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MODE-LOCKED LASERS

O. Svelto
Centro di Elett. Quantistica e Strumentazione Elett.
Istituto di Fisica del Politecnico
Milano, Italy

5 and 20 times the cavity photon decay time τ_c.* The time duration of the pumping pulse must therefore be approximately equal to this buildup time. With the conditions considered above, the peak inversion may range between 4 and 10 times the threshold value so that a laser pulse of high peak power and short duration can be produced.

The most common example of a gain-switched laser is the electrically pulsed TEA (transversely excited at atmospheric pressure; see Section 6.3.3.1) CO_2 laser. Taking a typical cavity length of $L = 1$ m, a 20% transmission of the output mirror, and assuming that the internal losses arise only from this mirror's transmission, we get $\gamma \cong 0.1$ and $\tau_c = L/c\gamma \cong 30$ ns. Assuming that the laser buildup occurs in a time ten times longer, we see that the duration of the pumping pulse should last ~300 ns, in agreement with experimental results. Note finally that any laser can in principle be gain-switched given a sufficiently fast and intense pump pulse, as for instance obtained by pumping with another laser. As examples we mention the case of dye lasers pumped by the fast (~0.5 ns) pulse of an atmospheric pressure N_2 laser or the case of a semiconductor diode laser pumped by a very short (~0.5 ns) current pulse.

5.4.5. *Mode Locking*[26,27]

The technique of mode locking allows the generation of laser pulses of ultrashort duration (from a few tens of femtoseconds to a few tens of picoseconds). Mode locking refers to the situation where the cavity modes are made to oscillate with comparable amplitudes and with locked phases.

As a first example we will consider the case of $2n + 1$ longitudinal modes oscillating with the same amplitude E_0 (Fig. 5.37a). We will assume the phases ϕ_l of the modes in the output beam to be locked according to the relation

$$\phi_l - \phi_{l-1} = \phi \tag{5.106}$$

where ϕ is a constant. The total electric field $E(t)$ of the e.m. wave, at a given point in the output beam, can be written, apart from a constant value for the total phase, as

$$E(t) = \sum_{-n}^{n}{}_l E_0 \exp\{i[(\omega_0 - l\Delta\omega)t + l\phi]\} \tag{5.107}$$

where ω_0 is the frequency of the central mode and $\Delta\omega$ is the frequency difference between two consecutive modes. For simplicity, we have considered the field

* It should be noted that the build-up time in Fig. 5.24 corresponds to a much longer time than these quoted values because Fig. 5.24 refers to the case of a three-level laser and also to a situation where pumping exceeds the threshold value by only a modest amount.

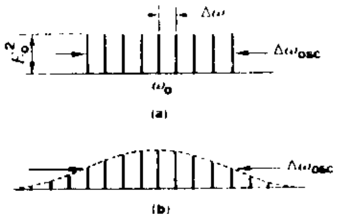


FIG. 5.37. Mode amplitude (represented by vertical lines) versus frequency for a mode-locked laser. (a) Uniform amplitude. (b) Gaussian amplitude distribution over a bandwidth (FWHM) $\Delta\omega_{osc}$.

at that point where the phase of the center mode is zero. According to (5.107) the total electric field of the wave, $E(t)$, can be written as

$$E(t) = A(t)\exp(i\omega_0 t) \tag{5.108}$$

where

$$A(t) = \sum_{-n}^{n}{}_l E_0 \exp[il(\Delta\omega t + \phi)] \tag{5.108a}$$

Equation (5.108) shows that $E(t)$ can be represented in terms of a sinusoidal carrier wave at the center-mode frequency ω_0 whose amplitude $A(t)$ is time modulated. If we now change to a new time reference t' such that $\Delta\omega t' = \Delta\omega t + \phi$, (5.108a) transforms to

$$A(t') = \sum_{-n}^{n}{}_l E_0 \exp(il\Delta\omega t') \tag{5.109}$$

and the sum appearing in the right-hand side can easily be recognized as a geometric progression with a ratio equal to $\exp(i\Delta\omega t')$. $A(t')$ is then readily calculated to give

$$A(t') = E_0 \frac{\sin[(2n+1)\Delta\omega t'/2]}{\sin[\Delta\omega t'/2]} \tag{5.110}$$

To understand the physical significance of this expression, we have plotted in Fig. 5.38 the quantity $A^2(t')$, which is proportional to the beam intensity,

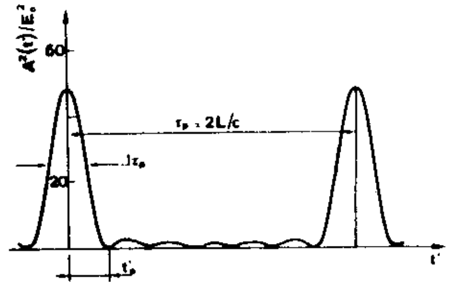


FIG. 5.38. Time behavior of the squared amplitude of the electric field for the case of seven oscillating modes with locked phases and equal amplitude.

versus time t' for $2n + 1 = 7$ oscillating modes. We see that, as a result of the phase-locking condition (5.106), the oscillating modes interfere to produce a train of evenly spaced light pulses. The pulse maxima occur at those times for which the denominator of (5.110) vanishes. In the new time reference t' a maximum thus occur for $t' = 0$. For $t' = 0$ the numerator of (5.110) also vanishes and $A^2(0)$ is seen to be equal to $(2n+1)^2E_0^2$. The next pulse will occur when the denominator of (5.110) vanishes again. This occurs at a time t' such that $(\Delta\omega t'/2) = \pi$. Two successive pulses are therefore separated by a time

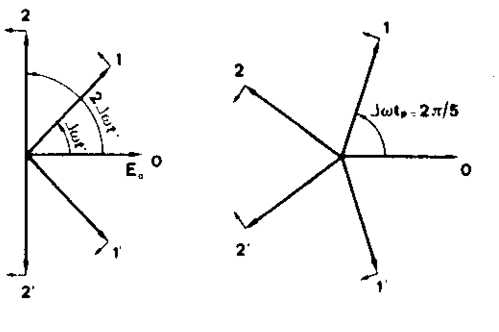
$$\tau_p = 2\pi/\Delta\omega \tag{5.111}$$

For $t' > 0$, the first zero for $A^2(t')$ in Fig. 5.38 occurs when the numerator of (5.110) again vanishes. This occurs at a time t'_p such that $[(2n+1)\Delta\omega t'_p/2] = \pi$. Since the width $\Delta\tau_p$ (FWHM) of $A^2(t)$ (i.e., of each laser pulse) is approximately equal to t'_p we thus have

$$\Delta\tau_p \cong 2\pi/(2n+1)\Delta\omega = 1/\Delta\nu_{osc} \tag{5.112}$$

where $\Delta\nu_{osc} = (2n+1)\Delta\omega_{osc}/2\pi$ is the total oscillating bandwidth (see Fig. 5.37a).

The mode-locking behavior of Fig. 5.38 can be readily understood if we consider the various modes to be represented by vectors in the complex plane. The lth mode would thus correspond to a complex vector of amplitude E_0 and rotating at the angular velocity $(\omega_0 + l\Delta\omega)$. If we now refer to axes rotating at angular velocity ω_0, the central mode will appear fixed relative to these axes and the lth mode rotating at velocity $l\Delta\omega$. At time $t' = 0$, according to (5.109), all vectors will have zero phase and thus lie in the same direction which we will assume to be the horizontal direction in Fig. 5.39. The total field will in this case be $(2n+1)E_0$. For $t' > 0$, while the vector corresponding to the central mode remains fixed, the vectors of the modes with $\omega > \omega_0$ will



(a) (b)

FIG. 5.39. Representation of cavity modes in the complex plane (five modes). Figure (b) depicts the time instant in which the sum of the five modes is zero.

rotate in one direction (e.g., counterclockwise) while the vectors of the modes with $\omega < \omega_0$ will rotate in the opposite sense (clockwise). Therefore, for the case, e.g., of five modes, the situation at some later time t', will be as indicated in Fig. 5.39a. If now the time t' is such that mode 1 has made a 2π rotation (which occurs when $\Delta\omega t' = 2\pi$), mode 1' will also have rotated (clockwise) by 2π, while modes 2 and 2' will have rotated by 4π. All these vectors will therefore be aligned again with that at frequency ω_0, and the total electric field will again be $(2n+1)E_0$. Thus the time interval τ_p between two consecutive pulses must be such that $\Delta\omega\tau_p = 2\pi$, as indeed shown by (5.111). Note that, in this picture, the time instant t'_p at which $A(t)$ first vanishes (see Fig. 5.38) corresponds to the situation where all of the vectors are equally spaced in angle (Fig. 5.39b). To achieve this condition, mode 1 must have made a rotation of only $2\pi/5$, or more generally, for $2n+1$ modes, of $2\pi/(2n+1)$. The time t'_p and hence the pulse duration $\Delta\tau_p$ thus turn out to be given by (5.112).

Before proceeding it is worthwhile to summarize and comment on the main results that have been obtained so far. We have found that the mode-locking condition given by (5.106) gives an output beam that consists of a train of mode-locked pulses, the duration of each pulse $\Delta\tau_p$ being about equal to the inverse of the oscillating bandwidth $\Delta\nu_{osc}$. This result can be readily understood by noting that the time behavior of the pulse is just the Fourier transform of its frequency spectrum. Now, since $\Delta\nu_{osc}$ can be of the order of the width of the gain line $\Delta\nu_0$, very short pulses (down to a few picoseconds) can be expected to result from mode-locking of solid-state or semiconductor lasers. For dye lasers the gain linewidth is about 10^2 times larger than that of a solid-state laser material, and very much shorter pulsewidths are possible and have indeed been obtained (down to about 30 fs). In the case of gas lasers, on the other hand, the gain linewidth is much narrower (up to a few gigahertz) and relatively long pulses are generated (down to ~100 ps). We now recall that two consecutive pulses are separated by a time τ_p given by (5.111). Since $\Delta\omega = 2\pi\Delta\nu = \pi c/L$, where L is the cavity length, τ_p turns out to be equal to $2L/c$, which is just the cavity round trip time. The oscillating behavior inside the laser cavity can therefore be visualized as being due to an ultrashort pulse of duration $\Delta\tau_p$ given by (5.112) which propagates back and forth in the cavity. In such a case, in fact, the output beam from one mirror is obviously a pulse train with a time separation between two consecutive pulses equal to the cavity round trip time. Some typical numbers bear out this picture since the spatial extent Δz of a pulse of duration $\Delta\tau_p = 1$ ps, say, is $\Delta z = c_0\Delta\tau = 0.3$ mm, i.e., much shorter than a typical length of the laser cavity.

Before proceeding it is perhaps worth pointing out what happens in the case of random phases. Figure 5.40 shows the time behavior of $|A(t)|^2$ for the case of seven oscillating modes evenly spaced in frequency by $\Delta\omega$, with the same amplitude E_0 and with random values for their phases. We see that the output beam, unlike the mode-locked case considered above, now consists of an irregular sequence of light pulses. As expected from the general properties

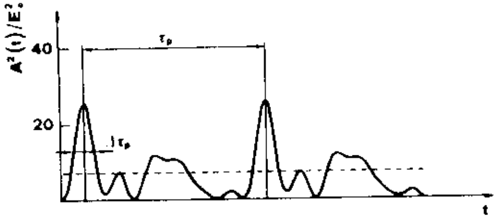


FIG. 5.40. Time behavior of the squared amplitude of the electric field for the case of seven oscillating modes with equal amplitude and phases chosen at random ($\phi_1 = 2.4789$, $\phi_2 = 2.3316$, $\phi_3 = 5.5959$, $\phi_4 = 4.3687$, $\phi_5 = 0.6872$, $\phi_6 = 0.7608$, $\phi_7 = 1.5217$, radians).

of Fourier series, however, each light pulse still has a duration $\Delta\tau_p$ roughly equal to $1/\Delta\nu_{osc}$, where $\Delta\nu_{osc}$ is the total oscillating bandwidth, the average time between pulses is roughly equal to $\Delta\tau_p$, and the pulse sequence repeats itself after a period $\tau_p = 2\pi/\Delta\omega$. Note that, since the response time of a conventional electronic detector is usually much larger than $\Delta\tau_p$, one does not resolve this complex time behavior in the output of a multimode non-mode-locked laser, but rather its average value is monitored. This value is simply the sum of powers in the modes and hence is proportional to $(2n+1)E_0^2$. Since, in the case of mode-locking, the peak power is proportional to $(2n+1)^2E_0^2$, we see that mode-locking is useful for producing pulses not only of very short duration but also of high peak power. In fact, according to the discussion above, the ratio between the peak pulse power in the mode-locked case and the average power in the non-mode-locked case is equal to the number, $(2n+1)$, of oscillating modes, which, for solid-state or liquid lasers, may be quite high (10^3–10^4).

So far we have restricted our considerations to the rather unrealistic case of an equal-amplitude mode-spectrum (Fig. 5.37a). In general the mode spectrum is expected to have a bell-shaped form. To see what happens in this case we will assume the mode spectrum to be given by a Gaussian distribution (Fig. 5.37b). We can therefore write down the amplitude E_l of the lth mode as

$$E_l^2 = E_0^2 \exp\left[-\left(\frac{2l\Delta\omega}{\Delta\omega_{osc}}\right)^2 \ln 2\right] \tag{5.113}$$

where $\Delta\omega_{osc}$ is the spectral bandwidth (FWHM). If we again assume that the phases are locked according to (5.106) and that the phase of the central mode is equal to zero we again find that $E(t)$ can be expressed as in (5.108) where the amplitude $A(t)$ in the time reference t' is given by

$$A(t') = \sum_{-\infty}^{+\infty}{}_l\, E_l \exp i(l\Delta\omega t') \tag{5.114}$$

If the sum is approximated by an integral [i.e., $A(t) = \int E_l \exp i(l\Delta\omega t)\, dl$], the field amplitude $A(t)$ is seen to be proportional to the Fourier transform of the spectral amplitude E_l. We then get

$$A^2(t) \propto \exp\left[-\left(\frac{2t}{\Delta\tau_p}\right)^2 \ln 2\right] \tag{5.115}$$

where the pulsewidth $\Delta\tau_p$ (FWHM) is

$$\Delta\tau_p = 2\ln 2/\pi\Delta\nu_{osc} = 0.441/\Delta\nu_{osc} \tag{5.116}$$

As a conclusion to the two examples given above, we can say that, when the mode-locking condition (5.106) holds, the field amplitude is proportional to the Fourier transform of the magnitude of the spectral amplitude. The pulsewidth $\Delta\tau_p$ is related to the width of the spectral intensity $\Delta\nu_{osc}$ by the relation $\Delta\tau_p = k/\Delta\nu_{osc}$, where k is a numerical factor (of the order of unity), which depends on the particular shape of the spectral intensity distribution. A pulse of this sort is said to be *transform-limited.*

Under locking conditions different from (5.106) the output pulse may be far from being transform-limited. If for instance we take

$$\phi_l = l\phi_1 + l^2\phi_2 \tag{5.117}$$

[note that (5.106) can be written as $\phi_l = l\phi$] and again assume a Gaussian amplitude distribution such as in (5.113), the Fourier transform of the spectrum can again be analytically calculated and $E(t)$ can, in this case, be written as

$$E(t) \propto \exp[-\alpha t^2]\exp[i(\omega_0 t + \beta t^2)] \tag{5.118}$$

We see that the beam intensity, which is proportional to $|E(t)|^2$, is still described by a Gaussian function whose pulsewidth $\Delta\tau_p$, in terms of the parameter α appearing in (5.118), is equal to

$$\Delta\tau_p = (2\ln 2/\alpha)^{1/2} \tag{5.118a}$$

Note, however, that, owing to the presence of the phase term $l^2\phi_2$ in (5.117), which is quadratic in the mode index l, $E(t)$ has now a phase term, βt^2, which is quadratic in time. This means that the carrier frequency $\omega_0 + 2\beta t$ of the wave now has a linear frequency sweep. The value of β and hence the magnitude of this sweep depends upon the value of ϕ_2 in (5.117), and its specific expression is not given here since it will not be needed in what follows. What is important to notice, however, is that a frequency chirped pulse of the form given in (5.118) can indeed be obtained under the particular mode-locking conditions given by (5.117). It can now be readily shown that a pulse of the type (5.118) is not transform-limited. To show this we can easily calculate its spectral bandwidth by taking the Fourier transform of (5.118). The oscillating bandwidth is, in this case, found to be

$$\Delta\nu_{\text{osc}} = \frac{0.441}{\Delta\tau_p}\left[1 + \frac{(\beta\Delta\tau_p^2)^2}{2\ln 2}\right]^{1/2} \tag{5.119}$$

where (5.118a) has also been used. We see from (5.119) that for $\beta\Delta\tau_p^2 \gg 1$, i.e., for sufficiently large values of the frequency chirp, the product $\Delta\tau_p\Delta\nu_{\text{osc}}$ becomes much larger that 1. The physical reason for this can be understood if we notice that the spectral broadening now arises both from the amplitude modulation of $E(t)$ [which accounts for the first term on the right-hand side of (5.119)] and frequency sweep $2\beta t$ [which accounts for the second term on the right-hand side of (5.119)].

5.4.5.1. *Methods of Mode Locking*

Mode-locking methods can be divided into two categories: (1) Active mode-locking, in which the loss or gain of the laser is modulated by an external driving source, and (2) passive mode-locking, usually achieved by a suitable saturable absorber.

As a first example of active mode-locking, suppose we insert in the cavity a modulator, driven by an external signal, which produces a sinusoidal time-varying loss at frequency $\Delta\omega'$. If $\Delta\omega' \neq \Delta\omega$, this loss will simply amplitude modulate the electric field $E(t)$ of each cavity mode to give

$$E_l(t) = E_0(1 + \delta\cos\Delta\omega' t)\cos(\omega_l t + \phi_l) \tag{5.120}$$

where δ is the depth of modulation, ω_l is the mode frequency, and ϕ_l its phase. Note that, owing to the presence in (5.120) of the term

$$E_0\delta\cos\Delta\omega' t\cos(\omega_l t + \phi_l) = (E_0\delta/2)\{\cos[(\omega_l + \Delta\omega')t + \phi_l] + \cos[(\omega_l - \Delta\omega')t + \phi_l]\}$$

$E_l(t)$ actually contains two terms oscillating at the two frequencies $\omega_l \pm \Delta\omega'$ (modulation side-bands). If now $\Delta\omega' = \Delta\omega$, these modulation side-bands will coincide with the adjacent mode frequencies of the resonator, which are equal to $\omega_l \pm \Delta\omega$. These two side-band terms will thus give contributions to the two field equations for the adjacent cavity modes at frequencies $\omega_l \pm \Delta\omega$. The cavity mode equations thus becomes coupled, in the sense that the field equation of a given cavity mode will contain two terms arising from the modulation of the two adjacent modes. This mode-coupling mechanism can be shown to lock the mode phases according to (5.106) if the modulator is placed very close to one of the end mirrors. This type of mode-locking is often referred to as amplitude-modulation (AM) mode locking.

The operation of the AM-type of mode locking can perhaps be more readily understood by considering its behavior in the time domain rather than in the frequency domain. Figure 5.41a shows the time behavior of the cavity losses γ which are modulated at frequency $\Delta\omega'$. We will assume the modulator to be placed at one end of the cavity. If $\Delta\omega' = \Delta\omega$, the modulation period T' will be equal to the cavity round-trip time $2L/c$. In this case light pulses will develop in the cavity as indicated in Fig. 5.41a. In fact, if a pulse is assumed to pass through the modulator at a time t_m of minimum loss it will return to the modulator after a time $(2L/c)$ when the loss is again at a minimum. If, on the other hand, the pulse is assumed to initially pass through the modulator at a time, e.g., slightly before t_m (solid-line pulse in Fig. 5.41b) the leading edge of the pulse will suffer more attenuation by the modulator's time-varying loss γ_m than the pulse trailing edge (see dashed-line pulse in Fig. 5.41b). Thus, after passing through the modulator, the time at which the pulse peak occurs will have been advanced in such a way that, during the next passage, the peak will arrive closer to t_m. This shows that the situation depicted in Fig. 5.41a corresponds to the stable mode-locked condition.

After these preliminary considerations of AM mode locking, we can go on to consider the physical phenomena that determine the time duration of the mode-locked pulses. These phenomena are quite different depending on whether the laser line is homogeneously or inhomogeneously broadened. For an inhomogeneously broadened line and if the laser is sufficiently far above

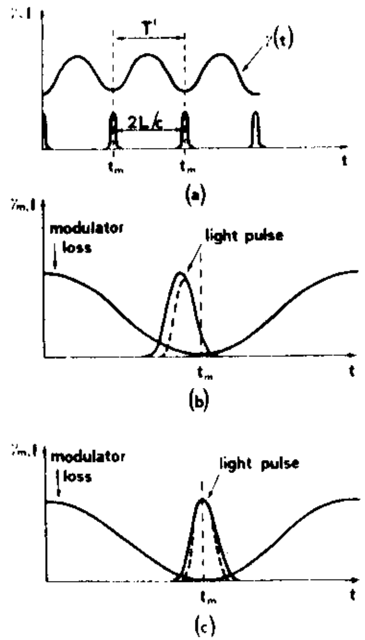


FIG. 5.41. Time-domain description of AM locking: (a) steady-state condition; (b) light pulse arriving before the time t_m of minimum loss; (c) pulse shortening occurring when the pulse arrives at time t_m.

threshold, the oscillating bandwidth $\Delta\nu_{osc}$ tends to cover the whole laser bandwidth $\Delta\nu_0^*$. Assuming a Gaussian distribution for the mode amplitudes, we then get from (5.116)

$$\Delta\tau_p \cong 0.44/\Delta\nu_0^* \tag{5.121}$$

In the case of a homogeneous line, as explained in Section 5.3.5.1, the width of the oscillating spectrum tends to be concentrated in a narrow region around the central frequency ν_0. In this case, the oscillating bandwidth and hence the laser pulsewidth is determined by a different physical mechanism. With reference to Fig. 5.41c suppose that a light pulse of finite duration passes through the modulator at the time t_m of minimum loss. After passing through the modulator, the output pulse (dotted line) is narrower than the incident pulse (solid line) since both the leading and trailing edge of the pulse are somewhat attenuated while the peak of the pulse passes without attenuation. However, this pulse narrowing is counteracted by pulse broadening which occurs when the pulse passes through the active material. As remarked above a homogeneously broadened gain line tends to reduce the oscillating bandwidth of the pulse and hence increase its temporal width. The steady state pulse profile, which is established by these two competing effects of pulse narrowing (in the modulator) and pulse broadening (in the amplifier), can be described analytically in a simple way and to a good approximation (see Appendix C). The intensity profile turns out in fact to be well described, under typical conditions, by a Gaussian function whose width τ_p (FWHM) is given by

$$\Delta\tau_p \cong 0.45/(\nu_m\Delta\nu_0)^{1/2} \tag{5.122}$$

where ν_m is the modulation frequency of the modulator ($\nu_m = c/2L$). If the pulsewidth expressions for the inhomogeneous (5.121) and homogeneous (5.123) gain lines are compared for the same value of the laser linewidth (i.e., for $\Delta\nu_0^* = \Delta\nu_0$), we get

$$\frac{(\Delta\tau_p)_{\text{hom}}}{(\Delta\tau_p)_{\text{inhom}}} \cong \left(\frac{\Delta\nu_0}{\nu_m}\right)^{1/2} \tag{5.123}$$

Since usually $(\Delta\nu_0/\nu_m) = (\Delta\nu_0 L/2c) \gg 1$, we see that much longer mode-locked pulses result in the homogeneous case than in the inhomogeneous case. As a final comment on this topic, we point out that the pulse-narrowing mechanism depicted in Fig. 5.41c does not play an appreciable role in the case of an inhomogeneous line, although it is still obviously present also in this case. In this case the short pulsewidth is in fact established by the inverse of the gain bandwidth, and the main role of the modulator is to establish synchronism between the oscillating modes so that the laser pulses pass through the modulator at the time of minimum loss (Fig. 5.41a).

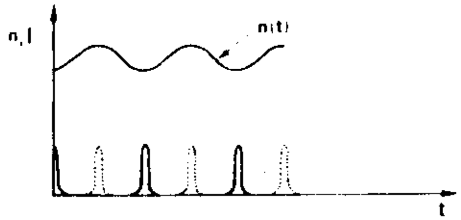


FIG. 5.42. FM locking. Time behavior of modulator refractive index n and of output intensity I.

As a second example of active mode-locking, suppose one inserts in the cavity a modulator, driven by an external signal, whose refractive index n is modulated at frequency $\Delta\omega'$. Again, if the modulator is placed at one cavity end and if $\Delta\omega' = \Delta\omega$, the phases of the modes become locked, although with a different relationship from that given in (5.106). Nevertheless, one again obtains short pulses whose duration is of the order of the inverse of the oscillating bandwidth. Since the optical length of the modulator is* $L'_{opt} = n(t)L'$, where L' is its true length, this type of modulator produces a modulation of the effective cavity length. As a result, the cavity resonance frequencies are modulated, and this method of locking is often referred to as frequency-modulation (FM) mode-locking. In the time domain, the FM mode-locking behavior can be described as indicated in Fig. 5.42. Note that, in this case, two stable mode-locking states can occur, where the light pulses pass through the modulator either at each minimum of $n(t)$ (solid-line pulses) or at each maximum (dotted-line pulses). Actually, switching between these two states is often found to occur in practice. To gain a deeper understanding of what happens in this case is a more complicated task than for the AM case. Since this type of locking is much less frequently used in practice, we will not consider it any further here. We merely limit ourselves to pointing out that our cavity is equivalent in its effect to a cavity without a modulator but where the position of one cavity mirror is made to oscillate at frequency $\Delta\omega$. According to the situation shown in Fig. 5.42, the mode-locked pulses tend to strike this mirror when it is at either of its extreme positions (i.e., when the mirror is stationary).

As a third example of active mode locking we will consider the case where the laser gain is modulated rather than the laser losses. In the case of a laser being pumped by another laser this is commonly achieved by pumping with another mode-locked laser and adjusting the length L of the second laser cavity so that the pulse repetition period of this second laser, $(2L/c)$, is equal to that of the pump laser. The mode-locked pulses of the second laser are then in synchronism with those of the pump laser, and this method is usually

* The optical length L'_{opt} is here defined such that the phase shift of a wave in passing through the modulator can be written as $\phi = (2\pi/\lambda_0)L'_{opt}$.

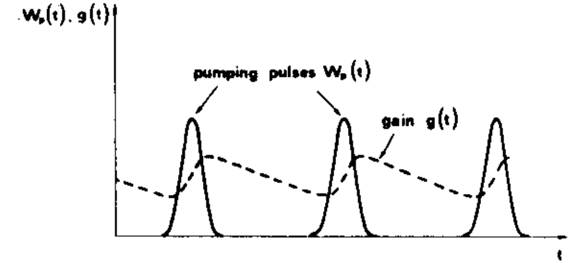


FIG. 5.43. Time behavior of the pump rate $W_p(t)$ and of the laser gain $g(t)$ in a synchronously mode-locked laser.

referred to as mode locking by synchronous pumping. This same type of pumping can also be achieved in a semiconductor laser by pulsing the pumping current through the diode junction at a repetition rate equal to $c/2L$, where L is the semiconductor cavity length. For both cases, the time behavior of the laser gain resulting from this pulsed pumping can be described as indicated by the dashed line in Fig. 5.43. From the discussion of AM mode locking, we can readily appreciate that the mode-locked pulses (not shown in Fig. 5.43) will tend to pass through the active material at the time of maximum gain. Note that, for this scheme to work, the decay time of the inversion, for the synchronously pumped laser, must be fast enough (i.e., of the order of the cavity transit time) so that the corresponding gain can actually be significantly modulated. The method is therefore often used for dye, color center, and semiconductor lasers which have short lifetimes (a few nanoseconds) for the upper state.

The last case we will consider is passive mode locking by a saturable absorber. We consider an absorber with a transition frequency that coincides with the laser frequency, with a low saturation intensity, and whose relaxation time is much shorter than the cavity round trip time (fast saturable absorber). To understand how such an absorber can lead to mode-locking we will consider a time domain description. Let us suppose then that the absorber is contained in a thin cell placed in contact with one of the cavity mirrors (Fig. 5.44a). If the modes are initially unlocked, the intensity of each of the two traveling

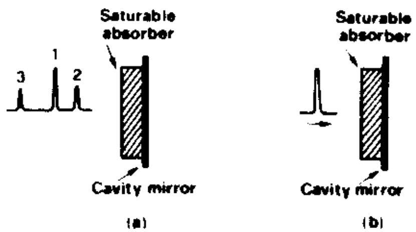


FIG. 5.44. Time-domain description of passive mode locking.

waves in the cavity will be made up of a random sequence of light bursts (indicated as 1, 2, and 3 in Fig. 5.44a; see also Fig. 5.40). As a result of absorber saturation, pulse 1 in the figure, being the most intense, will suffer the least attenuation in the absorber. This pulse will grow faster than the others, and after many round trips the situation depicted in Fig. 5.44b will eventually be established where a single intense mode-locked pulse remains. Actually the saturable absorber only works in the way described above provided that its decay time τ is shorter than or at least comparable to the time separation between two consecutive noise pulses of Fig. 5.44a (typically a few tens of picoseconds). In the case of a slow absorber (which typically means τ of the order of a few nanoseconds) the saturation induced in the absorber by, e.g., light pulse 1 in Fig. 5.44a will not have decayed appreciably by the time pulse 3 arrives at the absorber and the process of selecting the most intense pulse will no longer have effect.

Although many passively mode-locked lasers make use of a fast saturable absorber, under special circumstances, slow saturable absorbers can also lead to mode locking. This may occur when the saturation energy of the gain medium is comparable to, although a little larger, than that of the saturable absorber. The physical phenomena that lead to mode-locking are rather subtle, in this case,[28] and will be described with the help of Fig. 5.45. For simplicity we will consider both the saturable absorber and the active material to be placed together in a single cell at one end of the laser cavity. Before the arrival of the mode-locked pulse the gain is assumed to be smaller than the losses, so that the early part of the leading edge of the pulse will suffer a net loss. At some time during the leading edge of the pulse, when the accumulated energy density of the pulse becomes comparable to the saturation energy density of the absorber, saturation of the saturable absorber will begin to occur. The absorber loss can thus become smaller than the gain, and, if the energy of the

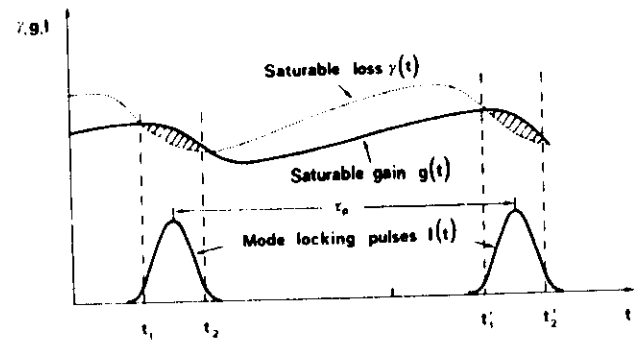


FIG. 5.45. Continuous-wave mode locking by means of a slow saturable absorber. Note that the figure is not to scale since the time duration of a mode-locked pulse is typically smaller than 1 ps while the time interval τ_p between two consecutive pulses, i.e., the cavity round trip time is typically a few nanoseconds.

pulse is sufficient, this can occur at some time during the leading edge of the pulse (times t_1 and t_1' in Fig. 5.45). Starting from this time, the pulse will be amplified rather than attenuated. However, if the saturation energy density of the gain medium is only slightly higher than that of the saturable absorber, gain saturation will also be produced at some later time during the pulse evolution. The gain can thus become smaller than the loss at some time during the trailing edge of the pulse (times t_2 and t_2' in Fig. 5.45). Under the above conditions, the pulse will see a net gain in its central part (i.e., for $t_1 < t < t_2$) and a net loss in its wings (i.e., for $t < t_1$ and $t > t_2$). Upon passing through the cell the pulse will therefore be shortened and amplified. This process of pulse shortening and amplification will cease when the pulse duration becomes comparable to the inverse of the gain bandwidth $\Delta\nu_0$. Therefore, in this case, the pulse duration $\Delta\tau_p$ is expected to be roughly equal to $1/\Delta\nu_0$. Note, finally, that, after the mode-locked pulse has passed and before the arrival of the next one, the saturable loss recovers to the initial value by spontaneous (radiative + nonradiative) decay. During the same time interval, the saturable gain recovers to its initial value by means of the pumping process. For the latter circumstance to occur it is necessary that the recovery time of the gain medium (i.e., its upper state lifetime) be comparable to the cavity round trip time. So, this type of mode-locking can be made to occur with short-lifetime (of the order of a few nanoseconds) gain media such as dyes or semiconductors, but does not occur with long-lifetime (of the order of 1 ms) gain media such as Nd:YAG or CO_2. When the delicate conditions for this type of mode locking can be met, however, very short light pulses down to the inverse of laser linewidth can be obtained. In fact cw passive mode-locked dye lasers have in this way produced the shortest pulses (~25 fs for the rhodamine 6G laser mode-locked by a DODCI saturable absorber).

5.4.5.2. *Mode-Locking Systems*

Mode-locked lasers can be operated either with a pulsed or cw pump (Fig. 5.46). In the pulsed case, active mode locking is commonly achieved either by means of a Pockels cell electro-optic modulator or an acousto-optic modulator. A possible configuration for a Pockels cell modulator is that of Fig. 5.28a, where the voltage to the Pockels cell is sinusoidally modulated from zero to a fraction of the $\lambda/4$ voltage. Passive mode locking in pulsed lasers is usually achieved by fast saturable absorbers. In pulsed mode locking the overall duration $\Delta\tau_p'$ of the train of mode-locked pulses is in some cases determined by the duration of the pump pulse. This is, for instance, true for gain materials with fast recovery time (e.g., dye lasers), and in this case $\Delta\tau_p$ may typically be a few microseconds. For gain materials with slow recovery time (e.g., solid-state lasers) and when a saturable absorber is used for mode-locking, the presence of the saturable absorber will result not only in mode-locked but also in Q-switched operation. In this case the duration $\Delta\tau_p'$ of the

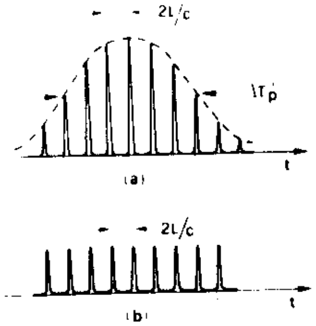


FIG. 5.46. (a) Pulsed and (b) cw mode locking.

mode-locked train will be given by the duration $\Delta\tau_p$ of the Q-switched pulse as calculated in Section 5.4.3.3 (a few tens of nanoseconds). Note that, when a slow saturable absorber (τ of about a few nanoseconds) is used with a slow gain material,* passive Q switching with single mode selection rather than mode locking will tend to occur, as explained in Section 5.4.3.1.

In the case of mode locking with a cw pump, the output beam consists of a continuous train of mode-locked pulses, two consecutive pulses being separated by the cavity round trip time $2L/c$ (see Fig. 5.46b). Active mode locking is usually achieved either by a Pockels cell modulator or, more commonly, because of its lower insertion loss, by an acousto-optic modulator. An acousto-optic modulator used for mode locking differs from that used for Q switching (see Fig. 5.30) since the face to which the transducer is bonded and the opposite face of the optical material are now cut parallel each other. The sound wave launched into the material by the transducer is now reflected back by the opposite face of the material. If the length of the optical block is equal to an integer number of half-wavelengths of the sound wave, an acoustic standing wave pattern will be produced. Under these conditions, if the sound wave is oscillating at frequency ω, the diffraction loss will be modulated at frequency 2ω. In fact, the diffraction loss reaches a maximum whenever a maximum amplitude of the standing wave pattern occurs. Consider now a

* It must be noted that we have been using the terms "fast" and "slow" in regard to recovery time in a way that is different for the cases of absorber and gain medium. The recovery time of a saturable absorber is considered to be slow when its value (typically a few nanoseconds) is comparable to a typical cavity round trip time. This value of lifetime is typical for absorbers whose decay is determined by spontaneous emission via an electric-dipole-allowed transition. The recovery time is considered to be fast (a few picoseconds) when it is comparable to a typical duration of a mode-locked pulse. Such fast recovery times usually arise from fast nonradiative decay in the absorber. By contrast, the lifetime of a gain material is considered to be fast when comparable to the cavity round-trip time. This occurs for an electric-dipole-allowed laser transition. The lifetime of a gain material is considered to be slow when it corresponds to an electric-dipole-forbidden transition (τ of the order of a millisecond).

standing sound wave of the form $S = S_0(\sin\omega t)(\sin kz)$. The maximum amplitude of the standing wave is S_0 and this maximum is reached twice in an oscillation period (i.e., at $t = 0$ and $t = \pi/\omega$). The modulator loss is thus modulated at frequency 2ω and mode locking is achieved when (1) the modulator is put as near as possible to one cavity mirror, (2) the modulator frequency 2ω is set equal to $2\pi(c/2L)$ and, accordingly, the transducer is driven at a frequency ν equal to $c/4L$ (e.g., $\nu = 50$ MHz for $L = 1.5$ m).

As discussed in the previous section, passive mode locking with cw lasers can be achieved, under special circumstances, using slow absorbers combined with fast laser gain media (notably dye lasers).

As a conclusion to this section as well as to the entire section on mode-locking, Table 5.1 summarizes the operating conditions for some of the most commonly used mode-locked lasers. For a detailed description of each of these lasers, the reader is referred to the next chapter. We merely wish to note here that, when mode-locked by an acousto-optic modulator, cw Ar^+ and Nd:YAG lasers give comparable pulsewidths $\Delta\tau_p$ although the laser linewidth for Nd:YAG ($\Delta\nu_0 \cong 150$ GHz) is much larger than that for the argon laser ($\Delta\nu_0^* = 3.5$ GHz). This is due to the fact that the laser line is homogeneously broadened

TABLE 5.1. Mode-Locking Systems

Active material	Mode-locking element[a]	Type of operation	$\Delta\tau_p$
Gas			
He-Ne	Quartz AOM	cw	1 ns
He-Ne	Neon cell SA	cw	350 ps
He-Ne	Cresyl violet SA	cw	220 ps
Ar^+	Quartz AOM	cw	150 ps
CO_2	Germanium AOM	cw	10-20 ns
(low pressure)	SF_6 SA	cw	10-20 ns
CO_2 (TEA)	Germanium AOM	Pulsed	1 ns
	SF_6 SA	Pulsed	1 ns
Solid			
Nd:YAG	Quartz AOM	cw	100 ps
Nd:YAG	$LiNbO_3$ EOM	Pulsed	40 ps
Nd:glass	Kodak 9860 or 9840 SA	Pulsed	5 ps
Ruby	DDI SA	Pulsed	10 ps
GaAs	SA	cw	5 ps
Color center	Sync. pump	cw	5 ps
Liquid (dye lasers)			
Rhodamine 6G	DODCI SA	cw, Ar^+ pumped	25 fs
Rhodamine 6G	DODCI SA	Flash pumped	1 ps
Rhodamine 6G	Sync. pump	cw, Ar^+ pumped	0.5 ps

[a] AOM, acousto-optic modulator; SA, saturable absorber; EOM, electro-optic modulator.

for Nd:YAG while it is inhomogeneously broadened in the case of the argon laser. Note also that the cw mode-locking of a rhodamine 6G dye laser by the DODCI saturable absorber has produced the shortest pulse (25 fs).

5.4.6. Cavity Dumping

The technique of cavity dumping allows the energy contained in the laser to be coupled out of the cavity in a time equal to the cavity round-trip time. The principle of this technique can be followed with the help of Fig. 5.47, where the laser cavity is considered to be made of two 100% reflecting mirrors and the output beam is taken from an output coupler of a special kind. The reflectivity $R = R(t)$ of this coupler is in fact zero up to a given time instant and then suddenly becomes equal to 100%. This coupler will thus dump out of the cavity, in a double transit, whatever power is circulating in the laser cavity. Alternatively, if the reflectivity R of the output coupler is switched to a value less than 100%, the cavity dumper will still work correctly, coupling out the fraction R of the circulating power, if the coupler reflectivity is switched to its high value for a time equal to the round-trip time and then returned to its zero value. Cavity dumping is a general technique that can be used to advantage whether the laser be mode-locked, or cw or Q switched. In the discussion that follows we will limit ourselves to considering the case of cavity dumping of a mode-locked laser, since this is the case where cavity dumping is used most often in practice.

For pulsed mode-locked lasers, cavity dumping is usually carried out at the time when the intracavity mode-locked pulse reaches its maximum value (see Fig. 5.46a). In this way a single ultrashort pulse of high intensity is coupled out of the laser cavity. Note that, if the output coupler switches to a nominally 100% reflectivity, cavity dumping is simply achieved by merely switching the coupler to its 100% reflectivity state. This type of dumping is often obtained by a Pockels cell electro-optic modulator used in a configuration that is similar to that used for Q switching (Fig. 5.28). In the configuration shown in Fig. 5.28a, the voltage in the Pockels cell is pulsed to its $\lambda/4$ voltage at the instant when cavity dumping is required, and the reflected beam from the polarizer is taken as output.

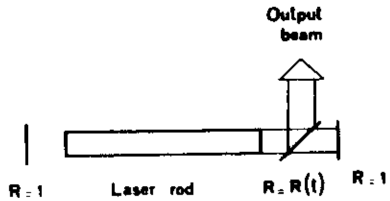


FIG. 5.47. Principle of laser cavity dumping.