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**SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY**

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# TO GRAND UNIFICATION AND BEYOND

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Please note: These are preliminary notes intended for internal distribution only.

## Standard Model


- $SU(2)_L \times U(1)_Y \times SU(3)^c \xrightarrow[\sim 300\text{ GeV}]{\langle\phi\rangle} U(1)_{em} \times SU(3)^c$
- 3 families of fermions $(e, \mu \,\&\, \tau)$

$$\begin{array}{c} \leftarrow SU(3)^{Color} \rightarrow \\ SU(2)_L \updownarrow \begin{pmatrix} u_r \\ d_r \end{pmatrix}_L \begin{pmatrix} u_y \\ d_y \end{pmatrix}_L \begin{pmatrix} u_b \\ d_b \end{pmatrix}_L \\ U(1)_Y \rightarrow \quad 1/3 \quad 1/3 \quad 1/3 \end{array} \quad \vdots \quad \begin{array}{c} \leftarrow SU(3)^{col} \rightarrow \\ (u_r \quad u_y \quad u_b)_R \\ \leftarrow Y_W = 4/3 \cdots \rightarrow \\ (d_r \quad d_y \quad d_b)_R \\ \leftarrow Y_W = -2/3 \rightarrow \end{array}$$

$$SU(2)_L \updownarrow \begin{pmatrix} \nu^e \\ e^- \end{pmatrix}_L \rightarrow Y_W = -1 \quad \vdots \quad e^-_R \rightarrow Y_W = -2$$

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Same for $\mu$ and $\tau$- families

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- one Higgs doublet.

→ Strikingly successful in describing Low Energy Physics ($E_{CM} \lesssim 100$ GeV)

→ But can not be the whole story (1972).

## Motivations for Going Beyond The Standard Model


### (A). CONCEPTUAL SHORTCOMINGS

(i) Why there are quarks and leptons?

(ii) Why there are weak, EM & Strong Int., Weak int. being universal w.r.t. $q$ & $\ell$, while strong int. are not?

→ 3 gauge couplings $g_3, g_2, g_1$

(iii) Arbitrariness in the choice of $Y_W$ ?

(iv) $\boxed{\text{Why } Q_{em} \text{ quantized ?}}$

(v) Why $e^-$ and $p$ same sign of Long. Pol. in $\beta$-decay rather than $e^+$ & $p$?

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(vi) Arbitrariness in Higgs Sector

(vii) WHY $N = 3$ or $N > 3$ Families?

## (B) Large No. of Parameters

| | | |
|---|---|---|
| Gauge Couplings $(g_1, g_2, g_3)$ | $\longrightarrow$ | 3 |
| Fermion masses $(u,d,e),(c,s,\mu),(t,b,\tau)$ | $\longrightarrow$ | 9 |
| KMC angles $(\theta_{1,2,3})$ | $\longrightarrow$ | 3 |
| $m_W, m_\phi$ | $\longrightarrow$ | 2 |
| CP phase $(\delta)$ | $\longrightarrow$ | 1 |
| $\bar{\Theta} = \Theta_{QCD} - \Theta_{WK}$ | $\longrightarrow$ | 1 |
| | | 19 |

$\Rightarrow$ $\boxed{\text{There must exist fundamental new physics beyond standard model}}$

$\rightarrow$ At what energies should one expect to see at least some signals of such new physics.?



## Remark: Diversities in Particle Physics

### (i) Among Particles:

In terms of $Q_{em}$ $\rightarrow$ $\begin{pmatrix}\nu\\ e^-\end{pmatrix}\begin{matrix}Q_{em}\\ 0\\ -1\end{matrix}$, $\begin{pmatrix}u\\ d\end{pmatrix}\begin{matrix}Q_{em}\\ \rightarrow 2/3\\ \rightarrow -1/3\end{matrix}$.

In terms of SU(3)-color charge $\Big\}$ $\begin{pmatrix}u_r & u_y & u_b\\ d_r & d_y & d_b\end{pmatrix}$, $\begin{pmatrix}\nu\\ e^-\end{pmatrix}$

| | $\underset{\sim}{3}^c$ | $\underset{\sim}{1}^c$ |
|---|---|---|
| $B \rightarrow$ | $1/3$ | 0 |
| $L \rightarrow$ | 0 | 1 |

### (ii) Among Forces

Strong $\rightarrow g^2/4\pi \sim 1$

EM $\rightarrow e^2/4\pi \sim 1/137$

Weak $\rightarrow G_F m_p^2 \sim 10^{-5}$

Gravity $\rightarrow G_N m_p^2 \sim 10^{-39}$

### (iii) Among Scales: From $M_{Planck}$ to $m_\nu$

$$\boxed{\left(M_{Pl}/m_{\nu_e}\right) \gtrsim 10^{27}}$$

## Diversity in Scales

$>10^{27}$

$M_{Planck} \sim 10^{19}$ GeV

$10^8$ ↓

$M_{Intermediate} \sim 10^{11} - 10^{13}$ GeV → SUSY Breaking; → Inflation / B-Excess; → P-Q Breaking

$10^8$ ↓

SUSY Breaking Mass splittings

$\delta m_S = |m_{Boson} - m_{Fermion}| \sim 1$ TeV

↓

$m_{W,Z} \sim 100$ GeV

↓

$m_{top} \sim 60$ GeV ?

↓

$m_b \sim 5$ GeV $\longleftrightarrow$ $m_\tau \sim 1.8$ GeV

↓

$m_c \sim 1.3$ GeV

↓

$m_s \sim 150$ MeV $\longleftrightarrow$ $m_\mu \sim 106$ MeV

↓

$m_u \sim 4$ MeV

↓

$m_d \sim 9$ MeV $\longleftrightarrow$ $m_e \sim 0.5$ MeV

↓

$m_{\nu_e} < 30$ eV



## Going Beyond The Standard Model

- Diversity among the Particles
- Diversity among the Forces (S, E, W)
- The major Conceptual Shortcomings (i) — (V)

↓ Resolved

## Grand Unification:

$F = \{q, \ell\}$

↓

Symmetry $G \supset SU(2)_L \times U(1)_Y \times SU(3)^C$

↓

Will need to go "beyond" grand unif. one way or another.

↓

Supersymmetry; supergravity

+ ?

Composite $q, \ell, \phi$

→ Superstrings

↓

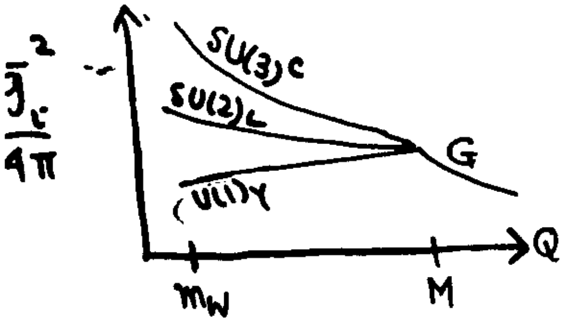
• Unify gravity with W, E, S

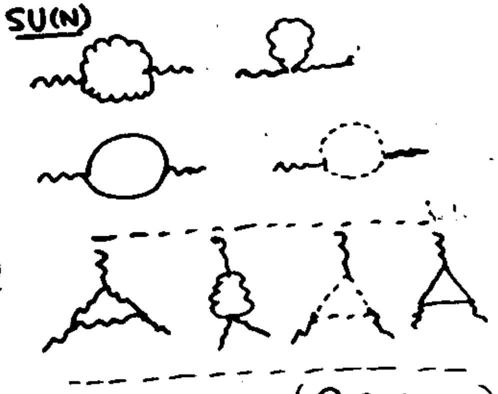
## Towards Grand Unification

Unite $\{q, \ell\} = F \longrightarrow$ Symm. Group $G$.

Generate $\{S, E, W\}$ by gauging $G$. (JCP, AS 72, 73) (G & G, 74)

$$G \xrightarrow[M \gg 1\,\mathrm{TeV}]{\langle \Sigma \rangle} SU(2)_L \times U(1)_Y \times SU(3)^c \xrightarrow[\sim 300\,\mathrm{GeV}]{\langle \Phi \rangle} U(1)_{em} \times SU(3)^c$$




(GQW, 74)

$$\boxed{\frac{1}{g_i^2(Q)} = \frac{1}{g_i^2(M)} - 2 b_i \ln\frac{M}{Q}} \qquad i = 1, 2, 3$$

$$SU(N): \quad b_N = \frac{1}{16\pi^2}\left[\frac{11N}{3} - \frac{2}{3} n_f - \frac{1}{3} t_2(s)\right]$$

$$\frac{1}{g_3^2(Q)} - \frac{1}{g_2^2(Q)} = -2\underbrace{(b_3 - b_2)}_{>0} \ln\frac{M}{Q} \qquad (Q < M)$$

$$Q \sim m_W \ll M \longrightarrow g_3(m_W) \gg g_2(m_W)$$

$$\boxed{M = m_W \exp\left[\frac{|O(1)|}{\alpha_{em}}\right] \gg m_W}$$



## To Grand Unification with SU(4)-Color

(JCP, AS, 72)

$$q_L^e = \overset{\longleftarrow\ SU(3)\text{-Col}\ \longrightarrow}{\begin{bmatrix} u_{red} & u_{yel} & u_{bl} \\ d_{red} & d_{yel} & d_{bl} \end{bmatrix}_L} \updownarrow SU(2)_L, \quad \ell_L^e = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \rightarrow 8$$

$$(u_{r,y,b})_R\,;\ (d_{r,y,b})_R\,;\ e_R^- \longrightarrow 7$$

15

Naturally Suggest $\Big\downarrow$ $(q, \ell)$ unification

$$F_L^e = \overset{\longleftarrow\ SU(4)\text{-Color}\ \longrightarrow}{\begin{bmatrix} u_r & u_y & u_b & u_{lilac} = \nu^e \\ d_r & d_y & d_b & d_{lilac} = e^- \end{bmatrix}_L} \updownarrow SU(2)_L \rightarrow 8$$

$$F_R^e = \begin{bmatrix} u_r & u_y & u_b & u_{lilac} = \nu^e \\ d_r & d_y & d_b & d_{lilac} = e^- \end{bmatrix}_R \updownarrow SU(2)_R \rightarrow 8$$

16

$$\boxed{\mathcal{G}_0 = SU(2)_L \times SU(2)_R \times SU(4)^{col}_{L+R}}$$

$$F_L^e = (2_L, 1, 4^c)\,; \quad F_R^e = (1, 2_R, 4^c)$$

- No abelian U(1)-Factor
- Matter Left-Right Symmetric ($\nu_L \leftrightarrow \nu_R$)
- $\boxed{\text{LEPTON NO. IS THE 4TH COLOR}}$

- $$\boxed{\mathcal{G}_0 = SU(2)_L \times SU(2)_R \times SU(4)^c_{L+R}}$$

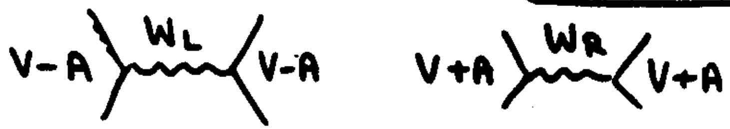
$$(\vec{W}_L) \qquad (\vec{W}_R) \qquad V(\underline{15}) \to \text{21 gauge Particles}$$

$$\vec{W}_L \leftrightarrow \vec{W}_R \; ; \; F_L \leftrightarrow F_R \; ; \; g_{2L} = g_{2R}$$

$$g_2 \left[ W_L \cdot (V-A) + W_R \cdot (V+A) \right].$$

$\Rightarrow$ Parity (P) and Charge Conj. (C) are exact Symm. of the Basic Lagrangian

$\to$ Spont. Symm. Breaking $\to$ $\boxed{m_{W_R} \gg m_{W_L}}$




$$g_2^2 \left[ \frac{(V-A)^2}{m^2_{W_L}} + \frac{(V+A)^2}{m^2_{W_R}} \right]. \quad \text{at } Q \ll m_{W_{L,R}}$$

$\to$ (V−A) Int. dominate at low energies

$\to$ Parity (P), C spont'ly violated

$\to$ At High "Energies" & temperatures $> m_{W_R}$ P and C restored.



Reminder

$\longleftarrow$ SU(4)-Color $\longrightarrow$

$$F_L^e = \begin{bmatrix} u_r & u_y & u_b & u_\ell = \nu^e \\ d_r & d_y & d_b & d_\ell = e^- \end{bmatrix}_L \quad \updownarrow SU(2)_L$$

$$F_R^e = \begin{bmatrix} u_r & u_y & u_b & u_\ell = \nu^e \\ d_r & d_y & d_b & d_\ell = e^- \end{bmatrix}_R \quad \updownarrow SU(2)_R$$

$$\boxed{\mathcal{G}_0 = SU(2)_L \times SU(2)_R \times SU(4)^{col}_{L+R}}$$

Lepton no. is the 4th color.

Has many desirable features even without embedding $\mathcal{G}_0$ in a simple group $G$. It explains:

✓ • Why $q$ and $\ell$ and why 3 forces with weak int. Universal w.r.t. $q$ & $\ell$ but Strong int. not.

✓ • Removes arbitrariness in choice of $Y_W$. Now, $\boxed{Y_W = I_{3R} + \sqrt{2/3}\, F_{15}}$

✓ • Explains quantization of electric charge

$$Q_{em} = I_{3L} + I_{3R} + \sqrt{2/3}\, F_{15}$$

$$\boxed{Q_{em} = I_{3L} + I_{3R} + \frac{B-L}{2}}$$

$Q_{e^-} = -\tfrac{1}{2} + 0 - \tfrac{1}{2} = -1$ ; $Q_p = +\tfrac{1}{2} + \tfrac{1}{2} = +1$

✓ $\boxed{Q_{e^-} = -Q_p}$

✓ • Explains why $e^-$ & $p$ (not $e^+$ & $p$) have same sign of long. pol. in $\beta$-decays.

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• $\mathcal{G}_0$ as a group is anomaly-free

• other elegant features

(a) • $L \leftrightarrow R$ Symmetry $\rightarrow$ P & C exact in Basic Lag $\rightarrow$ Violated spontaneously.

(b) • CP also violated spont'ly (Mohapatra, JCP)

(c) • Natural origin for $\nu$-masses due to compelling need for $\nu_R$ together with $\nu_L$, as well as a simple mechanism for smallness of $m(\nu_L^i) \ll m_{Dirac}(u^i)$.



## Spontaneous Breaking of $\mathcal{G}_0$

$$\mathcal{G}_0 = \underset{\downarrow \atop (W_L^\pm, W_L^3)}{SU(2)_L} \times \underset{\downarrow \atop (W_R^\pm, W_R^3)}{SU(2)_R} \times \underset{\downarrow \atop V(\underset{\sim}{15})}{SU(4)^C}$$

$$V(\underset{\sim}{15}) \; \left[\begin{array}{c:c} \underset{\text{gluons}}{V(\underset{\sim}{8})} - \frac{(\text{Const})}{3} S^0 (\mathbb{1})_{3\times 3} & \begin{matrix} X_1 \\ X_2 \\ X_3 \end{matrix} \\ \hdashline \bar{X}_1 \quad \bar{X}_2 \quad \bar{X}_3 & (\text{Const}).S^0 \end{array}\right] \rightarrow \underbrace{8}_{\substack{\downarrow \\ SU(3)^{color}}} + 3 + \bar{3} + 1$$

$$V(\underset{\sim}{8})^i_\mu \longrightarrow \bar{q}_\alpha \gamma_\mu \frac{(\lambda^i)_{\alpha\beta}}{2} q_\beta \quad (\alpha, \beta \rightarrow r, y, b)$$

$$X_\mu^{(\alpha)} \longrightarrow \bar{q}_\alpha \gamma_\mu \ell$$

$$\rightarrow S^0_\mu \; [\; \underbrace{\bar{q}_r \gamma_\mu q_r + \bar{q}_y \gamma_\mu q_y + \bar{q}_b \gamma_\mu q_b}_{\substack{\downarrow \\ B_q}} \underbrace{- 3 \bar{\ell} \gamma_\mu \ell}_{\substack{\downarrow \\ -3L}} \;]$$

$$F_{15} = \frac{1}{2\sqrt{6}} \begin{array}{c} \begin{matrix} r & \;\; y & \;\; b & \;\; \ell \end{matrix} \\ \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -3 \end{bmatrix} \end{array} \propto B - L$$

$B_q = 1$ for $q$, $B = \tfrac{1}{3}$ for $q$ $\rightarrow B_q = 3B$

$$I_{3L} = \begin{array}{cc} u_L & d_L \\ \end{array}\begin{bmatrix} 1/2 & 0 \\ 0 & -1/2 \end{bmatrix}, \quad I_{3R} = \begin{array}{cc} u_R & d_R \\ \end{array}\begin{bmatrix} 1/2 & 0 \\ 0 & -1/2 \end{bmatrix}$$

$$F_{15} = \frac{1}{2\sqrt{6}} \begin{array}{cccc} r & y & b & \ell \end{array} \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -3 \end{bmatrix} \begin{array}{l} \propto B_q - 3L \\ \propto B-L \end{array}$$

$$B-L = \begin{bmatrix} 1/3 & & & \\ & 1/3 & & \\ & & 1/3 & \\ & & & -1 \end{bmatrix}$$

$$SU(2)_L \times \underbrace{SU(2)_R \times SU(4)^{col}_{L+R}}$$

$$\Big\downarrow \; \langle \Sigma_R \rangle \gg 1\,\text{TeV}, \quad m_X \neq 0, \quad m_{W_R} \neq 0, \quad \cancel{P}, \cancel{C}$$

$$SU(2)_L \times U(1)_Y \times SU(3)^{col}$$

$$\Big\downarrow \; \langle \Phi \rangle = (2_L, 2_R, 1^c)$$

$$U(1)_{em} \times SU(3)^{col}$$

$$\boxed{Q_{em} = I_{3L} + \underbrace{I_{3R} + \frac{B-L}{2}}_{(Y_W/2)}}$$

$$\left(\frac{Y_W}{2}\right)_{st.\,Model} = I_{3R} + \frac{B-L}{2}$$



## Choice of $\Sigma_R$ and Neutrino Masses :

A specially desirable choice for $\Sigma_R$

$$\boxed{\Delta_R \sim (1_L, \underset{\sim}{3}_R, \underset{\sim}{10}^c)} \text{ of } SU(2)_L \times SU(2)_R \times SU(4)$$

$$\mathcal{G}_0 = SU(2)_L \times \underbrace{SU(2)_R \times SU(4)^c_{L+R}}$$

$$\Big\downarrow \; \langle \Delta_R \rangle = \langle \Delta_{44}^{I_{3R}=+1} \rangle \equiv v_R \neq 0$$

Only the component with quantum no. of "$\nu_R \nu_R$" is electrically neutral

- Breaks B−L
- Gives masses to X, $W_R$'s
- $\boxed{\text{Gives Majorana Mass to } \nu_R}$

$$\mathcal{G}_{st} = SU(2)_L \times U(1)_Y \times SU(3)^{col}$$

$$\boxed{h_M \left( \underbrace{F_R^T C^{-1} F_R}\langle \Delta_R^\dagger \rangle + L \leftrightarrow R \right) + h.c}$$

$$\left\{ (1, 2_R, 4^c) \otimes (1, 2_R, 4^c) \right\}_{Symm}$$

$$= (1_L, 3_R, 10^c) + (1_L, 1_R, 6^c)$$

$$\Big\downarrow \qquad |\Delta L| = 2$$

$$h_M \nu_R^T C^{-1} \nu_R \langle v_R \rangle + h.c \qquad \text{(MAJORANA MASS)}$$

$$\boxed{M_R = h_M v_R} \gg 1\,\text{TeV}$$

## Dirac Masses

$$h_D^i \; \overline{\nu_L^i} \, \nu_R^i \, \langle \phi \rangle + h.c. \qquad \underline{\lfloor i = e, \mu, \tau}$$

$$\downarrow$$

$$m(\nu^i)_D \sim h_D^i \langle \phi \rangle \sim m(u^i)$$

$$\simeq \underset{\substack{\uparrow \\ \nu_e}}{\text{few MeV}}, \; \underset{\substack{\uparrow \\ \nu_\mu}}{\text{few hundred MeV}}, \; \underset{\substack{\uparrow \\ \nu_\tau}}{\text{Tens of GeV}}$$

## See-Saw Mechanism For $\nu$-Masses

Gell-Mann, Ramond, Slansky // Yanagida // ...
Mohapatra, Senjanovic

$$\begin{array}{c|cc} & \nu_L & \bar{\nu}_R \\ \hline \nu_L & m_L \simeq 0 & m_D \\ \bar{\nu}_R & m_D & m_R \end{array}$$

$$\Big\downarrow \; m_R \gg m_D \gg m_L$$

$$\begin{bmatrix} -m_D^2/m_R & 0 \\ 0 & m_R \end{bmatrix}$$



$\Rightarrow$ A light eigenstate which is essentially $\nu_L$

$$m(\nu^i) \simeq -\left[(m_D^i)^2/m_R\right] \rightarrow$$

Take $m^i_{Dirac} \simeq (1-2)$ MeV $\longrightarrow \nu^e$

$\simeq (300-500 \text{ MeV}) \longrightarrow \nu^\mu$

$\simeq (10-20 \text{ GeV}) \longrightarrow \nu^\tau$

| | $m(\nu^e_L)$ | $m(\nu^\mu_L)$ | $m(\nu^\tau_L)$ |
|---|---|---|---|
| $m_R \simeq 10^4$ GeV | $(1-4)\times 10^{-1}$ eV | $(1-4)\times 10^{4}$ eV | $(1-4)\times 10$ MeV |
| $\simeq 10^{10.5}$ GeV | $(3-12)\times 10^{-8}$ eV | $(3-10)\times 10^{-3}$ eV | $(3-15)$ eV |
| $\simeq 10^{14}$ GeV | $(1-4)\times 10^{-11}$ eV | $(1-4)\times 10^{-6}$ eV | $(1-4)\times 10^{-3}$ eV |

Expt. $\nu_L^e \longrightarrow < 30$ eV

$\nu_L^\mu \longrightarrow < 500$ KeV

$\nu_L^\tau \longrightarrow < 40$ MeV

**Any proof of non-zero $\nu$-mass $\longrightarrow$ A strong signal for $L \leftrightarrow R$ Symmetry.**

## Embedding of $\mathcal{G}_0 \to G$ and alternative Grand Unif. Symmetries

$$\mathcal{G}_0 = \underbrace{SU(2)_L \times SU(2)_R}_{\downarrow} \times \underbrace{SU(4)^c}_{\downarrow}, \quad F_L = (2_L, 1, 4^c), \quad F_R = (1, 2_R, 4^c)$$

$$\sim \quad SO(4) \times SO(6)$$

$$\xrightarrow{\text{simplest embedding}} \quad SO(10)$$

Anomaly-free (Georgi, FM)

Rep Representations: $\underset{\sim}{1}$, $\underset{\sim}{10}$, $\underset{\sim}{16}$, ..., $\underset{\sim}{45}$, $\underset{\sim}{120}$, $\underset{\sim}{126}$, .....

$\underset{\sim}{16}$ Spinor rep. of $SO(10)$ $\longrightarrow \boxed{2^{\frac{10}{2}-1} = 2^4 = 16}$

$$\underset{\sim}{16}_L = \begin{pmatrix} F_L \\ \hline F^c{}_L = (F_R)^c \end{pmatrix} \begin{matrix} \dashrightarrow 8 \\ \dashrightarrow 8 \end{matrix}$$

$$\psi^c \equiv C \bar{\psi}^T ;$$



$$SU(2)_L \times SU(2)_R \times SU(4)^c \longrightarrow SO(10) \longrightarrow E_6 \; (\underset{\sim}{27}_L)$$

$$(8_L + 8_R) \qquad (\underset{\sim}{16})_L \longrightarrow SU(16) \quad \underset{\sim}{16}_L + \overset{*}{16}_L$$

(Mirrors) Needed To Cancel anomalies

$$(\underset{\sim}{27})_{E_6} = (\underset{\sim}{16} + \underset{\sim}{10} + \underset{\sim}{1})_{SO(10)}$$

$$(\underset{\sim}{16}_L)_{SO(10)} = (2_L, 1, 4^c)_L + (1, 2_R, \overset{*}{4}{}^c)_L$$

$SU(16)$ $(\underset{\sim}{16})$ $\longrightarrow$ $SO(10)$ $(\underset{\sim}{16})$

$SU(16)$ $(\underset{\sim}{16})$ $\dashrightarrow$ $SU(8) \times SU(8)$ $(8_L + 8^*_L)$

$SO(10)$ $(\underset{\sim}{16})$ $\longrightarrow$ $SU(5)$ $(5^* + 10 + 1)$

$SO(10)$ $(\underset{\sim}{16})$ $\longrightarrow$ $SU(2)_L \times SU(2)_R \times SU(4)^c$ $(2_L, 1, 4^c) + (1, 2_R, 4^{*c})$

$SU(8) \times SU(8)$ $\dashrightarrow$ $SU(2)_L \times SU(2)_R \times SU(4)^c$

$SU(5)$ $(5^* + 10 + 1)$, $1 \leftarrow (\nu_R)$ $\longrightarrow$ $SU(2)_L \times U(1)_Y \times SU(3)^c$

$SU(2)_L \times SU(2)_R \times SU(4)^c$ $\longrightarrow$ $SU(2)_L \times U(1)_Y \times SU(3)^c$

$$(8_L + 7_R + \nu_R)$$

## SU(5) - The Minimal Grand Unif. Symm.

Georgi, Glashow 74.

$$(\underset{\sim}{16})_{SO(10)} = (5^* + 10 + 1)_{SU(5)}$$

$$(\underset{\sim}{10})_{SO(10)} = (5^* + 5)_{SU(5)}$$

$$SU(5) \longrightarrow SU(3)^c \times SU(2)_L \times U(1)_Y$$

$$\underset{\sim}{5} = (\underset{\sim}{3}^c, 1) + (1^c, 2)$$

$$5^* = (3^{*c}, 1) + (1^c, 2^*)$$

$$5 \times 5 = (10)_{\text{Antisymm}} + (15)_{\text{Symm.}}$$

$$5 \times 5^* = \underset{(\text{Adjoint})}{\underset{\sim}{24}} + 1$$

$$5^* \rightarrow \begin{pmatrix} d^c_{red} \\ d^c_{yel} \\ d^c_{bl} \\ \hline e^- \\ -\nu_e \end{pmatrix}_L ; \quad \underset{\sim}{10} \rightarrow \begin{bmatrix} 0 & u^c_b & u^c_y & -u_r & -d_r \\ -u^c_b & 0 & u^c_r & -u_y & -d_y \\ -u^c_y & -u^c_r & 0 & -u_b & -d_b \\ u_r & u_y & u_b & 0 & -e^+ \\ d_r & d_y & d_b & e^+ & 0 \end{bmatrix}_L$$

$$= \begin{pmatrix} \nu_e & \bar{d} \\ e^- & \end{pmatrix}_L \qquad = \begin{bmatrix} e^+ & \begin{matrix} u \\ d \end{matrix} & \bar{u} \end{bmatrix},$$



$$\text{Anomaly}(\underset{\sim}{5}^*) + \text{Anomaly}(\underset{\sim}{10}) = 0$$

For SU(5), need to put members of one family (not counting $\nu_R$) in 2 separate reps. $5^* + 10$.

$$SU(5) \xrightarrow{\langle \Sigma_{\underset{\sim}{24}} \rangle = v_\Sigma \left(\begin{array}{ccc|cc} 1 & & & & \\ & 1 & & & 0 \\ & & 1 & & \\ \hline & 0 & & -3/2 & \\ & & & & -3/2 \end{array}\right)} SU(2)_L \times U(1) \times SU(3)^c \xrightarrow{\langle \Phi_{\underset{\sim}{5}} \rangle = v_\phi \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}} U(1)_{em} \times SU(3)^c.$$

$$V = \sum_{a=1}^{24} V^a \lambda^a/2$$

$$\frac{1}{\sqrt{2}} \left[\begin{array}{ccc|cc} & & & X_1 & Y_1 \\ & V(\underset{\sim}{8}) & & X_2 & Y_2 \\ & \text{Color gluons} & & X_3 & Y_3 \\ \hline \bar{X}_1 & \bar{X}_2 & \bar{X}_3 & W^3/\sqrt{2} & W^+ \\ \bar{Y}_1 & \bar{Y}_2 & \bar{Y}_3 & W^- & -W^3/\sqrt{2} \end{array}\right] + \frac{B^0}{\sqrt{60}} \left[\begin{array}{ccc|cc} -2 & & & & \\ & -2 & & & 0 \\ & & -2 & & \\ \hline & 0 & & 3 & \\ & & & & 3 \end{array}\right]$$

# Renormalization Group Eqns For Running Coupling Constants in Grand Unification

$$G = SU(5),\ SO(10),\ SU(16) \text{ or } E_6$$

$$\big\downarrow \ \langle \Sigma \rangle = M$$

$$SU(2)_L \times U(1)_Y \times SU(3)^C$$

$$\big\downarrow \ \langle \phi \rangle \simeq 300 \text{ GeV}$$

$$U(1)_{em} \times SU(3)^C$$

Recall: For $SU(N)$

$$\beta_N = \frac{d\bar{g}}{dt} = -b_N \bar{g}^3 \quad ; \quad \boxed{t = \tfrac{1}{2} \ln Q^2/\mu^2}$$

$$\boxed{b_N = \frac{1}{16\pi^2}\left[\frac{11N}{3} - \frac{2}{3} n_f - \frac{1}{3} t_2(s)\right]}$$

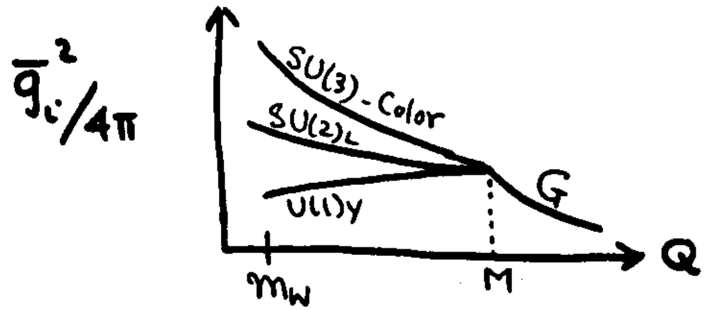
gauge fld contrib — fermion Contrib. — Spin-0 Contrib.

$$\int_{g(\mu^2)}^{g(Q^2)} \frac{d\bar{g}}{\bar{g}^3} = -b \int_{t'=0}^{t'=t} dt'$$

$$\frac{1}{g^2(Q^2)} = \frac{1}{g^2(\mu^2)} + 2bt$$

$$= \frac{1}{g^2(\mu^2)} - 2b \ln \frac{\mu}{Q}$$






Georgi, Quinn, Weinberg.

Choose $\mu = M$ and $Q < M$ : $\boxed{g_1 = g_2 = g_3 \text{ at } M}$

consq. of Grand Unif.

$$\frac{1}{g_i^2(Q)} = \frac{1}{g_i^2(M)} - 2b_i \ln(M/Q)$$

$$b_3 = \frac{11}{48\pi^2}(3) + b_F + b_H^{(3)}$$

$$b_2 = \frac{11}{48\pi^2}(2) + b_F + b_H^{(2)}$$

$$b_1 = 0 + b_F + b_H^{(1)}$$

Ignore (Very Small)

$$\Rightarrow \frac{1}{g_2^2(Q)} - \frac{1}{g_1^2(Q)} = -2(b_2 - b_1) \ln M/Q$$

$$\Rightarrow (e^2/g_2^2)_Q - (e^2/g_1^2)_Q = -\frac{11e^2}{12\pi^2} \ln M/Q$$

$$\Rightarrow \left\{\sin^2\theta_W - \left[\frac{3}{5}(1 - \sin^2\theta_W)\right]\right\}_Q = -\frac{11e^2}{12\pi^2} \ln M/Q$$

$$\Rightarrow \left[\frac{8}{5}\sin^2\theta_W - \frac{3}{5}\right] = -\frac{11e^2}{12\pi^2} \ln M/Q$$

$$\Rightarrow \boxed{\sin^2\Theta_W = \frac{3}{8} - \frac{55e^2}{96\pi^2}\ln\frac{M}{Q}} \quad \text{-----} \ (1)$$

Note $\sin^2\Theta_W^0 \equiv (\sin^2\Theta_W)_{Q=M} = 3/8 \leftarrow$ A general group prop for a no. of symm. gr.

From $g_3$ & $g_2$ eqns.

$$\frac{1\times e^2}{g_3^2(Q)} - \frac{1\times e^2}{g_2^2(Q)} = -2\left(\frac{11\times e^2}{48\pi^2}\right)\ln M/Q$$

$$\Rightarrow \frac{\alpha}{\alpha_s} - \sin^2\Theta_W = \frac{-22\alpha}{12\pi}\ln M/Q$$

$$\underset{\text{sing(1)}}{\Rightarrow} \boxed{\frac{\alpha}{\alpha_s} = \frac{3}{8} - \frac{11\alpha}{24\pi}\ln M/Q} \quad \text{--->} \ (2)$$

Put $Q = m_W$, $\alpha_s(m_W) = .107 {+.013 \atop -.006}$

$$\alpha(m_W) = \frac{1}{127.8 \pm .3}$$

$$\Rightarrow \boxed{\begin{array}{l} M = \left(2 {+2.1 \atop -1.0}\right)\times 10^{14}\ \text{GeV} \\ \sin^2\Theta_W = .214 {+.003 \atop -.004} \end{array}}$$

$\Lambda_{\overline{MS}} = 150 {+150 \atop -75}$ MeV, 3 families

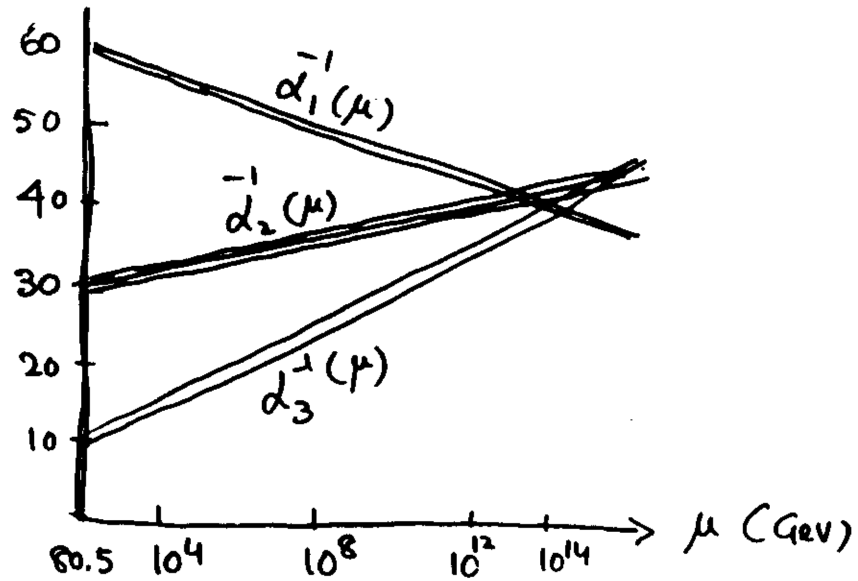
$m_t \simeq 50$ GeV.



Expt. $\boxed{\sin^2\Theta_W(m_W) = .228 \pm .0044.}$

$\uparrow$

Inconsistent with minimal SU(5)




Will see that the value of M is also inconsistent with lower limit on proton life-time.

a

**Excercise**

$$SU(5) \longrightarrow SU(3)^c \times SU(2)_L \times U(1)_Y$$
$$(g_2) \quad (g')$$

$$\frac{A}{e} = \frac{W_3}{g_2} + \frac{B^0}{g'} \rightarrow \boxed{\frac{1}{e^2} = \frac{1}{g_2^2} + \frac{1}{g'^2}}$$

$$Q_{em} = I_{3L} + \frac{Y_W}{2}$$

$$\underset{\sim}{5} \rightarrow \begin{bmatrix} d^c_r \\ d^c_y \\ d^c_b \\ e^- \\ \nu \end{bmatrix} \rightarrow Q_{em} = \begin{bmatrix} 1/3 & & & & \\ & 1/3 & & & \\ & & 1/3 & & \\ & & & -1 & \\ & & & & 0 \end{bmatrix}$$

$$= I_{3L} + Y_W/2 \leftarrow SU(2)_L \times U(1)$$

Normalization : $Tr\,(F_i F_j) = \frac{1}{2}\delta_{ij}$

$$\left(\frac{Y_W}{2}\right)_{SM} = \begin{bmatrix} 1/3 & & & & \\ & 1/3 & & & \\ & & 1/3 & & \\ & & & -1/2 & \\ & & & & -1/2 \end{bmatrix}$$

$$Tr\left(\frac{Y_W}{2}\right)^2_{SM} = \frac{1}{9} \times 3 + \frac{1}{4} \times 2 = 5/6$$

$$\Rightarrow \boxed{[Y']_{SU(5)} = \sqrt{3/5}\,[Y_W/2]_{SM}}$$

b

$$g' B^0_\mu \left(J_\mu^{(Y_W/2)}\right) = g'\left[\sqrt{5/3}\,\left(J_\mu^{(Y')}\right)_{SU(5)}\right] \cdot B^0_\mu$$

$$= \left(g'\sqrt{5/3}\right)\left[\left(J_\mu^{(Y')}\right)_{SU(5)} B^0_\mu\right]$$

$$\uparrow$$
Normalized

$$\Rightarrow (g_1)_{SU(5)} = \sqrt{5/3}\, g'$$

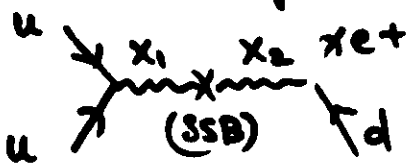
$$\Rightarrow \frac{1}{e^2} = \frac{1}{g_2^2} + \frac{5}{3}\frac{1}{g_1^2} = \frac{8}{3}\left[\frac{1}{g_2^2}\right]_M$$

$$\Rightarrow \sin^2\theta_W^0 = \left(\frac{e^2}{g_2^2}\right)_M = \frac{3}{8}$$

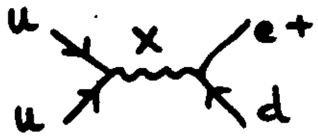
$$\boxed{\frac{e^2}{g_1^2} = \frac{3}{5}\left(1-\sin^2\theta_W\right)}$$

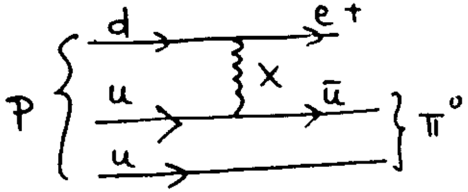
**B, L Violations**: With $(q, \ell)$ unification through a gauge symmetry, B, L cons. not absolute → either violated explicitly or spontaneously

In max. symm. like SU(16), B, L exact in basic lag. but violated spo t'ly




In non-maximal symm like SO(10) or SU(5), B, L violated explicitly.




To illustrate, Consider SU(4)-Color gauging.

$(q_r \; q_y \; q_b \; \ell)$

$$\to g V_\mu^{15} \left[ \underbrace{\bar{q}_r \gamma_\mu q + \bar{q}_y \gamma_\mu q_y + \bar{q}_b \gamma_\mu q_b}_{B_q} - \underbrace{3 \bar{\ell} \gamma_\mu \ell}_{-3L} \right]$$



Thus $g V_\mu^{15} J_\mu^{B-L} \longrightarrow$ Couples to B-L.

⇒ B-L exact and $V_\mu^{15}$ massless in basic lag.

→ By Eötvos-type experiments, Coupling much less than gravitational

⇒ Such Coupling not permitted unless $V_\mu^{15}$ acquires mass spont'ly.

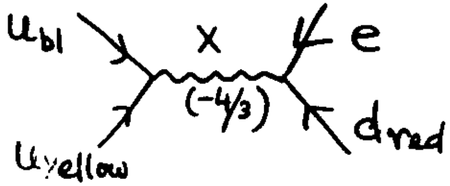
⇒ $\boxed{\text{Corresponding Charge B-L must be Violated spontaneously}}$

→ Argument holds if any linear Combination of B and L gauged (SU(16))

→ MUST BE VIOLATED SPONT'LY.

## For SU(5)

$$\mathcal{L}_{int} = g\, X_\mu \left[ \overline{d^c_{rL}}\, \gamma_\mu e_L + \overline{u^c_{bL}}\, \gamma_\mu u_{yellow} \right].$$




$\Rightarrow p \to e^+ \pi^0$

Similarly for SO(10)

$$\rightarrow \quad \Gamma(p \to e^+\pi^0)^{-1}_{\binom{\text{minimal}}{SU(5)}} \simeq 4 \times 10^{29 \pm 0.7} \left[\frac{M_X}{2\times 10^{14}\,\text{GeV}}\right]^4$$

$$\lesssim 3.3 \times 10^{31}\ \text{yr}$$

Using uncertainty in $M_X$ and also in theoretical estimate of relevant matrix element.

IMB, KAMIOKANDE $\boxed{\Gamma(p \to e^+\pi^0)^{-1}_{expt} > 5 \times 10^{32}\ \text{yr}}$

Excludes Minimal SU(5) ;

Also excludes one-step breaking of SO(10), SU(16) which yield shorter life-time.



## Branching Ratios

| | | | |
|---|---|---|---|
| $p \to$ | $e^+\pi^0$ | $(e^+\omega, e^+\rho)$ | $e^+\eta$ |
| | $\cdot 40$ | $\cdot 30$ | $\cdot 01$ |

| | | | | |
|---|---|---|---|---|
| $p \to$ | $\bar\nu_e \pi^+$ | $\bar\nu_e \rho^+$ | $\mu^+ K^0$ | $\bar\nu_\mu K^+$ |
| | $\cdot 16$ | $\cdot 04$ | $\cdot 03$ | $\cdot 02$ |

| | | | | |
|---|---|---|---|---|
| $n \to$ ($\Delta B \neq 0$) | $e^+\pi^-$ | $e^+\rho^-$ | $\bar\nu_e \pi^0$ | $\bar\nu_\mu K^0$ |
| | $\cdot 80$ | $\cdot 05$ | $\cdot 08$ | $\cdot 02$ |

Sources of departures from the predictions of the Minimal SU(5) model

- Additional Higgs like $\underset{\sim}{45}$ of SU(5)
- Bigger Gauge Symmetry like SO(10) with a two-step breaking.

  $G \xrightarrow{M} SU(2)_L \times SU(2)_R \times SU(4)^c \xrightarrow{M_I} SU(2) \times U(1) \times SU(3)^c$
- Supersymmetry

## Two-Step Breaking of SO(10), SU(6) – Proton Decay, $\sin^2\theta_W$, $\nu$-Masses etc.

$$
\begin{tikzcd}
 & SO(10) \arrow[dl, "M_U"', "(A)"] \arrow[dr, "M_U", "(B)"'] & \\
SU(2)_L \times SU(2)_R \times SU(4)^C \arrow[dr, "M_I"'] & & SU(2)_L \times SU(2)_R \times SU(3)^C \times U(1)_{B-L} \arrow[dl, "M_I"] \\
 & SU(2)_L \times U(1)_Y \times SU(3)^C &
\end{tikzcd}
$$

$M_U$ is larger & $M_I$ lower than before.
$\sin^2\theta_W$ is larger than before.

1)

| $\sin^2\theta_W$ | $M_I$ (GeV) | $M_U$ (GeV) | $\Gamma(p\to e^+\pi^0)^{-1}$ |
|---|---|---|---|
| .217 | $1.0\times10^{14}$ | $1.0\times10^{14}$ | $.3\times10^{29\pm1.7} \lesssim 1.5\times10^{[illegible]}$ yr |
| .225 | $8\times10^{12}$ | $4.3\times10^{14}$ | $7.2\times10^{30\pm1.7} \lesssim 3.5\times10^{[illegible]}$ yr |
| .23 | $1.2\times10^{12}$ | $8\times10^{14}$ | $7.2\times10^{31\pm1.7} \lesssim 3.5\times10^{32}$ yr |
| .235 | $1.2\times10^{11}$ | $1.2\times10^{15}$ | $3.6\times10^{32\pm1.7} \lesssim 1.7\times10^{34}$ yr |



## Selection Rules For B, L Violations

$$\left.\begin{array}{l} p\to e^+\pi^0 \\ \quad\to e^+\omega \\ \quad\to \bar\nu\,\pi^+ \end{array}\right\} \left.\begin{array}{l}\Delta B=-1,\ \Delta L=-1 \\ \Delta(B-L)=0\end{array}\right\} M_c\sim 10^{14}-10^{15}\,\text{GeV}$$

$$\left.\begin{array}{l} p\to e^-\pi^+\pi^+ \\ \quad\to \mu^-\pi^+\pi^+ \\ n\to e^-\pi^+ \\ \quad\to \mu^-\pi^+ \end{array}\right\} \left.\begin{array}{l}\Delta B=-1,\ \Delta L=+1 \\ \Delta(B+L)=0\end{array}\right\} M_c\sim 10^{11}-10^{12}\,\text{GeV}$$

$$\left.\begin{array}{l} p\to e^-(e^+\nu)\pi^+ \\ p\to \mu^-(e^+\nu)\pi^+ \\ p\to \nu\,(e^+\nu) \\ n\to e^-(e^+\nu) \end{array}\right\} \begin{array}{c}\text{Same}\\ \text{Selection}\\ \text{Rules.}\end{array} \quad \begin{array}{c}\text{Same charac.}\\ \text{Mass Scale.}\end{array}$$

$$p\to 3\ell+\pi\text{'s}\ \}\ \Delta\left(B+\tfrac{L}{3}\right)=0\ \}\ M_c\sim 10^5\,\text{GeV}$$

$$p\to 3\bar\ell+\pi\text{'s}\ \}\ \Delta\left(B-\tfrac{L}{3}\right)=0\ \}\ M_c\sim 10^5\ \text{''}$$

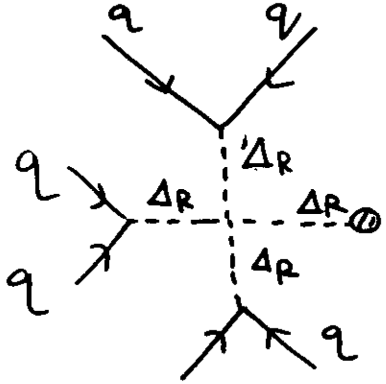
EUTRON OSCIL. $n\leftrightarrow\bar n$ $\left\}\begin{array}{l}|\Delta B|=2,\ \Delta L=0\\ \Delta(B-L)=\pm2\end{array}\right\}$ $M_c\sim 10^4-10^5$ GeV

IEUTRINO-ESS DOUBLE $\beta$-decay. $nn\to ppe^-e^-$ $\left\}\begin{array}{l}\Delta B=0,\ \Delta L=2\\ \Delta(B-L)=-2\end{array}\right\}$ ''

## With Scalar Higgs Exchanges

$\Delta(B-L) \neq 0$ modes can become important and even dominant compared to $\Delta(B-L)=0$ modes.

e.g.



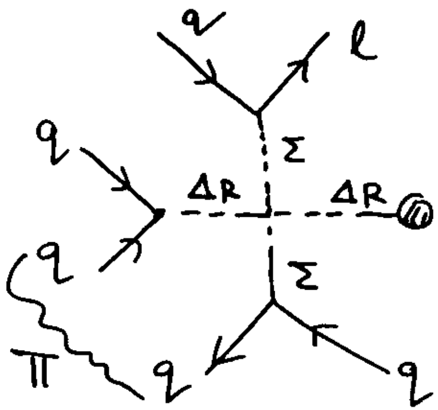

$6q \to$ vacuum

$n \leftrightarrow \bar{n}$

Mohapatra, Marshak

$\tau_{osc}(n \leftrightarrow \bar{n}) \gtrsim 10^6$ sec

free neutrons




$3q \to l\,\pi$

$p \to e^- \pi^+ \pi^+$

$n \to e^- \pi^+$

$\boxed{\Delta(B-L) = -2}$

JCP, AS, Sen



## Supersymmetric Grand Unif. Models

Fermion $\leftrightarrow$ Boson

RGE $\to$ $\beta$-functions are altered

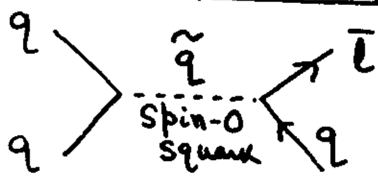
$\Rightarrow M_X \simeq (\tfrac{1}{2} - 2) \times 10^{16}$ GeV $\quad \boxed{\delta m_{SUSY} \simeq m_W}$

Minimal SUSY-GU model

$\Rightarrow$ Gauge Boson Mediated Proton decay too slow to be observable.

$$\boxed{\sin^2\theta_W(m_W) = .237^{+.003}_{-.004} - \frac{4}{15}\frac{\alpha}{\pi}\ln\left(\frac{\delta m_S}{m_W}\right)}$$

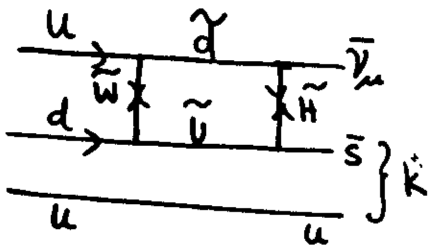
### Scalar Exchanges:




Dim. 4

$\downarrow$

Can be forbidden by discrete symm.




Dim. 5

$\downarrow$

Typically $\tau_p \lesssim 10^{31}$ yrs.

expt $\gtrsim 7 \times 10^{31}$ yrs.

In general, there is Considerable uncertainty in predictions of SUSY models regarding proton decay.

→ So also in Superstring inspired models.

- Flipped SU(5)

$SU(5)' \times U(1)$

Antoniadis, Ellis, Hagelin, Nanopoulos

$$\underline{\underline{5^*}} \quad \begin{pmatrix} \nu_e & \bar{u} \\ e^- & \end{pmatrix}_L \qquad \underset{\sim}{10} \quad \begin{pmatrix} \bar{\nu} & \begin{matrix} u \\ d \end{matrix} & \bar{d} \end{pmatrix}_L \qquad \underset{\sim}{1} \quad e^+_L$$

Higgs $(\underset{\sim}{1}, \underset{\sim}{5}, \underset{\sim}{10})$.



## Cosmological Baryon Asymmetry

$$\boxed{\frac{n_B - n_{\bar{B}}}{n_\gamma} \simeq 10^{-10}} \; ; \; n_{\bar{B}} \lll n_B.$$

A strong signal that B, L violations were responsible in the early universe (e.g. $t \sim 10^{-35}$ sec.) leading to this asymmetry ($n_B \neq n_{\bar{B}}$) ← Assuming $n_B = n_{\bar{B}}$ at $t=0$

- $\boxed{\text{Need } \Delta B \neq 0 \text{ processes}}$ (Provided Most Naturally by Grand Unif.).
- C & CP violations
- Thermodynamic Non-Equilibrium of $\Delta B \neq 0$ Processes.

## In Summary

- "Grand" Unification an attractive Underlying Concept – The only (Known) Consistent framework to understand
  - (i) Quantization of Electric charge
  - (ii) Existence of quarks & leptons
  - (iii) (Existence of) Strong, EM & weak forces

$$g_3 \gg g_2 \sim g_1 \quad \text{at low Energies}$$

→ Thus Very likely to be part of an underlying fundamental theory (as in Superstring theories $E_8 \times E_8$ ; $E_8 \supset E_6 \supset SO(10) \ldots$).

→ Should emerge at least effectively as a broken theory at low energies with all the underlying Constraints of Grand unification.



→ This is what happens in superstring theories

→ This can also happen in a class of Supersymmetric Composite models in which $q$, $\ell$ and $\phi$'s are formed as Composites at a high scale $\Lambda_M \gg 1\,\text{TeV}$.

→ Few Central Features are Common
- B, L Violations
- $q - \ell$ distinction
- $g_3 \gg g_2 \sim g_1$ at low energies.
- Quantization of $Q_{em}$

→ But details involving $\sin^2\theta_W$, proton decay etc. Vary.

(I) Need to Probe into Proton-decays & other (B, L) violations.

Need proton-decay searches sensitive to $\Gamma(p \to e^+ \pi^\circ)^{-1} \lesssim 10^{34}$ years.

Need Fiducial Volume } 20 - 40 Kilotons
$\sim (1 - 2) \times 10^{34}$ Nucleons.

Now

IMB 3300 tons
Kamiokande 880 " → SUPERNOVA 1987 A.
Frejus 550 "

- - - - - - - - - - - - -

Super Kamiokande (Proposed) ⟶ 20,000 tons

Moon - Colony ⟶ $\nu$ - Background lower by at least a factor of 200.