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# SUPERSYMMETRY AND SUPERGRAVITY

## GRAND UNIFICATION

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## IC. SUSY Mass Spectrum Predictions

[R.A., Nath]

Consider now what predictions can be made concerning SUSY mass spectrum within supergravity GUTs. We will examine this within framework that allows for SU(5)-type proton decay. We allow the parameters $m_0, m_{1/2}, A_0, \tan\beta$ to vary arbitrarily for fixed $m_t$ subject to following constraints:

(1) Radiative breaking of $SU(2)\times U(1)$ occurs.

(2) Current experimental bounds on SUSY masses obeyed

(3) Impose two theoretical constraints:

(i) No extreme fine tuning of parameters:

$$m_0, m_{\tilde{g}} < 1 \text{ TeV}$$

(ii) $M_{H_3} = O(M_G)$ which we quantify as:

$$M_{H_3} < 3 M_G$$

(perturbative GUT sector)

The proton decay constraint then reads

$$B < 293 \text{ GeV}^{-1} \text{ for } \beta_p = 5.6\times10^{-3}\text{ GeV}^3$$



Procedure of calculation

(i) Run 1-loop SUSY RGE from $M_G$ to $M_Z$ where radiative breaking occurs. As first approx. neglect loop correction to Higgs extrema eqns. [Discuss this correction later]

(ii) Calculate all 31+1 SUSY masses (including 1-loop correction to $m_h$) at fixed $m_t$ as a function of $m_0, m_{1/2}, A_0, \tan\beta$

(iii) Impose experimental bounds on SUSY masses to limit parameter space

(iv) Impose p-decay bound $B < 293\text{ GeV}^{-1}$ which further limits parameter space

(v) Allowing parameters to vary over remaining allowed region obtain bands or bounds on SUSY masses. These are the predictions of supergravity grand unification [Note low energy or global SUSY cannot make predictions.]

[In calculations:

Assume $\rho_2/\rho_3 = -1$ as get least constraint on SUSY masses

Use "central" values of CKM elements:

$$V_{31} = 0.011,\ V_{32} = -0.042\ ;\ V_{13} = -0.002 \,]$$

Keep only t-quark Yukawa ($\tan\beta \le 10$)]

Proton decay formula for $B$ very complicated. However, can get semi-quantitative picture by considering only the $2^{nd}$ generation part and in the limit of large $m_0$. Then $B$ greatly simplifies:

$$B_2 \simeq - \frac{2\alpha_2}{\alpha_3 \sin 2\beta} \frac{m_{\tilde g}}{m_{\tilde q}^2} \; ; \; m_{\tilde q}^2 \cong m_0^2 + a\, m_{\tilde g}^2$$
$$a \simeq 0.7$$

Thus an upper bound on $|B_2|$ implies p-decay favors

(i) small $m_{\tilde g}$

(ii) large $m_0$

(iii) small $\tan\beta$ (i.e. large $\sin 2\beta$)

Fig: $B$ vs $m_0$; $m_t = 125$ GeV, $m_{\tilde g} = 160$ GeV, $\tan\beta = 1.73$, $\mu < 0$

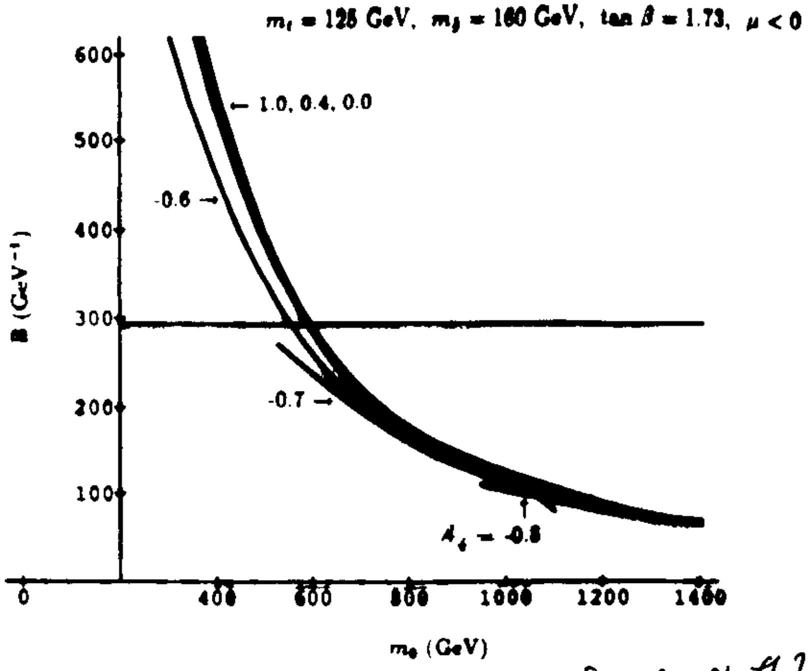
Solns. exist satisfying all constraints for wide range of Polonyi constants. The p-decay constraint strongly limits parameter space:

$m_0 \gtrsim 550$ GeV need LHC or SSC

In general one finds

$-1.2 \lesssim A_t \lesssim 1.2$ , $A_t m_0$ = Polonyi

Low $m_0$ cut off for $A_t < 0$ due to $\tilde t_1$ turning tachyonic




[R.A., Nath]

$A_t \longrightarrow A_t m_0$

Fig. 1

Fig.: B vs $m_{\tilde{g}}$; $m_t = 125$ GeV, $A_t = -0.6$, $\tan\beta = 1.73$, $\mu < 0$

Again see p-decay sharply limits the parameter space and B decreases with increasing $m_0$ and decreases with decreasing $m_{\tilde{g}}$. If impose the fine tuning constraint that $m_0 < 1$ TeV get

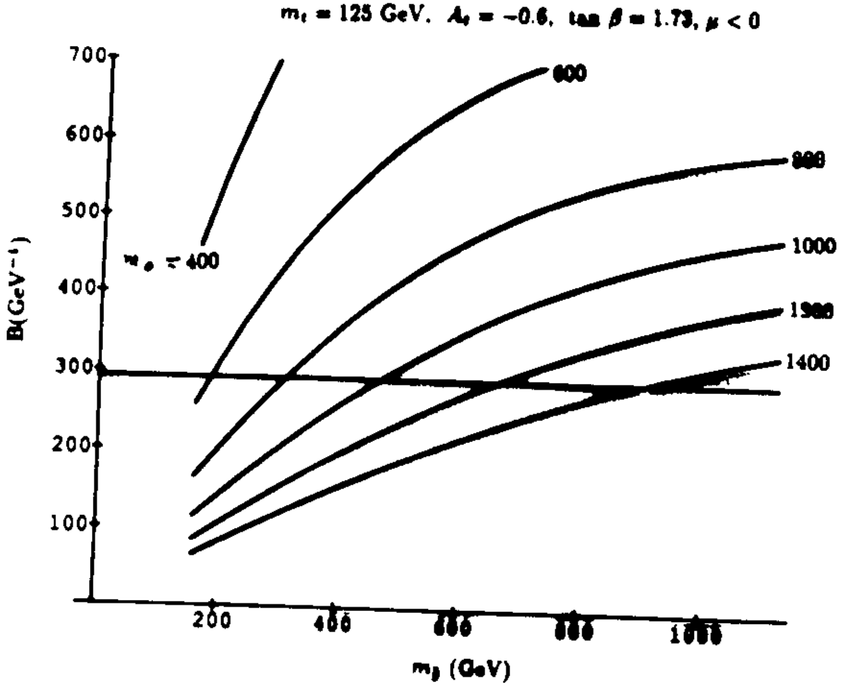
$$m_{\tilde{g}} \lesssim 450 \text{ GeV}$$

Super-Kamiokande plans to be sensitive to the $p \to \bar{\nu} K$ mode up to $(2\text{-}3) \times 10^{33}$ yr. This would correspond to a bound on B of $B \lesssim 60$ GeV. Thus if one does not wish to severely fine tune parameters, or use very large values of $m_H$, one would expect to see the $p \to \bar{\nu} K$ mode if SU(5)-type supergravity GUT is right.

Fig: $m_{\tilde{g}}$ vs $m_0$; $m_t = 125$ GeV, $A_t = -0.6$, $\mu < 0$

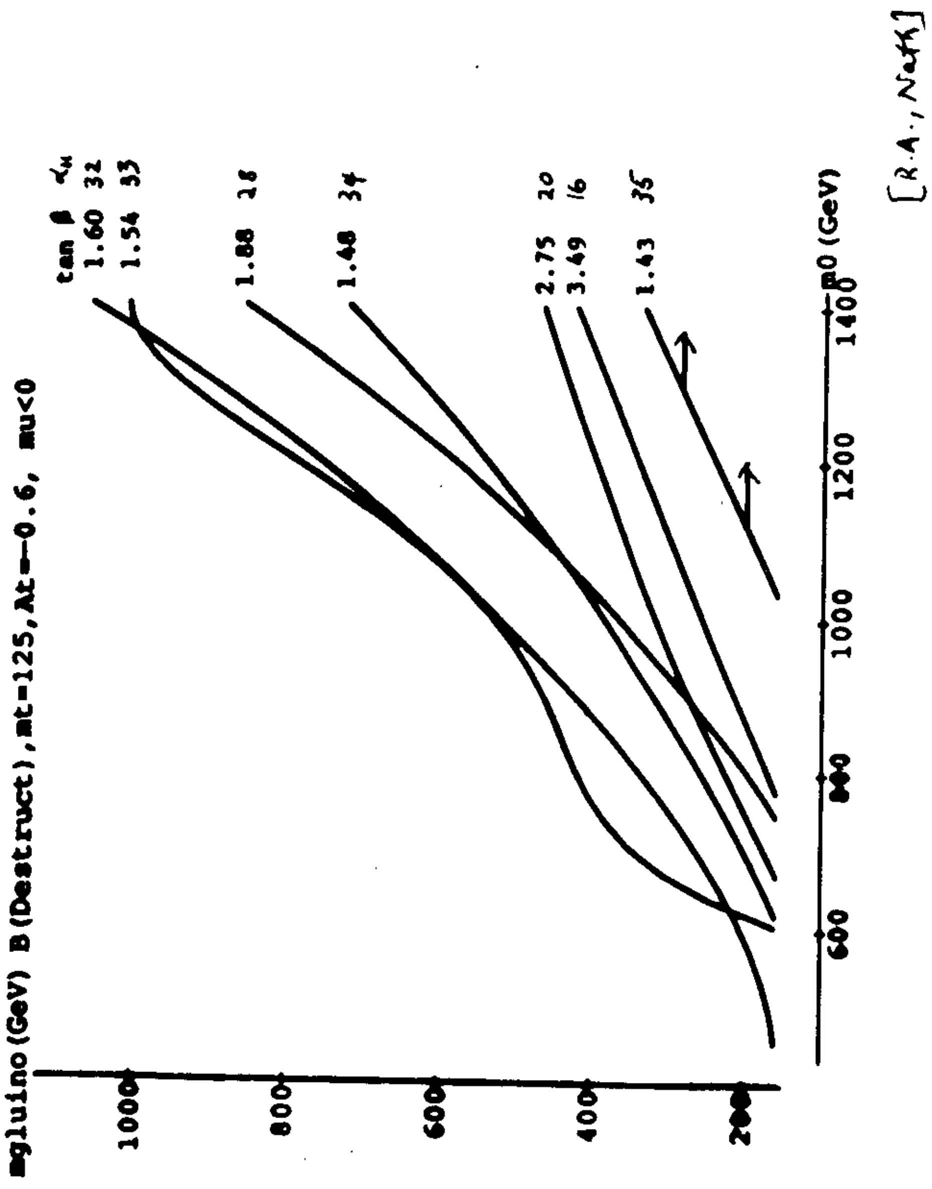
Shows region allowed by p-decay implies $m_{\tilde{g}} < m_0$. Large $\tan\beta$ cut-off due to p-decay, small $\tan\beta$ due to $\tilde{t}_1$ becoming tachyonic. In general find

$$1.1 \lesssim \tan\beta \lesssim 4.7$$




[P.A., Nath]

Fig. 2




## Charginos and Neutralinos

Fig : $m_{\tilde{W}_1}, m_{\tilde{Z}_1}$ vs $m_{\tilde{g}}$ ; $m_t = 125$ GeV, $A_t = -0.6$, $\tan\beta$, $\mu < 0$

The mass bands are very narrow and hence $m_{\tilde{W}_1}$, $m_{\tilde{Z}_1}$ essentially independent of $m_0$. Also a slowly varying fnc. of $A_t$ and $\tan\beta$. Thus there is a strong correlation between neutralino and chargino masses and gluino mass. One finds numerically in the allowed region of parameter space

$$m_{\tilde{Z}_1} \cong \frac{1}{2} m_{\tilde{W}_1}$$

$$m_{\tilde{Z}_2} \cong m_{\tilde{W}_1}$$

$$m_{\tilde{W}_2} \cong m_{\tilde{Z}_3} \cong m_{\tilde{Z}_4}$$

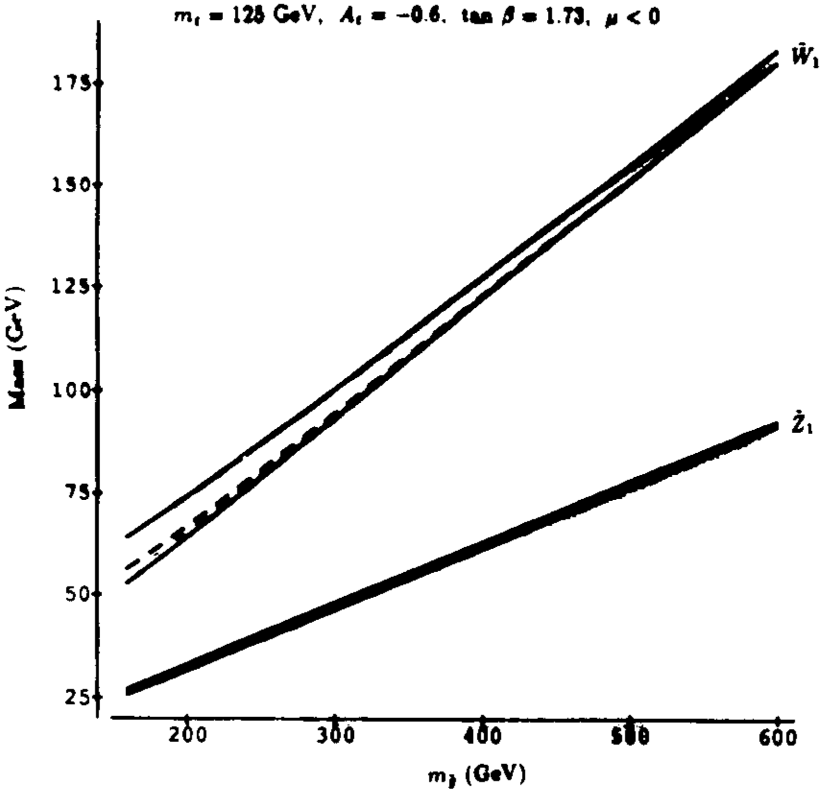
and

$$m_{\tilde{W}_1} \simeq \frac{1}{3} m_{\tilde{g}} \quad , \quad \mu < 0$$

$$m_{\tilde{W}_1} \simeq \frac{1}{4} m_{\tilde{g}} \quad , \quad \mu > 0 \qquad \text{(R.A., Nath]}$$

Thus the gluino determines the positions of all the [redacted] light Wino and Zinos, and the heavy particles are degenerate.

One can understand these remarkable results in following way:




[R.A, Nath]

Fig. 3

The p-decay constraint generally requires $m_0 \lesssim 550$ GeV and this in turn implies $\mu^2 \gg M_Z^2, \tilde m_2^2$. The $\tilde W$ mass formula was

$$m_{\tilde W_{1,2}} = \frac{1}{2}\left| [4\nu_+^2 + (\mu - \tilde m_2)^2]^{\frac12} \mp [4\nu_-^2 + (\mu + \tilde m_2)^2]^{\frac12} \right|$$

$$\sqrt{2}\,\nu_\pm = M_W(\sin\beta \pm \cos\beta)$$

Expanding gives

$$m_{\tilde W_1} \cong \tilde m_2 - \frac{M_W^2 \sin 2\beta}{\mu} \; ; \; m_{\tilde W_2} \cong \mu + \frac{M_W^2}{\mu}$$

Since $\tilde m_2 = \frac{\alpha_2}{\alpha_3} m_{\tilde g} \cong 0.294\, m_{\tilde g}$ we see that for $\mu < 0$, $m_{\tilde W_1} > \tilde m_2$ (i.e. increasing to approx $\frac13 m_{\tilde g}$) and for $\mu > 0$ $m_{\tilde W_1} < \tilde m_2$ (i.e. decreasing to approx $\frac14 m_{\tilde g}$).

The 4 neutralino masses are the 4 solns. of the quartic secular det. eqn. ($M_{\tilde Z_i} = |\lambda_i|$):

$$\lambda^4 - \lambda^3[\tilde m_1 + \tilde m_2] - \lambda^2[\mu^2 + M_Z^2 - \tilde m_1 \tilde m_2]$$
$$+ \lambda[\mu^2(\tilde m_1 + \tilde m_2) - \mu M_Z^2 \sin 2\beta + M_Z^2\, m_{\tilde\gamma}]$$
$$+ [-\mu^2 \tilde m_1 \tilde m_2 + \mu M_Z^2\, m_{\tilde\gamma} \sin 2\beta] = 0$$

and expanding for $\mu^2 \gg M_Z^2, \tilde m_2^2$ gives

$$M_{\tilde Z_1} \cong \tilde m_1 - (M_Z^2/\mu)\sin 2\beta \sin^2\theta_W$$
$$M_{\tilde Z_2} \cong \tilde m_2 - (M_Z^2/\mu)\sin 2\beta \cos^2\theta_W$$
$$M_{\tilde Z} \cong \left|\mu + \frac12 \frac{M_Z^2}{\mu}(1 \pm \sin 2\beta)\right|$$

We see in this limit

$$\frac{M_{\tilde{Z}_1}}{M_{\tilde{Z}_2}} \cong \frac{\tilde{m}_1}{\tilde{m}_2} = \frac{\alpha_1}{\alpha_2} = 0.508$$

and

$$M_{\tilde{Z}_2} \cong M_{\tilde{W}_1}$$

$$M_{\tilde{W}_2} \cong M_{\tilde{Z}_3} \cong M_{\tilde{Z}_4} \cong |\mu|$$

These various relations and degeneracies will effect the phenomenology of the system.

## t – Mass Limit

As one increases the value of $m_t$, the allowed region in parameter space shrinks, leading to a maximum value of $m_t$. One finds numerically

$$m_t \lesssim 180 \text{ GeV}$$

[R.A., Nath]

which is in excellent agreement with the LEP and other data analysis of $m_t = 130^{+25}_{-28}$ GeV [Ellis, Fogli, Lisi].



The bound on $m_t$ arises as follows. Recall that p-decay produces an upper bound of $\tan\beta \leq 4.7$ (since $\Gamma(p \to \bar{\nu} K) \sim \frac{1}{\sin^2 2\beta}$). Now as $m_t$ increases the L-R mixing in the stop mass matrix increases:

$$\begin{pmatrix} m_{\tilde{t}_L}^2 & m_t(A_t + \mu\,\mathrm{ctn}\beta) \\ m_t(A_t + \mu\,\mathrm{ctn}\beta) & m_{\tilde{t}_R}^2 \end{pmatrix}$$

eventually making $m_{\tilde{t}_1} < 45$ GeV (LEP bound) and subsequently tachyonic. Thus as $m_t$ grows, the parameter space shrinks pushing $A_t$ and $\mathrm{ctn}\beta \to 0$. The confluence of this plus the p-decay upper bound on $\tan\beta$ leads to the maximum value of $m_t$ above.

## Higgs Masses

Recall Higgs masses

$$m_{h,H}^2 = \frac{1}{2}\left[ M_Z^2 + m_A^2 + \epsilon \mp \{ (M_Z^2 + m_A^2 + \epsilon)^2 - 4 m_A^2 M_Z^2 \cos^2 2\beta + \epsilon_1 \}^{1/2} \right]$$

$$m_A^2 = m_1^2 + m_2^2 \quad ; \quad m_{H^\pm} = m_A^2 + M_W^2$$

and

$$\epsilon, \epsilon_1 \sim \frac{3\alpha_2}{4\pi} \frac{m_t^4}{M_W^2 \sin^2\beta} \ln \frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{m_t^4} + \cdots$$

Since in general $m_A^2$ is large one has

$$m_A \cong m_H \cong m_{H^\pm}$$

One can see that $m_h$

(i) increases with $m_0$ ( $m_{\tilde{t}_{1,2}}$ increase with $m_0$)

(ii) increases with $m_t$ ( $m_t^4/M_Z^2$ factor )

(iii) increases with $\tan\beta$ (from $\cos^2 2\beta$ factor)

Figs. $m_h$ vs $m_{\tilde{g}}$ ; $m_t = 125$ GeV, $A_t = -0.6$, $\mu < 0$

$m_h$ increases with $m_0$, $\tan\beta$; $m_h < 95$ GeV

$m_h$ vs $m_{\tilde{g}}$ ; $m_t = 150$, $A_t = 0.0$, $\mu < 0$

$m_h$ increases with $m_t$; regions of $m_h > 95$ GeV

Figs. $m_H$ vs $m_{\tilde{g}}$ ; $m_t = 125$, $A_t = -0.6$, $\alpha_H = 30$

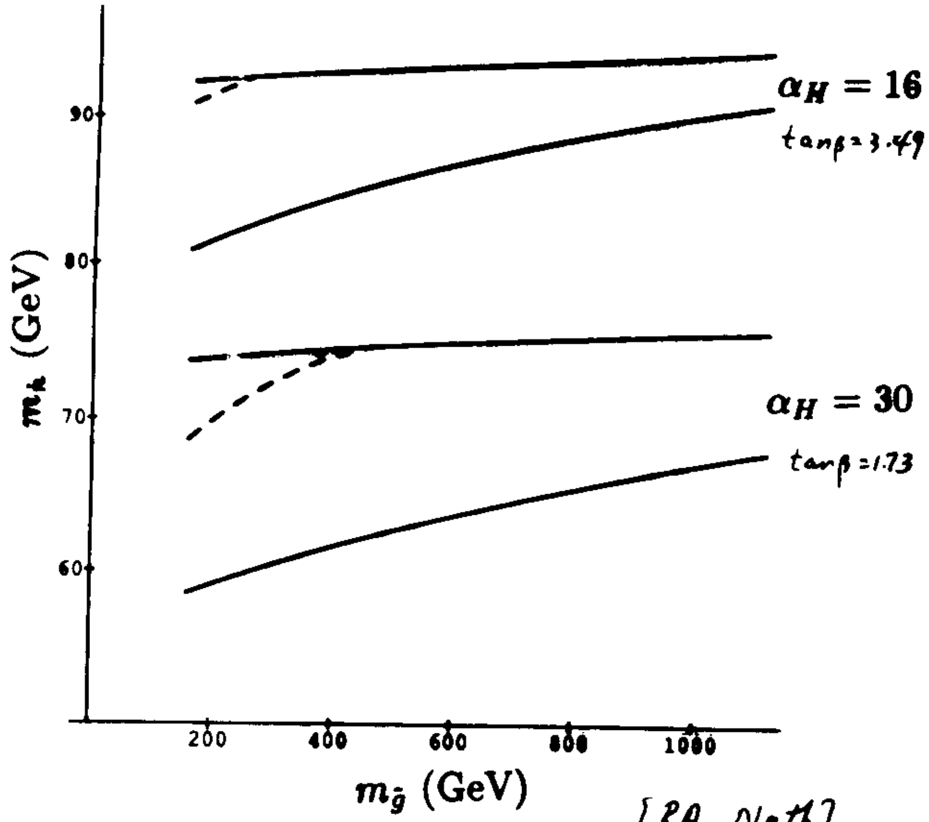
$m_t = 135$, $A_t = -0.6$, $\alpha_H = 16$

$m_t = 125$, $A_t = -0.6$, composite

$m_H$ decreases with $\tan\beta$, increases with $m_0$.

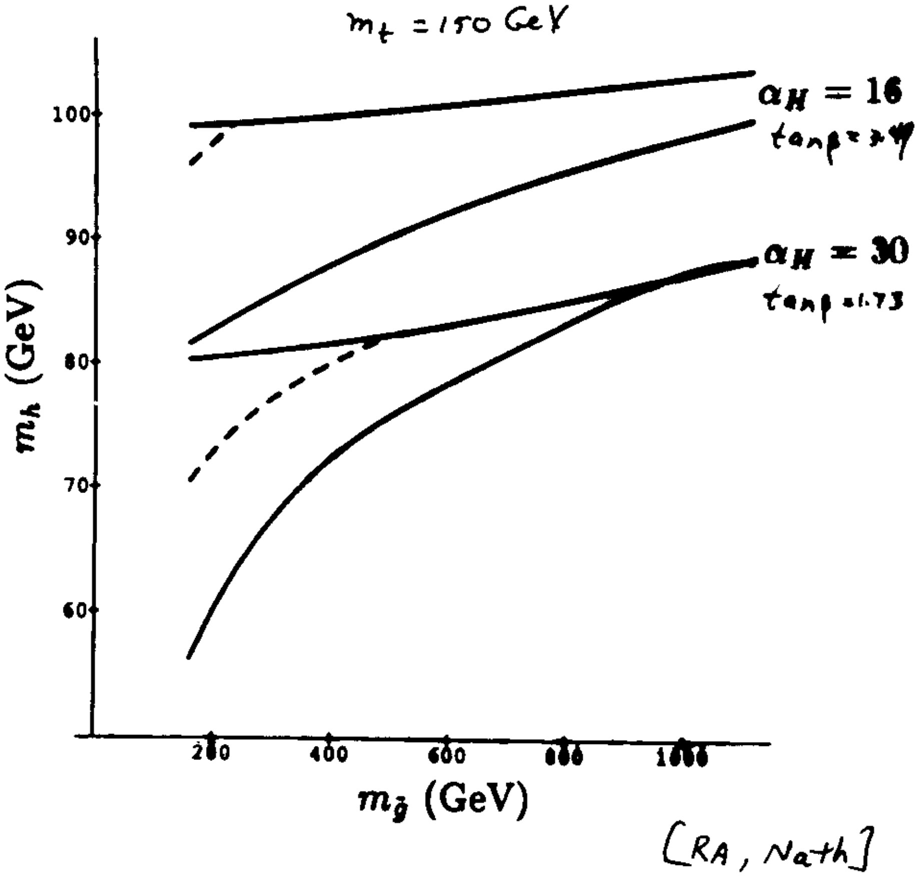
LEP2 can see the $h$ boson if $m_h \lesssim 95$ GeV [Haber] and the $\tilde{W}_1$ if $m_{\tilde{W}_1} \lesssim 100$ GeV [Baer et al]. One has

$m_t = 125$ GeV ; $\tan\beta = \frac{1}{\tan\gamma_H}$

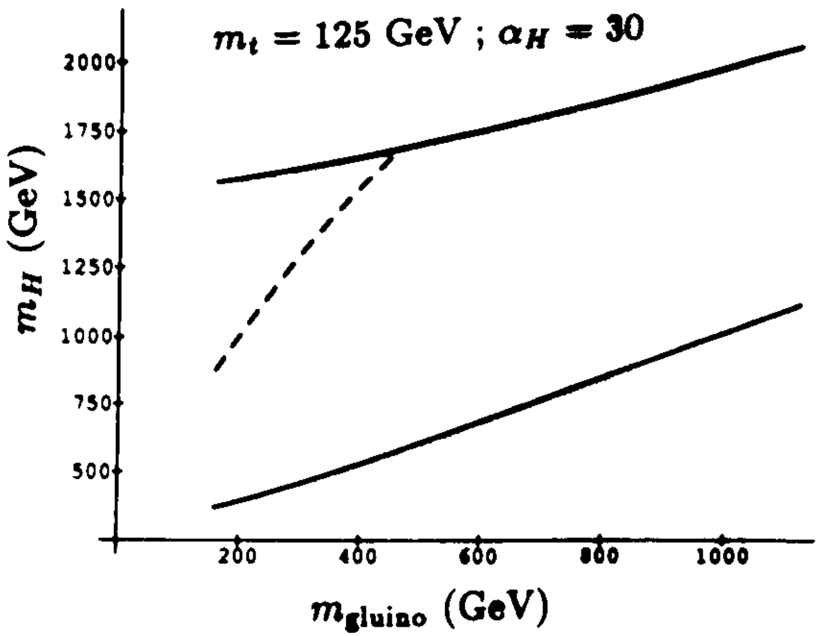



[RA, Nath]

Fig. 1

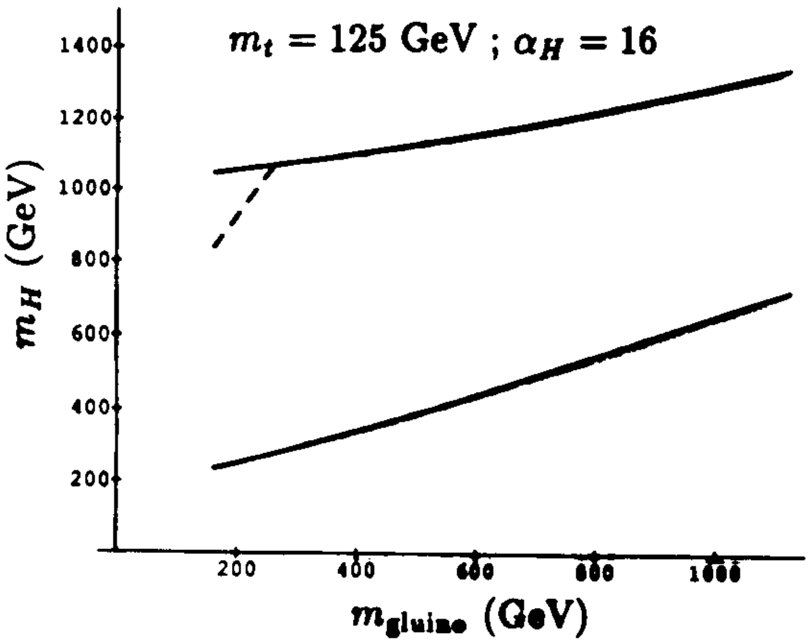


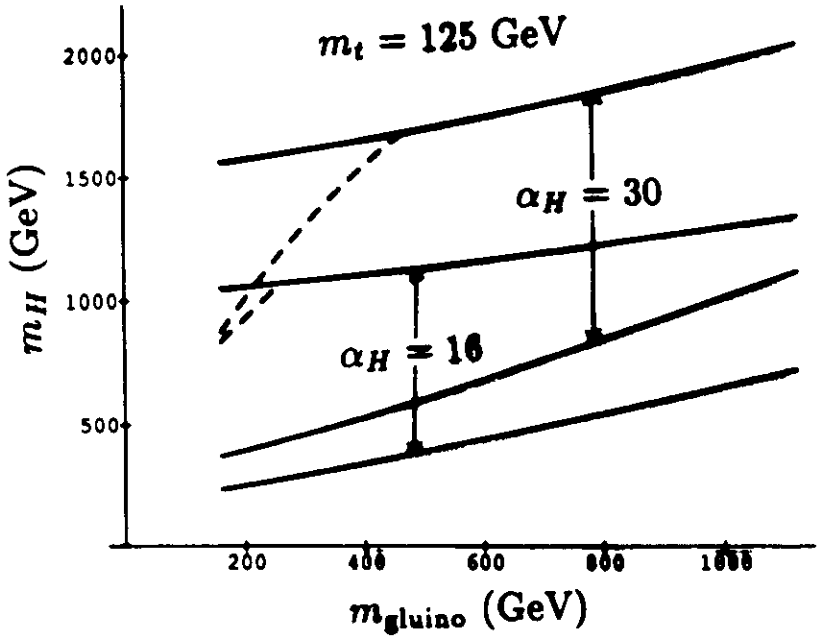
$m_t$ = 150 GeV
100
90
80
70
60
$m_h$ (GeV)
$\alpha_H = 16$
tanβ = 3.4
$\alpha_H = 30$
tanβ = 1.73
200
400
600
800
1000
$m_{\tilde{g}}$ (GeV)
[RA, Nath]

$m_t = 125$ GeV ; $\alpha_H = 30$
2000
1750
1500
1250
1000
750
500
$m_H$ (GeV)
200
400
600
800
1000
$m_{\text{gluino}}$ (GeV)


Fig. 2


$m_t = 125$ GeV ; $\alpha_H = 16$
1400
1200
1000
800
600
400
200
$m_H$ (GeV)
200
400
600
800
1000
$m_{\text{gluino}}$ (GeV)


$m_t = 125$ GeV
2000
1500
1000
500
$m_H$ (GeV)
$\alpha_H = 30$
$\alpha_H = 16$
200
400
600
800
1000
$m_{\text{gluino}}$ (GeV)

1

1

$m_t = 125$ GeV

$m_h < 95$ GeV for $\alpha_H \gtrsim 16°$ ($\tan\beta \lesssim 3.5$)

most of parameter space

$m_t = 150$ GeV

$m_h < 95$ GeV for $\alpha_H \gtrsim 18°$ ($\tan\beta \lesssim 3.1$)

part of parameter space

$m_t = 170$ GeV

$m_h < 95$ GeV for $\alpha_H \gtrsim 20°$ ($\tan\beta \lesssim 2.7$)

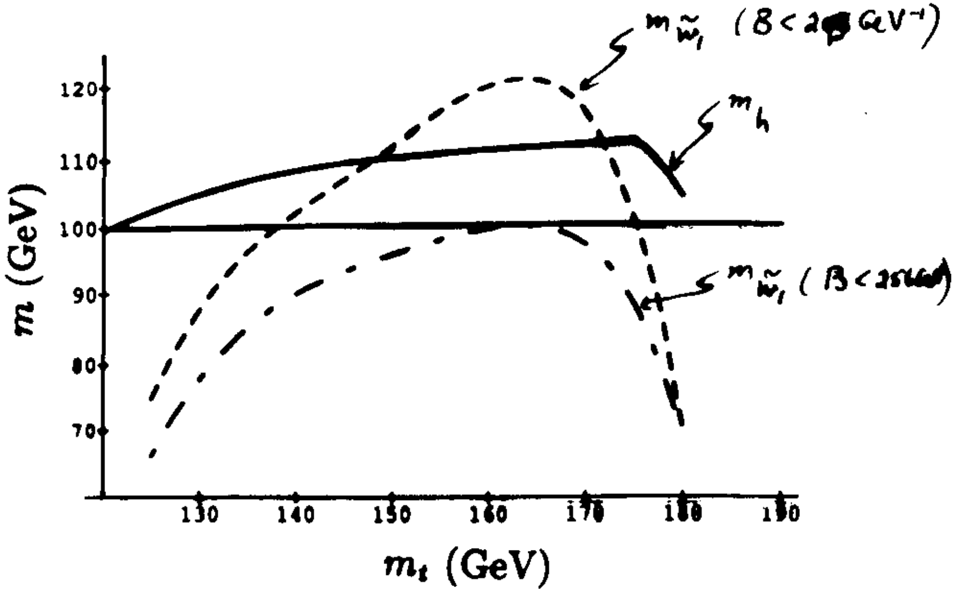
very little of parameter space

Thus the chances of seeing the light Higgs at LEP 2 depends on the t-quark mass.

However, a larger Higgs mass requires ~~smaller~~ larger $\tan\beta$ and hence smaller $\sin 2\beta$. Since p-decay B:

$$B \sim \frac{m_{\tilde{g}}}{\sin 2\beta} < 293 \text{ GeV}^{-1}$$

the larger $m_h$ needs smaller $m_{\tilde{g}}$ to satisfy p-decay constraints and hence smaller value of $m_{\tilde{W}_1} \simeq (\frac{1}{4} - \frac{1}{3}) m_{\tilde{g}}$.




[R.A., Nath]

**Fig. 3**

Fig: Max $m_h$, $m_{\tilde{W}_1}$ vs $m_t$

——— : max. value $m_h$ as fnc. of $m_t$

- - - - : max. value of $m_{\tilde{W}_1}$ when $m_h > 95$ GeV for $B < 293$ GeV$^{-1}$

—·—·— : max. value of $m_{\tilde{W}_1}$ when $m_h > 95$ GeV for $B < 256$ GeV$^{-1}$

Thus for $m_t \lesssim 140$ GeV, either $h$ or $\tilde{W}_1$ or both will be observable at LEP2 when p-decay amplitude $B < 293$ GeV$^{-1}$

and

for all $m_t$, either $h$ or $\tilde{W}_1$ or both will be observable at LEP2 when p-decay amplitude $B < 256$ GeV$^{-1}$ [$\tau(p \to \bar{\nu} K^+) > 1.3 \times 10^{32}$ yr]



## Summary

When the proton decay constraint is added to radiative breaking conditions, minimal supergravity GUT makes already strong predictions concerning the SUSY mass spectrum. These include:

$$m_0 \gtrsim 550\,\text{GeV}\ ; \quad m_{\tilde{g}} \lesssim 450\,\text{GeV}$$

$$m_{\tilde{Z}_1} \cong \tfrac{1}{2} m_{\tilde{W}_1}\ ; \quad m_{\tilde{Z}_2} \cong m_{\tilde{W}_1}\ ; \quad m_{\tilde{W}_2} \cong m_{\tilde{Z}_3} \cong m_{\tilde{Z}_4}$$

$$m_{\tilde{W}_1} \simeq \tfrac{1}{8} m_{\tilde{g}}\ ,\ \mu < 0;\ \text{or}\ m_{\tilde{W}_1} \simeq \tfrac{1}{4} m_{\tilde{g}}\ ,\ \mu > 0$$

$$m_H \cong m_A \cong m_{H^\pm}$$

Either $h$ or $\tilde{W}_1$ observable at LEP2 for $m_t \lesssim 140$ GeV (and possibly for all $m_t$) and possibly both.

In addition one has:

$$m_t \lesssim 180\ \text{GeV}$$

and the decay $p \to \bar{\nu} K^+$ is expected in the next round of proton decay experiments

Note: Some of above predictions depend upon the theoretical requirements imposed of $m_0, m_{\tilde{g}} \lesssim 1$ TeV (no extreme fine tuning) and $M_{H_3} < 3 M_G$ (no anomalously large Higgs triplet mass). If these constraints are relaxed some of above predictions will be modified.

## 11. No Scale Model

One particularly interesting supergravity model is the No-scale model which we define here as

$$m_0 = 0 = A_0$$

Thus after radiative breaking, the theory depends only on two parameters:

$$m_{1/2}\ ,\ \tan\beta$$

where $m_{1/2}$ is the basic 'germ' of supersymmetry breaking. These models have recently been analysed by a number of groups [Kelley et al, Roberts and Ross, Inoue et al] where constraints on particle spectrum have been examined

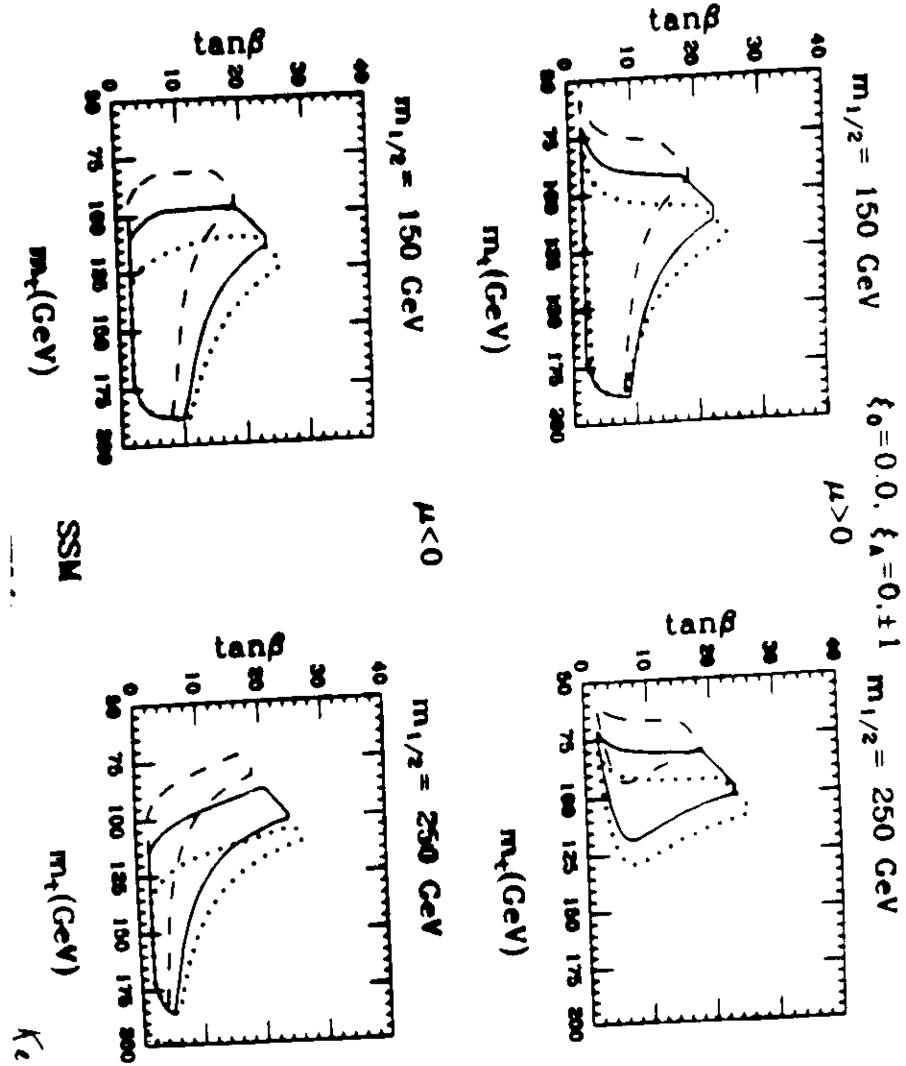
Fig: Solid curves: No-scale curves in $\tan\beta - m_t$ space for $m_{1/2} = 150, 250$

We assume here that the model is of an SU(5)-type with normal SUSY p-decay allowed. We chose

$$P_2 / P_3 = -1\ ,\ \mu > 0$$

and take the maximum (90% CL) values of CKM matrix:

$$V_{31} = 0.019\ ;\ V_{32} = -0.058\ ;\ \text{and}\ V_{13} = -0.007$$




Kelley et al

This maximizes the amount of cancellation between 3rd and 2nd generation contributions to p-decay, and hence maximizes the viability of the theory.

Fig $B$ vs. $m_{1/2}$ : $\tan\beta = 2, \cdots 10$. $B$ lies far above the allowed p-decay limit $B < 293$ GeV for $m_t = 125$ GeV

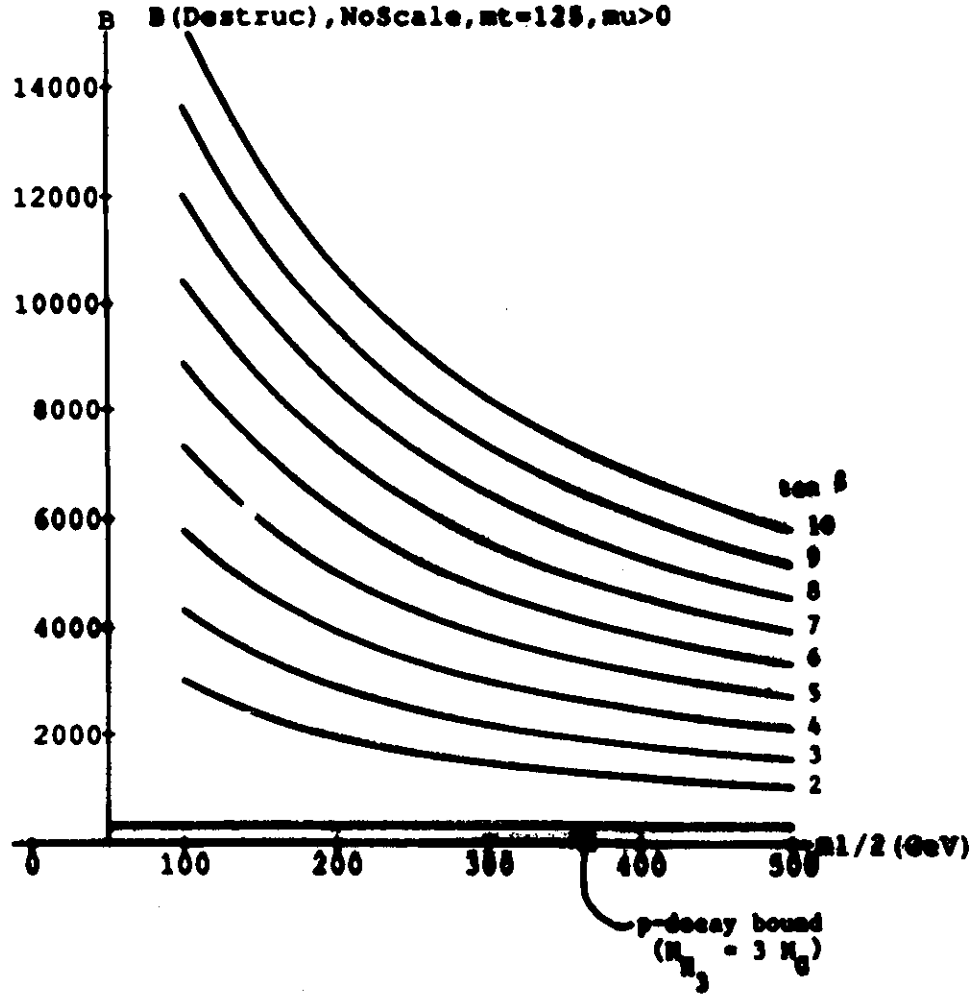
Fig $B$ vs. $m_{1/2}$ : $\tan\beta = 1.1$ to $2$. ($\tan\beta = 1.1$ is min. allowed value) $B$ is much greater than limit $B < 293$ GeV for $m_t = 125$ GeV.

Fig. $B$ vs. $\tan\beta$ : $m_t$ dependence for $90 < m_t < 165$. $m_t$ variation is very slow. ($m_{1/2} = 200$ GeV)

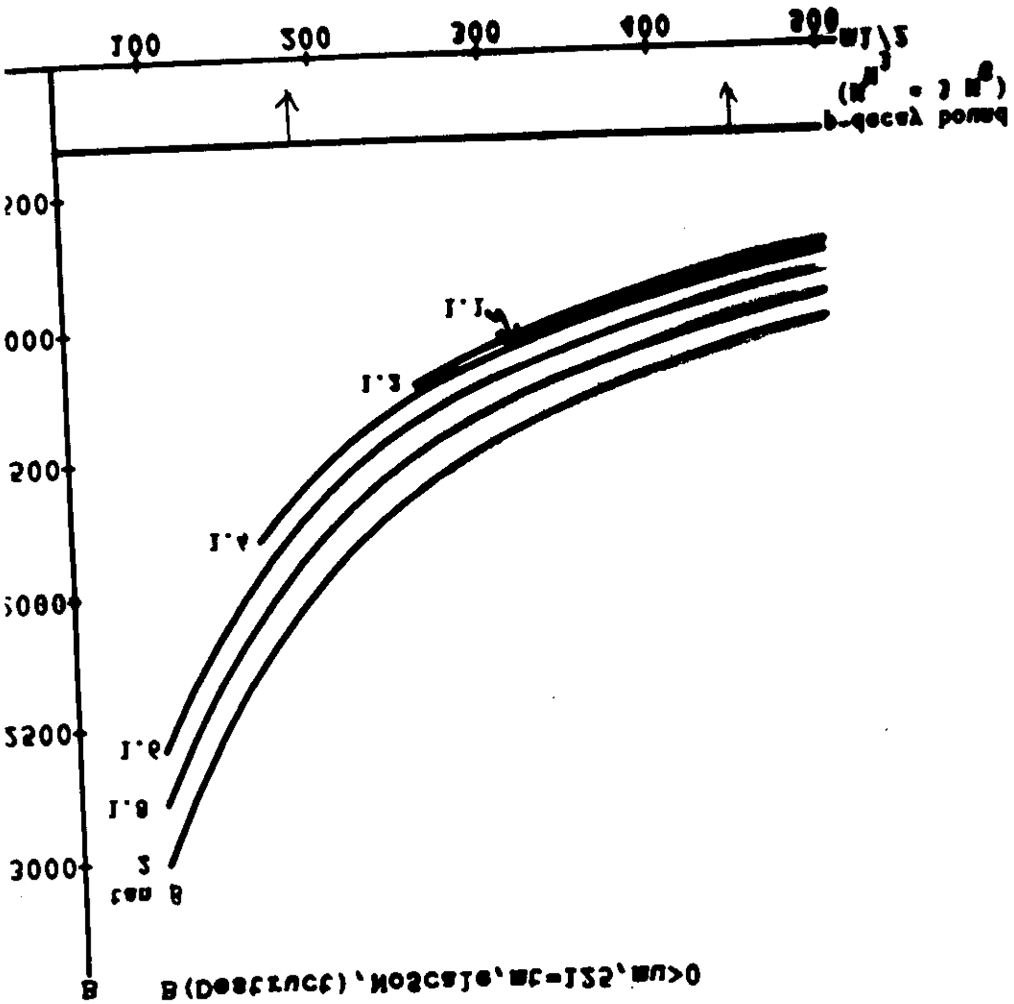
One finds over entire permissible domain of $m_t$, the predictions of No-scale model for $B$ are in large disagreement with experimental upper limit.

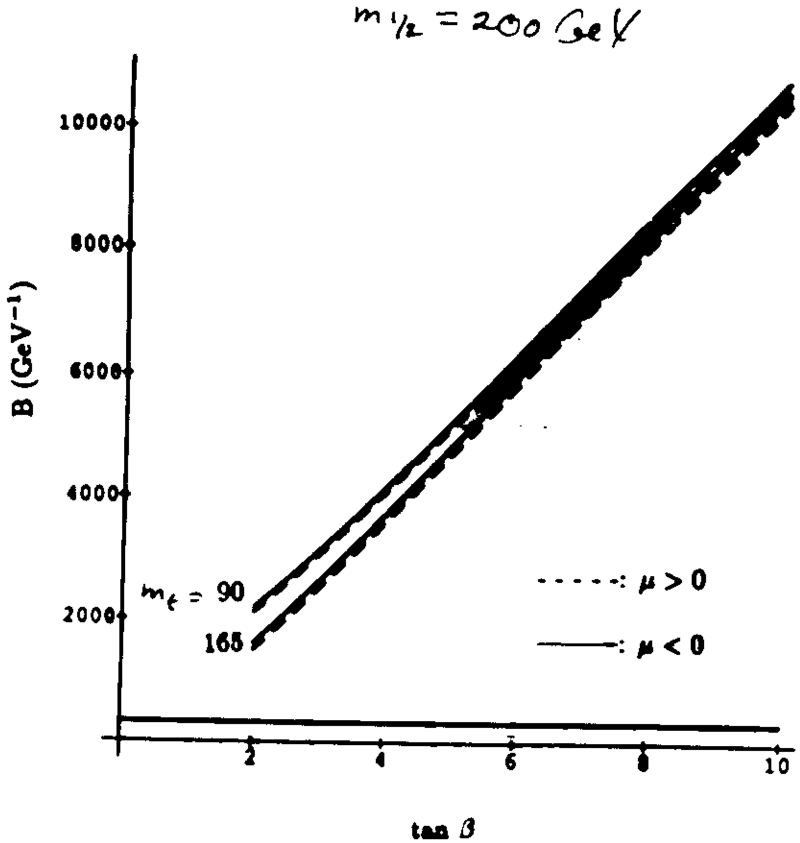
There is one version of No-scale model where this difficulty disappears: shift gauge group

$SU(5) \longrightarrow$ Flipped $SU(5) \times U(1)$




[R.A., Nath]





$m_{1/2} = 200$ GeV
B (GeV$^{-1}$)
10000
8000
6000
4000
2000
$m_t = 90$
165
- - - - -: $\mu > 0$
——: $\mu < 0$
2
4
6
8
10
tan $\beta$


FIG. 3

Then the SUSY p-decay mode $p \to \bar{\nu} K^+$ is suppressed (leaving only $p \to e^+ \pi^0$) and so constraints from p-decay are removed. There is some loss of predictive power, eg., there is an extra U(1) coupling constant (need not equal the SU(5) coupling at $M_G$), $m_b/m_\tau$ relation need not hold, and one must chose GUT sector carefully to preserve large neutrino masses (4 extra gauge singlets). However, basic question is the experimental one of whether the next round of p-decay experiments see $p \to \bar{\nu} K^+$ occur.



12a. O(10) MODELS

If one imposes O(10) symmetry one gets an additional constraint on Yukawa couplings not found in SU(5):

$$\lambda_t = \lambda_b = \lambda_\tau \qquad \mu = M_G$$

Since

$$m_t = \lambda_t(m_t)\langle H_2 \rangle \ ; \ m_b = \lambda_b(m_b)\langle H_1 \rangle$$

one has

$$\tan\beta \simeq \frac{m_t}{m_b} \simeq 35-40 \gg 1$$

Properties of these models have been analysed [Ananthanarayan, Lazarides, Shafi]. A number of the results obtained

$$m_t = 142^{+26}_{-49} \text{ GeV}$$

$$m_h = 104^{+14}_{-12} \text{ GeV}$$

In general the SUSY spectrum is quite heavy compared to other models, and some of the particles may be difficult to see experimentally:

$$1\,\text{TeV} \lesssim m_{\tilde q} \lesssim 3.2\,\text{TeV}$$
$$1.2\,\text{TeV} \lesssim m_{\tilde g} \lesssim 3.3\,\text{TeV}$$
$$320\,\text{GeV} \lesssim m_{\tilde W_1} \lesssim 910\,\text{GeV}$$
$$200\,\text{GeV} \lesssim m_{\tilde Z_1} \lesssim 550\,\text{GeV}$$ [Arnowitt et al.]

An important question in the model is p-decay. Recall that for usual type of p-decay

$$|B| \approx 2\,\frac{\alpha_2}{\alpha_3}\,\frac{1}{\sin 2\beta}\,\frac{m_{\tilde g}}{m_{\tilde q}^2}\times 10^6 \lesssim 293\,\text{GeV}^{-1}$$

which favors small $\tan\beta$, small $m_{\tilde g}$, ~~small~~ large $m_{\tilde q}$. A very rough estimate indicates that large $\tan\beta$, large $m_{\tilde g}$ will violate the current experimental bounds (though more detailed calculations needed). Thus the full GUT model is needed to know the structure of p-decay for this model.

[A similar danger exists for the model of Drees and Nojiri who impose $B_0 = A_0 - m_0$ and then require $\tan\beta \approx 25-30$.]



## 12. Loop Corrections To Radiative Breaking

Previous discussions have neglected the loop corrections in electroweak radiative breaking eqns. Recall

$$\frac{1}{2}M_Z^2 = \frac{\mu_1^2 - \mu_2^2\tan^2\beta}{\tan^2\beta - 1}\ ; \quad \sin 2\beta = \frac{2m_3^2}{\mu_1^2+\mu_2^2}$$

where $\mu_a^2 = m_a^2 + \Sigma^a$ and $\Sigma^a$ is the 1-loop term

$$\Sigma^a = \frac{1}{32\pi^2}\sum_i (-1)^{2j_i}\, n_i M_i^2 \ln\left[\frac{M_i^2}{e^{1/2}Q^2}\right]\frac{\partial M_i^2}{\partial v_a}$$

$j_i$ = spin, $n_i$ = no. of helicity states, $M_i(v_a)$ = mass

There are large number particles entering sum in $\Sigma^a$: 31 SUSY + t-quark + h + W + Z etc. and if each gave $(\frac{1}{2}-1)\%$ correction could generate (20-30)% change from tree values. However, for significant part of parameter space, $\Sigma^a$ has large cancellations significantly reducing its size. These cancellations mainly not due to supersymmetry (which is badly broken anyway).

Consider two cases: (i) $m_0^2 \gg M_Z^2, m_{1/2}^2$ and (ii) $m_{1/2}^2 \gg m_0^2, M_Z^2$

(i) $m_0^2 >> M_Z^2, m_{1/2}^2$

Consider 1st two generations of squarks

$$\tilde{q}_i = \tilde{u}_L, \tilde{d}_L, \tilde{u}_R, \tilde{d}_R \quad ; \quad i = 1,2$$

One has

$$\Sigma^a_{\tilde{q}_i} = -\frac{s_a}{8\pi \cos^2\theta_W}\left[\sin^2\theta_W Y_i - \cos^2\theta_W T_{3i}\right] M^2_{\tilde{q}_i} \ln\left(\frac{M^2_{\tilde{q}_i}}{e^{1/2}G^2}\right)$$

$$s_a = (1,-1)$$

The RGE give for the squark masses

$$M^2_{\tilde{q}_i} = m_0^2 + \gamma_i m_{1/2}^2 + \underbrace{(\sin^2\theta_W Y_i - \cos^2\theta_W T_{3i}) M_Z^2 \cos 2\beta}_{\text{D-term}}$$

The leading piece of $\gamma_i$ from SU(3) interactions and same for all squarks. Remaining parts $\gamma_i'$ small

$$\gamma_i = \gamma + \gamma_i' \quad ; \quad \gamma_i' << \gamma \quad (\text{from } SU(2)\times U(1))$$

Now expand

$$M_{\tilde{q}_i}^2 = m_0^2 + \Delta_i^2 \quad , \quad \Delta_i^2 << m_0^2$$

Get:

$$\Sigma^a = -\frac{s_a}{8\pi\cos^2\theta_W}\left(\sin^2\theta_W Y_i - \cos^2\theta_W T_{3i}\right)\Big[m_0^2 \ln\left(\frac{m_0^2}{e^{1/2}G^2}\right)$$
$$+ \Delta_i^2\left\{1 + \ln\left(\frac{m_0^2}{e^{1/2}G^2}\right)\right\} + O\left(\frac{\Delta_i^4}{m_0^2}\right)\Big]$$

Define Tr = sum over $i$. Since $\mathrm{Tr}\, Y_i = 0 = \mathrm{Tr}\, T_{3i}$, leading $m_0^2$ and $\gamma m_{1/2}^2$ part of $\Delta_i$ cancel leaving only small electroweak and D-term part of $\Delta_i$.

Same happens for 3 generations of sleptons.



Get then

$$\Sigma^a (2 \text{ gen. } \tilde{q}, 3 \text{ gen. } \tilde{l}) \cong$$
$$-\frac{s_a \alpha_2}{8\pi}\left(2.32 \cos 2\beta\, M_Z^2 - 0.26\, m_{1/2}^2\right)\left(1 + \ln\left(\frac{m_0^2}{e^{1/2}G^2}\right)\right)$$
$$+ O\left(\frac{\Delta_i^4}{m_0^2}\right)$$

The third squark generation is distorted by large L-R mixing of t-squarks and so here cancelation reduced. Supersymmetry gives partial cancellation among charginos + charged Higgs and neutralinos + neutral Higgs.

Table 1 $\quad m_0 = 800\,\text{GeV}$, $m_{\tilde{g}} = 160\,\text{GeV}$, $A_t = -0.6$, $\tan\beta = 1.88$
$m_t = 125\,\text{GeV}$
Without cancellations $R_\mu \equiv \delta\mu/\mu$
moves from 0.32% to 5.76%

Table 2 $\quad m_0 = 1000\,\text{GeV}$, $m_{\tilde{g}} = 160\,\text{GeV}$, $A_t = -0.8$, $\tan\beta = 47$
$m_t = 125\,\text{GeV}$
without cancellations $R_\mu$ moves from 18.5% to 89.2%!

(ii) $m_{1/2}^2 >> m_0^2, M_Z^2$

For squarks analysis as before with expansion

$$M^2_{\tilde{q}_i} = \gamma m_{1/2}^2 + \Delta_i' \quad ; \quad \Delta_i' << \gamma m_{1/2}^2$$
$$\gamma_i = \gamma + \gamma_i'$$

and similar cancelation.

Table 1. Contributions to one loop corrections $\Sigma^1$ and $\Sigma^2$ for $m_{\tilde{g}}$ = 100 GeV, $m_{\tilde{q}}$ = 800 GeV, $A_t = -0.6$, $\alpha_H = 28°$ ($\tan\beta = 1.88$), $\mu < 0$, and $m_t$ = 125 GeV. $TR$ is the sum over all states.

| Particle | Mass (GeV) | $\Sigma^1$ (GeV$^2$) | $\Sigma^2$ (GeV$^2$) |
|---|---|---|---|
| $\tilde{u}_L$ (gen. 1,2) | 808.9 | 1502.4 | -1502.4 |
| $\tilde{u}_R$ (gen. 1,2) | 808.7 | 677.9 | -677.9 |
| $\tilde{d}_L$ (gen. 1,2) | 811.1 | -1854.2 | 1854.2 |
| $\tilde{d}_R$ (gen. 1,2,3) | 809.3 | -330.6 | 330.6 |
| $\Sigma_q$ (gen. 1+2) | | -36.9 | 36.9 |
| $\tilde{e}_L$ (gen. 1,2,3) | 801.7 | -1137.2 | 1137.2 |
| $\tilde{e}_R$ (gen. 1,2,3) | 801.0 | -992.4 | 992.4 |
| $\tilde{\nu}$ (gen. 1,2,3) | 799.5 | 2117.1 | -2117.1 |
| $\Sigma_l$ (gen. 1+2+3) | | -37.8 | 37.8 |
| $\tilde{b}_L$ | 637.2 | 1001.6 | -1001.6 |
| $\tilde{t}_1$ | 380.8 | -3872.0 | -343.0 |
| $\tilde{t}_2$ | 674.1 | [illegible] | 13730.7 |
| $t$ | 0 | -17.0 | -17.0 |
| $\tilde{W}_1$ | 54.6 | 3.7 | 0.9 |
| $\tilde{W}_2$ | 648.1 | -6448.2 | -7313.6 |
| $H^{\pm}$ | 1161.7 | 8107.8 | 8107.8 |
| $W + Z$ | | -40.0 | -40.0 |
| $\tilde{Z}_1$ | 25.9 | 0.06 | 0.02 |
| $\tilde{Z}_2$ | 55.1 | 4.8 | 1.9 |
| $\tilde{Z}_3$ | 630.9 | 2113.8 | -1330.2 |
| $\tilde{Z}_4$ | 640.7 | -6010.8 | -3406.8 |
| $h$ | 68.1 | 6.1 | -3.4 |
| $H$ | 1161.6 | [illegible] | 1006.3 |
| $A$ | 1100.0 | 0 | 0 |
| $TR\ \Sigma^a$ | | [illegible] | [illegible] |
| $TR\ \vert\Sigma^a\vert$ | | [illegible] | [illegible] |
| $\mu_1 = -630.73$ ; $\mu_2 =$ [illegible] ; $R_\mu =$ [illegible] | | | |



Table 2. Contributions to one loop corrections $\Sigma^1$ and $\Sigma^2$ for $m_{\tilde{g}}$ = 100 GeV, $m_{\tilde{q}}$ = 1000 GeV, $A_t = -0.8$, $\alpha_H = 12°$ ($\tan\beta = 4.70$), $\mu < 0$ and $m_t$ = 125 GeV.

| Particle | Mass (GeV) | $\Sigma^1$ (GeV$^2$) | $\Sigma^2$ (GeV$^2$) |
|---|---|---|---|
| $\tilde{u}_L$ (gen. 1,2) | 1006.7 | 2500.7 | 2500.7 |
| $\tilde{u}_R$ (gen. 1,2) | 1006.8 | 1100.6 | -1100.6 |
| $\tilde{d}_L$ (gen. 1,2) | 1009.6 | -3196.8 | 3196.8 |
| $\tilde{d}_R$ (gen. 1,2,3) | 1007.6 | -586.0 | 586.0 |
| $\Sigma_q$ (gen. 1+2) | | -47.0 | 47.0 |
| $\tilde{e}_L$ (gen. 1,2,3) | 1001.8 | -1961.1 | 1961.1 |
| $\tilde{e}_R$ (gen. 1,2,3) | 1001.1 | -1730.2 | 1730.2 |
| $\tilde{\nu}$ (gen. 1,2,3) | 999.9 | 3687.5 | -3687.5 |
| $\Sigma_l$ (gen. 1+2+3) | | -71.3 | 71.3 |
| $\tilde{b}_L$ | 821.9 | 1916.5 | -1916.5 |
| $\tilde{t}_1$ | 557.8 | -9040.5 | -9870.5 |
| $\tilde{t}_2$ | 846.2 | 37009.0 | 34941.4 |
| $t$ | 138.0 | 0 | -13.9 |
| $\tilde{W}_1$ | 52.4 | 19.5 | 0.15 |
| $\tilde{W}_2$ | 294.1 | -140.3 | -837.5 |
| $H^{\pm}$ | 1061.5 | 6567.0 | 6567.0 |
| $\tilde{Z}_1$ | 25.8 | -0.11 | 0.02 |
| $\tilde{Z}_2$ | 52.6 | -4.4 | 0.23 |
| $\tilde{Z}_3$ | 283.1 | 995.0 | -220.6 |
| $\tilde{Z}_4$ | 300.3 | -1373.0 | -285.6 |
| $h$ | 94.8 | 8.8 | -3.2 |
| $H$ | 1060.2 | 15704.1 | 33.6 |
| $A$ | 1058.5 | 0 | 0 |
| $W + Z$ | | -40.0 | -40.0 |
| $TR\ \Sigma^a$ | | 41701.8 | [illegible] |
| $TR\ \vert\Sigma^a\vert$ | | 100770.8 | 74097.0 |
| $\mu_1 = -272.43$ ; $\mu_2 = -300.00$ ; $R_\mu = -10.40\%$ | | | |

For sleptons, $\gamma = 0$ (no SU(3) coupling) but slepton contribution is small anyway. In general largest contributions come from heaviest particles i.e. $\tilde{t}_i$, $\tilde{W}_2$, $\tilde{Z}_4$.

Tables No-scale model: $m_{1/2} = 200$ GeV, $\tan\beta = 5$, $\mu < 0$
$m_t = 125$

without cancellations $R_\mu$ goes from 5.61% to 13.0%

Summary

While there are places in parameter space where loop correction can be quite large in general this does not occur. For most of parameter space in case where $m_0$ is large has $R_\mu \lesssim (5$[illegible]$)\%$ and in case where $m_{1/2}$ is large $R_\mu \lesssim 10\%$.

Table 3. Contributions to one loop corrections $\Sigma^1$ and $\Sigma^2$ for No-scale model: $m_{1/2} = 200$ GeV ($m_{\tilde{g}} = 580.8$ GeV), $\tan\beta = 5.0$, $\mu < 0$ and $m_t = 125$ GeV.

| Particle | Mass (GeV) | $\Sigma^1$ (GeV$^2$) | $\Sigma^2$ (GeV$^2$) |
|---|---|---|---|
| $\tilde{u}_L$ (gen. 1,2) | 456.3 | 300.1 | -300.1 |
| $\tilde{u}_R$ (gen. 1,2) | 440.1 | 163.5 | -163.5 |
| $\tilde{d}_L$ (gen. 1,2) | 462.7 | -506.2 | 506.2 |
| $\tilde{d}_R$ (gen. 1,2,3) | 430.8 | -81.6 | 81.6 |
| $\Sigma_{\tilde{q}}$ (gen. 1+2) | | -88.4 | 88.4 |
| $\tilde{e}_L$ (gen. 1,2,3) | 148.2 | -0.8 | 0.8 |
| $\tilde{e}_R$ (gen. 1,2,3) | 87.9 | 0.23 | [illegible] |
| $\tilde{\nu}$ (gen. 1,2,3) | 126.0 | 0.3 | -0.3 |
| $\Sigma_{\tilde{l}}$ (gen. 1+2+3) | | -1.3 | 1.3 |
| $\tilde{b}_L$ | 435.9 | 480.7 | -480.7 |
| $\tilde{t}_1$ | 348.9 | 4087.5 | -1064.3 |
| $\tilde{t}_2$ | 400.0 | -11175.0 | 7313.0 |
| $t$ | 125.0 | 0 | -66.5 |
| $\tilde{W}_1$ | 157.0 | -800.0 | 53.0 |
| $\tilde{W}_2$ | 261.8 | 2306.0 | -1080.4 |
| $H^\pm$ | 276.4 | 280.9 | [illegible] |
| $W + Z$ | | -9.0 | -4.4 |
| $\tilde{Z}_1$ | 87.4 | 0.06 | 0.00 |
| $\tilde{Z}_2$ | 155.8 | 4.7 | -2.4 |
| $\tilde{Z}_3$ | 256.4 | 120.2 | -60.8 |
| $\tilde{Z}_4$ | 241.3 | 1430.0 | 206.1 |
| $h$ | 91.4 | -0.12 | 0.04 |
| $H$ | 267.3 | 844.0 | 3.0 |
| $A$ | 264.5 | 0 | 0 |
| $TR\,\Sigma^s$ | | -1604.0 | [illegible] |
| $TR\,\vert\Sigma^s\vert$ | | [illegible] | [illegible] |

$\mu_1 = -280.34$ ; $\mu_2 = -310.00$ ; [illegible]

Table 4. Contributions to one loop corrections $\Sigma^1$ and $\Sigma^2$ for No-scale model: $m_{1/2} = 450$ GeV ($m_{\tilde{g}} = 1306.8$ GeV), $\tan\beta = 9.0$, $\mu < 0$ and $m_t = 125$ GeV.

| Particle | Mass (GeV) | $\Sigma^1$ (GeV$^2$) | $\Sigma^2$ (GeV$^2$) |
|---|---|---|---|
| $\tilde{u}_L$ (gen. 1,2) | 1031.7 | 3007.4 | -3007.4 |
| $\tilde{u}_R$ (gen. 1,2) | 992.5 | 1261.3 | -1261.3 |
| $\tilde{d}_L$ (gen. 1,2) | 1034.7 | -3706.3 | -1261.3 |
| $\tilde{d}_R$ (gen. 1,2,3) | 966.4 | -604.4 | 3706.3 |
| $\Sigma_q$ (gen. 1+2) | | -104.1 | 604.4 |
| $\tilde{e}_L$ (gen. 1,2,3) | 321.0 | -110.9 | 104.1 |
| $\tilde{e}_R$ (gen. 1,2,3) | 170.7 | -17.3 | 119.3 |
| $\tilde{\nu}$ (gen. 1,2,3) | 311.2 | 205.0 | 17.3 |
| $\Sigma_l$ (gen. 1+2+3) | | 206.1 | -205.0 |
| $\tilde{b}_L$ | 1001.5 | 3409.1 | -206.1 |
| $\tilde{t}_1$ | 892.5 | 83411.4 | -3409.1 |
| $\tilde{t}_2$ | 1009.5 | -114315.0 | -38109.2 |
| $t$ | 125.0 | 0 | 80001.6 |
| $\tilde{W}_1$ | 200.1 | -13010.0 | 0.77 |
| $\tilde{W}_2$ | 416.5 | 30411.6 | 666.8 |
| $H^\pm$ | 484.2 | 926.1 | -910.4 |
| $W + Z$ | | -4.4 | 926.1 |
| $\tilde{Z}_1$ | 102.6 | -18.1 | [illegible] |
| $\tilde{Z}_2$ | 207.5 | 32.0 | -9.0 |
| $\tilde{Z}_3$ | 416.3 | 492.7 | -80.2 |
| $\tilde{Z}_4$ | 326.1 | 2141.5 | -251.9 |
| $h$ | 93.1 | -0.04 | 156.2 |
| $H$ | 447.0 | 2007.0 | 0.31 |
| $A$ | 447.2 | 0 | 0.46 |
| $TR\,\Sigma^a$ | | [illegible] | 0 |
| $TR\,\vert\Sigma^a\vert$ | | [illegible] | [illegible] |

$\mu_1 = -317.20$ ; $\mu_2 = -$[illegible] ; [illegible]



## 13. Fine Tuning

The original fine tuning problem began in Standard Model GUTS with the Higgs mass:

$$m_H^2 = m_0^2 + c\,\frac{\tilde{\alpha}}{4\pi}\Lambda^2 \quad , \quad \Lambda = M_G$$

Thus to maintain $m_H \approx M_Z$, $m_0^2$ must be chosen to precisely cancel the $\Lambda^2$ term. Can formally define criteria [Barbieri, Guidice]

$$c \equiv \frac{m_0^2}{m_H^2}\frac{\partial m_H^2}{\partial m_0^2} = \frac{m_0^2}{m_H^2} \approx \frac{c\,\frac{\tilde{\alpha}}{4\pi}\Lambda^2}{M_Z^2} \approx 10^{24}$$

i.e. must know $m_0^2$ to 24 decimal places.

The fine tuning problem resurfaces in SUSY GUTS in the radiative breaking condition

$$\frac{1}{2}M_Z^2 = \frac{m_1^2 - m_2^2\tan^2\beta}{\tan^2\beta - 1} \quad ; \quad \sin 2\beta = \frac{2m_3^2}{m_1^2 + m_2^2}$$

Here $m_i^2$ depend on GUT scale parameters $m_0, m_{1/2}$ etc. e.g.

$$m_1^2 = m_0^2 + g(t)\,m_{1/2}^2 + \mu^2$$

$$m_2^2 = h(t)\,m_0^2 + e(t)\,m_{1/2}^2 + f(t)A_0 m_{1/2} - k(t)\,A_0 m_0 + \mu^2$$

and if $m_0^2$, $m_{1/2}^2$ etc. get too large, have to have a fine tuning cancellation to get small mass $\frac{1}{2}M_Z^2$. Thus roughly speaking a constraint on fine tuning leads to a bound on SUSY masses.

For the many SUSY GUT parameters can define fine tuning conditions

$$c_i = \frac{a_i}{M_Z^2}\frac{\partial M_Z^2}{\partial a_i} \quad ; \; a_i = m_0^2,\, m_{1/2}^2,\, A_0,\, \mu_0^2$$

and require $c_i < \Delta_i$. Thus $\Delta \approx 10$ crudely means no more than 1% fine tuning. The trouble with This is no fundamental definition of $a_i$ (until one knows fundamental theory of their origin!)

For example chose a new linear combination

$$a_i' = A_{ij}\, a_j$$

Then

$$c_i' = \sum_{l,j} \frac{A_{il}\, a_l\, A^{-1}_{ji}}{a_j}\, c_j$$

and

$$\sum c_i' = \sum c_i$$

Thus can re-apportion The $c_i$'s and if one is too big can suppress it. Alternately make non-linear transformation and increase or decrease them.

As an example write

$$M_Z^2 = \sum b_i\, \lambda_{ij}\, b_j \qquad b_i = m_0^2, m_{1/2}^2 \; ; \; A_0 = \tilde{A}_0 m_0$$

and make orthogonal transf. That diagonalizes $\lambda_{ij}(\tilde{A}_0)$:

$$b_i' = O_{ij}\, b_j$$



Then

$$M_Z^2 = \sum \lambda_i\, b_i' \qquad \lambda_i = \text{eigenvalues}$$

There are regions in parameter space where $\lambda_2 = 0$ and so $b_2'$ can get arbitrarily large without violating a fine tuning constraint. Thus There is a level of arbitraryness in The SUSY fine tuning condition.

When one is fine tuning to 24 decimal places, as in The non-SUSY Standard GUT model, these ambiguities don't matter as they usually are factors of $\times 2-5$ in $\Delta$. However, when one is fine tuning to 1 or 2 decimal places as a criteria of acceptability such ambiguities can modify results. In the previous discussions we've side stepped This question by imposing a physical constraint on measureable squark and gluino masses i.e.

$$m_{\tilde{g}},\; m_0 < 1\ \text{TeV}$$

which roughly is the bound for detecting These particles at LHC or SSC.

# 4. Quark Masses

GUT theories also can make some predictions on quark masses. These arise from the group symmetry G which imposes constraints on the Yukawa couplings at GUT scale. Thus SU(5) gives

$$\lambda_\tau(0) = \lambda_b(0) \; ; \; \mu = M_G \quad ; \quad \lambda_{\tau,b} = \text{Yukawa couplings}$$

By using RG eqns. one can then predict mass ratios i.e.

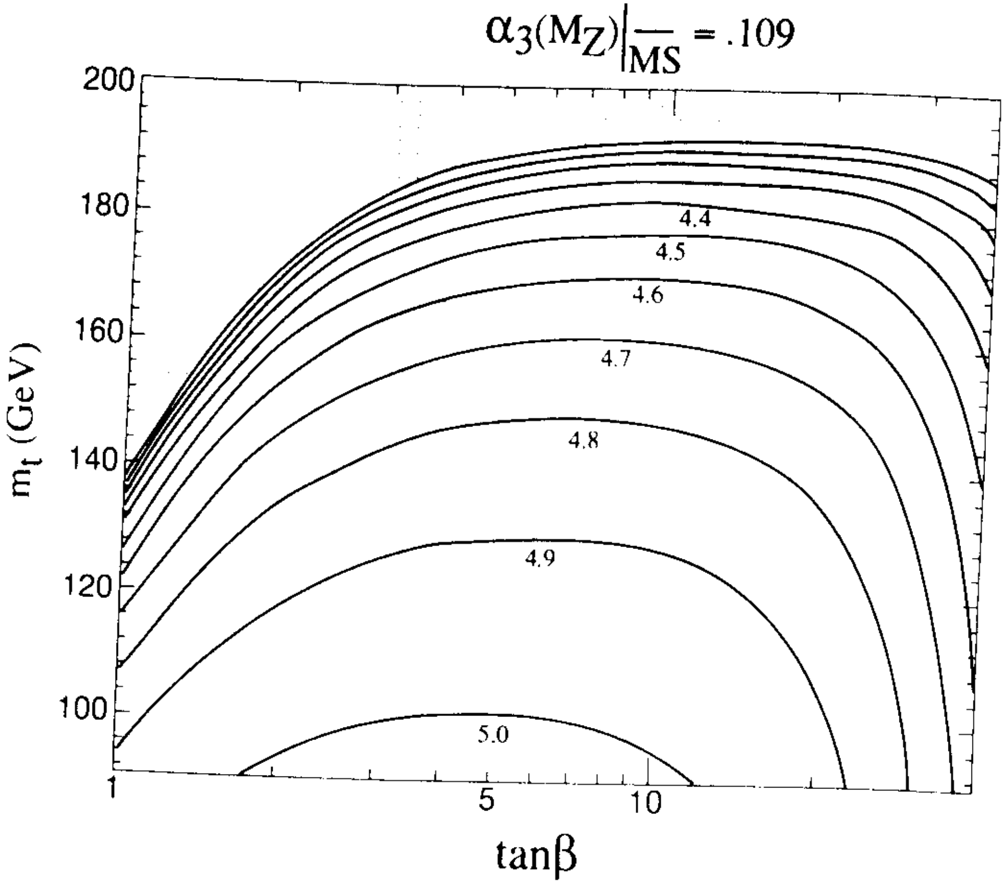
$$\frac{m_b}{m_\tau} = \frac{\lambda_b(m_b)\langle H_1\rangle}{\lambda_\tau(m_\tau)\langle H_1\rangle} = \frac{\lambda_b(m_b)}{\lambda_\tau(m_\tau)}$$

The predictions don't work for non-SUSY GUT but is in reasonable agreement for supergravity SU(5)

Fig. $m_t$ vs $\tan\beta$ for $\alpha_3 = 0.109$
Curves of different $m_b(m_b)$
For $\tan\beta < 5$, $m_b > 4.3$ GeV : $m_t \leq 180$ GeV

Fig. $m_t$ vs $\tan\beta$ for $\alpha_3 = 0.117$
For $\tan\beta < 5$, $m_b > 4.3$ GeV : $m_t \lesssim 190$ GeV

Note however: $M_S = M_Z$




Ellis et al

Figure 2a

<u>However</u>: The <u>SU(5)</u> coupling constant relations also <u>predicts</u> The following <u>wrong</u> <u>mass relations</u>:

$$\frac{m_s}{m_d} = \frac{m_\mu}{m_e}$$

$$20 \qquad 200$$

One may take two viewpoints on This:

(i) SU(5) symmetry at GUT scale is incorrect and needs modification [Dimopoulos et al; Babu, Shafi]:

(a) Georgi – Jarlskog type GUT model [Dimopoulos et al]

$$\lambda^{(u)}_{ij} = \begin{pmatrix} 0 & C & 0 \\ C & 0 & B \\ 0 & B & A \end{pmatrix}; \quad \lambda^{(d)}_{ij} = \begin{pmatrix} 0 & Fe^{i\varphi} & 0 \\ Fe^{-i\varphi} & E & 0 \\ 0 & 0 & D \end{pmatrix}; \quad \lambda^{(l)}_{ij} = \begin{pmatrix} 0 & F & 0 \\ F & -3E & 0 \\ 0 & 0 & D \end{pmatrix}$$

$$Q = M_G$$

(b) SO(10) type GUT [Babu, Shafi]

$$\lambda^{(u)}_{ij} = \begin{pmatrix} 0 & A' & 0 \\ A' & 0 & B' \\ 0 & B' & C' \end{pmatrix}; \quad \lambda^{(d)}_{ij} = \begin{pmatrix} 0 & A & 0 \\ 0 & De^{i\alpha} & B \\ 0 & B & C \end{pmatrix}; \quad \lambda^{(l)}_{ij} = \begin{pmatrix} 0 & A & 0 \\ A & -3De^{i\alpha} & -3B \\ 0 & -3B & C \end{pmatrix}$$

$$Q = M_G$$



These relns. hold at $M_{GUT}$ scale and will have RG corrections as one goes down to lower scales. However, The give now

$$\lambda_b = \lambda_\tau \qquad Q = M_G$$

and

$$\frac{m_\mu}{m_e} = 9\,\frac{m_s}{m_d} \qquad \left[\, m_s = \frac{m_\mu}{3},\ m_d = 3\,m_e \right]$$

which is <u>experimentally good</u>. A large number of relns. concerning CKM matrix elements can be derived which appear promising.

To analyse more completely one would like a full GUT model exhibited with explicit GUT sector

(ii) SU(5) is OK but one can expect small corrections to impinge on GUT physics from The Planck scale [Ellis, Gaillard; Nanopoulos, Srednicki]

Thus one needs really only a small correction ($\approx 100$ MeV) in $1^{st}$ and $2^{nd}$ generation relns. and This might ~~come~~ come from "Planck slop". One model proposes That each scale by $(M_{Pl})^{3-i}$, $i$ = generation in Yukawa couplings [Nanopoulos, Srednicki] i.e.

$i=3$: usual renormalizable cubic Yukawa

$i=2$: $\frac{\lambda}{M_{Pl}} \bar{H}_{2x} \Sigma^{x}_{Y} M_2^{Y2} \bar{M}_{2\ell} + \cdots$

$i=1$: $\frac{\lambda'}{(M_{Pl})^2} \bar{H}_{2x} (\Sigma^2)^{x}_{Y} M_1^{Y\ell} \bar{M}_{1\ell} + \cdots$

Thus masses are reduced for $i = 2,1$

$i=2$: $\frac{\langle \Sigma \rangle}{M_{Pl}} \approx 10^{-2} \simeq \frac{m_c}{m_t}$

$i=1$: $\frac{\langle \Sigma \rangle^2}{M_{Pl}^2} \approx 10^{-4} \simeq \frac{m_u}{m_t}$

However, this requires a more detailed analysis than has yet been done.

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