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SUPERSYMMETRY AND SUPERGRAVITY

GRAND UNIFICATION

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6. RADIATIVE BREAKING

[Ibañez, Lopez; Alvarez-Gaumé et al] etc.

One of the remarkable features of supergravity grand unification is that it offers natural explanation of electroweak breaking. In SM one imposes electroweak breaking by inserting by hand negative $(\text{mass})^2$. In supergravity one deduces this from radiative effects.

We consider the "minimal" supergravity model which at the GUT scale, after the GUT group G has broken to $SU(3)\times SU(2)\times U(1)$, is described by the superpotential

$$W = W_Y^{(3)} + W^{(2)}$$
$$= \left[\lambda_{ij}^{(u)} q_i H_2 u_j^c + \lambda_{ij}^{(d)} q_i H_1 d_j^c + \lambda_{ij}^{(e)} l_i H_1 e_j^c\right] + \mu_0 H_1 H_2$$

and effective potential

$$V^{eff} = \left[\sum \left|\frac{\partial W}{\partial z_a}\right|^2 + V_D\right] + \left[m_0^2 z_a z_a^* + (A_0 W_Y^{(3)} + B_0 W^{(2)} + h.c.)\right]$$

and gaugino mass

$$\mathcal{L}^{\lambda}_{mass} = -m_{1/2}\bar\lambda^\alpha \lambda^\alpha$$

This form depends on following assumptions:

(1) A hidden sector exists which breaks supersymmetry which is a gauge group G singlet

(2) A GUT sector exists which breaks G to $SU(3)\times SU(2)\times U(1)$ at scale M_G

(3) The only light particles remaining after integrating out the superheavy particles is the SUSY Standard Model spectrum.

Aside from the Yukawa couplings (which exist also in the S.M.) there are

$$m_{1/2},\ m_0,\ A_0,\ B_0\ ;\ \mu_0;\ \alpha_G,\ M_G$$

This is only two more than in the non-SUSY Standard Model

$$\underbrace{m^2\ ,\ \lambda}_{\text{Higgs potential}}\ ,\ \alpha_1,\ \alpha_2,\ \alpha_3$$

and we'll see that radiative breakings allows one combination to be expressed in terms of M_Z. Further, α_G and M_G have now been "measured" by LEP.

Thus the minimal supergravity GUT models depend on 4 new arbitrary constants. These 4 constants then determine the masses of 32 additional particles:

31 SUSY particles + light Higgs (h)

Thus theory should have a large predictive power to correlate the SUSY masses. In order to implement this, must use Renormalization Group Eqns. (RGE) to take theory from GUT scale down to electroweak scale.

Radiative Breaking Equations

At GUT scale, each neutral Higgs boson receives a $(\text{mass})^2$ of $m_0^2 > 0$. One may then use the RGE to calculate the running mass and if one $(\text{mass})^2$ turns negative it signals the breaking of $SU(2)\times U(1)$ at a lower mass scale. The Higgs part of the RG improved effective potential is:

$$V_H = \underbrace{V_0}_{\text{tree}} + \underbrace{\Delta V_1}_{1\text{-loop}}$$

$$V_H = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 - m_3^2 (H_1 H_2 + h.c.)$$

$$+ \underbrace{\tfrac{1}{8}(g_2^2 + g_Y^2)(|H_1|^2 - |H_2|^2)^2}_{V_0 \sim \lambda\phi^4} + \Delta V_1$$

where [Coleman, E. Weinberg; S. Weinberg]

$$\Delta V_1 = \frac{1}{64\pi^2} \mathrm{STr}\left[M^4 \left[\ln \frac{M^2}{Q^2} - \frac{3}{2} \right]\right] \; ; \quad \overline{MS} \text{ scheme}$$

$M(v_1, v_2)$ = tree mass matrix

$v_i \equiv \langle H_i \rangle$

or

$$\Delta V_1 = \frac{1}{64\pi^2} \sum_a (-1)^{2s_a} n_a M_a^4 \ln\left(\frac{M_a^2}{e^{3/2} Q^2}\right)$$

s_a = spin of particle of type a

n_a = no. of helicity states

Each $m_i(t)$, $g_2(t)$, $g_Y(t)$ are running parameters at scale Q where $t = \ln(M_G^2/Q^2)$. One has

$$m_i^2(t) = m_{H_i}^2(t) + \mu^2(t) \quad ; \; i = 1,2$$

$$m_3^2(t) = -B(t)\,\mu(t)$$

The RGE are thus subject to b.c. at $Q = M_G$:

$$m_i^2 = m_0^2 + \mu_0^2 \; ; \quad m_3^2 = -B_0 \mu_0 \; ; \quad \alpha_2 = \alpha_G \text{ etc.}$$

List here some of 1-loop RGE [Ibañez, Lopez, etc. ... Ellis, Zwirner] keeping only large t-quark Yukawas

Gauge couplings:

$$\frac{d\tilde{\alpha}_i}{dt} = -\frac{b_i}{4\pi}\tilde{\alpha}_i^2 \;;\quad \tilde{\alpha}_i = \frac{\alpha_i}{4\pi} \;;\quad \alpha_3(0) = \alpha_2(0) = \tfrac{3}{5}\alpha_1(0) = \alpha_G$$

$b_i = (11, 1, -3)$

$\alpha_1 \equiv \alpha_Y$ here

Gaugino masses:

$$\frac{d\tilde{m}_i}{dt} = -\frac{b_i}{4\pi}\tilde{\alpha}_i\tilde{m}_i \;;\quad \tilde{m}_i(0) = m_{1/2}$$

Higgs masses:

$$\frac{dm_{H_1}^2}{dt} = \left(3\tilde{\alpha}_2\tilde{m}_2^2 + \tilde{\alpha}_1\tilde{m}_1^2\right)$$

$$\frac{dm_{H_2}^2}{dt} = \left(3\tilde{\alpha}_2\tilde{m}_2^2 + \tilde{\alpha}_1\tilde{m}_1^2\right) - 3Y_t\left(m_Q^2 + m_U^2 + m_{H_2}^2 + A_t^2\right)$$

$$m_{H_i}^2(0) = m_0^2$$

t Yukawa:

$$\frac{dY_t}{dt} = Y_t\left(\frac{16}{3}\tilde{\alpha}_3 + 3\tilde{\alpha}_2 + \frac{13}{9}\tilde{\alpha}_1 - 6Y_t\right)$$

$$Y_t = \frac{h_t^2}{(4\pi)^2} \;;\quad m_t \equiv h_t(m_t)\langle H_2\rangle$$

t Polonyi

$$\frac{dA_t}{dt} = \left(\frac{16}{3}\tilde{\alpha}_3\tilde{m}_3 + 3\tilde{\alpha}_2\tilde{m}_2 + \frac{13}{9}\tilde{\alpha}_1\tilde{m}_1\right) - 6Y_tA_t$$

$$A_t(0) = A_0$$

Squark masses (3rd generation)

$$\frac{dm_Q^2}{dt} = \left(\frac{16}{3}\tilde{\alpha}_3\tilde{m}_3^2 + 3\tilde{\alpha}_2\tilde{m}_2^2 + \frac{1}{9}\tilde{\alpha}_1\tilde{m}_1^2\right) - Y_t\left(m_{H_2}^2 + m_Q^2 + m_U^2 + A_t^2\right) \;;\quad Q \equiv \tilde{Q}_L$$

$U \equiv \tilde{U}_R$

$D \equiv \tilde{D}_R$

$$\frac{dm_U^2}{dt} = \left(\frac{16}{3}\tilde{\alpha}_3\tilde{m}_3^2 + \frac{16}{9}\tilde{\alpha}_1\tilde{m}_1^2\right) - 2Y_t\left(m_{H_2}^2 + m_Q^2 + m_U^2 + A_t^2\right)$$

$$m_{Q,U}^2(0) = m_0^2$$

[For gen. 1,2 set $Y_t \to 0$. For D set $Y_t \to 0$, $\frac{16}{9}\tilde{\alpha}_1\tilde{m}_1^2 \to \frac{4}{9}\tilde{\alpha}_1\tilde{m}_1^2$ in m_U^2 eqn.]

Polonyi B

$$\frac{dB}{dt} = \left(3\tilde{\alpha}_2\tilde{m}_2 + \tilde{\alpha}_1\tilde{m}_1\right) - 3Y_tA_t$$

$$B(0) = B_0$$

μ Parameter

$$\frac{d\mu^2}{dt} = \left(3\tilde{\alpha}_2 + \tilde{\alpha}_1 - 3Y_t\right)\mu^2$$

$$\mu(0) = \mu_0$$

One starts at $Q = M_G$ and integrates the RGE down to electroweak scale. At some point where a $(\text{mass})^2$ turns negative. Then VEVs of $H_{1,2}$ grow and $SU(2)\times U(1)$ breaking occurs. (Occurs due to t-quark Yukawa couplings.) Considering only V_0 first this occurs when

$$\mathcal{D} \equiv m_1^2 m_2^2 - m_3^4 < 0$$

To have a valid minimum, also require potential bounded from below. This condition is

$$\mathcal{L} \equiv m_1^2 + m_2^2 - 2|m_3^2| \geq 0 \quad ; \text{ stability}$$

Minimizing the total V_H one finds ($\frac{\partial V_H}{\partial v_i} = 0$, $v_i \equiv \langle H_i \rangle$):

$$\frac{1}{2} M_Z^2 = \frac{\mu_1^2 - \mu_2^2 \tan^2\beta}{\tan^2\beta - 1} \quad ; \quad \sin 2\beta = \frac{2 m_3^2}{\mu_1^2 + \mu_2^2}$$

where

$$\tan\beta \equiv \frac{v_2}{v_1} \; ; \; \mu_i^2 = m_i^2 + \Sigma_i \; ; \; m_3^2 = -B\mu$$

and Σ_i is 1-loop correction

$$\Sigma_i = \frac{1}{32\pi^2} \sum_a (-1)^{2S_a} n_a M_a^2 \ln\left(\frac{M_a^2}{\sqrt{e}\, Q^2}\right) \frac{\partial M_a^2}{\partial v_i}$$

Gamberini, Ridolfi, Zwirner argue that the extrema eqns, when the 1-loop corrections are included are approximately independent of Q^2 and so in following we will set

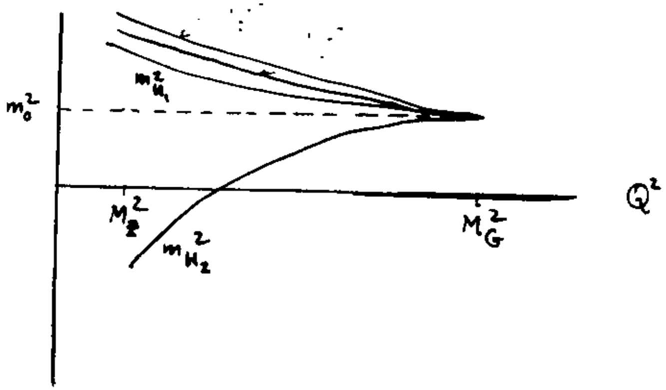
$$Q^2 = M_Z^2$$

Then M_Z^2 is the physical Z-mass ($M_Z^2(M_Z^2)$) and all other masses we discuss will be running masses at M_Z. Actually, things are simpler: It turns out (discussed later) that for physically interesting cases there is a large amount of cancellation in Σ_i so 1-loop corrections are generally small. Numerically, one gets a good approximation by considering only V_0, and one finds that the Q^2 dependence is indeed small over almost all of parameter space.

Fine points: We've assumed that in running RGE starting at M_G, it is Higgs $(\text{mass})^2$ that turns negative. It is possible that one of the other squarks or slepton (mass)^2 might turn negative breaking color or electric charge. A number of necessary conditions for this not to happen are known (though sufficiency conditions are not known). These constraints limit the parameter space and roughly require Polonyi constants to be bounded e.g.

$$|A_t| \lesssim 3 m_0 \text{ or } 3 m_{1/2}$$

since Polonyi terms make negative contributions to squark/slepton masses. The physically acceptable situation is:



Necessary to Prevent Charge or Color Breaking

Generations 1, 2

$$A_u^2 < 3(m_{\tilde{u}_L}^2 + m_{\tilde{u}_R}^2 + \mu_2^2)$$

$$A_d^2 < 3(m_{\tilde{d}_L}^2 + m_{\tilde{d}_R}^2 + \mu_1^2)$$

Generation 3

$$A_t^2 < 3(m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2 + \mu_2^2)$$

$$A_b^2 < 3(m_{\tilde{b}_L}^2 + m_{\tilde{b}_R}^2 + \mu_1^2)$$

Sleptons

$$A_e^2 < 3(m_{\tilde{e}_L}^2 + m_{\tilde{e}_R}^2 + \mu_1^2)$$

$$m_{H_1}^2 + m_{\tilde{e}_L}^2 > 0$$

[Frere, Jones, Raby]
[Drees, Glück, Grassie; Komatsu; etc.]

Returning to extrema eqns

$$\frac{1}{2} M_Z^2 = \frac{\mu_1^2 - \mu_2^2 \tan^2\beta}{\tan^2\beta - 1} \quad ; \quad \mu_1^2 = m_{H_1}^2 + \mu^2 + \Sigma_1$$

$$\mu_2^2 = m_{H_2}^2 + \mu^2 + \Sigma_2$$

$$\sin 2\beta = \frac{2 m_3^2}{\mu_1^2 + \mu_2^2} \quad ; \quad m_3^2 = -B\mu$$

One can express μ_i^2, m_3^2 in terms of GUT scale parameters via RGE, and use these to eliminate two of them. The convenient choice is to eliminate μ_0 and B_0 in terms of $\tan\beta$ and M_Z and remaining parameters. Since μ^2 enters in 1st eqn. there a two solns:

$$\mu < 0 \quad ; \quad \mu > 0$$

One has then the following: The low energy depends on the 4 parameters

$$m_{1/2},\ m_0,\ A_0,\ \tan\beta \left(\equiv \frac{1}{\tan\alpha_H}\right)$$

The ~~the~~ 31+1 SUSY particles can be expressed in terms of the 4 parameters, and parametrically on m_t, allowing for a sizable amount of predictiveness in the system.

7. SUSY Mass Formulae and MSSM

[Ibañez, Lopez, Mu… R.A., Chamseddine, Na…]

One may use the RGE to express all the SUSY particles masses in terms of the 4 remaining parameters. We will see that this leads to the MSSM as an approximate low energy theory.

Gaugino masses

Integrating the RGE for $\tilde{m}_i$:

$$\tilde{m}_i(t) = \frac{\alpha_i(t)}{\alpha_G} m_{1/2} \quad ; \quad i = 1,2,3 \quad t = \ln(M_G^2/Q^2)$$

Thus for the SU(3), SU(2), U(1) gauginos have [R.A., Chamseddine, Nath; Ibañez, Lopez]

$$\tilde{m}_3 : \tilde{m}_2 : \tilde{m}_1 = \alpha_3 : \alpha_2 : \alpha_1$$

which is postulated in MSSM.

Squarks, generations $i=1,2$

$$m_{\tilde{u}_{Li}}^2 = m_0^2 + \underset{\text{quark mass}}{m_{u_i}^2} + \left(\tfrac{8}{3}\tilde{\alpha}_3 f_3 + \tfrac{3}{2}\tilde{\alpha}_2 f_2 + \tfrac{1}{18}\tilde{\alpha}_1 f_1\right) m_{1/2}^2 + \underbrace{\left(\tfrac{1}{2} - \tfrac{2}{3}\sin^2\theta_W\right)\cos 2\beta\, M_Z^2}_{\text{D term}}$$

$$m_{\tilde{d}_{Li}}^2 = m_0^2 + m_{d_i}^2 + \left(\tfrac{8}{3}\tilde{\alpha}_3 f_3 + \tfrac{3}{2}\tilde{\alpha}_2 f_2 + \tfrac{1}{18}\tilde{\alpha}_1 f_1\right) m_{1/2}^2 + \left(-\tfrac{1}{2} + \tfrac{1}{3}\sin^2\theta_W\right)\cos 2\beta\, M_Z^2$$

$$m_{\tilde{u}_{Ri}}^2 = m_0^2 + m_{u_i}^2 + \left(\tfrac{8}{3}\tilde{\alpha}_3 f_3 + \tfrac{8}{9}\tilde{\alpha}_1 f_1\right) m_{1/2}^2 + \left(\tfrac{2}{3}\sin^2\theta_W\right)\cos 2\beta\, M_Z^2$$

$$m_{\tilde{d}_{Ri}}^2 = m_0^2 + m_{d_i}^2 + \left(\tfrac{8}{3}\tilde{\alpha}_3 f_3 + \tfrac{2}{9}\tilde{\alpha}_1 f_1\right) m_{1/2}^2 + \left(-\tfrac{1}{3}\sin^2\theta_W\right)\cos 2\beta\, M_Z^2$$

Sleptons

$$m_{\tilde{e}_{Li}}^2 = m_0^2 + m_{e_i}^2 + \left(\tfrac{3}{2}\tilde{\alpha}_2 f_2 + \tfrac{1}{2}\tilde{\alpha}_1 f_1\right) m_{1/2}^2 + \left(-\tfrac{1}{2} + \sin^2\theta_W\right)\cos 2\beta\, M_Z^2$$

$$m_{\tilde{e}_{Ri}}^2 = m_0^2 + m_{e_i}^2 + 2\tilde{\alpha}_1 f_1 m_{1/2}^2 - \sin^2\theta_W \cos 2\beta\, M_Z^2$$

$$m_{\tilde{\nu}_{Li}}^2 = m_0^2 + \left(\tfrac{3}{2}\tilde{\alpha}_2 f_2 + \tfrac{1}{2}\tilde{\alpha}_1 f_1\right) m_{1/2}^2 + \tfrac{1}{2}\cos 2\beta\, M_Z^2$$

$$f_i = t\,\frac{2 - \beta_i t}{(1+\beta_i t)^2}$$

$[\alpha_1 = \alpha_Y \text{ here}]$ $\qquad \beta_i \equiv b_i \tilde{\alpha}_i$

$\tfrac{5}{3}\tilde{\alpha}_1 = \tilde{\alpha}_2 = \tilde{\alpha}_3 = \dfrac{\alpha_G}{4\pi}$ $\qquad b_i = (11, 1, -3)$

Note following:

1) $m_{\tilde{c}}^2 - m_{\tilde{u}}^2 = m_c^2 - m_u^2$

Super GIM mechanism for suppressing FCNC

2) For $m_0^2, m_{1/2}^2 > M_Z^2$, the 1st two generations of squarks degenerate nearly and sleptons nearly degenerate $\Longrightarrow$ MSSM

3) Generally expect $m_{\tilde{q}_i}^2 > m_{\tilde{e}_i}$

Squarks, 3rd generation

$$m_{\tilde{b}_R}^2 = m_0^2 + m_b^2 + \left(\tfrac{8}{3}\tilde{\alpha}_3 f_3 + \tfrac{2}{9}\tilde{\alpha}_1 f_1\right) m_{1/2}^2 - \tfrac{1}{3}\sin^2\theta_W \cos 2\beta\, M_Z^2$$

$$m_{\tilde{b}_L}^2 = \tfrac{1}{2} m_0^2 + m_b^2 + \tfrac{1}{2} m_U^2 + \left(\tfrac{4}{3}\tilde{\alpha}_3 f_3 + \tfrac{1}{9}\tilde{\alpha}_1 f_1\right) m_{1/2}^2 + \left(-\tfrac{1}{2} + \tfrac{1}{3}\sin^2\theta_W\right)\cos 2\beta\, M_Z^2$$

The Polonyi and t-quark Yukawas cause mixing between $\tilde{t}_L, \tilde{t}_R$:

$$\begin{pmatrix} m_{\tilde{t}_L}^2 & m_t(A_t + \mu\cot\beta) \\ m_t(A_t + \mu\cot\beta) & m_{\tilde{t}_R} \end{pmatrix}$$

Eigenvalues: $\tilde{t}_1, \tilde{t}_2$; $m_{\tilde{t}_1}^2 < m_{\tilde{t}_2}^2$

211a

$$m_{\tilde t_L}^2 = m_Q^2 + m_t^2 + \left(-\tfrac{1}{2} + \tfrac{2}{3}\sin^2\theta_W\right) M_Z^2 \cos 2\beta$$

$$m_{\tilde t_R}^2 = m_U^2 + m_t^2 + \left(-\tfrac{2}{3}\sin^2\theta_W\right) M_Z^2 \cos 2\beta$$

where

$$m_U^2 = \tfrac{1}{3} m_0^2 + \tfrac{2}{3} A_0 m_{1/2} f - \tfrac{2}{3} k A_0^2 + \tfrac{2}{3} h m_0^2$$
$$+ \left[\tfrac{2}{3} e + \tfrac{2}{3\pi}\alpha_G f_3 - \tfrac{1}{4\pi}\alpha_G f_2 + \tfrac{1}{12\pi}\alpha_G f_1\right] m_{1/2}^2$$

$$m_Q^2 = \tfrac{2}{3} m_0^2 + \tfrac{1}{3} A_0 m_{1/2} f - \tfrac{1}{3} k A_0^2 + \tfrac{1}{3} h m_0^2$$
$$+ \left[\tfrac{1}{3} e + \tfrac{2}{3\pi}\alpha_G f_3 + \tfrac{1}{4\pi}\alpha_G f_2 - \tfrac{1}{60\pi}\alpha_G f_1\right] m_{1/2}^2$$

and functions $f(t)$, $k(t)$, $h(t)$, $e(t)$ [defined in Ibañez et al Nuc. Phys. B256, 218 (1985)] are functions of $t = \ln(M_G^2/Q^2)$ and t-quark Yukawa coupling constants, but independent of $m_0, m_{1/2}, A_0, B_0, \mu_0$.

Charginos (Winos)

$$m_{\tilde W_{1,2}} = \tfrac{1}{2}\left| \left[4\nu_+^2 + (\mu - \tilde m_2)^2\right]^{\frac{1}{2}} \mp \left[4\nu_-^2 + (\mu + \tilde m_2)^2\right]^{1/2}\right|$$

$$\sqrt{2}\,\nu_\pm = M_W(\sin\beta \pm \cos\beta)$$

Neutralinos (Zinos)

$$\begin{pmatrix} m_{\tilde\gamma} & m_{\tilde\gamma\tilde Z} & 0 & 0 \\ m_{\tilde\gamma\tilde Z} & m_{\tilde Z} & M_Z & 0 \\ 0 & M_Z & \mu\sin 2\beta & -\mu\cos 2\beta \\ 0 & 0 & -\mu\cos 2\beta & -\mu\sin 2\beta \end{pmatrix}$$

$$\Rightarrow \tilde Z_i,\ i = 1\cdots 4 \ ; \quad m_{\tilde Z_i} < m_{\tilde Z_j} \ ; \ i < j$$

$$m_{\tilde\gamma} = \tilde m_1 \cos^2\theta_W + \tilde m_2 \sin^2\theta_W$$
$$m_{\tilde Z} = \tilde m_1 \sin^2\theta_W + \tilde m_2 \cos^2\theta_W \ ; \quad m_{\tilde\gamma\tilde Z} = \cos\theta_W \sin\theta_W (\tilde m_1 - \tilde m_2)$$

Higgs Bosons

$$m_A^2 = \mu_1^2 + \mu_2^2 = \frac{m_3^2}{\sin 2\beta}, \quad \text{CP odd Higgs}$$

$$m_{H^\pm}^2 = m_A^2 + M_W^2, \quad \text{Charged Higgs}$$

For CP even neutral Higgs, loop correction to the light Higgs (h) are important. [Okada, Yamaguchi, Yanagida; Ellis, Ridolfi, Zwirner; Haber, Hempfling]

One finds keeping top sector in loops only

$$m^2_{h,H} = \frac{1}{2}\left[M_Z^2 + m_A^2 + \epsilon \mp \left\{ (M_Z^2 + m_A^2 + \epsilon)^2 - 4 m_A^2 M_Z^2 \cos^2 2\beta + \epsilon_1 \right\}^{\frac{1}{2}} \right]$$

where

$$\epsilon = \mathrm{Tr}\,\Delta \;;\quad \epsilon_1 = -4\left(\mathrm{Tr}\,\nu\Delta + \det\Delta\right)$$

$$\nu_{11} = s^2 M_Z^2 + c^2 m_A^2 \;;\quad \nu_{22} = c^2 M_Z^2 + s^2 m_A^2 \;;\quad \nu_{12} = \nu_{21} = sc(M_Z^2 + m_A^2)$$

$$\Delta_{11} = x\mu^2 y^2 z \,,\quad \Delta_{12} = x\mu y(\omega + A_t y z) = \Delta_{21}$$

$$\Delta_{22} = x\,(\upsilon + 2A_t y\omega + A_t^2 y^2 z)$$

and

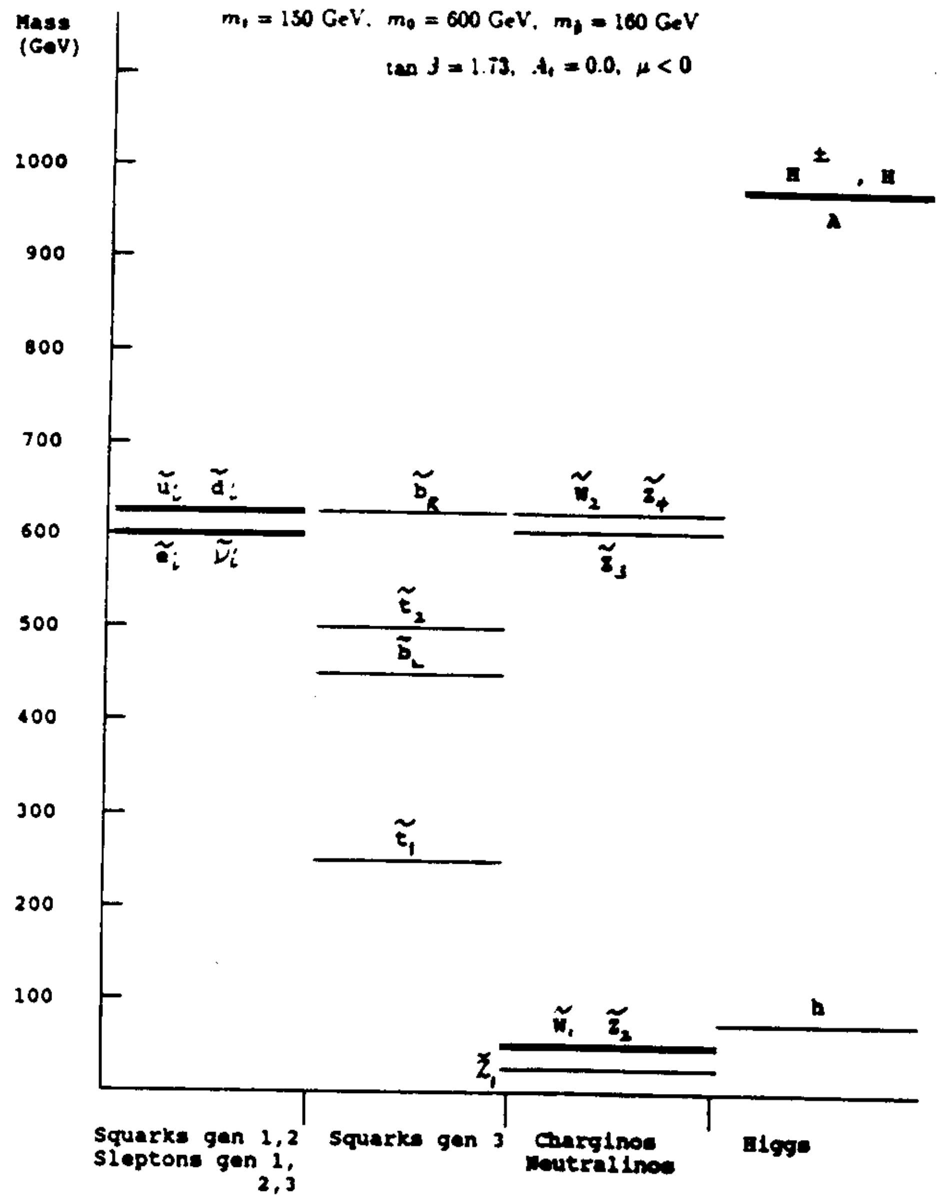
$$x = \frac{3\alpha_2}{4\pi}\,\frac{m_t^4}{M_W^2 s^2} \,,\quad y = \frac{A_t + \mu\cot\beta}{m_{\tilde t_1}^2 - m_{\tilde t_2}^2} \,,\quad z = 2 - \omega\,\frac{m_{\tilde t_1}^2 + m_{\tilde t_2}^2}{m_{\tilde t_1}^2 - m_{\tilde t_2}^2}$$

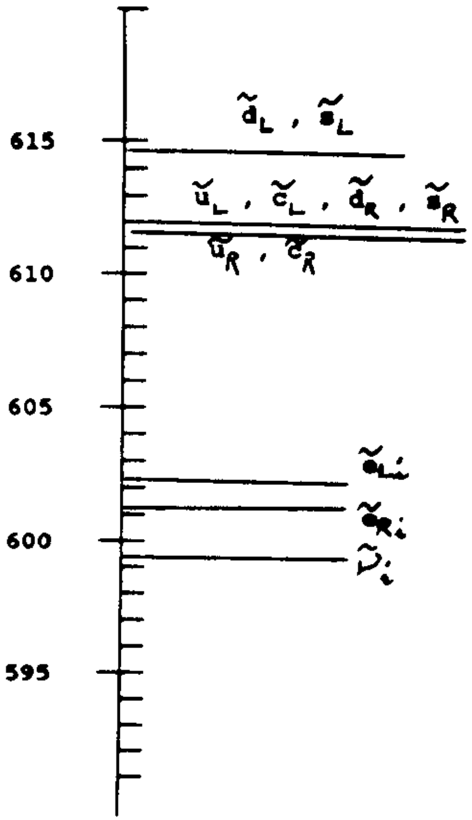
$$\omega = \ln\frac{m_{\tilde t_1}^2}{m_{\tilde t_2}^2} \;;\quad \upsilon = \ln\left(\frac{m_{\tilde t_1}^2 m_{\tilde t_2}^2}{m_t^4}\right) \;;\quad (s,c) = (\sin\beta, \cos\beta)$$

The m_t^4 in x can produce unexpectedly large effects for the light Higgs h particle.

There are loop corrections for m_A, $m_{H^\pm}$, but these effects ~~unless~~ are small unless A, $H^\pm$ are very light.

For a given set of GUT theory parameters (m_0, $m_{1/2}$, A_0, $\tan\beta$) one can predict the positions of all the SUSY particles.





$m_t = 150$, $m_0 = 600$, $m_{\tilde{g}} = 160$, $\tan\beta = 1.73$

$\tilde{\ell}_L = 602.2$

$\tilde{e}_R = 601.2$

$\tilde{\nu} = 599.5$

$\tilde{u}_L = 612.0$

$\tilde{u}_R = 611.7$

$\tilde{d}_L = 614.6$

$\tilde{d}_R = 612.3$

$\tilde{b}_L = 465.0$

$\tilde{b}_R = 612.3$

$\tilde{t}_1 = 241.6$

$\tilde{t}_2 = 500.9$

$\tilde{W}_1 = 55.4$

$\tilde{Z}_2 = 56.1$

$\tilde{Z}_1 = 26.0$

$\tilde{Z}_3 = 601.9$

$\tilde{Z}_4 = 612.7$

$\tilde{W}_2 = 610.8$

$h = 72.3$

$H = 983.7$

$A = 980.1$

$H^{\pm} = 983.4$

8. Gauge Unification Revisted

Since one now has a well defined theory one can reconsider the question of threshold effects on the unification of the gauge coupling constants. Consider first the low energy SUSY thresholds. One might start with specific GUT model with fixed parameters $m_{1/2}$, m_0 etc. One can then calculate the SUSY spectrum. However, one needs to know M_G and α_G to actually do this. One procedure might be iterative:

First neglect mass splitting and set all SUSY particles at M_S (as Amaldi et al do). Then calculate (zero'th approx.) $M_G^{(0)}$, $\alpha_G^{(0)}$. Using these calculate SUSY spectrum. Include in SUSY thresholds in RGE and calculate first approx $M_G^{(1)}$, $\alpha_G^{(1)}$, and iterate until convergence.

The above is a rather lengthy analysis and has not been carried out yet.

Threshold effects at GUT scale more problematic as physics there not understood. Consider, as example SU(5) D.-G./S. model of GUT interactions:

$$W_G = \lambda_1\left[\tfrac{1}{3}\mathrm{Tr}\,\Sigma^3 + \tfrac{1}{2} M\,\mathrm{Tr}\,\Sigma^2\right] + \lambda_2 H_{2x}\left(\Sigma^x_Y + 3M\,\delta^x_Y\right)H_1^Y \; ; \quad \Sigma = 24,\; H_1 = 5;\; H_2 = \bar{5}$$

Spontaneous breaking $SU(5) \to SU(3)\times SU(2)\times U(1)$ leads to VEVs for Σ^x_Y:

$$\langle \Sigma^x_Y \rangle = M\,\mathrm{diag}(2,2,2,-3,-3) \; ; \; M = O(M_G)$$

which causes mass growth of vector bosons, H_1^a, H_{2a} = Higgs color triplets (Higgs doublets remain massless) and components of Σ^x_Y.

In general, massive states are not degenerate, and don't fall into SU(5) multiplets.

For above case have following massive states [R.A., Nath]:

Higgs triplet (chiral) multiplet:

$$M_{H_3} = \frac{5}{2}\lambda_2 \langle\Sigma\rangle \quad ; \quad \langle\Sigma\rangle \equiv 2M$$

Massive vector multiplet:

Massive vector boson, Dirac spinor, hermitian scalar with $SU(3)_c \times SU(2)$ quantum nos: $(3,2) + (\bar{3},2)$

$$M_V = \frac{5}{2} g \langle\Sigma\rangle \quad ; \quad g = \text{GUT scale gauge coupling const.}$$

9 Chiral multiplets from Σ^X_Y

$$M_\Sigma = \frac{5}{4}\lambda_1 \langle\Sigma\rangle$$

1 Chiral multiplet from Σ^X_Y

$$M_{\Sigma'} = \frac{1}{4}\lambda_1 \langle\Sigma\rangle$$

Can impose upper bounds on mass ratios by requiring that GUT physics remain in perturbative domain. Thus if require

$$\alpha_{\lambda_2} = \frac{\lambda_2^2}{4\pi} < \frac{1}{3}$$

$$\alpha_{\lambda_1} = \frac{\lambda_2^2}{4\pi} < \frac{1}{3}$$

$$\frac{M_{H_3}}{M_V} = \frac{\lambda_2}{g} \lesssim 3 \quad ; \quad \alpha_G = \frac{g^2}{4\pi} \cong (1/25.7)$$

$$\frac{M_\Sigma}{M_V} = \frac{1}{2}\frac{\lambda_1}{g} \lesssim 1.5$$

$$\frac{M_{\Sigma'}}{M_V} = \frac{1}{10}\frac{\lambda_1}{g} \lesssim 0.3$$

If associate M_V with $M_G = 10^{16.1}$ GeV, we see that the other masses cannot get significantly above M_G

Heavy thresholds such as above produce effects of size $\approx$ 1 std in gauge coupling unification analysis. However, a theory with a complex GUT sector might produce large threshold effects.

9. Proton Decay

One may vary the 4 parameters $m_{1/2}, m_0, A_0, \tan\beta$ determining the masses of the 31+1 SUSY particles subject to the constraints: (i) Radiative breaking of $SU(2)\times U(1)$ occurs and (ii) SUSY masses stay within the current experimental bounds. In this way one may predict theoretically allowed ranges of SUSY masses which could be tested experimentally. These predictions can be greatly sharpened if the model possesses the SUSY proton decay modes, as p-decay is a strong constraint on any GUT theory.

Experimental bounds on partial lifetimes (90% CL) are

$$\tau(p\to e^+\pi^0) > 5.5\times10^{32}\ yr \qquad [IMB]$$

$$\tau(p\to \bar\nu K^+) > 1\times10^{32}\ yr \qquad [Kamiokande]$$

The $p\to e^+\pi^0$ decay in SUSY models proceeds as in non-SUSY GUT via the exchange of the superheavy vector bosons. The only difference is that M_G is now larger ($M_G \simeq 10^{16}$ GeV) and using LEP data estimate

$$\tau(p\to e^+\pi^0) \simeq 10^{35\pm2}\ yr$$

Super-Kamiokande expects to achieve sensitivity in this mode up to $\simeq 1\times10^{34}$ yr, and so possibly observable then.

The $p\to\bar\nu K$ mode [Weinberg; Sakai; Ellis, Nanopoulos, Rudaz; Chadha, Daniels; Campbell, Ellis, Nanopoulos; R.A. Chamseddine, Nath] is more specifically supersymmetric in its origin. Consider a GUT ~~group~~ G which is SU(5) or contains SU(5). LEP data implies that only 2 Higgs doublets interact with matter and if these are embedded in SU(5) $5+\bar 5$ then

$$W_Y = \lambda^1_{ij}\,\epsilon_{XYZWU}\,\underset{5}{H_1^X}\,\underset{10}{M_i^{YZ}}\,\underset{10}{M_j^{WU}} + \lambda^2_{ij}\,\underset{\bar 5}{\bar H_{2X}}\,\underset{\bar 5}{\bar M_{iY}}\,\underset{10}{M_j^{XY}}$$

One requires that the GUT physics makes the color triplets of H_1, $\bar{H}_2$ superheavy, and integrating them out leads to quartic superpotential $W^{(4)} \sim 1/M_{H_3}$ possessing model independent p-decay interactions for mode $p \to \bar{\nu} K^+$. The p-decay interactions correspond to exchange of superheavy color triplet Higgsinos coupled to a gaugino "dressing".

Total decay rate is the sum

$$\Gamma(p \to \bar{\nu} K^+) = \sum_i \Gamma(p \to \bar{\nu}_i K^+) \ ; \quad i = e, \mu, \tau$$

(the rate for $p \to \bar{\nu}_e K^+$ is negligible). Further the CKM matrix elements in Wino aloop allow all three generations to enter (again first generation gives negligible contribution):

$$\Gamma(p \to \bar{\nu}_i K) = C \left(\frac{\beta_p}{M_{H_3}} \right) |A|^2 \, |B_i|^2$$

$$B_i = \frac{m_i^d V_{i1}^\dagger}{m_2^d V_{21}^\dagger} \left[P_2 B_{2i} + \frac{m_t V_{31} V_{32}}{m_c V_{21} V_{22}} P_3 B_{3i} \right] \frac{1}{\sin 2\beta}$$

$$P_i = e^{i\alpha_i} = \text{CP violating phases}$$

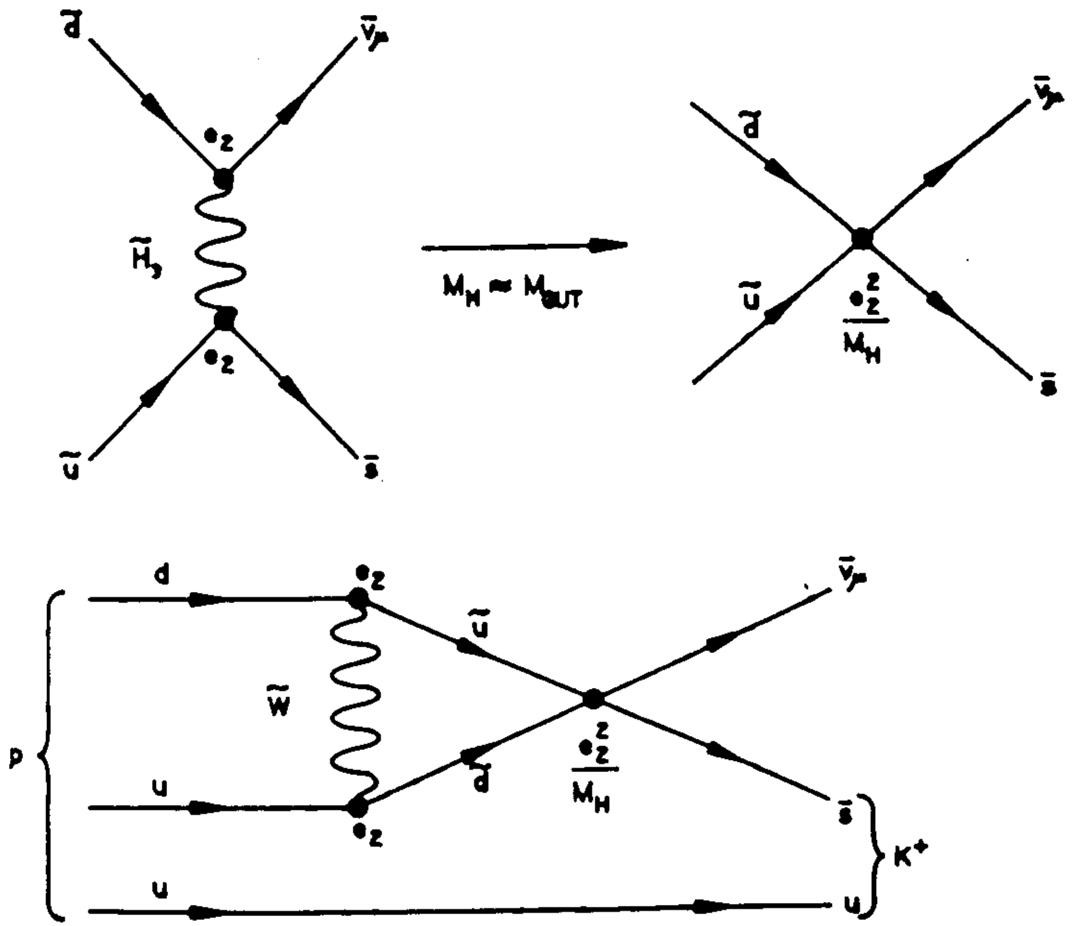


Fig. 1

Proton decay formulae for $p \to \bar{\nu} K$

[R.A., Nath]

$$\Gamma(p \to \bar{\nu} K^+) = \sum \Gamma(p \to \bar{\nu}_i K^+)$$

$$\Gamma(p \to \bar{\nu}_i K^+) = C \left(\frac{\beta_p}{M_{H_3}}\right)^2 |A|^2 \, |B_i|^2$$

where

$$C = \frac{m_N}{32\pi f_\pi^2} \left[\left(1 + \frac{m_N (D+F)}{m_B}\right)\left(1 - \frac{m_K^2}{m_N^2}\right)\right]^2$$

$D = 0.76$, $F = 0.48$, $f_\pi = 139$ MeV
$m_N = 938$ MeV, $m_B = 1154$ MeV
$m_K = 495$ MeV

$$A = \frac{\alpha_2^2}{2M_W^2} m_s m_c V_{21}^\dagger V_{21} A_L A_S ;$$

$A_L = 0.283$, $A_S = 0.833$
[R.A., Nath]

$$B_i = \frac{m_i^d V_{i1}^\dagger}{m_s V_{21}^\dagger} \left[P_2 B_{2i} + \frac{m_t V_{31} V_{32}}{m_c V_{21} V_{22}} P_3 B_{3i} \right] \frac{1}{\sin 2\beta}$$

$$B_{ji} = F(\tilde{u}_i, \tilde{d}_i, \tilde{W}) + (\tilde{d}_i \to \tilde{e}_i)$$

$$F(\tilde{u}_i, \tilde{d}_j, \tilde{W}) = \Big[E \cos\gamma_- \sin\gamma_+ \tilde{f}(\tilde{u}_i, \tilde{d}_j, \tilde{W}_1) + \cos\gamma_+ \sin\gamma_- \tilde{f}(\tilde{u}_i, \tilde{d}_j, \tilde{W}_2)$$
$$- \frac{1}{2} \frac{m_i^u \sin 2\delta_{ui}}{\sqrt{2} M_W \sin\beta} \Big[\big\{ E \sin\gamma_- \sin\gamma_+ f(\tilde{u}_{i1}, \tilde{d}_j, \tilde{W}_1)$$
$$- \cos\gamma_- \cos\gamma_+ f(\tilde{u}_{i2}, \tilde{d}_j, \tilde{W}_2) \big\} - \{ \tilde{u}_{i1} \to \tilde{u}_{i2} \} \Big]$$

$$\tilde{f}(\tilde{u}_i, \tilde{d}_j, \tilde{W}_k) = \sin^2\delta_{ui} f(\tilde{u}_{i1}, \tilde{d}_j, \tilde{W}_k) + \cos^2\delta_{ui} f(\tilde{u}_{i2}, \tilde{d}_j, \tilde{W}_k)$$

$$f(a,b,c) = \frac{m_c}{m_b^2 - m_c^2} \left[\frac{m_b^2}{m_a^2 - m_b^2} \ln\left(\frac{m_a^2}{m_b^2}\right) - (m_b \to m_c) \right]$$

$\gamma_\pm = \beta_+ \pm \beta_-$ where

$$\sin 2\beta_\pm = \frac{(\mu \mp \tilde{m}_2)}{\left[4\nu_\pm^2 + (\mu \mp \tilde{m}_2)^2\right]^{\frac{1}{2}}} \; ; \; \sqrt{2}\, \nu_\pm = M_W (\sin\beta \pm \cos\beta)$$

$$E = \begin{cases} +1 & \sin 2\beta > \mu \tilde{m}_2 / M_W^2 \\ -1 & \sin 2\beta < \mu \tilde{m}_2 / M_W^2 \end{cases}$$

$$\sin 2\delta_{u3} = \frac{-2(A_t + \mu \cot\beta) m_t}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}$$

β_p is defined by

$$\beta_p U_L^\gamma = \epsilon_{abc} \epsilon_{\alpha\beta} \langle 0 | d_{aL}^\alpha u_{bL}^\beta u_{cL}^\gamma | p \rangle$$

U_L^γ = p-wave fnc

$$\beta_p = (5.6 \pm 0.8) \times 10^{-3} \text{ GeV}^3$$ [Gavela et al]

There are two extremes;

$$\frac{\beta_3}{\beta_2} = -1 \qquad \text{Destructive interference}$$

$$\frac{\beta_3}{\beta_2} = +1 \qquad \text{Constructive interference}$$

Define total amplitude

$$B \equiv \left[|B_2|^2 + |B_3|^2 \right]^{\frac{1}{2}} \left[\frac{M_S\,(\mathrm{GeV})}{10^{2.5}} \right]^{0.3} \times 10^6 \,\mathrm{GeV}^{-1}$$

and using (LEP results):

$$M_G \cong 10^{16.1}\,\mathrm{GeV} \left[\frac{10^{2.5}}{M_S\,(\mathrm{GeV})} \right]^{0.3}$$

the experimental bound on $\tau(p \to \bar{\nu} K^+)$ becomes a bound on B:

$$B < 293 \left(\frac{5.6 \times 10^{-3}}{\beta_p\,(\mathrm{GeV})^3} \right) \left(\frac{M_{H_3}}{3 M_G} \right) \mathrm{GeV}^{-1}$$

where the theoretical formula for B is a function SUSY masses (and hence by RGE the SUSY GUT parameters)