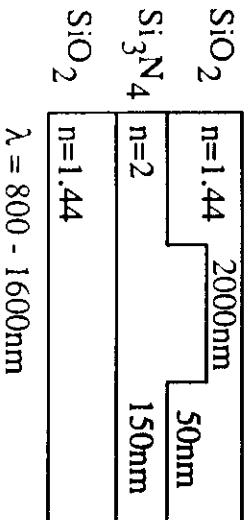
Vectorial FDM :



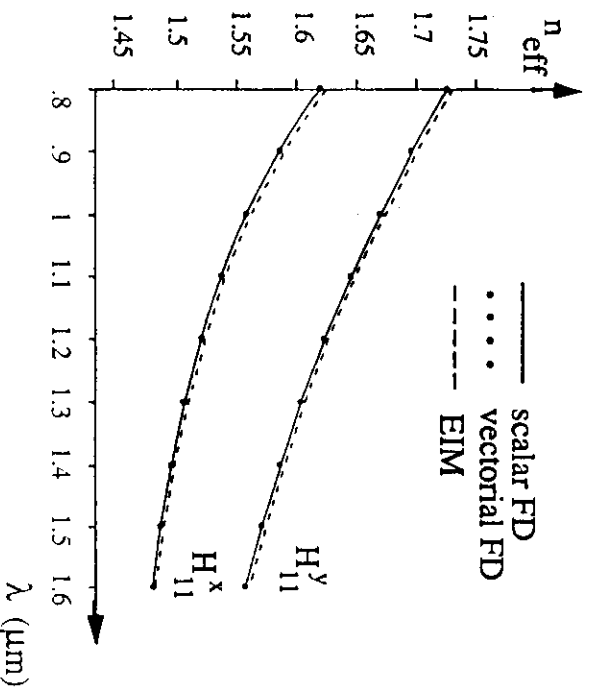
MODELING AND DESIGN OF INTEGRATED OPTIC CIRCUITS  
3d mode solvers

## Vectorial Finite Difference Method : Example : rib waveguide 2

waveguide geometry



without material dispersion

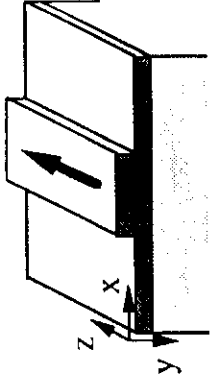


Conclusion : EIM overestimates  $n_{eff}$

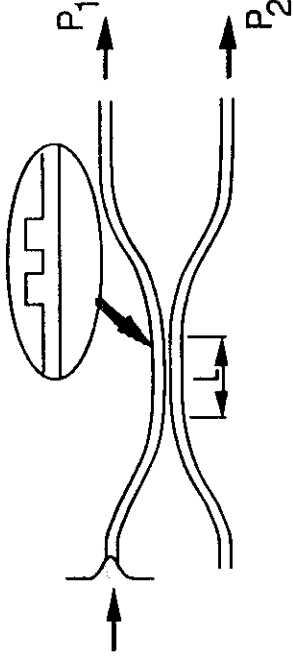


## Beam Propagation Method : Introduction

- Longitudinally invariant structure

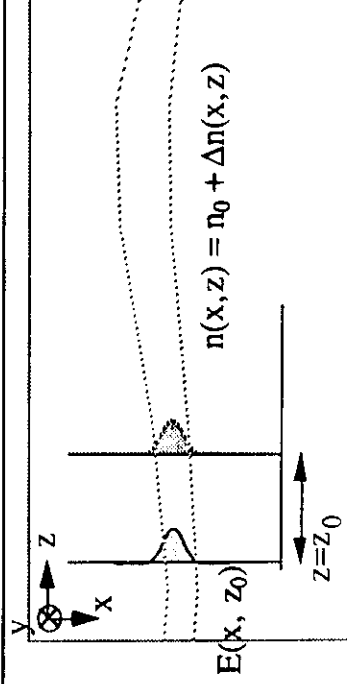


- Longitudinally variant structure



- Coupling to radiation modes
- *Beam Propagation Method (BPM)*

## Beam Propagation Method : Formulation of Problem



- Problem :

Given  $E(x, z) |_{z=z_0} \longrightarrow$  find  $E(x, z) |_{z=z_0+\Delta z}$

- BPM operator :

$$E(x, z_0+\Delta z) = P_{\text{BPM}} \langle E(x, z_0) \rangle$$

$$P_{\text{BPM}} = ?$$

(can be used in subsequent steps along z)

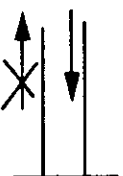
## Beam Propagation Method : Restrictions

- Simple solution possible if

- propagation field is scalar

- $|\frac{\Delta n}{n_0}| \ll 1$

- no backward reflections



- angular spectrum limited

