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INTRODUCTION TO FUNCTIONAL METHODS, GAUGE THEORIES AND QUANTIZATION

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Please note: These are preliminary notes intended for internal distribution only.

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U.V. Divergences: from the loop integrations

$$\Gamma^{(N)}(p) \sim \int_{\text{Euclidean}} \prod_1^L d^d k_\ell \prod_1^I \frac{1}{(\Sigma k' + \Sigma p')^2 + m^2} \qquad d = 4 - \varepsilon$$

Overall divergence: rescale $k_\ell = Q\, k_\ell^0$

write $1 = \int d^d \bar{K}\, \delta^{(d)}(\bar{K} - \sum_1^L k_\ell)$ here

then rescale $\bar{K}_\mu = Q\, \bar{K}_\mu^0 \qquad ((\bar{K}_\mu^0)^2 = 1)$

$$\Rightarrow \Gamma^{(N)}(p) \sim \int \frac{dQ}{Q}\, Q^{Ld - 2I} \cdot \left(P_0(p) + P_2(p) \cdot Q^{-2} + \dots \right) =$$

$$= \frac{P_0(p)}{2I - Ld} + \frac{P_2(p)}{2I + 2 - Ld} + \dots$$

divergent for $L \cdot d - 2I = n$ integer $(n = 0, 2, \dots)$

$$= \frac{1}{\varepsilon} \frac{P_0(p)}{L} \qquad \text{for } L \cdot 4 - 2I = 0 \quad (\text{log div})$$

$$= \frac{1}{\varepsilon} \frac{P_2(p)}{L} \qquad \text{for } L \cdot 4 - 2I = 2 \quad (\text{quadratic div})$$

Theorem: the divergences for $\varepsilon \to 0$ are of the form

$$\Gamma^{(N)}(p) \sim \frac{P(p)}{\varepsilon^n}$$

where $P(p)$ is a polynomial in the external momenta p_μ whose non negative degree is given by dimensionality

examples ($\lambda \phi^4$ in $d = 4$)

$[\Gamma^{(2)}(p)] = p^2$ then $\Gamma^{(2)}(p) \sim \frac{\alpha p^2 + \beta M^2}{\varepsilon^n}$

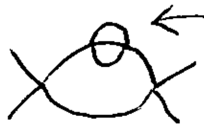
$[\Gamma^{(4)}(p)] = 1$ then $\Gamma^{(4)}(p) \sim \frac{c}{\varepsilon^n}$

$[\Gamma^{(n>4)}(p)] = p^{-(n-4)} \Rightarrow \Gamma^{(n>4)}$ = convergent

here the coupling constant is dimensionless $[\lambda] = 1$ therefore the conclusion is independent of the perturbation theory order λ^ℓ: renormalizable theory

(if $[\lambda] = p^{-2}$ then unrenormalizable theory!)

Of course there are also subdivergences

example  subdivergent

iterative pattern: we assume subdivergencies have already taken care by at the previous orders.

In renormalizable theories the divergent terms in Γ have the same form as the classical Lagrangian:

$$\Gamma_c^{(2)} = p^2 - m_0^2 \qquad \Gamma_c^{(4)} = -\lambda_0$$

$\Rightarrow$ Take in the classical Lagrangian all the terms $\Gamma_c^{(n)}$ whose dimensionality is nonnegative:

$$\mathcal{L} = \frac{1}{2}\left((\partial\varphi)^2 - m_0^2\varphi^2\right) - \frac{\lambda_0}{4!}\varphi^4 + \cancel{\frac{g_0}{6!}\varphi^6}$$

because $[g_0] = p^{-2}$

Make redefinitions:

$$\varphi = Z^{1/2}\varphi_R \qquad \lambda_0 = \lambda_R \mu^\varepsilon Z_1 Z^{-2} \qquad m_0^2 = m_R^2 + \delta m^2$$

(for $d = 4-\varepsilon$ $[\lambda_0] = p^\varepsilon$ $[\lambda_R] = 1$)

Rewrite thus the renorm Lagrangian:

$$\mathcal{L} = \frac{1}{2}\left\{(\partial\varphi_R)^2 - m_R^2\varphi_R^2\right\} - \frac{\lambda_R\mu^\varepsilon}{4!}\varphi_R^4$$

$$+ \frac{1}{2}(Z-1)\left\{(\partial\varphi_R)^2 - m_R^2\varphi_R^2\right\} - \frac{Z\delta m^2}{2}\varphi_R^2 - \frac{\lambda_R\mu^\varepsilon}{4!}(Z_1-1)\varphi_R^4$$

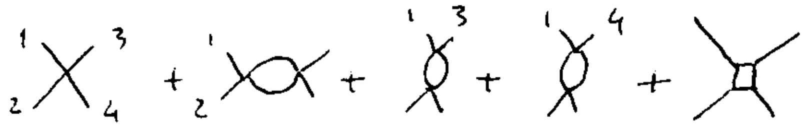
expand

$$\left.\begin{array}{c} Z-1 \\ Z_1-1 \\ Z\delta m^2 \end{array}\right\} = \sum_{n\geq 1}^{\infty} \lambda_R^n a_n$$

use perturbation expansion in λ_R

fix a_n requiring cancellation of poles in ε

Examples:

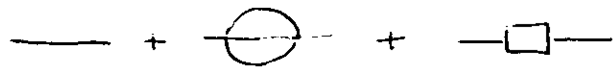
$\Gamma_R^{(4)} =$  $=$

$-\lambda_R \quad \underbrace{\qquad\qquad}_{O(\lambda_R^2)} \quad \underbrace{-\lambda_R\,(Z_1-1)}_{O(\lambda_R^2)}$ $\qquad Z_1 - 1 = \lambda_R\, a_1(Z_1$

$$= -\lambda_R + \cancel{\lambda_R^2 \frac{c}{\varepsilon}} + \lambda_R^2 f_R(p) - \cancel{\lambda_R^2 a_1(Z_1)}$$

$$= -\lambda_R + \lambda_R^2 f_R(p) \quad \text{finite} \qquad \left(\frac{\mu^\varepsilon}{\varepsilon} \to \frac{1}{\varepsilon} + \ln\mu \right)$$

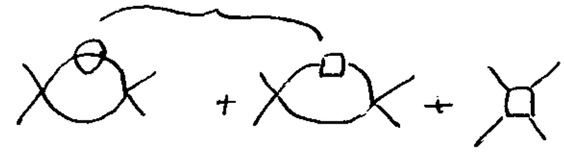
remember ...

$\Gamma_R^{(2)} =$ 

$$p^2 - m_R^2 + \cancel{\lambda_R^2 \frac{\alpha p^2 + \beta m^2}{\varepsilon}} + \cancel{(Z-1)} p^2 - \cancel{Z\delta m^2}$$
$$+ \lambda_R^2\, g_R(p^2)$$

$$= p^2 - m_R^2 + \lambda_R^2\, g_R(p^2)$$

use here

At next order $O(\lambda_R^4)$ $\qquad \Gamma_R^{(4)} =$ 

$$= \cancel{\lambda_R^4 \frac{c}{\varepsilon^2}} + \lambda_R^4\, F_R(p) + \cancel{\lambda_R^4 a_3(Z_1)}$$

now

from $\lambda_R(Z_1-1) = \lambda_R^2 a_1 + \lambda_R^4 a_3 + \dots$

Every step fixes a new term

$a_1, a_2, a_3, \dots$

$\Rightarrow \Gamma_R^{(N)} = \text{finite}$ $\qquad$ (the physical $m^2 = m_R^2$ + finite correction computable

etc.)

Gauge theories

Covariant derivative $\nabla(A)\psi = (\partial + \hat{A})\psi$

$\hat{A} = i t_\alpha A_{\alpha\mu}$

gauge transf $\psi^\Omega = \Omega\psi$ $\Omega = e^{i\omega_\alpha t_\alpha}$ $\omega = \omega(x)$ local

require $\nabla(A^\Omega)\psi^\Omega = \Omega\,\nabla(A)\psi \Rightarrow \hat{A}^\Omega = \Omega\hat{A}\Omega^{-1} + \Omega\partial\Omega^{-1}$

$\bar{\psi}^\Omega = \bar{\psi}\Omega^{-1}$ $\bar{\psi}\gamma\nabla(A)\psi$ = invariant

Infinitesimal form: $\delta\psi = i\omega t\psi$ $[t_\alpha t_\beta] = i f_{\alpha\beta\gamma} t_\gamma$

our convention $\mathrm{Tr}(t_\alpha t_\beta) = \frac{1}{2}\delta_{\alpha\beta}$

$f_{\alpha\rho\sigma}$ = completely antisymm $= i(\bar{\theta}_\alpha)_{\rho\sigma}$ $[\bar{\theta}_\alpha \bar{\theta}_\beta] = i f_{\alpha\beta\gamma}\bar{\theta}_\gamma$

adjoint repr.

$\Rightarrow \delta A_\alpha = i\omega_\gamma(\bar{\theta}_\gamma)_{\alpha\beta}A_\beta - \partial\omega_\alpha = -(\partial + i\bar{\theta}_\beta A_\beta)\omega = -\nabla(A)\omega$

we can also introduce other fields ϕ, $\delta\phi = i\theta\varphi$

$\hat{F}_{\mu\nu} = [\nabla_\mu(A)\nabla_\nu(A)] = \partial\hat{A} - \partial\hat{A} + [\hat{A}\hat{A}] = (\partial A - \partial A - f AA)it$

$= it_\alpha F^\alpha_{\mu\nu}$

$\hat{F}^\Omega = \Omega F \Omega^{-1}$

$\Rightarrow \mathcal{L} = -\frac{1}{2g^2}\mathrm{Tr}\,\hat{F}^2 + \bar{\psi}(i\gamma\nabla(A) - m)\psi$ invariant

(funct. int. measure also invariant)

$Z = N\int \mathcal{D}A\mathcal{D}\psi\mathcal{D}\bar{\psi}\, e^{iS(A+\bar{\psi})}$

starting point ↑ : <u>propagator</u> a problem with the gauge field

$$\mathcal{L}(A) = -\frac{1}{4}(k_\mu A_\nu - k_\nu A_\mu)^2 = -\frac{1}{2} K^2 A_\mu \underbrace{\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)}_{P^\perp_{\mu\nu}} A_\nu \qquad P^L_{\mu\nu} = \frac{k_\mu k_\nu}{k^2}$$

$$P^\perp + P^L = 1$$

if I had $\quad A_\mu\left(\alpha P^\perp_{\mu\nu} + \beta P^L_{\mu\nu}\right)A_\nu$

$$P^\perp P^\perp = P^\perp \qquad P^\perp P^L = 0$$
$$P^L P^L = P^L$$

then $\quad (\alpha P^\perp_{\mu\nu} + \beta P^L_{\mu\nu})^{-1} = \frac{1}{\alpha} P^\perp_{\mu\nu} + \frac{1}{\beta} P^L_{\mu\nu}$

in our case $\beta = 0$ $(\alpha = K^2)$ therefore $(\alpha P^\perp + \beta P^L)^{-1} = \,?$

$\Rightarrow$ The functional integration over A_μ is divergent

$$N \int \mathcal{D}A_\mu \, e^{-\alpha A^{\perp 2}} = \text{divergent}$$

there are redundant degrees of freedom (gauge)

In order to fix them (reabsorb them into N)

write

$$1 = \int d\Omega \, \delta(\partial A^\Omega - B)\, \Delta(A^\Omega) \qquad \forall B$$

($f(A^\Omega)$ in general)

$$A^\Omega = A^{\bar\Omega} - \nabla(A^{\bar\Omega})\omega \qquad \text{for } \Omega = (1 + it\omega)\bar\Omega \qquad (d\Omega \approx d\omega)$$

$$\Rightarrow \left.\frac{\delta\, \partial A^\Omega}{\delta \omega}\right|_{\Omega=\bar\Omega} = -\partial \nabla(A^\Omega) \qquad \Rightarrow \quad \Delta(A^\Omega) = |\det \partial \nabla(A^\Omega)|$$

then $(\forall B)$

$$Z = N\int \mathcal{D}A\,\mathcal{D}\psi\,\mathcal{D}\bar\psi\; e^{iS(A,\psi,\bar\psi)} = N\int \underbrace{\mathcal{D}A\,\mathcal{D}\psi\,\mathcal{D}\bar\psi\; e^{iS(A,\psi,\bar\psi)}}\, d\Omega\,\delta(\partial A^\Omega - B)\cdot\Delta(A^\Omega)$$

$$= N\int \underbrace{\mathcal{D}A^\Omega\,\mathcal{D}\psi^\Omega\,\mathcal{D}\bar\psi^\Omega\; e^{iS(A^\Omega\psi^\Omega\bar\psi^\Omega)}}\; d\Omega\;\delta(\partial A^\Omega - B)\,\Delta(A^\Omega) =$$

because it is invariant   Call $A \equiv A^\Omega$ etc
(ok for sources if gauge invariant)

$$= \underbrace{\left(N\int d\Omega\right)}_{N'} \int \mathcal{D}A\,\mathcal{D}\psi\,\mathcal{D}\bar\psi\; e^{iS(A\psi\bar\psi)}\,\delta(\partial A - B)\,\Delta(A) \qquad \forall B$$

$$Z = \text{cost}\int \mathcal{D}B\, e^{\frac{i}{2\alpha}\int B^2}\cdot Z = \text{cost}\int \mathcal{D}A\,\mathcal{D}\psi\,\mathcal{D}\bar\psi\; e^{iS(A\psi\bar\psi)+\frac{i}{2\alpha}\int(\partial A)^2}\,\Delta(A)$$

$$\Delta(A) = \int \mathcal{D}c\,\mathcal{D}\bar c\; e^{i\int \bar c\,\partial\nabla(A)c} = \qquad c_\alpha, \bar c_\beta \text{ anticommuting}$$

$$= \det \partial\nabla(A)$$

$$\Rightarrow Z = \text{cost}\int \mathcal{D}A\,\mathcal{D}\psi\,\mathcal{D}\bar\psi\,\mathcal{D}c\,\mathcal{D}\bar c\; e^{iS_{gf}(A,\psi,\bar\psi,c,\bar c)}$$

$$S_{gf} = -\frac{1}{4g^2}F_{\mu\nu\alpha}F^{\mu\nu\alpha} + \frac{1}{2\alpha}\left(\partial_\mu A^\alpha_\mu\right)^2 + \bar\psi\left(i\gamma\nabla(A) - m\right)\psi\} + \bar c\,\partial_\mu\left(\partial_\mu + i\bar\theta_\gamma A_{\mu\gamma}\right)c$$

Now the quadratic part in A^α_μ is:

$$-\frac{1}{2}K^2 A_\mu P^\perp_{\mu\nu}A_\nu + \frac{1}{2\alpha}A_\mu P^L_{\mu\nu}A_\nu$$

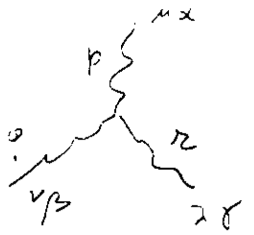
propagator $\quad {}^{\alpha}_{\mu}\!\sim\!\sim\!\sim{}^{\beta}_{\nu} = -i\,\delta_{\alpha\beta}\left(g_{\mu\nu} - (1+\alpha)\frac{K_\mu K_\nu}{K^2}\right)\frac{1}{K^2} = -\frac{i\delta_{\alpha\beta}g_{\mu\nu}}{K^2}$

Feynman gauge for $\alpha = -1$

the result is α indep. (for gauge inv. quantities)

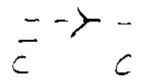
Other F-rules (rescale $A \to gA$ so that $-\frac{1}{4} F_0^2$ + interacts)

$$i\mathcal{L} = + \dots + \frac{1}{2} g\, f_{\alpha\beta\gamma} (\partial_\mu A_{\nu\alpha} - \partial_\nu A_{\mu\alpha}) A_{\mu\beta} A_{\nu\gamma} - \frac{1}{4} g^2 f f A A$$

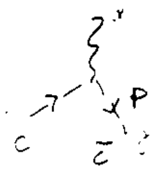


$$= g\, f_{\alpha\beta\gamma} \left\{ (r-p)_\nu\, g_{\lambda\mu} + (p-q)_\lambda\, g_{\mu\nu} + (q-r)_\mu\, g_{\nu\lambda} \right\}$$

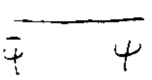
$$= -i g^2 f f \,(g g - g g) + \text{perms})$$



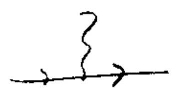
ghost prop $\frac{i}{p^2}$



$g\, p_\mu f_{\alpha\beta\gamma}$



fermion $\frac{i}{\gamma p - m}$



$= -i g \gamma_\mu t_\alpha$

$\Rightarrow$ Compute Loops (remember a (−) for loops of anticomm fields, ψ and c)

U.V. divergences: since we have "broken" gauge invariance, how can we be sure that we do not need counterterms else than Lagrangian?

BRS invariance: (29) ~~(23)~~

def of BRS transf $\qquad$ $\delta\lambda$ global anticomm $(\delta\lambda^2 = 0)$

$$\delta A_\alpha = -\nabla(A)(\delta\lambda c_\alpha) = i\vartheta\delta\lambda c A - \partial\delta\lambda c$$

$$\delta\psi = +it(\delta\lambda c)\psi$$
$$\delta\bar\psi = +\bar\psi(-it)(\delta\lambda c)$$
$$\delta c_\alpha = -\delta\lambda \tfrac{1}{2} f_{\alpha\beta\gamma} c_\beta c_\gamma$$

$$\frac{\delta u}{\delta\lambda} = F(u)$$

property: $\delta F(u) = 0$

$$\delta\bar c = \delta\lambda \tfrac{1}{\alpha}\partial A \qquad (\text{else } \delta\bar c = \delta\lambda B \qquad \delta B = 0)$$

$$\mathcal{L}_{gf} = \mathcal{L}_0 + \tfrac{1}{2\alpha}(\partial A)^2 + \bar c\,\partial\nabla c = \mathcal{L} + B\partial A - \tfrac{\alpha}{2}B^2 + \bar c\,\partial\nabla c$$

$\Rightarrow$ funct. int. measure invariant.

Properties 1) representing BRS by Q then $Q^2 = 0$

2) $\delta S_{gf} = 0$

Introduce ad hoc sources: $K^A, K^\psi, K^{\bar\psi}, L$

$$S_{TOT} = S_{gf} + \int K^A\cdot(\nabla(A)\,c) + \int K^\psi\cdot(tc\psi) + \int(\bar\psi tc)K^{\bar\psi} + \int L\, ffcc$$

$\delta S_{TOT} = 0$ (from 1) and 2))

and also $\displaystyle\sum_{\varphi = A,\psi,\bar\psi} \frac{\delta S_T}{\delta\varphi}\frac{\delta S_T}{\delta K^\varphi} + \frac{\delta S_T}{\delta c}\frac{\delta S_T}{\delta L} - \frac{\delta S_T}{\delta\bar c}\frac{1}{\alpha}\partial A = 0$

$\uparrow$

Quantum theory: introduce sources for generating Green functions

$$Z(J_\varphi\, \eta\, \bar\eta) = \int \mathcal{D}\varphi\, \mathcal{D}c\, \mathcal{D}\bar c\; e^{i(S_T + J_\varphi \varphi + \bar\eta c + \bar c \eta)}$$

make a BRS change of variables

$u' = u + \delta u$ only the sources term $J_\varphi \varphi + \bar\eta c + \bar c \eta$ change

$$\Rightarrow \langle J_\varphi \delta\varphi + \bar\eta \delta c + \delta \bar c\, \eta \rangle = 0 \qquad (*)$$

$W = -i \ln Z \quad \Rightarrow \quad \sum_\varphi J_\varphi \frac{\delta W}{\delta \varphi} + \bar\eta \frac{\delta W}{\delta L} - \eta \frac{1}{\alpha} \langle \partial A \rangle = 0$

$W = W(J_\varphi\, \eta\, \bar\eta ; K^\varphi L)$

Define $\Gamma = W - J_\varphi \varphi - \bar\eta c - \bar c \eta \qquad \Gamma(\varphi, c, \bar c ; K^\varphi L)$

$$\Rightarrow \sum_\varphi \frac{\delta\Gamma}{\delta\varphi}\frac{\delta\Gamma}{\delta K^\varphi} + \frac{\delta\Gamma}{\delta c}\frac{\delta\Gamma}{\delta L} - \frac{\delta\Gamma}{\delta \bar c}\frac{1}{\alpha}\partial A = 0$$

(now A is like φ, expansion parameter)

The quantum action Γ is also BRS inv:
this eq. encodes all the S.T.W identities.

Example: from $(*)$ take only J_A and $\eta \neq 0$:

$$\int dx\, J_A(x) \langle \nabla(A) c \rangle + \frac{1}{\alpha}\int dx\, \eta(x) \langle \partial A(x) \rangle = 0$$

taking $\frac{\delta}{\delta J_A(y)}$ and $\frac{\delta}{\delta \eta(z)}$ and then $J_A = \eta = 0$, further $\frac{\partial}{\partial y}$

$$\underbrace{\langle \bar c(z)\, \partial\nabla c(y) \rangle}_{i\delta(z-y)} + \frac{1}{\alpha}\langle \partial A(z)\, \partial A(y) \rangle = 0 \Rightarrow p_\mu p_\nu \langle A_\mu A_\nu \rangle = i\alpha$$

Since $p_\mu p_\nu \langle A_\mu A_\nu \rangle_{\text{tree}} = i\alpha$

$\Rightarrow \quad p_\mu p_\nu \langle A_\mu A_\nu \rangle_{\text{Loop}} = 0$ the quantum corrections to the A propagator are transverse.

Possible divergent part of Γ: by dim analysis and global invariances

(example L: gh # = -2 dim = 2)

$$\Gamma_{div} = Q_L \, L_\alpha f_{\alpha\beta\gamma} c_\beta c_\gamma + K^A (Q_1 \partial + Q_A \, i \theta A) c +$$

$$+ K^\Psi Q_\Psi \, i t c \Psi + \bar\Psi i t c \, Q_{\bar\Psi} K^{\bar\Psi} +$$

$$+ \tilde\Gamma_{div} (A \Psi \bar\Psi c \bar c)$$

a general polyn compatible with the global invariances & dimensionality

Require BRS identities for Γ_{div}:

$$\Rightarrow \quad Q_\Psi = Q_{\bar\Psi} = Q_A = Q_L$$

$$\Rightarrow \tilde\Gamma_{div} = Q_1 \bar c (\partial + i \frac{Q_L}{Q_1} \theta A) c + \frac{1}{2\alpha} (\partial A)^2 + \Gamma_{div}(A \Psi \bar\Psi)$$

$\Rightarrow \quad \Gamma_{div}(A, \Psi, \bar\Psi)$ is gauge inv with respect to the covariant derivative $\partial + i \frac{Q_L}{Q_1} \theta A$

$$\partial + i \frac{Q_L}{Q_1} t A$$

like a redefinition of the normalisation of the structure constants $\partial = Q_L/Q_1$.

$\Rightarrow$ By rescaling $A \to gA$ as usual,

$\tilde{\Gamma}_{div}$ has the same form as $\left(Q_0 \equiv \frac{Q_c}{Q_1} \right)$

$$\mathcal{L} = \frac{Z}{4} \left(\partial A - \partial A - Q_0 g f A A \right)^2 + \tilde{Z} \bar{c}\, \partial (\partial + i Q_0 g \bar{\theta} A) c$$

$$+ Z_\psi \bar{\psi} \left[i\gamma (\partial + i Q_0 g t A) - (m + \delta m) \right] \psi$$

$$= -\frac{Z}{4} (\partial A - \partial A)^2 + \frac{Z_1 g}{2} (\partial A - \partial A) f A A + \frac{Z_2 g^2}{4} f A A f A A$$

$$+ \tilde{Z} \bar{c}\, \partial \partial c - \tilde{Z}_1 g\, \partial \bar{c}\, i \bar{\theta} A c$$

$$+ Z_\psi \bar{\psi}\, i \partial\!\!\!/ \psi + Z_{1\psi} g\, \bar{\psi}\, i t A\!\!\!/ \psi - \bar{\psi} (m + \delta m) \psi$$

$$Z_1 = Z Q_0 \qquad \tilde{Z}_1 = \tilde{Z} Q_0 \qquad Z_{1\psi} = Z_\psi Q_0 \qquad Z_2 = Z Q_0^2$$

$\Rightarrow$ identities $\quad \dfrac{Z_1}{Z} = \dfrac{\tilde{Z}_1}{\tilde{Z}} = \dfrac{Z_{1\psi}}{Z_\psi}$

$\mathcal{L}$ can be obtained from the Lagrangian containing the <u>bare</u> fields and couplings

$$A_B = Z^{1/2} A \qquad c_B = \tilde{Z}^{1/2} c \qquad \psi_B = Z_\psi^{1/2} \psi$$

$$g_B = Z_1 Z^{-3/2} g = Z_{1\psi} Z_\psi^{-1} Z^{-1/2} g = \tilde{Z}_1 \tilde{Z}^{-1} Z^{-1/2} g$$

$$\alpha_B = Z \alpha$$

To be used for computing β functions and then making predictions for experimental observations.