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"Interaction Between Surface Waves & Mean Flow"

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INTERACTION BETWEEN SURFACE WAVES AND MEAN FLOW

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Chapter 1

Introduction

The purpose of these lectures is to introduce the basic mechanisms of the interaction between surface waves and ocean currents and to show its practical relevance.

On one side the ocean currents affect the surface waves because their presence modifies the medium across which the waves propagate. The speed of the mean current enters explicitly the wave dispersion relation, and therefore current variations modify the frequency, the wavelength and the amplitude of the surface wave. Moreover, in shallow seas, the dependence on the variable water depth, that is associated with tidal currents, must be added to the explicit dependence on the current velocity. The equations describing the dependence of frequency and wavelength on the current are derived by the standard Ray-theory (see, Whitham 1974). To complete the description of the effect of the currents on the waves an equation for the wave amplitude is needed. This is provided by the wave action conservation equation (Bretherton and Garrett, 1969). Wave action conservation is more fundamental than energy or momentum convervation, because it holds also in not homogeneous and not stationary media, where, respectively, momentum and energy are respectively not conserved. These topics are introduced in the second section.

On the other side there is also an effect of the surface waves on the mean flow and on the mean sea level, because waves have energy, momentum and they transport mass. Therefore the fluxes of energy, momentum and mass, that are associated with the presence of surface waves, enter the overall energy, momentum and mass conservation equations. Average energy, momentum and mass transport in a surface wave and their effect on the mean current and on the mean surface displacement are introduced in the third section.

As an example of the practical implications of this the "set-up" is discussed. This is a phenomenon which takes place near the coast, when the loss of momentum associated with the wave breaking is balanced by a gradient of the mean sea leavel. The "set-up" is important during storms, when it gives a relevant contribution to the sea level increment at the coast.

The prediction of the "set-up" needs the availability of the wave spectrum offshore. This can be provided by a wave model. The principles of wave modelling are briefly presented in the last section, where as a practical application, a computation of the "set-up" in front of Venice, in the North Adriatic Sea, during a very intense storm, is also given.

Chapter 2

Surface waves in presence of currents

The fundamental equations describing the surface wave motion are the mass conservation equation for an incompressible fluid,

$$\nabla u = 0, \tag{2.1}$$

and the momentum equation where viscosity, earth rotation, buoyancy are neglected and the motion is assumed irrotational ($\nabla \wedge u = 0$).

$$\frac{\partial \vec{u}}{\partial t} + \nabla\frac{|u|^2}{2} + \nabla(\frac{p}{\rho} + gz) = 0. \tag{2.2}$$

As the motion is assumed irrotational, the velocity can be expressed as the gradient of the velocity potential φ,

$$u = \nabla\varphi, \tag{2.3}$$

reducing the continuity equation (2.1) to the Laplace's equation

$$\nabla^2\varphi = 0, \tag{2.4}$$

which holds in the region occupied by the fluid, i.e. for $H < z < \eta$, where H is the sea bottom and η the free surface. The Laplace equation has a unique solution, that is determined by the boundary conditions. At the bottom the velocity component normal to the bottom vanishes. For a flat bottom this gives

$$\phi_z = 0 \text{ if } z = -H \tag{2.5}$$

Here and in the following expressions underscripts denote derivation. At the free surface the situation is more complicated because the position of the surface itself is actually a part of the solution. Therefore two conditions are needed to close the system. The first condition, called kinematic boundary condition, requires that the component of the fluid speed normal to the free surface vanishes:

$$\phi_z - \nabla_H\phi\nabla_H\eta = \eta_t, \tag{2.6}$$

where ∇_H denotes the horizontal gradient $i\partial/\partial x + j\partial/\partial y$. The second condition, called dynamic boundary condition, is derived by integrating the momentum equation (2.2). It requires that the pressure on both sides of the air-sea interface is the same:

$$\phi_t + \frac{|\nabla\phi|^2}{2} + g\eta = p_{atm} + p_{st}. \tag{2.7}$$

Note that the surface boundary conditions (2.6) and (2.7) imply that the problem is nonlinear. Anyway considering only small amplitude waves, i.e. retaining only the linear terms in

(2.6) and (2.7) the system of equations becomes:

$$\nabla^2\phi = 0 \quad \text{if } 0 > z > -H \tag{2.8}$$
$$\phi_z = 0 \quad \text{if } z = -H \tag{2.9}$$

$$\left.\begin{aligned} \phi_z &= \eta_t \\ \phi_t + g\eta &= -\sigma\eta_{xx} \end{aligned}\right\} \text{if } z = 0 \tag{2.10}$$

where $\sigma\eta_{xx}$ is the contribution of the surface tension to the pressure and σ is the surface tension coefficient. Seeking a solution in the form

$$\phi = \phi_o\hat{\phi}(z)\exp(i(kx - \omega t)), \tag{2.11}$$
$$\eta = \eta_o\exp(i(kx - \omega t)), \tag{2.12}$$

the Laplace's equation (2.4) and the bottom boundary condition (2.5) give

$$\hat{\phi}(z) = \cosh(k(z + H)). \tag{2.13}$$

Substituting (2.11), (2.12) and (2.13) in the surface boundary condition one obtains the dispersion relation

$$\omega^2 = gk(1 + B)\tanh(kH), \tag{2.14}$$

where

$$B = \frac{\sigma k^2}{g\rho}, \tag{2.15}$$

which is a measure of the relative importance of gravity and surface tension as restoring force (in the gravity wave limit $B \to 0$). The phase speed C_{ph} is given as

$$C_{ph}^2 = \frac{g}{k}(1 + B)\tanh(kH) \tag{2.16}$$

and the group velocity C_g is given as

$$C_g = \frac{C_{ph}}{2}\left(\frac{2kH}{\sinh 2kh} + \frac{1 + 3B}{1 + B}\right) \tag{2.17}$$

Until now the effect of currents has not been taken into account. Obviously in an inertial reference frame moving with the current U the solution for a surface wave is the same as we already found. But to a stationary observer the wave appears to have a frequency ω, which is called absolute frequency,

$$\omega = \omega_o + Uk, \tag{2.18}$$

where ω_o is called relative frequency, or Doppler shifted frequency, and it is given by (2.14). The variation of wave frequency and wavenumber in the presence of a current varying in space and time can be described by the Ray-theory, approximating the solution $\eta(x,t)$ as a sinusoidal wave whose amplitude, wavenumber and frequency vary slowly in space and time:

$$\eta(x,t) = \eta_o(x,t)\exp(i\Theta(x,t)), \tag{2.19}$$

with wavenumber and frequency defined as

$$k = \nabla\Theta, \tag{2.20}$$
$$\omega = -\Theta_t \tag{2.21}$$

satisfying the consistency relation

$$k_t + \nabla\omega = 0. \tag{2.22}$$

Interpreting k as a crest density and ω as a crest flux, equation (2.22) states that the number of crest is conserved. Using the dispersion relation (2.14) one obtains the equation for the wavenumber and the frequency

$$\frac{d}{dt}k = -k_j\frac{\partial U_j}{\partial x_i} + \frac{\partial\sigma}{\partial H}\nabla H, \tag{2.23}$$
$$\frac{d}{dt}\sigma = +U_t k + \frac{\partial\sigma}{\partial H}H_t, \tag{2.24}$$

where the operator d/dt is

$$\frac{d}{dt} = \frac{\partial}{\partial t} + (C_g + U)\cdot\nabla. \tag{2.25}$$

The equations (2.24) and (2.24) describe the variation of frequency and wavenumber along a ray path given by the equation

$$\frac{dx}{dt} = C_g + U \tag{2.26}$$

If the medium is homogeneous in space, i.e. U and H do not depend on x, then the wavenumber is conserved along a ray; if the medium does not depend on time, i.e. U and H do not depend on t, then the frequency is conserved along a ray.

The use of the dispersion relation and the Ray theory does not provide any equation for the wave amplitude. As it will be shown in the next section the wave energy is proportional to the square of the wave amplitude. An equation for the wave amplitude could therefore be derived from an equation for the wave energy, but in general, the energy is not conserved for a wave traveling in a medium that varies in time. It has been proved that there is a quantity, the wave action, whose conservation is more fundamental, because it holds also for media that are not homogeneous and not stationary (Bretherton and Garret 1969). The wave action A is related to the wave energy E as $A = E/\omega$ and its conservation is described by the equation

$$\frac{d}{dt}A = 0, \tag{2.27}$$

which follows elegantly from the average Lagrangian approach introduced by Whitham. Its derivation is anyway beyond the scope of these lectures. Note that if the frequency ω is constant then the wave action conservation equation (2.27) reduces to the energy conservation equation. As is evident from equation (2.24) this is a case for a stationary medium.

The validity of equations (2.24), (2.24), (2.27) is restricted to slowly varying wave trains, i.e. when amplitude, wavenumber and frequency vary over a scale that is much larger then the wave period and the wavelength. If the medium varies over a scale comparable to the wavelength or the period of the sought sinusoidal wave solution, the idea of a sinusoidal solution itself is not applicable and the solution cannot be approximated by (2.19), with (2.20 and (2.21).

Chapter 3

Wave energy and momentum fluxes

In this section we investigate the properties of the surface waves, as they are described by the linear system (2.9), (2.10) and (2.9) that was introduced in the previous section. The solution of the linearized equations is:

$$\eta = \eta_o \exp i(kx - \omega t) \tag{3.1}$$

$$\phi = -i\varphi_o \cosh(k(H + z)) \exp i(kx - \omega t) \tag{3.2}$$

$$\varphi_o = \frac{g}{\omega} \frac{1 + B}{\cosh(kH)} \eta_o \tag{3.3}$$

The fundamental reason because surface waves affect the mean flow is that they transport energy, momentum and mass which enter the overall balance equation. Examining the properties of the flow associated with (3.1), (3.2), (3.3), it is clear that the local fluid energy, momentum and speed vary with the wave profile. Anyway we are not directly interested in the local values, but in the mean quantities which are left after averaging over the wave oscillations. Of course positive defined quantities, like the kinetic energy, have a non vanishing average value. It is less obvious that oscillating quantities, like the momentum and the mass flux do not average to zero, implying that waves have momentum and transport mass. To determine the average quantitities related to the presence of surface waves the Eulerian expressions have to be vertically integrated and, successively, they have to be averaged over the phase of the wave or, equivalently, over a wave period.

There are three contributions to the wave energy E: a kinetic energy contribution E_k, a potential energy contribution E_p and a surface tension contribution E_{st}. The wave kinetic energy is given by

$$E_k = < \int_{-H}^{\eta} \frac{1}{2} \rho |u|^2 dz >, \tag{3.4}$$

which, substituting the linear solution (3.1), with a little algebra gives

$$E_k = \rho g \eta_o^2 \frac{1 + B}{4}. \tag{3.5}$$

The contribution of the wave motion to the potential energy is

$$E_p = < \int_{-H}^{\eta} \rho g z dz > - \int_{-H}^{0} \rho g z dz = \frac{\rho}{4} g \eta_o^2. \tag{3.6}$$

Finally there is the contribution from the surface tension, acting as a massless elastic membrane (Morse and Ingard 1968):

$$E_{st} = \frac{1}{4} \rho g \eta_o^2 B \tag{3.7}$$

The total energy density is then given as

$$E = E_k + E_p + E_{st} = \frac{1}{2} \rho g \eta_o^2 (1 + B) \tag{3.8}$$

The same procedure can be applied to determine the momentum density P. By retaining the lowest order term one obtains:

$$P = < \int_{-H}^{\eta} \rho u dz > = < \int_{0}^{\eta} \rho u dz > \simeq < \rho u \eta > \tag{3.9}$$

Substituting the linear solutions (3.1) one obtains

$$P = \frac{1}{2C_{ph}} \rho g \eta_0^2 (1 + B) \frac{\vec{k}}{k} \tag{3.10}$$

The momentum is in the direction of the wave propagation. Comparing (3.8) and (3.10) one immediately notices that energy and momentum are related by $E = C_{ph} P$. Note that because of the oscillatory nature of the flow the only contribution to the total momentum comes from the region between crests and troughs, where there is no balance between forward and backward moving fluid. The wave momentum can be written as $P = \rho H U_M$, being in this way associated with a mean velocity U_M

$$U_M = \frac{g \eta_o^2}{2C_{ph} H} (1 + B). \tag{3.11}$$

This is not only a mathematical interpretation of the equation (3.10), because the motion of the fluid particles does not follow close orbits. In fact, since the horizontal velocity decays with the depth, a larger distance is covered during the upper part of the orbit than during the lower one. Therefore the average velocity of the fluid particles is not zero but it is given by the Stokes' drift velocity U_{Sd} :

$$U_{Sd} = \frac{\omega k}{2 \sinh^2(kH)} \eta_o^2 \cosh(2k(H + z)), \tag{3.12}$$

which, integrated over depth, averages to U_M. This average motion implies a transport of mass due to water waves.

As surface waves have momentum and energy, traveling across a medium, they produce momentum and energy fluxes. The energy and momentum fluxes can be obtained by repeating the previous procedure of vertically integrating and phase averaging in the momentum and energy equation. Consider the horizontal momentum equation

$$\rho \frac{\partial u_i}{\partial t} + \frac{\partial u_i}{\partial x_j} (p\delta_{ij} + \rho u_i u_j) + \frac{\partial}{\partial z} \rho u_i w = 0 \tag{3.13}$$

where w is the vertical velocity. The second and the third terms represent a momentum flux whose divergence balances the local variation of momentum. The contribution from wave motion to this flux is

$$T_{ij} = < \int_{-H}^{\eta} p\delta ij dz > + < \int_{-H}^{\eta} \rho u_i u_j) dz > + \Sigma_{ij} - \int_{-H}^{0} \rho g z dz \tag{3.14}$$

re Σ_{ij} represents the effect of capillarity, acting like a stretching membrane at the water face, and the last term is the hydrostatic pressure, which is subtracted because it is independent from the presence of the wave. Assuming that the wave propagates in the x-direction (Morse and Ingard 1968)

$$\Sigma_{ij} = \frac{B}{1+B}\frac{E}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{3.15}$$

The second term in (3.14) can be computed by direct substitution of the linear solution, replacing η with 0 as upper limit of integration, to retain only the lowest order contribution:

$$< \int_{-H}^{0} \rho u_i u_j dz >= \begin{pmatrix} E(\frac{1}{2} + \frac{kH}{\sinh(2kH)}) & 0 \\ 0 & 0 \end{pmatrix} \tag{3.16}$$

To determine the first contribution, note that, on the average, the flux of vertical momentum must be balanced by the weight of the fluid:

$$< p + \rho w^2 >= -\rho g z = p_o \tag{3.17}$$

therefore

$$< \int_{-H}^{o} (p - p_o) dz >=< -\int_{-H}^{o} \rho w^2 dz >= E(-\frac{1}{2} + \frac{kH}{\sinh(2kH)}). \tag{3.18}$$

Only the computation of $\int_0^\eta p dz$ is missing to conclude. At lowest approximation the pressure near the surface is the sum of the hydrostatic pressure and the surface tension contribution $\sigma \eta_{xx}$:

$$< \int_0^\eta p dz >\simeq< \int_0^\eta (\rho g(\eta - z) + \sigma k^2 \eta) dz >= E\frac{B+\frac{1}{2}}{B+1} \tag{3.19}$$

Adding all the contributions, the final expression for the flux of momentum is

$$T_{ij} = E\begin{pmatrix} \frac{2kH}{\sinh(2kH)} + \frac{1+3B}{2(1+B)} & 0 \\ 0 & \frac{kH}{\sinh(2kH)} \end{pmatrix} \tag{3.20}$$

The quantity T_{ij} is called radiation stress. Therefore surface waves are associated with a forward flux of forward momentum, represented by T_{11}, and sideways flux of sideways directed momentum, represented by T_{22}. Both are normal stresses. Note two limits: for deep water waves

$$T_{ij} = \delta_{1i} E\frac{1+3B}{2(1+B)} \tag{3.21}$$

and for gravity waves

$$T_{ij} = \begin{pmatrix} \frac{2kH}{\sinh(2kH)} + \frac{1}{2} & 0 \\ 0 & \frac{kH}{\sinh(2kH)} \end{pmatrix}. \tag{3.22}$$

Finally for deep water gravity waves

$$T_{ij} = T_{11} = C_g P \tag{3.23}$$

Analogously there is an energy flux Φ, which can be derived by repeating the previous procedure for the kinetic energy equation

$$\frac{\partial}{\partial t}(\frac{1}{2}\rho u^2 + \rho g z) + \frac{\partial}{\partial x_j}[(p + \frac{1}{2}\rho u^2 + r g z)u_j] = 0, \tag{3.24}$$

producing the final result

$$\Phi = C_g E. \tag{3.25}$$

As surface waves transport energy, mass and momentum, it is evident that there are two contributions to the overall conservation equations: one from the mean flow and one from the surface waves. Consider first the mass conservation

$$\nabla(U + u) = 0 \tag{3.26}$$

where U is the mean flow and u the flow associated with surface waves. Vertically integrating and averaging (3.26) gives

$$\rho\frac{\partial}{\partial t} < \eta > + \nabla_H(P + M) = 0 \tag{3.27}$$

where M is momentum of the current. This shows that variation in space of the wave momentum, because of the related mass transport, can produce variation of the mean water level.

Repeating this procedure with the overall horizontal momentum equation (3.13), integrating and averaging one obtains:

$$\frac{\partial}{\partial t}(P+M)_i + \frac{\partial}{\partial x_j}[T_{ij} + < \int_{-H}^{\eta} \rho(u_i U_j + u_j U_i) dz + \int_{-H}^{\eta} \rho U_i U_j dz >] = -\rho g(< eta > + H)\frac{\partial < \eta >}{\partial x_i} \tag{3.28}$$

This represents the overall momentum balance. To obtain the current momentum balance, the equation for the wave momentum alone must be subtracted from (3.28). The wave momentum balance is derived by the wave action conservation equation (2.27) and the wavenumber equation (2.24), using the relation $P = Ak$, which gives

$$\frac{\partial}{\partial t}P + \frac{\partial}{\partial x_j}(P(U + C_g)_j) = -P_j \frac{\partial}{\partial x_i} U_j \tag{3.29}$$

Subtracting (3.29) from (3.28) one obtains

$$\frac{\partial}{\partial t}M_i + \frac{\partial}{\partial x_j} < \int_{-H}^{\eta} U_i U_j dz >= -\rho g(< \eta > + H)\frac{\partial}{\partial x_i} < \eta > + \Gamma_i, \tag{3.30}$$

where

$$\Gamma_i = \frac{\partial}{\partial x_j}[(U + C_g)_j P_i - T_{ij} - < \int_{-H}^{\eta} (u_i U_j + u_j U_i)\rho dz > + P_j \frac{\partial}{\partial x_i} U_j] \tag{3.31}$$

represents the mean force exerted by the wave on the mean flow. This is the basic equation describing the effect of the surface waves on the mean flow.

As applications we consider a more simple, one dimensional situation, i.e. a situation which is homogeneous in the y-direction, in which there is no mean flow in the absence of waves. In this case the quantities like $< \eta >$ and U are of order η_o^2. Retaining only the lowest order terms, assuming the mean current, that is generated by the waves, to be depth independent, in steady conditions the momentum equation (3.28) becomes

$$\frac{\partial < \eta >}{\partial x} = -\frac{3}{2\rho g \eta}\frac{\partial E}{\partial x}. \tag{3.32}$$

When waves approach the shore they break and their height can be (rather euristically, on a dimensional argument) assumed proportional to the water depth: $\eta_o = \alpha H$. Therefore the wave energy is

$$E = \frac{1}{2}\rho g \alpha^2 H^2 \tag{3.33}$$

which, by substitution in (3.32) gives

$$\frac{\partial < \eta >}{\partial x} = -\frac{3}{2}\alpha^2\frac{\partial H}{\partial x}. \tag{3.34}$$

This equation implies that a rise of the mean sea level corresponds to the wave breaking near the shore. In practice the gradient of pressure associated with a slope in the sea surface must balance the loss of wave momentum.

When equation (3.32) is applied seaward the point where waves begin breaking, i.e. to the situation in which the waves experience the decreasing water depth, but they do not break, a decrement of the sea level is predicted. This is a consequence of the decrease of the group velocity as the wave approaches the shore. Another effect of the waves on the mean flow is the generation of a mean flow to compensate the mass that is transported by the waves towards the shore, determining a complicate situation in which concentrated "rip-currents" interfere with the incoming waves. In a bidimensional situation in which the angle of incidence of the incoming waves is not 90° also a longshore current is generated. According to (3.27), the variable radiation stress, that is associated with wave groups, produces variation of the mean sea level, decreasing it under the large waves and enhancing it between groups. These phenomena, very briefly mentioned in this last paragraph, are special cases of the general equations (3.28) and (3.27). Their detailed presentation can be found in the original paper by Longuett-Higgins and Stewart (1964) or in the book by Leblond and Mysak (1978), together with most of the material presented in these lectures.

Chapter 4

ocean wave modelling

This chapter aims to briefly introduce the instruments that can be used to describe the evolution of a wave field over an oceanic scale. This is a complementary topic with respect to the content of the previous chapters, in which the properties of the surface waves and their interaction with the mean flow have been presented.

Surface wave modelling does not provide a deterministic description of the wave filed, but a statistic one, describing the evolution of the wave spectrum. The two possibilities are to consider the energy spectrum, or - equivalently - the surface variance spectrum, or to consider the wave action spectrum. The surface variance spectrum is defined from the surface displacement $\eta(x,t)$ as

$$S(\bar{k},\omega) = \lim_{T\to\infty} \frac{1}{(2\pi)^3 L^2 T} \left| \int_{-T/2}^{T/2} \eta(x,t) \exp i(kx - \omega t) dt dx \right|^2, \tag{4.1}$$

which, for a stationary ergodic process, is the Fourier transform of the autocovariance $\Gamma(\bar{r},\tau) = <\eta(\bar{x},t)\eta(\bar{x} + barr, t+\tau)>$, i.e.

$$\Gamma(\bar{r},\tau) = \int S(\bar{k},\omega) \exp i(\bar{k}\bar{r} - \omega\tau) d\bar{k} d\tau \tag{4.2}$$

It follows then that

$$<\eta^2> = \int S(\bar{k},\omega) d\bar{k} d\omega \tag{4.3}$$

Because of the dispersion relation, the frequency can be expressed as function of the wavenumber and therefore the spectrum is actually bidimensional: $S \equiv S(\bar{k})$. Since the energy of a sinusoidal wave is proportional to the square of the amplitude the energy spectrum is actually proportional to the spectrum of the variance and the total energy E_T is

$$E_T = \int \frac{1}{2} \rho g S(\bar{k}) d\bar{k} \tag{4.4}$$

As already discussed in the previous sections, in the presence of variable depth and current the energy is not conserved, and the convenient conservation equation is the wave action conservation equation (2.27). But, during the development of the waves in the ocean, the wave action is not conserved too, because of interactions. There are interactions with external fields like the wind and the bottom producing respectively a gain and a loss of energy for the involved wave component. Wave breaking is another phenomenon producing the decay of the wave energy. Moreover the nonlinear terms, which are present in the equations (2.6) and (2.7) and that have been neglected in the following part of the presentation, determine an exchange of energy among wave components. The nonlinear interactions are energy conserving, i.e. they do not modify the overall energy content of the wave system, but they determine positive or negative transfer rates for each involved component. The equation to be solved for the wave action density

$$A(k) = \frac{1}{2}\rho g \frac{S(k)}{\omega} \quad (4.5)$$

has therefore the form

$$\frac{\partial}{\partial t}A(k) + \nabla[(C_g + U)A(k)] = G_{wi} + G_{nl} + G_{bf} + G_{br} \quad (4.6)$$

where the terms on the right hand side represent sources and sinks due, respectively, to the wind input, the nonlinear interactions, the bottom friction and the wave breaking. Each of them is actually a functional of $A(k)$ and of the external fields that are involved. The wavenumber equation (2.21)and the frequency equation (2.24) must be added to (4.6) in order to complete the system. Attempts to model equation (4.6), (2.21) and (2.24) are recent (Tolman ,1990). Previously the energy transport equation,

$$\frac{\partial}{\partial t}F(k) + \nabla[(C_g + U)S(k)] = S_{wi} + S_{nl} + S_{bf} + S_{br} \quad (4.7)$$

to which equation (4.6) reduces in absence of variable currents, was solved. The representation of the source functions and the methods to solve the balance equation depend on the specific model that is considered.

Although the energy transport equation (4.7) had been proposed already in 1957 (Gelci et al.), the lack of proper knowledge of the source functions prevented for a long time its satisfactory solution. First generation models, developed in the 60's and early 70's, mostly retained only the wind input source term in the form

$$S_{in} = A + BF(f, \theta, \bar{x}, t), \quad (4.8)$$

following the early theories of Phillips and Miles. The growth of each component was stopped when a saturation level, defined by a universal equilibrium distribution (Phillips 1958), was reached. First generation models were forced to overestimate S_{in} both with respect to theories and to observations in order to agree with the observed rate of growth of the low frequencies. When open field experiments, mainly JONSWAP, stressed the importance of the nonlinear interactions, it was realized that they contribute substantially to the growth of the low frequencies and that their proper inclusion in the computation could reduce the wind input in agreement with the observations. The implied coupling among different frequencies is the common characteristic of second generation models. While in first generation models the physics was incorrect, in second generation models the troubles came mostly from the numerics. In fact, although the expression of S_{nl} was known (Hasselmann 1961), unfortunately its exact computation is even nowadays too time consuming in the operational framework; therefore second generation wave models were forced to parametrize the spectral shape (e.g. JONSWAP spectrum) or the source function itself. This approach was not possible for a swell spectrum where the nonlinear coupling is negligible, and the wave energy propagates without relevant interactions. Consequently second generation models have difficulties in describing the transition from windsea to swell.

The flaws of first and second generation wave models were analysed in an intercomparison study (SWAMP 1985), after which an international group of scientists (the WAM Group) decided to develop a third generation model, where the energy transport equation was explicitly solved. To such purpose an efficient approximation of the nonlinear transfer S_{nl} and a specification of the substantially unknown dissipation S_{ds} were required. Here the structure of the source functions in the WAM model is briefly described.

The exact expression for the nonlinear energy transfer is given by the Boltzmann integral

$$\begin{aligned} S_{nl} = & \int \omega D\delta(\vec{k}_1 + \vec{k}_2 - \vec{k}_3 - \vec{k})\delta(\omega_1 + \omega_2 - \omega_3 - \omega) \\ & [(A_1 A_2(A_3 + A) - A_3 A(A_1 + A_2)]dk_1 dk_2 dk_3, \end{aligned} \quad (4.9)$$

where ω denotes the circular frequency, k the wavenumber, D the interaction coefficient, and A the wave action. In the WAM model an efficient way to evaluate the integral (4.10) is obtained substituting it by the discrete interaction approximation (Hasselmann and Hasselmann 1985) which retains the structure of the exact expression (4.10), but it limits the number of the configurations involved in the computation:

$$S_{nl} = \sum_{i=1,2} A_i\omega[(A'_1 A'_2(A'_3 A) - A'_3 A(A'_1 A'_2)]. \quad (4.10)$$

where the index i indicates summation over selected configurations.

The other required step was the specification of the dissipation source function which previously was not explicitly required in a wave model. An approach suitable for implementations in a wave model was proposed by Hasselmann (1974). Arguing that the process, although highly nonlinear locally, is weak in the mean, he proposed a quasi-linear source function, proportional to the spectrum with a coefficient depending on mean spectral parameters. Moreover from the small scale of the breaking event Hasselmann derived a quadratic dependence on the frequency. The final expression in the WAM model is

$$\begin{aligned} S_{ds} = & \quad \gamma\omega^2 F \\ \gamma = & \ -2.33 \cdot 10^{-5} \quad (\frac{\hat{a}}{\hat{a}_{PM}})\omega^{-1} \end{aligned} \quad (4.11)$$

where E is the total energy, **g** the constant of gravity, $\hat{a}_{PM} = 4.57 \cdot 10^{-3}$ and $\hat{a} = E\omega^4 g^2$. The coefficient γ was obtained by tuning the WAM model to reproduce a Pierson-Moskowitz spectrum as a final stage of the growth (Komen et al. 1984).

For the input source function the formula derived from measurements by Snyder et al. (1981) has been adopted, scaling it with the friction velocity $u_* = \sqrt{\tau\rho^{-1}}$ (Komen et al. 1984):

$$\begin{aligned} S_{in} = & \quad \beta F \\ \beta = & \ max \quad [0., 0.25\frac{\rho_a}{\rho_w}(28.\frac{u_*}{c}\cos\theta_{ww} - 1)] \end{aligned} \quad (4.12)$$

where θ_{ww} is the angle between wind direction and wave propagation direction, c is the wave phase speed, ρ_a and ρ_w are air and water density respectively. If the wind speed is used as input, then u_* is derived using the drag coefficient C_D :

$$u_*^2 = C_D U_{10}^2 \quad (4.13)$$

where U_{10} is the wind speed at 10m level.

Finally the bottom friction S_{bf} has been taken from the JONSWAP study

$$S_{bf} = -\frac{0.038\omega^2}{g^2 \sinh^2(kH)}F \quad (4.14)$$

The description of the WAM model can be found in the paper by the WAMDI group (1988) and exhaustive descriptions of first and second generation wave models can be found in SWAMP (1985). Without entering into any further detail about wave modelling the remaining aim of this section is to show the results of a wave model implementation in relation to the description of a "set-up" event in fron of Venice, in The Adriatic Sea.

A WAM model run has been carried out, producing the description of the evolution of the wave field over the Adriatic Sea. The spectra are computed in each point of a two-dimensional grid; they are representative of the wave condition in the open sea and not of the condition close to the shore. Starting from the hindcasted wave condition offshore, the "set-up" in front of one of the inlets of the Venice lagoon has been evaluated with a one dimensional model of the "set-up". The set up model assumes equilibrium conditions and it computes the sea level from the point where the wave spectrum has been produced by the wave model until the shore. This evaluation of the set-up has been added to sea level enhancement that was predicted by a storm surge model to obtain the overall sea level variation. Fig. a) shows the recorded and astronomical tide at Venice. Fig. b) shows the recorded storm surge level (difference between the two graphs in the previous figures) and the prediction of the storm surge model. Fig. c) shows the wave height produced by the WAM model, the evaluated and the recorded set-up at the lagoon inlet. Fig. d) shows the recorded storm surge level and the corresponding model result, obtained adding to the storm surge model prediction that is shown in b) the evaluated set-up that is shown in c).

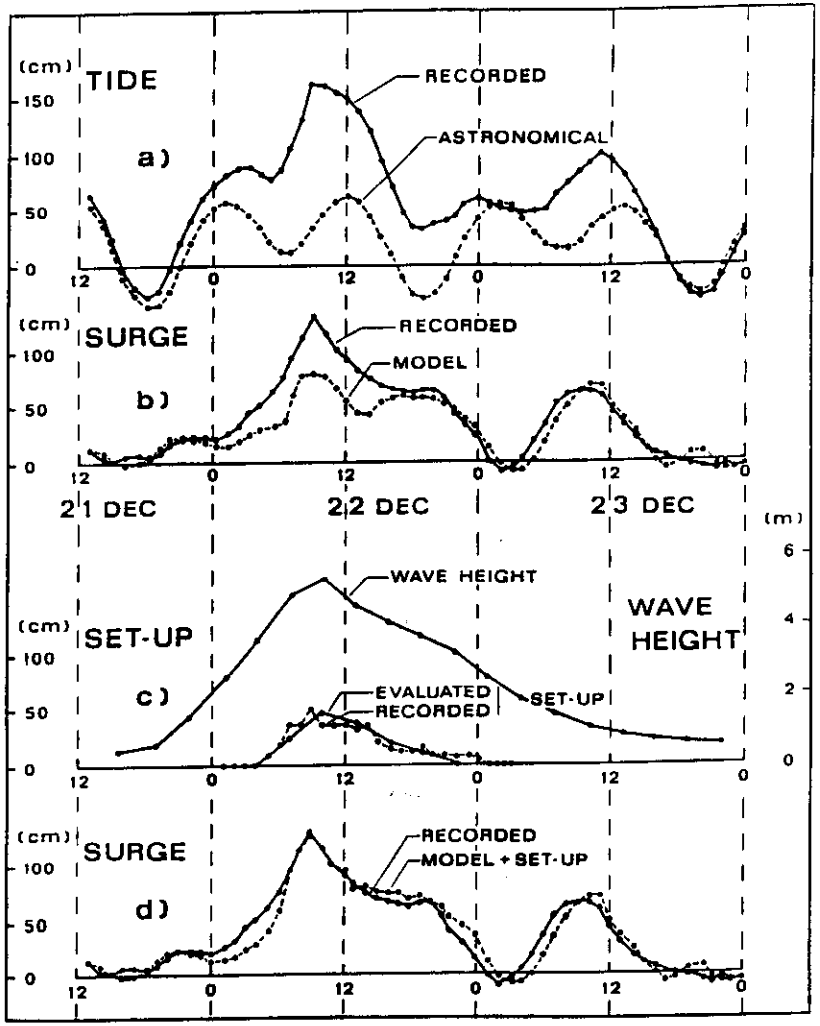

Fig. 4. a) Recorded and astronomical tide at Venice; b) recorded storm surge level (difference of the two graphs in a) and model prediction; c) wave height at the tower, evaluated and recorded set-up at the harbour entrance; d) recorded storm surge level (same as b) and corresponding model result (addition of prediction in b) and recorded set-up in c).

Bibliography

[1] Cavaleri L., L.Bertotti and P.Lionello (1990): 'Extreme storms in the Adriatic Sea', 22^{nd} Coastal Engineering Conference, Delft, The Netherlands, June 1990.

[2] Leblond P.H. and L.A. Mysak (1978): 'Waves in the ocean' Elsevier Oceanography Series, Amsterdam, PP.602

[3] Phillips O. M. (19): 'The sea surface", pp.229-237

[4] Tolman H.L. (1990) 'The influence of unsteady depths and currents of tides on wind wave propagation in shelf seas', J. Phys. Oceanogr., vol.20 pp.1166-1174

[5] Longuett Higgins M.S. and R.W.Stewart (1964): 'Radiation stress in water waves; a Physical discussion with applications', Deep-Sea Res., vol.11, pp529-551.

[6] Bretherton F.P. and C.J.Garret (1969): 'Wavetrains in inhomogeneous, moving media', Proc.Roy.Soc. A.302, pp.529-551.

[7] Whitham G.B. (1974) 'Linear and nonlinear waves', Wiley, New York, 636pp.

[8] The SWAMP group (1985): 'Ocean Wave Modeling', Plenum Press, pp.256.

[9] The WAMDI group 1988: 'The WAM model - A third generation ocean waves prediction model', J. Phys. Oceanogr. , vol.18, pp.1776-1810.

[10] Morse P.M. and K.U.Ingard (1968): "Theoretical Acoustic", Mc Graw-Hill, New York, 927pp.