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SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

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PRECISION TESTS OF THE ELECTROWEAK MODEL

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Precision Tests of Electroweak Theories-I

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Lecture 1

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Topics [Lectures I+II]

- Introduction – Standard Model
- 1-Loop Radiative Corrections
- Experimental Status
- Extrapolation to high energies, GUTS
- Oblique Radiative Corrections (Peskin-Takeuchi Method)
- Applications – Constraints on Technicolour Models
- LR Symmetric Models; SUSY
- Concluding Remarks

Precision Measurements

$$\alpha = \frac{1}{137.0359895(61)}$$

$$G_\mu = 1.16637(2) \times 10^{-5} \ \mathrm{GeV}^{-2}$$

$$\alpha_s(m_Z)_{\overline{MS}} = 0.12 \pm 0.01 \qquad (QCD)$$

$$m_Z(\mathrm{GeV}) = 91.175 \pm 0.021$$

$$m_W/m_Z = 0.8791 \pm 0.0034$$

$$N_\nu = 3.00 \pm 0.05$$

$$\sin^2\theta_W(m_Z)_{\overline{MS}} = 0.2326 \pm 0.0088$$

The Standard Electroweak Model

(Glashow; Salam; Weinberg)

$$\boxed{SU(2)_L \otimes U(1)_Y}$$

$$\mathcal{L} = \mathcal{L}_f + \mathcal{L}_G + \mathcal{L}_H$$

Fermions

Leptons: $\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L , \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L , \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L ; \ e_R, \mu_R, \tau_R$

Quarks: $\underbrace{\begin{pmatrix} u \\ d \end{pmatrix}_L , \begin{pmatrix} c \\ s \end{pmatrix}_L , \begin{pmatrix} t \\ b \end{pmatrix}_L}_{\text{Weak-SU(2) Doublets}} ; \ \underbrace{u_R, d_R, c_R, \cdots}_{\text{U(1) Singlets}}$

Gauge Bosons

4 Fields: $\underbrace{W^i_\mu \quad (i=1,2,3)}_{\text{SU(2) Triplet}} , \quad \underbrace{B_\mu}_{\text{U(1) Singlet}}$

Scalars

4 Fields: $\Phi = \underbrace{\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}}_{\text{SU(2) Doublet}} , \quad \Phi^\dagger = \underbrace{\begin{pmatrix} \phi^- \\ \bar{\phi}^0 \end{pmatrix}}$

(Higgs Bosons)

Table 1 Elementary Particles

Particle	Symbol	Spin	Charge	Color	Mass (GeV)	
Electron neutrino	ν_e	1/2	0	0	$< 0.94 \times 10^{-8}$	
Electron	e	1/2	-1	0	0.51×10^{-3}	1st
Up quark	u	1/2	2/3	3	5×10^{-3}	generation
Down quark	d	1/2	-1/3	3	9×10^{-3}	
Muon neutrino	ν_μ	1/2	0	0	$< 0.25 \times 10^{-3}$	
Muon	μ	1/2	-1	0	0.106	2nd
Charm quark	c	1/2	2/3	3	1.25	generation
Strange quark	s	1/2	-1/3	3	0.175	
Tau neutrino	ν_τ	1/2	0	0	< 0.035	
Tau	τ	1/2	-1	0	1.78	3rd
Top quark *	t	1/2	2/3	3	> 89	generation
Bottom quark	b	1/2	-1/3	3	4.3	
Photon	γ	1	0	0	0	
W boson	$W^\pm$	1	±1	0	80.14 ± 0.31	gauge
Z boson	Z	1	0	0	91.17 ± 0.02	bosons
Gluon	g	1	0	8	0	
Higgs scalar *	H	0	0	0	$48 \lesssim m_H \lesssim 1000$	

* Yet to be detected in direct Exptl. Searches

Interactions

$$\boxed{\mathcal{L}_f = \sum_j \bar{\psi}_L^j \not{D} \psi_L^j + \sum_j \bar{\psi}_R^j \not{D} \psi_R^j}$$

where

$$\psi_L^1 = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L ; \quad \psi_R^1 = e_R \quad \text{etc.}$$

Covariant Derivative

$$\not{D} = D_\mu \gamma^\mu$$

(Weak-isospin matrices)

$$D_\mu = \partial_\mu - i g \vec{W}_\mu \cdot \vec{T} - i g' B_\mu \frac{Y}{2}$$

$T^a = \sigma^a/2$; σ^a Pauli Matrices (Weak Isospin)

Y: (Weak Hypercharge)

- ψ_L^j + ψ_R^j are Eigenstates of El. Charge (Q), $T^{(3)}$ + Y

Gell-Mann-Nishijima Relation

$$\boxed{Q = I_3 + \frac{Y}{2}}$$

$$\boxed{\mathcal{L}_W = -\frac{1}{4} W_{\mu\nu}^a W^{\mu\nu, a} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}}$$

with

SU(2): $W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g (W_\mu \times W_\nu)^a$

$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ $\quad a = 1, \cdots, 3$

$$\boxed{\mathcal{L} \text{[illegible]}) + \mathcal{L}(\text{[illegible]})}$$

[illegible]

SM: $\Phi =$ [illegible]

$\Phi = ($[illegible]$)$, $\Phi^\dagger = ($[illegible]$)$

Quantum Numbers: [illegible]

Higgs Potential

$$\boxed{V(\Phi) = \frac{1}{2}\mu^2|\Phi|^2 + \frac{1}{4}\lambda \text{[illegible]}}$$

- μ, λ two unknown Coupling Constants
- $\lambda > 0$ [$V(\Phi)$ is bounded from below]

(i) $\mu^2 > 0 \Rightarrow V(\Phi)_{min} = 0$ at [illegible]

(ii) $\mu^2 <$ [illegible] V [illegible]

[illegible]

$\Rightarrow$ [illegible] $\sqrt{\lambda}$ [illegible] (iii)

SM: $\langle \Phi \rangle_0 = ($[illegible]$/\sqrt{2})$ H = [illegible]

Higgs Couplings with Fermions

$$\mathcal{L}(q, l, \phi) = -h_q\left[\bar{q}_L \Phi d_R + \bar{q}_L \Phi^c u_R\right]$$

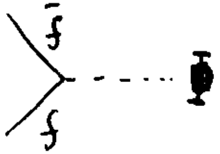
$$\boxed{\varphi^c = i\tau_2\phi^* = \begin{pmatrix} \bar{\varphi}^0 \\ -\varphi^- \end{pmatrix}} \quad - h_l\, \bar{l}_L \Phi e_R + \cdots\cdots$$

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix};$$ d_R, u_R, e_R isospin Singlets

After SSB: $\quad \Phi \to \frac{v}{\sqrt{2}} + \Phi'$

$SU(2)_L \otimes U(1)_Y \Rightarrow U(1)_{em} \Rightarrow$ $$\boxed{m_f = h_f \frac{v}{\sqrt{2}}} \qquad f = q, l$$

$\phi f\bar{f}$:



$\sim \sqrt{2}\, m_f / v$

Higgs Couplings with Gauge Bosons

$$\mathcal{L}(W, B, \phi) = \left|\left[i\partial_\mu - g\vec{W}_\mu \cdot \vec{T} - g'\frac{Y}{2}B_\mu\right]\phi\right|^2$$
$$= |D_\mu \phi|^2$$

After SSB:

$$\mathcal{L} = \frac{1}{4}v^2 g^2 W_\mu^+ W_\mu^- + \frac{1}{8}v^2(g^2 + g'^2) Z_\mu Z_\mu + \cdots$$

$$\Rightarrow \left.\begin{aligned} m_{W^\pm} &= \frac{1}{2}vg \\ m_Z &= \frac{1}{2}v(g^2 + g'^2)^{1/2} \end{aligned}\right\} \Rightarrow \frac{m_\phi^2}{m_W^2} = \frac{4\lambda}{\text{[illegible]}}$$

$m_\gamma = 0$

Electric Charge

$$e = \frac{g g'}{(g^2+g'^2)^{1/2}}$$

$$= g \sin\theta_W = g' \cos\theta_W$$

Exptl. Values: $e \simeq 0.3$, $g \simeq 0.6$, $g' \simeq 0.34$

$\alpha = 1/137$
$\sin^2\theta_W \simeq 0.23$

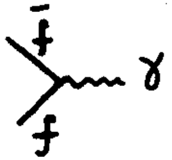
Neutral + Charged Currents

$$g W_\mu^a \tau^a + g' B_\mu Y \xRightarrow[\text{SSB}]{} \frac{e}{\sqrt{2}\sin\theta_W}(W_\mu^+ \tau^+ + W_\mu^- \tau^-)$$

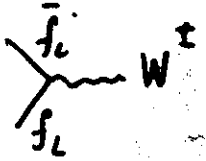
$$+ \frac{e}{\sin\theta_W \cos\theta_W} Z_\mu \left(I_L^3 - Q \sin^2\theta_W \right)$$

$$+ \quad e A_\mu Q$$

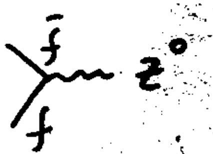
$\Rightarrow$



$: \quad e Q J_\mu^\gamma$



$: \quad \frac{e}{\sqrt{2}\sin\theta_W} J_{\mu,L}^\pm$



$: \quad \frac{e}{\sin\theta_W \cos\theta_W} J_\mu^Z$

$$\downarrow$$

$$= J_{\mu,L}^3 - \sin^2\theta_W J_\mu$$

Effective Interactions

$\mathcal{L} = 4G_F [J^+ . J^{-,\mu} + (J^3 - J\sin$

Standard Model Relations

$$W_\mu^a, B_\mu \xRightarrow[\text{SSB}]{} W_\mu^\pm, Z^0, \gamma$$

$$\phi^+, \phi^-, \phi^0, \bar{\phi}^0 \xRightarrow[\text{SSB}]{} \phi \text{ (or } H^0)$$

1 field
Remaining –
The Higgs Boson

$$W_\mu^\pm = \frac{W_\mu^1 \mp i W_\mu^2}{\sqrt{2}}$$

$$Z_\mu^0 = \frac{g W_\mu^3 - g' B_\mu}{\sqrt{g^2+g'^2}}$$

$$A_\mu = \frac{g' W_\mu^3 + g B_\mu}{\sqrt{g^2+g'^2}}$$

Weak Mixing Angle : θ_W

$$\cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}}$$

$$\sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}}$$

$$\Rightarrow \quad \frac{m_W^2}{m_Z^2} = \frac{g^2}{g^2+g'^2} = \cos^2\theta_W$$

Defining: $\rho \equiv \frac{m_W^2}{\ldots}$; $\boxed{\rho_{SM} = 1}$

Need for Radiative Corrections

$$\frac{G_\mu}{\sqrt{2}} = \frac{e^2}{8\sin^2\theta_W m_W^2} = \frac{g^2}{8m_W^2}$$

$$\cos\theta_W = \frac{g}{(g^2+g'^2)^{1/2}} = \frac{m_W}{m_Z}$$

$$\Rightarrow \quad m_W^2 = \frac{\pi\alpha}{\sqrt{2}G_\mu}\frac{1}{\sin^2\theta_W}$$

$$M_Z^2 = \frac{\pi\alpha}{\sqrt{2}G_\mu}\frac{1}{\sin^2\theta_W\cos^2\theta_W}$$

Present Status

$\alpha = 1/137.0359895(61)$ [Josephson effect; $(g-2)e$]

$G_\mu = 1.16637(2)\times 10^{-5}\ \mathrm{GeV}^{-2}$ [μ-decay]

$\sin^2\theta_W = 0.231 \pm 0.006$ (ν Expts.)

$\Rightarrow$ Theory (No RC)

$m_W^{(0)} = 77.6 \pm 1.0$ GeV

$m_Z^{(0)} = 88.5 \pm 0.8$ GeV

Expts. [Moriond '92]

$$\boxed{\begin{aligned} m_{W^\pm} &= 80.14 \pm 0.31\ \mathrm{GeV} \\ m_{Z^0} &= 91.175 \pm 0.021\ \mathrm{GeV}\end{aligned}}$$

$\Rightarrow$ [illegible] Need for RC

Radiative Corrections to EW Parameters

"[illegible] Approximation" $\mu^- \to e^-\bar{\nu}_e\nu_\mu$

$$\Rightarrow \tau_\mu^{-1} \Rightarrow \quad G_\mu = \frac{\pi\alpha}{\sqrt{2}m_W^2\sin^2\theta_W}$$

$O(\alpha)$ Corrections to Effective 4-Fermi Int.

$$\tau_\mu = \tau_\mu^{(0)}\left[1 + \frac{\alpha}{2\pi}\left(\frac{25}{4}-\pi^2\right)\right]$$ (Kinoshita, Sirlin, Berman)

$$\tau_\mu^{(0)} = \frac{G_\mu^2 m_\mu^5}{192\pi^3}\left(1 - 8m_e^2/m_\mu^2\right)$$

1-Loop Corrections in Standard EW Theory

$$\frac{G_\mu}{\sqrt{2}} = \frac{e_0^2}{8 s_W^{(0)2} M_W^{(0)2}}\left[1 + \frac{\Sigma^W(0)}{M_W^2} + (\text{Vertex, box})\right]$$

$$\boxed{s_W^{(0)2} = 1 - \frac{M_W^{(0)2}}{M_Z^{(0)2}}}$$

[illegible] by

$[\,e_0^2,\ M^2,\ \Sigma^W(q^2)]$

(Hollik)
CERN-TH. 5661/90

lready fixed the matrix element is finite when expressed in terms of the physical
ameters, i.e. all UV singularities associated with the regularization quantities
) μ are removed.

he vector boson masses and "Δr"

are prepared to attack the problem of the higher order contibutions to the mass
(52)(53) of section 2.3.

inally, the μ-lifetime τ_μ has been calculated within the framework of the effective
Fermi interaction. If we include the QED corrections

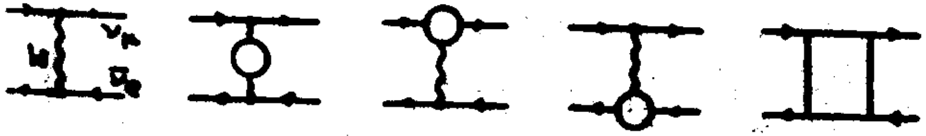
in the result

$$\frac{1}{\tau_\mu} = \frac{G_\mu^2 m_\mu^5}{192\pi^3}\left(1-\frac{8m_e^2}{m_\mu^2}\right)\left[1+\frac{\alpha}{2\pi}\left(\frac{25}{4}-\pi^2\right)\right]. \qquad (108)$$

ding 2nd order correction is obtained by replacing

$$\Longrightarrow \qquad \alpha \to \alpha\left(1+\frac{2\alpha}{3\pi}\log\frac{m_\mu}{m_e}\right).$$

rmula is used as the defining equation for G_μ in terms of the experimental μ-
. In lowest order, the Fermi constant is given by the Standard Model expression
the decay amplitude. In 1-loop order, $G_\mu/\sqrt{2}$ coincides with the calculable
on



$$\frac{G_\mu}{\sqrt{2}} = \frac{e_0^2}{8s_W^{02}M_W^{02}}\left[1+\frac{\Sigma^W(0)}{M_W^2}+(\text{vertex, box})\right]. \qquad (109)$$

ation contains the bare parameters with the bare mixing angle

$$s_W^{02} = 1-\frac{M_W^{02}}{M_Z^{02}}. \qquad (110)$$

m (vertex, box) schematically summarises the vertex corrections and box di-
in the decay amplitude, more explicitly shown in Figure 4. A set of infra-red
t "QED correction" graphs has been removed from this class of diagrams. These
diagrams, together with the real bremsstrahlung contributions, exactly repro-
QED correction factor of the Fermi model result in (108) and therefore have
ence on the relation between G_μ and the Standard Model parameters.

Vertices

Box diagrams

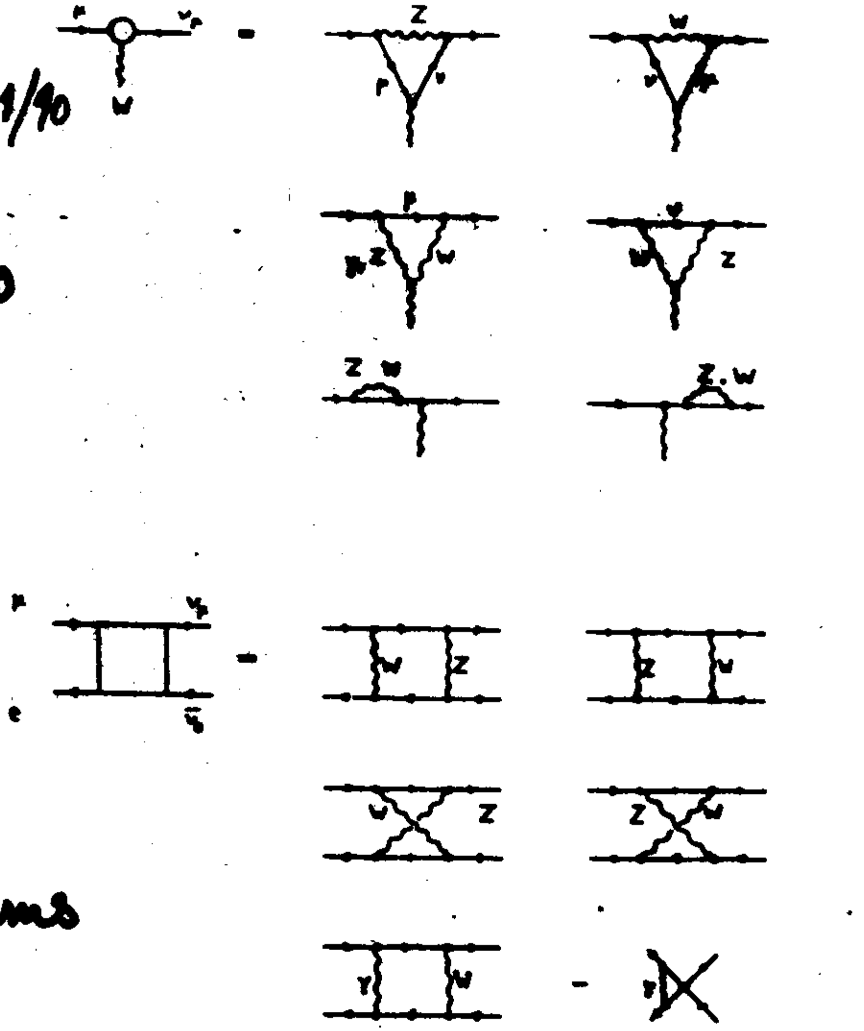


Fig. 4: Vertex corrections with external self energies and box diagrams contri the 1-loop amplitude for $\mu \to \nu_\mu e \bar{\nu}_e$. For the $We\nu$-vertex the analogous sample corrections is present as well. Omitted are the "QED" diagrams with a phot external charged lepton lines, and the photonic vertex correction to the Fermi a is subtracted from the box diagram with photon exchange

Renormalization of EW 'Parameters

$$e_{(0)}^2 = (e+\delta e)^2 = e^2\left(1+2\frac{\delta e}{e}\right)$$

$$M_W^{(0)\,2} = M_W^2\left(1+\delta M_W^2/M_W^2\right)$$

$$M_Z^{(0)\,2} = M_Z^2\left(1+\delta M_Z^2/M_Z^2\right)$$

Weak-angle Renormalization

Sirlin's Definition

$$S_W^{(0)\,2} = 1 - \frac{M_W^{(0)\,2}}{M_Z^{(0)\,2}} = 1 - \frac{M_W^2+\delta M_W^2}{M_Z^2+\delta M_Z^2}$$

$$= \underset{\substack{\| \\ S_W^2}}{\left(1-\frac{M_W^2}{M_Z^2}\right)} + \underset{\substack{\| \\ C_W^2}}{\frac{M_W^2}{M_Z^2}}\left(\frac{\delta M_Z^2}{M_Z^2}-\frac{\delta M_W^2}{M_W^2}\right) + \cdots$$

$$\boxed{S_W^{(0)\,2} = S_W^2 + C_W^2\left(\frac{\delta M_Z^2}{M_Z^2}-\frac{\delta M_W^2}{M_W^2}\right)}$$

$\Rightarrow$ $\frac{G_\mu}{\sqrt{2}} = \frac{e^2}{8S\cdots}\Big[1$ [illegible] $\Big(\delta M$ [illegible] $\delta\frac{M_W^2}{M_W^2}\Big)$ + [illegible]

Renormalization of Photon Propagator

Lowest order

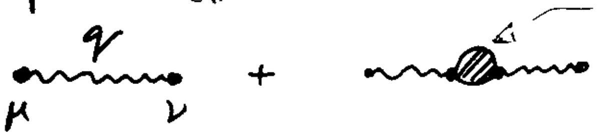
gauge parameter

$$D_{\mu\nu} = \left[-g_{\mu\nu} + (1-\xi)\frac{q_\mu q_\nu}{q^2}\right]\frac{i}{q^2+i\varepsilon}$$

$$= \frac{-i\,g_{\mu\nu}}{q^2+i\varepsilon} \qquad \left(\begin{array}{c}\text{Feynman gauge}\\ \xi = 1\end{array}\right)$$

Self-Energy Corrections

$$\Sigma^\gamma(q^2)$$



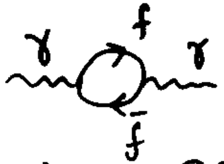
$$D_{\mu\nu}(q) = -\frac{i\,g_{\mu\nu}}{q^2+i\varepsilon} - i\,g_{\mu\nu}\,\frac{1}{(q^2+i\varepsilon)}\Sigma^\gamma(q^2)\frac{1}{(q^2+i\varepsilon)}$$

$$= -i\,\frac{g_{\mu\nu}}{q^2+i\varepsilon}\left[1-\Pi^\gamma(q^2)\right]$$

where $\Pi^\gamma(q^2) \equiv \dfrac{\Sigma^\gamma(q^2)}{q^2}$

↑ Photon Vacuum Polarization

$\Pi^{\gamma}(q^2)$ Fermionic Contributions



Using Dimensional Regularization

$$\Rightarrow \Pi^{\gamma}(q^2) = \frac{\alpha}{3\pi}\,\frac{1}{q^2}\Big\{ q^2\Big(\Delta - \ln\frac{m_f^2}{\mu^2}\Big) + (q^2 + 2m_f^2)\,B(q^2, m_f^2) - \frac{q^2}{3}\Big\}$$

$$\boxed{\Delta = \frac{2}{\epsilon} - \gamma_E + \ln 4\pi}$$

$\gamma_E = 0.577$, Euler's Constant

$\epsilon = 4 - D$; Dim. Regulator

- μ = an arbitrary scale, introduced due to the dim. of the Coupling Constant in D dimensions
- $$g = g_0\,\mu^{\epsilon}$$
  g_0 ← dimensionless

$$B(q^2, m_f^2) = -\int_0^1 dx\,\ln\Big(\frac{x^2q^2 - xq^2 + m_f^2}{m_f^2} - i\epsilon\Big)$$

$\Rightarrow$ $\Pi^{\gamma}(q^2)$ is divergent !

Properties of $\Pi^{\gamma}(q^2)$

- $q^2 = 0$

$$\Pi^{\gamma}(0) = \frac{\alpha}{3\pi}\Big(\Delta - \ln\frac{m_f^2}{\mu^2}\Big)$$

- Light fermions ($|q^2| >> m_f^2$)

$$\Pi^{\gamma}(q^2) = \frac{\alpha}{3\pi}\Big(\Delta - \ln\frac{m_f^2}{\mu^2} + \frac{5}{3} - \ln\frac{|q^2|}{m_f^2} + i\pi\theta(q^2)\Big)$$

Im. Part ↙

- Heavy fermions ($|q^2| << m_f^2$)

$$\Pi^{\gamma}(q^2) = \frac{\alpha}{3\pi}\Big(\Delta - \ln\frac{m_f^2}{\mu^2} + \frac{q^2}{5m_f^2}\Big)$$

Counter term

$$e_0 = e + \delta e$$

Renorm. Condition: $$i e \gamma_\mu \longrightarrow \underbrace{i\big[e + \delta e - \tfrac{1}{2} e\,\Pi^{\gamma}(0)\big]\gamma_\mu}_{= i e \gamma_\mu}$$

$$\Rightarrow \frac{\delta e}{e} = \frac{1}{2}\Pi^{\gamma}(0)$$

Ren. Vacuum Polarization

$$\boxed{\hat{\Pi}^{\gamma}(q^2) = \Pi^{\gamma}(q^2) - \Pi^{\gamma}(0)}$$

Properties of $\hat{\pi}^{\gamma}(q^2)$

- Vanishes for real Photons

$$\hat{\pi}^{\gamma}(0) = 0 \qquad \text{[Norm. Condition]}$$

- Light fermions ($|q^2| >> m_f^2$)

$$\hat{\pi}^{\gamma}(q^2) = \frac{\alpha}{3\pi}\left(\frac{5}{3} - \ln\left(\frac{|q^2|}{m_f^2}\right) + i\pi\theta(q^2)\right)$$

$\Rightarrow$ log. dependence on m_f

- Heavy fermions ($|q^2| << m_f^2$)

$$\hat{\pi}^{\gamma}(q^2) = \frac{\alpha}{3\pi}\,\frac{q^2}{5m_f^2}$$

$\Rightarrow$ Decoupling of heavy fermions

$$\boxed{\hat{\pi}^{\gamma}(m_Z^2) = \sum_f Q_f^2 N_f \frac{\alpha}{3\pi}\left(\frac{5}{3} - \ln\frac{m_Z^2}{m_f^2} + i\pi\right)}$$

- $\hat{\pi}^{\gamma}(m_Z^2)$ can be interpreted either as Correction to the γ-propagator or to the electric charge, e
- Running Coupling Constant. $e^*(q^2)$, $\alpha^*(q^2)$

$$\boxed{e^{*2}(q^2) = \frac{e^{*2}(q^2=0)}{1+\mathrm{Re}\,\hat{\pi}^{\gamma}(q^2)} = \frac{e^2}{1+\mathrm{Re}\,\hat{\pi}^{\gamma}(q^2)}}$$

Running QED Coupling Constant

$$\frac{1}{\alpha_*(m_Z^2)} - \frac{1}{\alpha} = \sum_f \Delta\alpha_*^{-1}(m_Z^2)_f$$

$$\Delta\alpha_*^{-1}(m_Z^2)_f = -\frac{1}{3\pi} Q_f^2 N_f \left[\ln\frac{m_Z^2}{m_f^2} - \frac{5}{3}\right]$$

$$= \mathrm{Re}\,\hat{\pi}^{\gamma}(m_Z^2)$$

Leptons

Fermion	e	μ	τ
Mass (MeV)	0.5	106	1784
$\Delta\alpha_*^{-1}(m_Z^2)_f$	2.4	1.3	0.7

$$\Rightarrow \boxed{\sum_\ell \Delta\alpha_*^{-1}(m_Z^2)_\ell = 4.4}$$

Quarks (Current Quark Masses)

Fermion	u	d	s	c	b
Mass (MeV)	5.5	8	150	1200	5000
$\Delta\alpha_*^{-1}(m_Z^2)_f$	2.5	0.6	0.4	1.0	0.1

$$\Rightarrow \sum_q \Delta\alpha_*^{-1}(m_Z^2)_q = 4.6$$

$$\Delta\alpha_*^{-1}(m_Z^2) = 9 \qquad \text{(Burkhardt et al.)}$$

- A better evaluation of $\hat{\pi}^{\gamma}(m_Z^2)$ hadronic

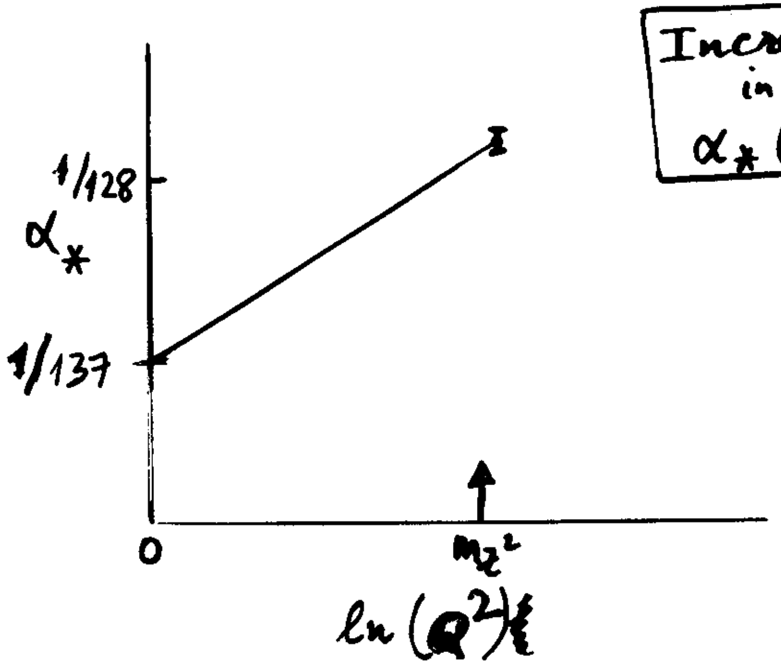
$$\hat{\pi}^{\gamma}_{had}(m_Z^2) = \frac{\alpha}{3\pi} m_Z^2 \int_{4m_\pi^2}^{\infty} ds' \frac{R^\gamma(s')}{s'(s'-m_Z^2-i\varepsilon)}$$

$$\hat{\Pi}^{\gamma}_{hadron}(m_Z^2) = -0.0288 \pm 0.0009$$

$$\Rightarrow \quad \mathrm{Re}\,\hat{\Pi}^{\gamma}(m_Z^2) = -0.0602 \pm 0.0009$$

(for $m_Z = 91.175$ GeV)

$$\boxed{\begin{array}{l} \alpha_*^{-1}(m_Z^2) = 128.77 \pm 0.12 \\[1em] \dfrac{\Delta\alpha_*^{-1}(m_Z^2)}{\alpha_*^{-1}(m_Z^2)} = 9 \times 10^{-4} \end{array}}$$



$$\boxed{\text{Increase in } \alpha_*(Q^2)}$$

Lumping the RC in One parameter

$$\Rightarrow \quad \boxed{\frac{G_\mu}{\sqrt{2}} = \frac{e^2}{8 s_W^2 M_W^2}\left[1 + \Delta r_W\right]}$$

$$\Delta r_W = \Delta r_W(M_W, M_Z, M_H, M_t, \alpha)$$

$$\boxed{\Delta r_W = \Delta\alpha - c_W^2/s_W^2\,\Delta\rho + (\Delta r)_{rem}}$$

<u>Light Fermion Contribution</u>

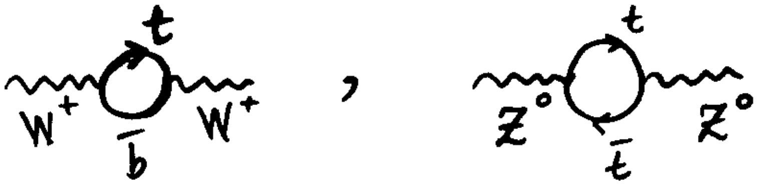
$$\Delta\alpha \equiv \Delta r_W\Big|_{\text{Light fermions}} = -\mathrm{Re}\,\hat{\Pi}^{\gamma}(m_Z^2) - \frac{\alpha}{3\pi}\,\frac{c_W^2 - s_W^2 \ln c_W^2}{4 s_W^4}$$

$$\Downarrow \qquad\qquad \uparrow$$

$$= \quad -0.0602 \pm 0.009 + 0.0054$$

Heavy Fermion Contribution

$\boxed{\Delta\rho}$



⇒ Isospin-breaking

$$(\Delta r_W)\Big|_{top} = -\frac{s_W^2}{c_W^2}\left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}\right)$$

$$\equiv -\frac{s_W^2}{c_W^2}\,\Delta\rho$$

$$\boxed{\Delta\rho \cong \frac{3\alpha}{16\pi s_W^2 c_W^2}\frac{m_t^2}{m_Z^2}}$$

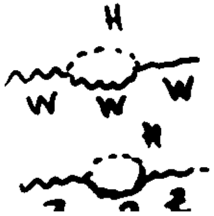
M. Veltman ('77)
Chanowitz et al. ('78)

⇒ Quadratic dependence on m_t !

$\boxed{(\Delta r)_{rem}}$ Contains all other remaining terms

- $$(\Delta r)^{top}_{rem} = -\frac{\alpha}{4\pi s_W^2}\left(\frac{c_W^2}{s_W^2} - \frac{1}{3}\right)\ln\frac{m_t}{m_Z} + \cdots$$
- Higgs Contribution

$$(\Delta r)^{Higgs}_{rem} = \frac{\alpha}{16\pi s_W^2}\,\frac{11}{3}\left(\ln\frac{m_H^2}{m_Z^2} - \frac{5}{6}\right)$$



Higher Order Corrections

- Replacing $1 + \Delta\alpha \to \frac{1}{1-\Delta\alpha}$ correctly takes into account all orders in leading logarithmic corrections $(\Delta\alpha)^n$ (Marciano, Sirlin)

⇒

$$\boxed{\begin{aligned} G_\mu &= \frac{\pi\alpha}{\sqrt{2}\, m_W^2 s_W^2}\,\frac{1}{1-\Delta\alpha} \\ &= \frac{\pi\alpha}{\sqrt{2}\, m_Z^2 c_W^2 s_W^2}\,\frac{1}{1-\Delta\alpha} \end{aligned}}$$

- For m_t large, $\Delta\rho$ is large

However, $(\Delta\rho)^n$ <u>not</u> correctly summed via $\frac{1}{1-\Delta r}$ prescription. Instead

(α^2)"

$$\boxed{\frac{1}{1-\Delta r} \to \frac{1}{1-\Delta\alpha}\,\frac{1}{1 + c_W^2/s_W^2\,\overline{\Delta\rho}} + (\Delta r)_{rem.}}$$

$$\overline{\Delta\rho} = N_c\frac{G_\mu m_t^2}{8\pi^2\sqrt{2}}\left[1 + \frac{G_\mu m_t^2}{8\pi^2\sqrt{2}}(19 - 2\pi^2)\right]$$

- QCD Corrections for heavy m_t (Van der Bij, Hoogeveen)

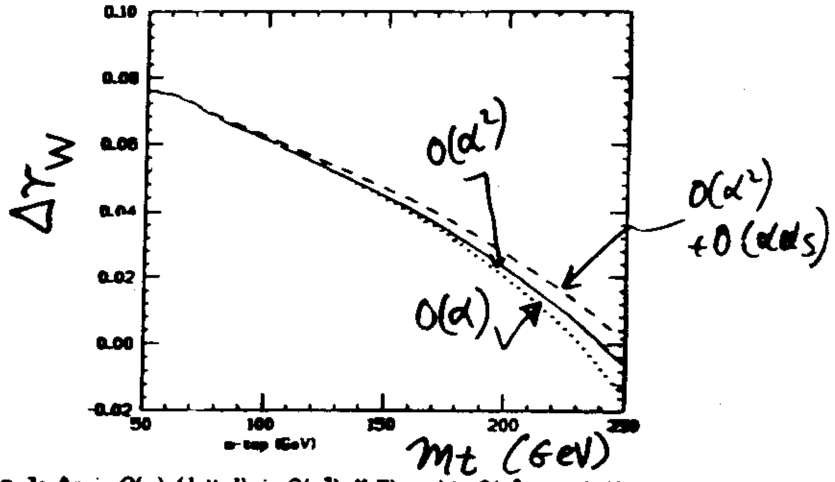


Fig. 1: Δr in $O(\alpha)$ (dotted), in $O(\alpha^2)$ (full), and in $O(\alpha^2 + \alpha\alpha_s)$ (dashed). $M_Z = 91.175$ GeV, $M_H = 300$ GeV.

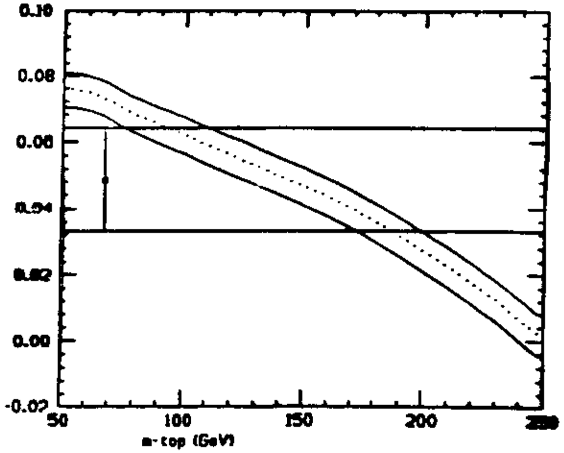


Fig. 2: Δr as a function of the top mass for M_H = 50, 300, 1000 GeV (lower, middle, upper line). $M_Z = 91.175 \pm .021$ GeV. 1σ bounds with $s_W^2 = 0.2273 \pm$ [illegible] from combined UA2 and CDF results[2].

$$\boxed{s_W \equiv 1 - \frac{m_W}{M_Z^2}}$$

$$\Rightarrow$$

$$A \equiv \frac{\pi\alpha}{\sqrt{2}\,G_\mu} = (37.2805 \pm 0.003)\ \text{GeV}$$

$$= \left(1 - \frac{m_W^2}{M_Z^2}\right)\left(1 - \Delta r_W\right)$$

$$\Delta r_W = \Delta\alpha - \frac{c_W^2}{s_W^2}\Delta\rho + (\Delta r_W)_{\text{rem.}}$$

$$\Rightarrow m_W^2/M_Z^2 = \frac{1}{2}\left(1 + \sqrt{1 - \frac{4A}{M_Z^2(1-\Delta r_W)}}\right)$$

$\overline{MS}$ definition $\quad s_W^2 = \frac{1}{2}\left(1 - \sqrt{\quad '' \quad}\right)$

$$\frac{\pi\alpha}{\sqrt{2}\,G_\mu} = \sin^2\hat{\theta}_W(\mu = M_Z)_{\overline{MS}}\left(1 - \Delta r(M_Z)_{\overline{MS}}\right)$$

with

$$\boxed{\sin^2\hat{\theta}_W(\mu)_{\overline{MS}} = s_W^{2\,(0)}\left\{1 + \frac{e_0^2\,\mu^{n-4}}{8\pi^2}\hat{\beta}\right\}}$$

$$\hat{\beta} = \left(\frac{11}{3} + \frac{19}{6\,s_W^{2\,(0)}}\right)\left(\frac{1}{n-4} + \frac{\gamma}{2} - \ln\sqrt{4\pi} + \cdots\right)$$

RGE: $\quad \mu\frac{\partial}{\partial\mu}\sin^2\hat{\theta}_W(\mu) = \frac{\alpha}{2\pi}\left[\frac{11}{3}\sin^2\hat{\theta}_W + \frac{19}{6}\right] + \cdots$

Marciano

BNL-45999

Table 4 Standard Model predictions (82) as a function of m_t, using $m_Z = 91.17$ GeV, $\alpha = 1/137.036$ and $G_\mu = 1.16637 \times 10^{-5}$ GeV^{-2} as input and assuming $m_H \simeq 100$ GeV.

m_t(GeV)	$10^2\Delta r_W$	$10^2\Delta r(m_Z)_{\overline{MS}}$	m_W(GeV)	$\sin^2\theta_W(m_Z)_{\overline{MS}}$
90	6.08	6.84	79.91	0.2336
100	5.76	6.88	79.97	0.2334
110	5.45	6.91	80.03	0.2331
120	5.13	6.94	80.08	0.2329
130	4.79	6.96	80.14	0.2326
140	4.44	6.98	80.20	0.2323
150	4.07	6.99	80.27	0.2319
160	3.68	7.01	80.33	0.2316
170	3.27	7.02	80.40	0.2312
180	2.83	7.03	80.48	0.2309
190	2.37	7.04	80.55	0.2304
200	1.87	7.05	80.63	0.2300
210	1.35	7.06	80.71	0.2296
220	0.80	7.07	80.80	0.2291
230	0.22	7.08	80.88	0.2286
240	−0.40	7.09	80.96	0.2282
250	−1.05	7.09	81.07	0.2276

a Scenario to be tested

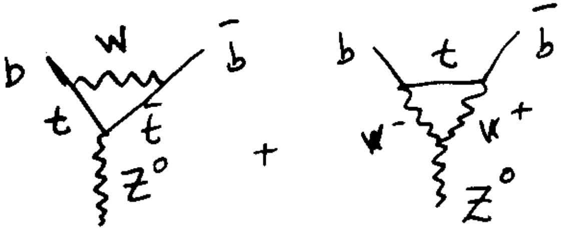
Table 5 Comparison of experimental decay rates found from averaging the four LEP experiments (84) with standard model predictions (83). The theoretical rates employ $m_Z = 91.17$ GeV and $\sin^2\theta_W(m_Z)_{\overline{MS}} = 0.2326$ which corresponds to $m_t \simeq 130$ GeV and $m_H \simeq 100$ GeV.

Experiment	Theory
$\Gamma(Z \to \text{all}) = 2487 \pm 9$MeV	2490 MeV
$\Gamma(Z \to \ell^+\ell^-) = 83.3 \pm 0.4$MeV	83.6 MeV
$\Gamma(Z \to \text{invisible}) = 493 \pm 10$MeV	500 MeV

(see Section 7). Unfortunately, we have reached a point where the theoretical uncertainties in extracting Γ_Z are becoming comparable to the errors in Table 5; so anticipated integrated luminosity increases will probably not significantly improve the situation.

Decay asymmetry measurements of the Z have also been carried out at LEP. The forward-backward decay asymmetries of various fermion pairs $\mu^+\mu^-$, $\tau^+\tau^-$, $b\bar{b}$ etc. have

- An Example Large RC ≠ RC|



Oblique

(Bardin et al.; Verzagnassi, Djouadi; Hollik, Kniehl; ....)

- Ignoring m_b ⇒ Mult. Ren. of Vertex $Z^0 \to b_L \bar{b}_R$

$$\mathcal{M} = -\frac{ie^2}{c_W s_W} Z_\mu (\bar{b}\gamma^\mu b_L)\left[\left(\frac{1}{2} - \frac{1}{3}s_W^2\right) - F\right]$$

$$\boxed{F = \frac{\alpha}{16\pi}\frac{1}{s_W^2}\frac{m_t^2}{m_W^2}}$$

⇒ $\Delta\Gamma(Z^0 \to b\bar{b})$ <u>not</u> obtained from Oblique RC ($\equiv \sim\!\bigcirc\!\sim$)

However $\frac{\Delta\Gamma(Z^0 \to b\bar{b})}{\Gamma} \simeq 1.5\%$ for $m_t = 100-200$ GeV

Difficult Expt!

Peskin
SLAC Report

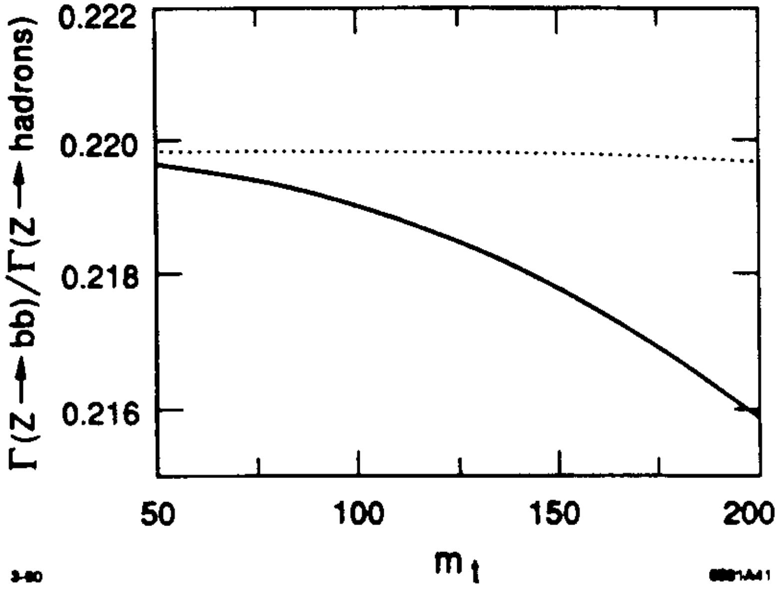


Figure 41. Dependence of the Z^0 width to $b\bar{b}$, as a fraction of the total Z^0 width to hadrons, as a function of m_t. The solid line includes the $b\bar{b}Z$ vertex corrections; the dashed line shows the result of omitting this effect, while retaining the top quark renormalization of $s_*^2(m_Z^2)$.

ratio of neutral to charged current cross sections

$$R^\nu = \int dx dy \frac{d\sigma(\nu, NC)}{dx dy} \Big/ \int dx dy \frac{d\sigma(\nu, CC)}{dx dy} , \qquad (5.61)$$

where x and y are the standard dimensionless kinematic variables and the integral is taken over the experimental acceptance. These cross sections are readily estimated in the naive parton model: If $f_q(x)$ is the parton distribution of the species q in the proton, the cross sections, for neutrino-proton scattering, at lowest order in weak interactions, are proportional to

$$\begin{aligned} \frac{d\sigma(\nu, CC)}{dx dy} &= \frac{G_F{}^2 sx}{\pi}\Big(f_d(x) + (1-y)^2 f_{\bar{u}}(x)\Big) \\ \frac{d\sigma(\nu, NC)}{dx dy} &= \frac{G_F{}^2 sx}{\pi}\Big([(\frac{1}{2} - \frac{2}{3}\sin^2\theta_w)^2 f_u(x) + (\frac{1}{2} - \frac{1}{3}\sin^2\theta_w)^2 f_d(x)] \\ &\quad + (1-y)^2[(\frac{2}{3}\sin^2\theta_w)^2 f_u(x) + (\frac{1}{3}\sin^2\theta_w)^2 f_d(x)] \\ &\quad + (1-y)^2[(\frac{1}{2} - \frac{2}{3}\sin^2\theta_w)^2 f_{\bar{u}}(x) + (\frac{1}{2} - \frac{1}{3}\sin^2\theta_w)^2 f_{\bar{d}}(x)] \\ &\quad + [(\frac{2}{3}\sin^2\theta_w)^2 f_{\bar{u}}(x) + (\frac{1}{3}\sin^2\theta_w)^2 f_{\bar{d}}(x)]\Big) , \end{aligned} \qquad (5.62)$$

plus contributions from heavier quark species. In the neutral current cross section, the two sets of terms for each quark refer to left- and right-handed species, respectively. The two prefactors are identical by virtue of (2.9). However, this is the only simplification available, and otherwise the integrands of (5.61) are complicated functions of x and y. When we include the QCD corrections to (5.62), these integrands will also depend on Q^2. How, then, can we extract any information to 1% accuracy?

The required strategy was set out in a beautiful paper by Llewellyn Smith.[51] In this paper, Llewellyn Smith encourages us to think about a world containing only u and d quarks. This allows three important simplifications in the computation

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Oblique Rad. Corr.

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$\overline{MS}$ Renorm. Scheme for RC

1) S. Sarantakos, A. Sirlin, W. Marciano, Nucl. Phys. B217 (1983) 84.
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1) T. Appelquist et al., Phys. Lett. B232 (1989) 211.
2) M. E. Peskin, T. Takeuchi, Phys. Rev. Lett. 65 (1990) 964
3) B. Holdom, J. Terning, Phys. Lett. B247 (1990) 88
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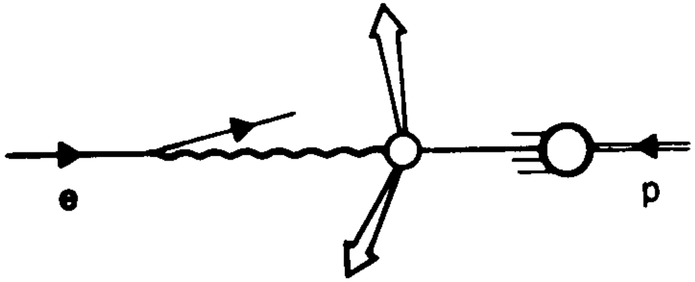
Recent analyses of the Electroweak data

- J. Ellis, CERN-TH. 6193/91
- The LEP Collaborations, CERN-PPE/91-232
- J. Ellis, G.L. Fogli & E. Lisi, CERN-TH. 6383/92
- G. Altarelli, Moriond Talk, CERN-TH-6525/92
- G. Altarelli et al., CERN-TH. 6051/91
- W. J. Marciano, BNL-45999 (1991)
- M. Peskin, T. Takeuchi, SLAC-PUB-5618 (1991)
- P. Langacker, Valencia Meeting, (1991) UPR-0492T

HERA PHYSICS

NC: $e + p \rightarrow e + X$

CC: $e + p \rightarrow \nu_e + Y$



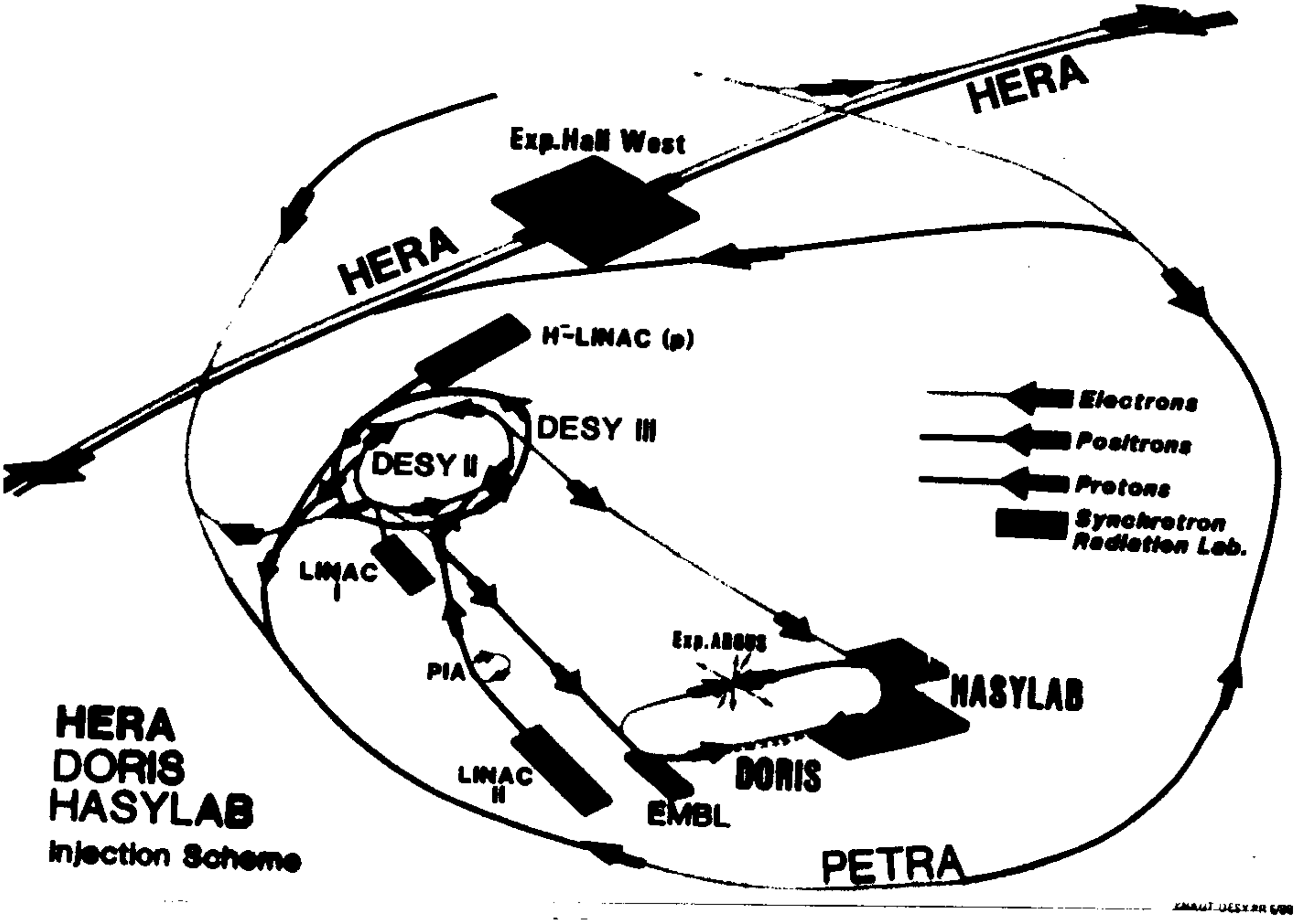
$E_e \simeq 30$ GeV

$E_p \simeq 820$ GeV

$\boxed{\sqrt{s} \simeq 314 \text{ GeV}}$

$\equiv$ 52 TeV for fixed Target ep Expts.

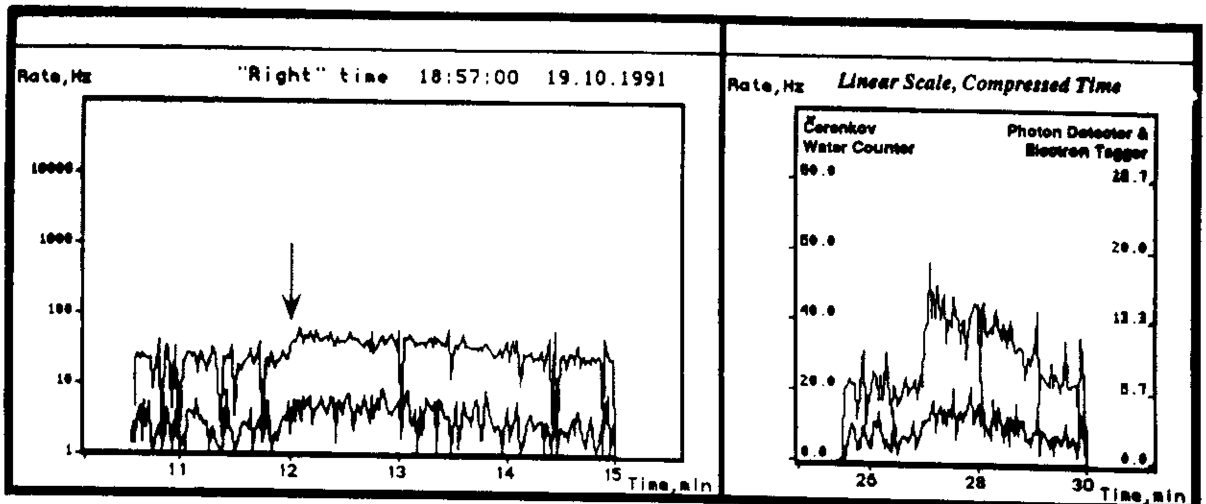
Figure 1



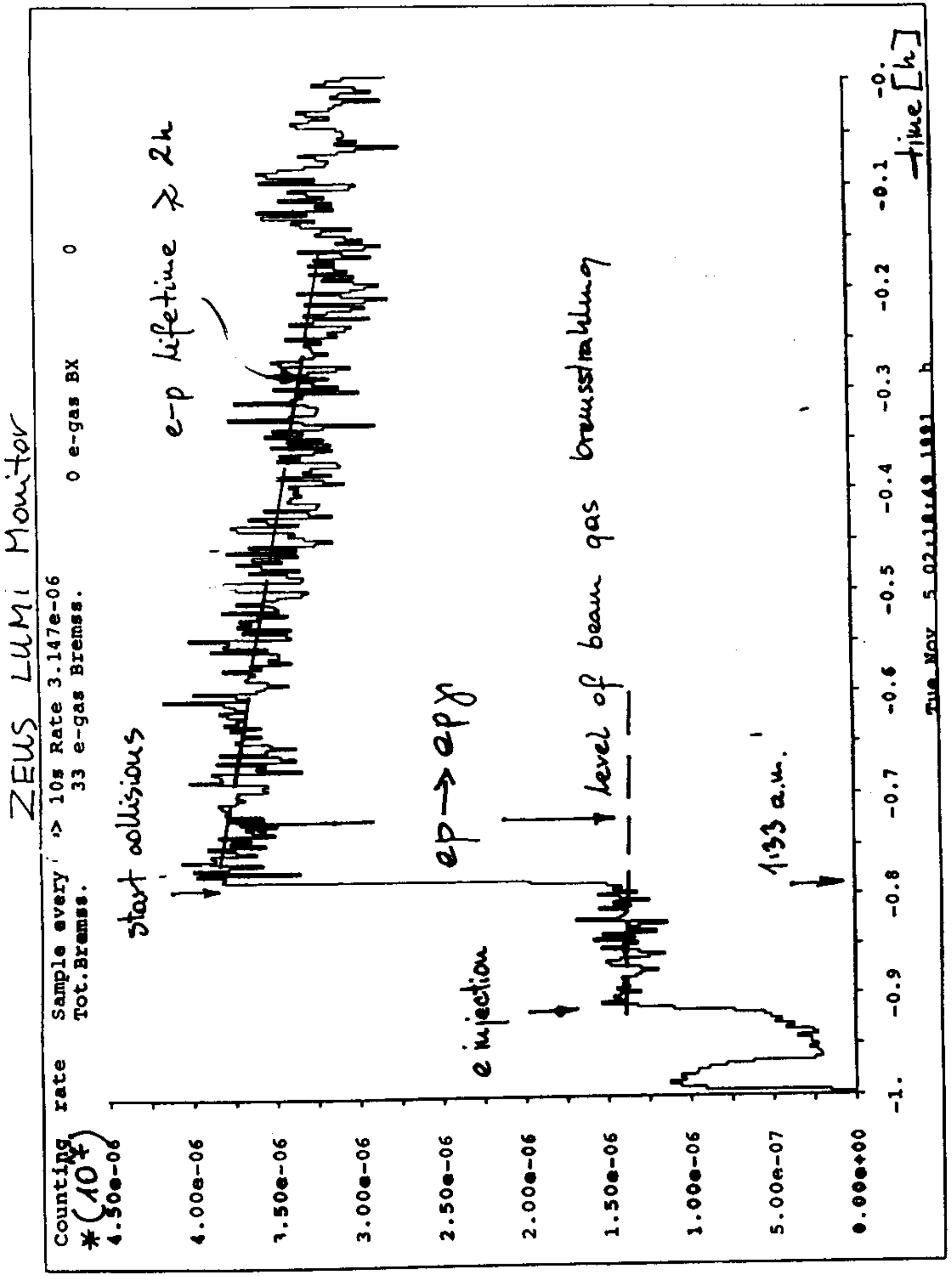
FIRST HERA e-p COLLISIONS

AS OBSERVED BY THE H1 LUMINOSITY-DETECTOR MONITORING SYSTEM

SATURDAY 19 OCTOBER 1991, 18:54



Electron Energy	12 GeV
Proton Energy	480 GeV
Expected Luminosity	$0.95 \times 10^{26} \pm 30\%\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$
Measured Luminosity	$1.03 \times 10^{26} \pm 13\%\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$



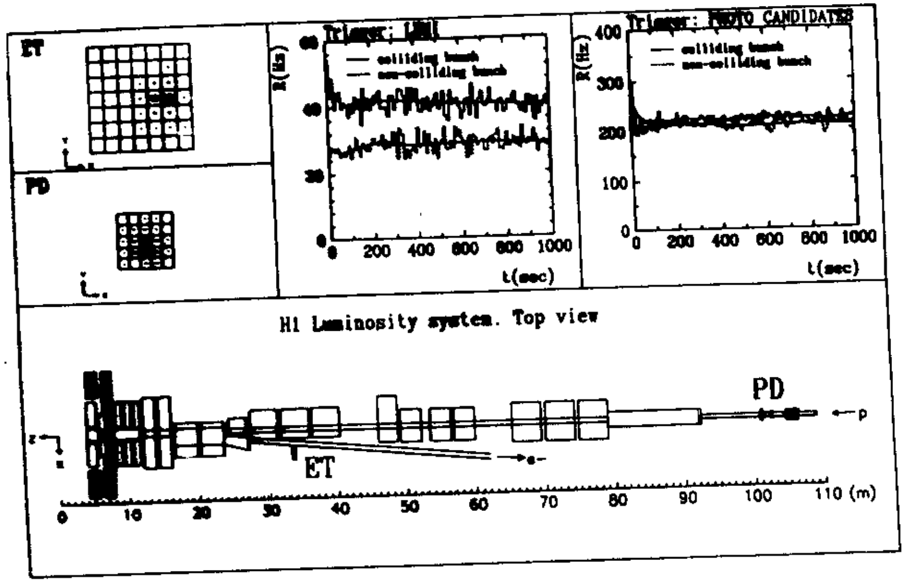


Figure 2: *General view of the H1 Luminosity System. Main H1 detector (around Inter- action Point [0,0]) is not shown. In the left upper corner a typical response of Electron Tagger(ET) and Photon Detector(PD) are shown for ep → eγp event. Two histograms present experimental results from the November'91 ep-run with 26.5 × 480 GeV collid- ing beams. These data correspond to the 2-bunch mode with full e⁻ intensity per bunch and ≈ 10% of proton intensity per bunch*

ep → epγ

↳ coincidence

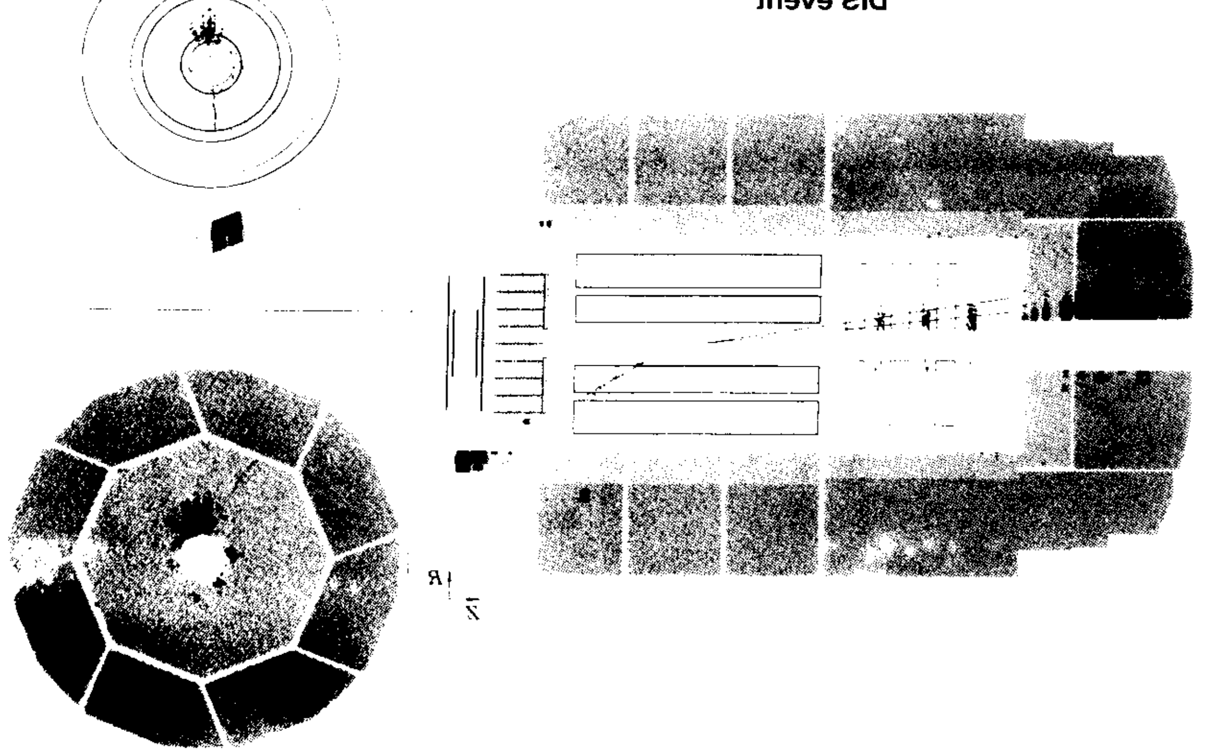
Run 25114 Event 15312 Class: 2 10 11 28 Date 7/07/1992

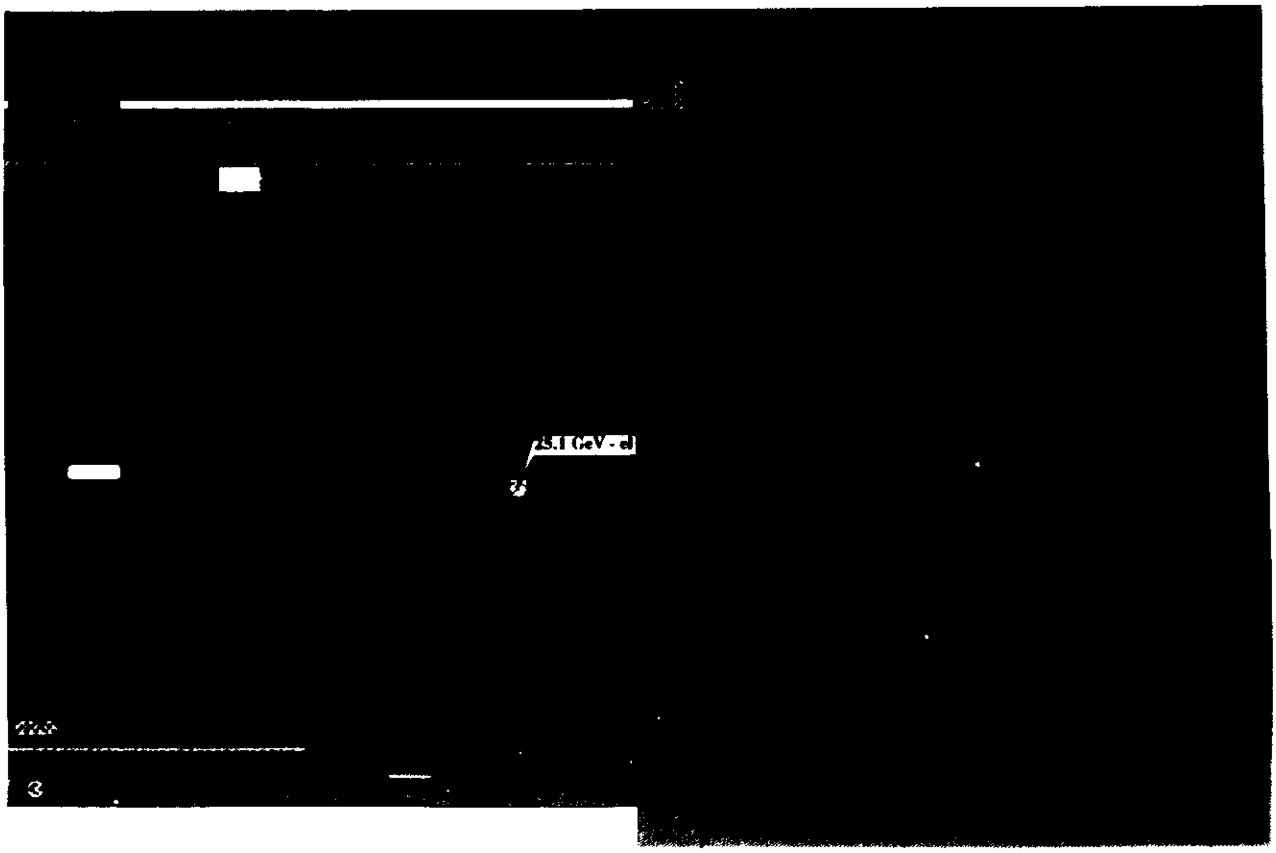
H1 Event Display 1.05/00
DSN=HERA01.H1SAMPLE.DATA

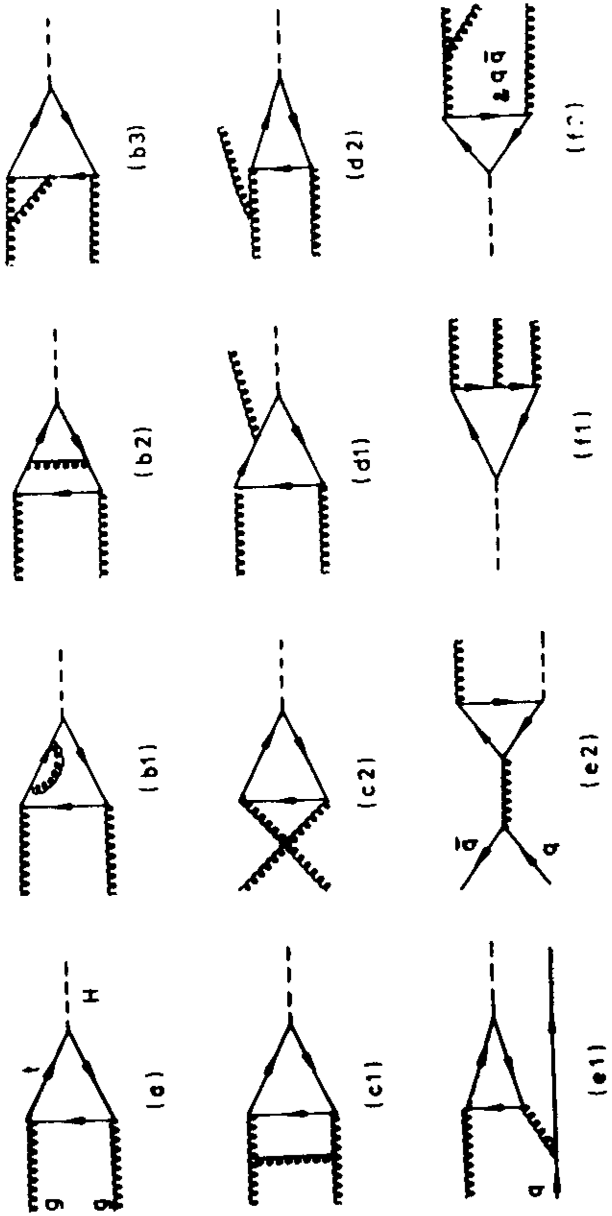
E= -26.7 x 819.9 GeV B=11.3 kG
Run date 92/07/04 04:30

Trigger information
AST bits = 56 59 60
RST bits = 48 56 59 60

DIS event

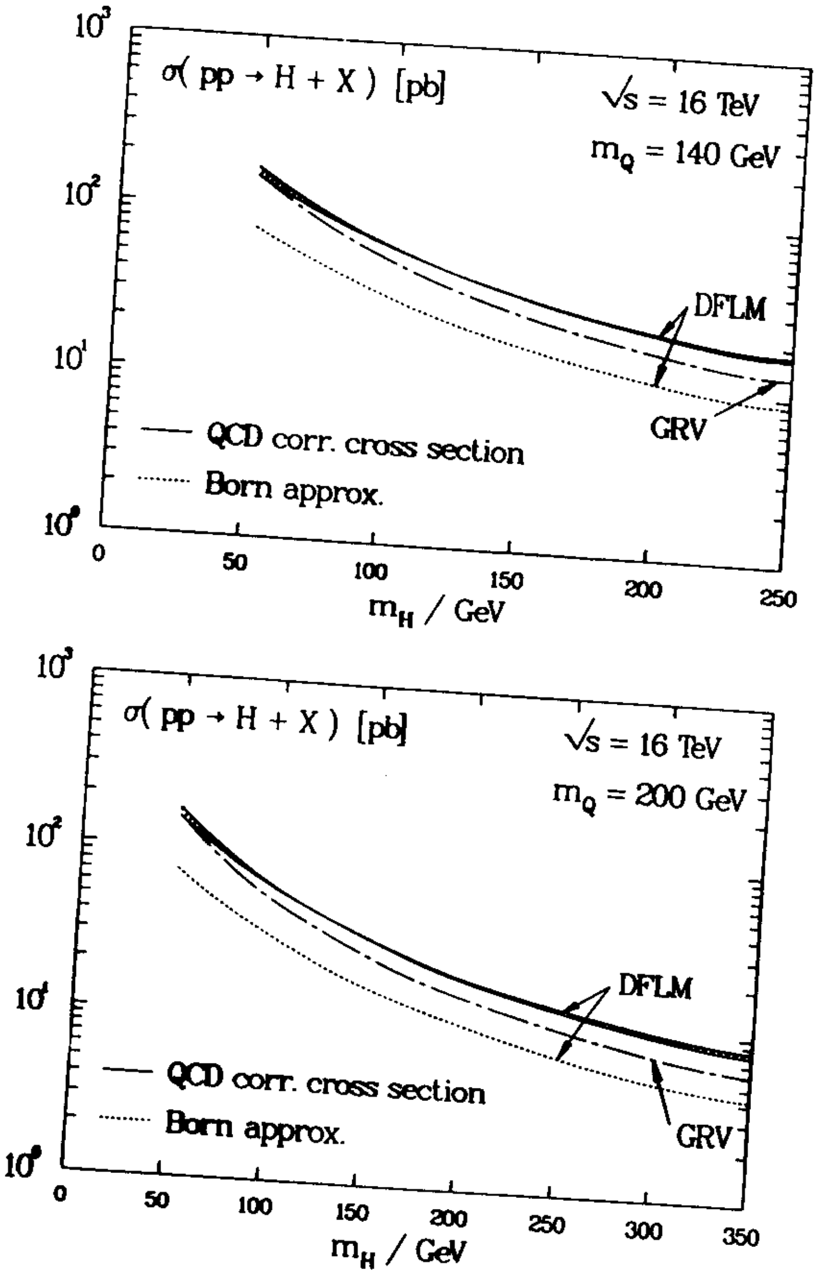






H
g
g
(a)
(b1)
(b2)
(b3)
(c1)
(c2)
(d1)
(d2)
q
(e1)
q
q̄
(e2)
(f1)
& q q̄
(f2)

Fig. 1



σ(pp → H + X) [pb]
√s = 16 TeV
m_Q = 140 GeV
10^3
10^2
10^1
10^0
DFLM
GRV
QCD corr. cross section
Born approx.
0
50
100
150
200
250
m_H / GeV
σ(pp → H + X) [pb]
√s = 16 TeV
m_Q = 200 GeV
10^3
10^2
10^1
10^0
DFLM
GRV
QCD corr. cross section
Born approx.
0
50
100
150
200
250
300
350
m_H / GeV

Fig. 2a

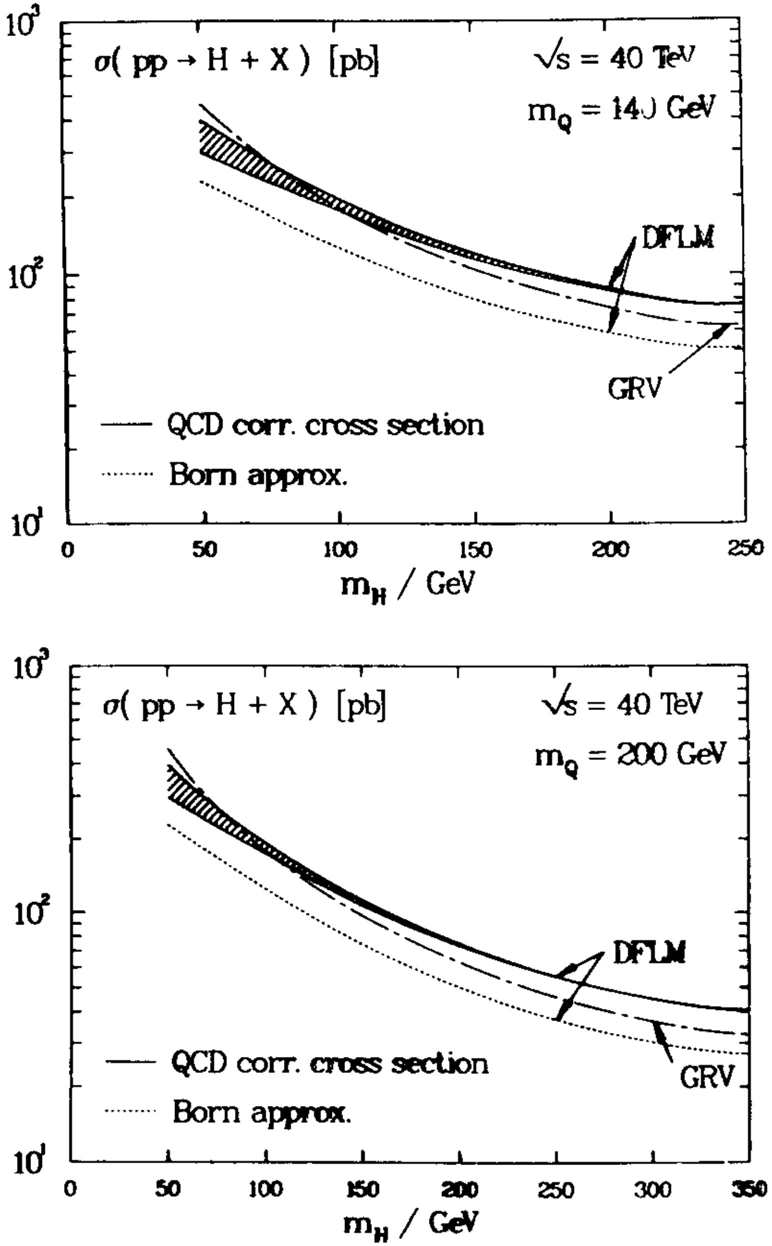


Fig. 2b

$e^+e^- \rightarrow W^+W^- + \gamma$

Beenakker et al.

(Leiden Report 19

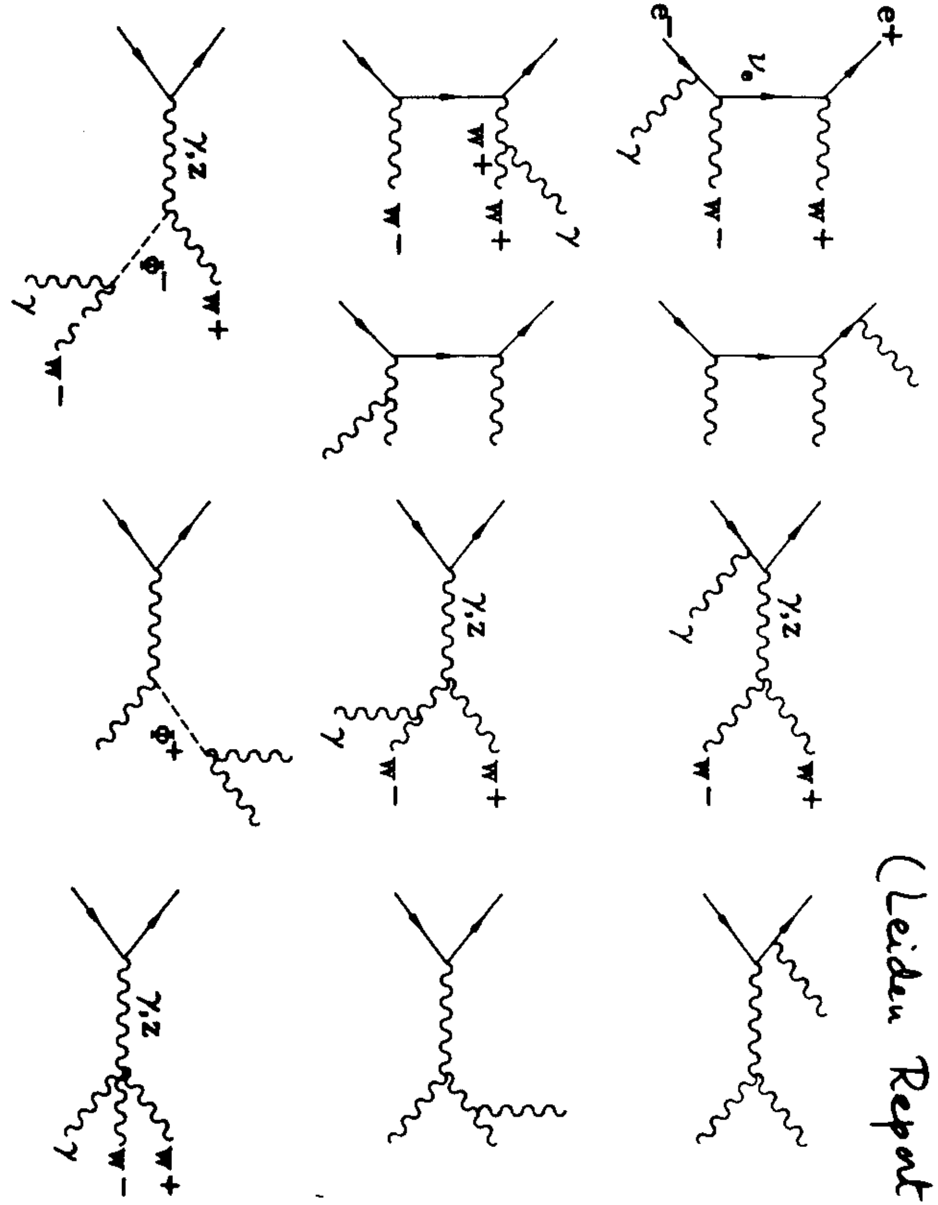


Fig. 1

Kunszt, Stirling, ETH-TH 90-50

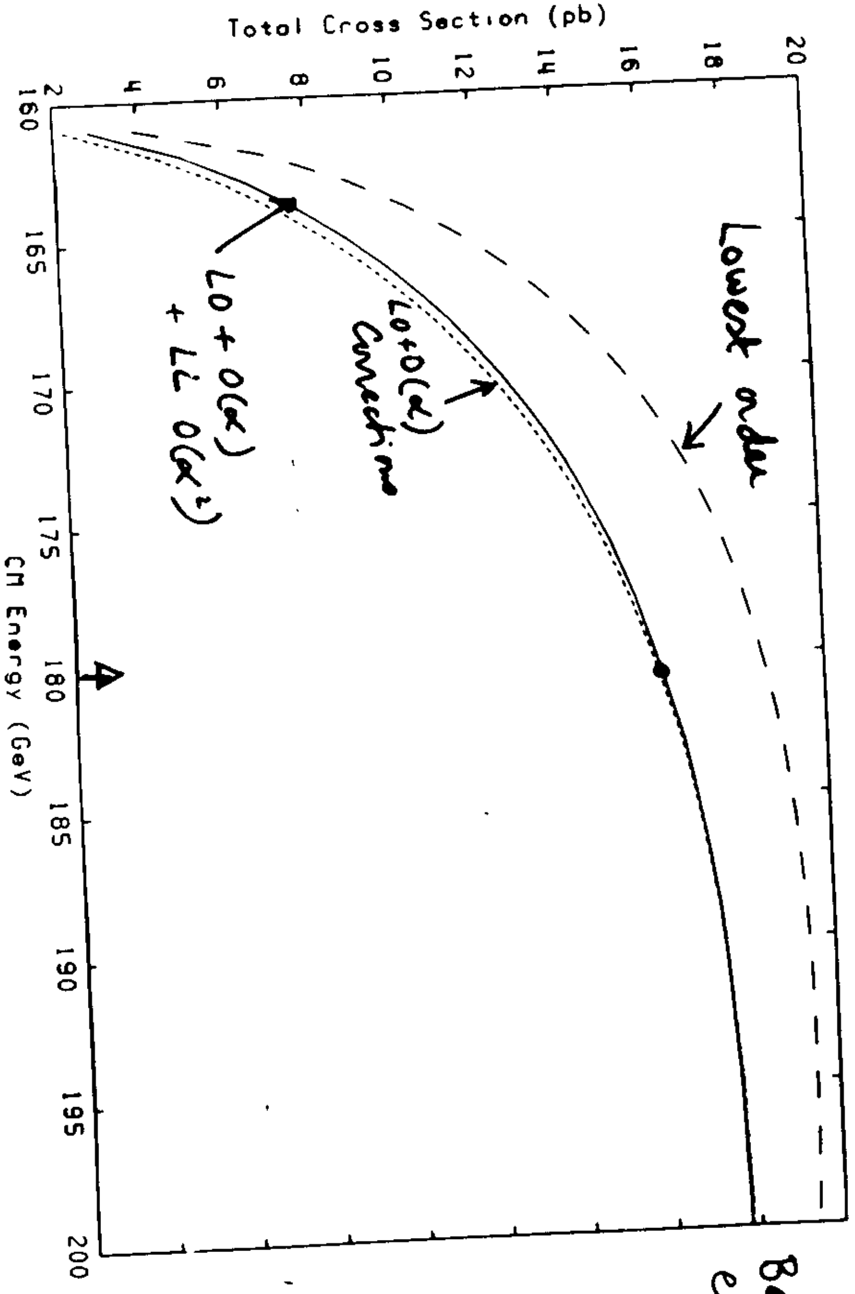


Fig. 3

$\sigma(e^+e^- \to W^+W^-) \sim 16$ Pb.

$\int \mathcal{L} dt = 1000 Pb^{-1} \Rightarrow 1.6 \times 10^4\ W^+W$

$\Delta M_W \sim \sigma(50$ MeV,

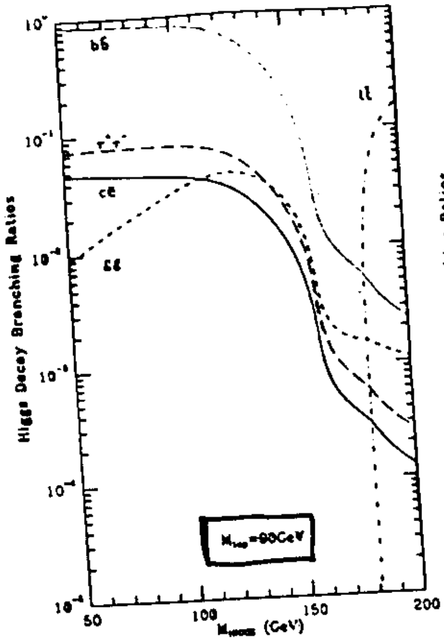


Fig. 1a

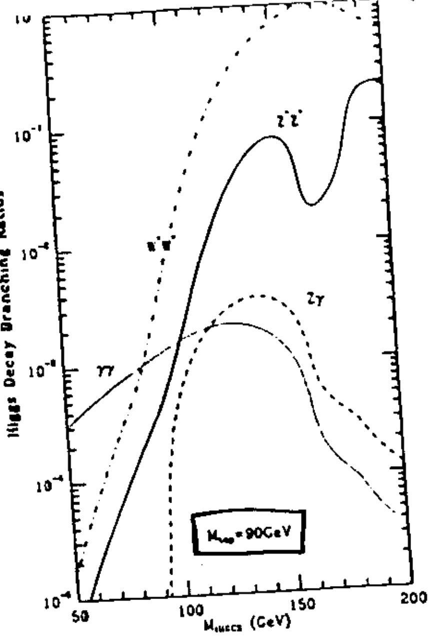
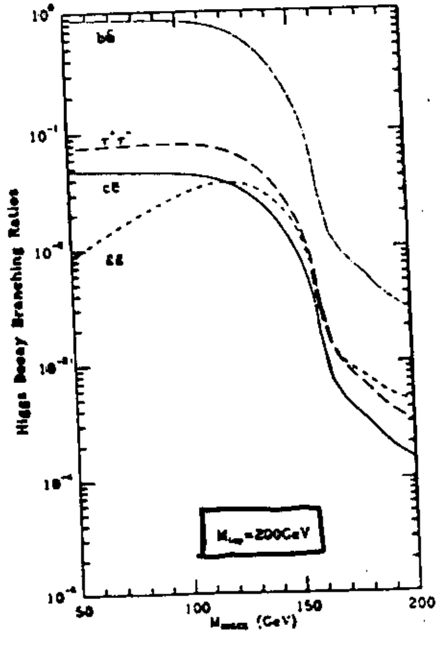
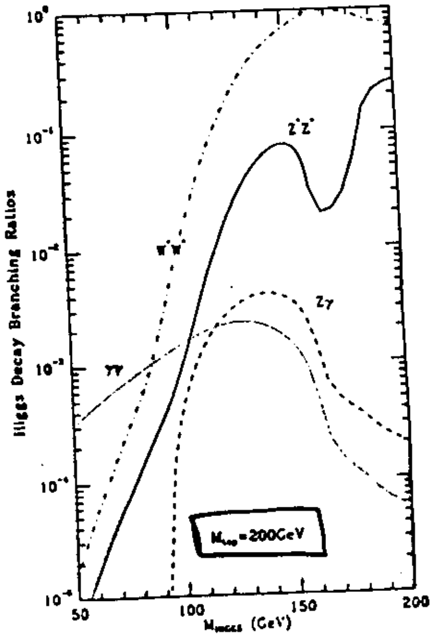


Fig. 1b





Higgs Production in Hadr. Collision

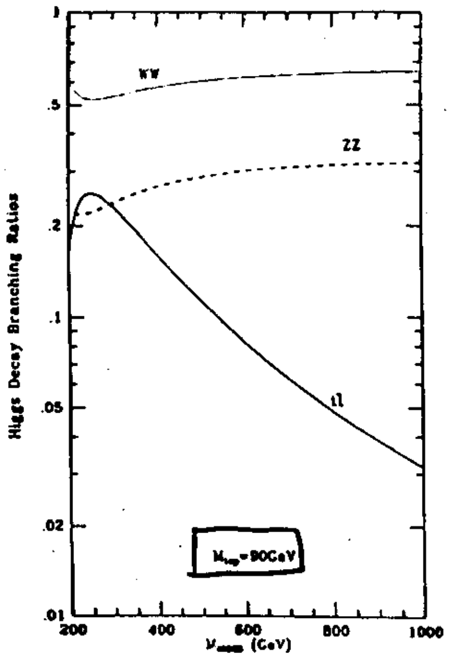


Fig. 3a

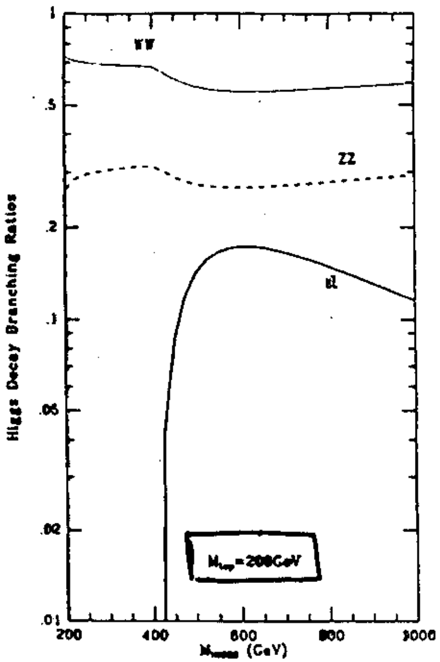
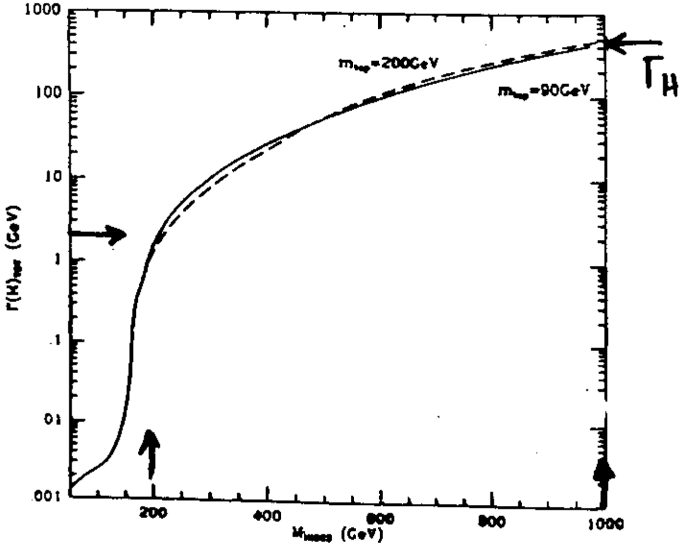


Fig. 3b



$\Gamma_H/M_H \simeq 10^{-2}$

$\Gamma_H/M_H \sim 1/2$

Fig. 4

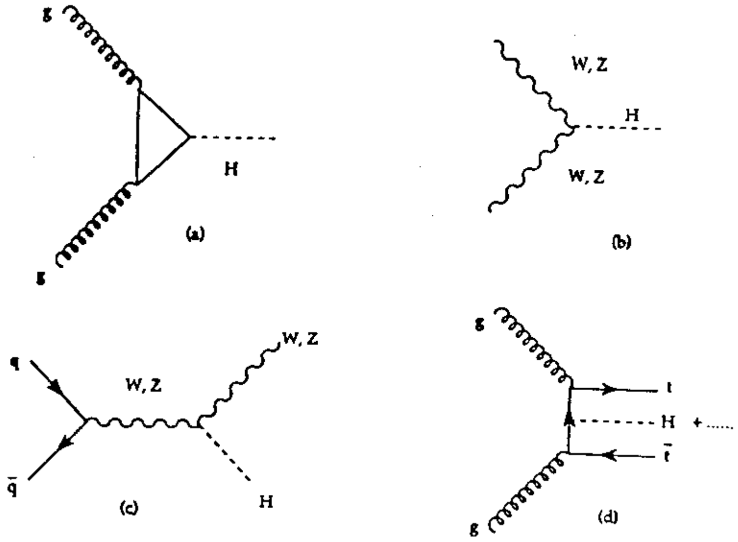
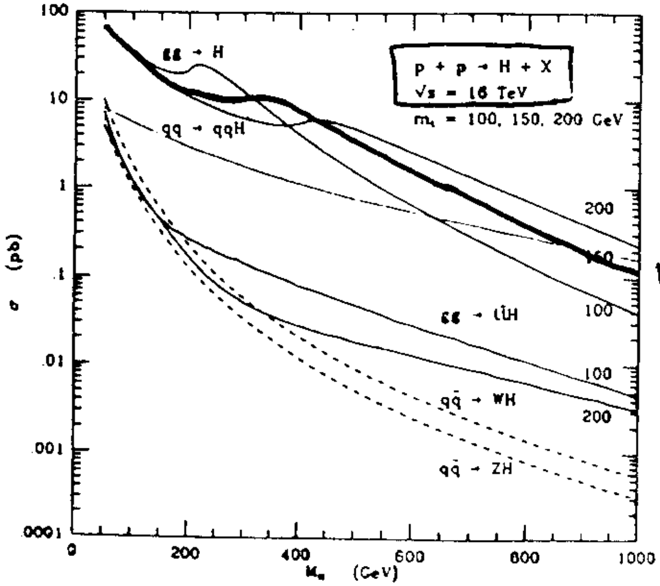


Fig. 5



LHC

$m_t = 150$ GeV

Fig. 6

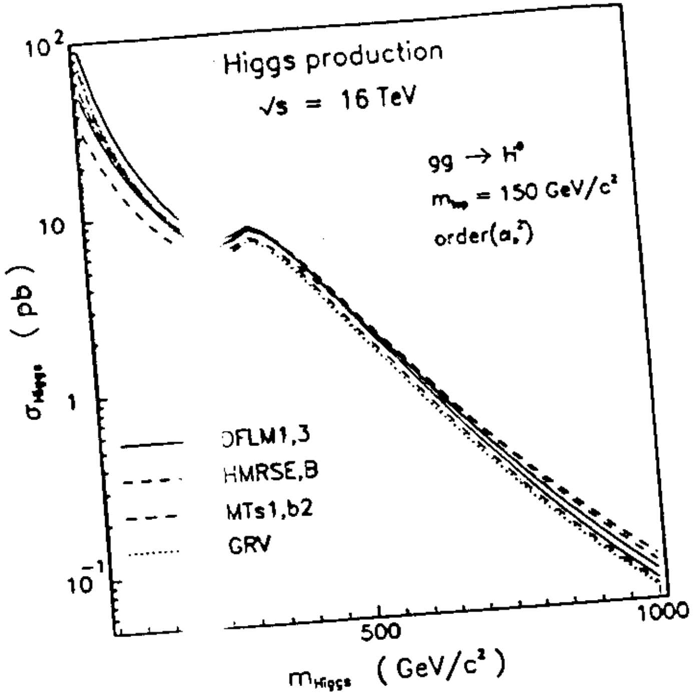
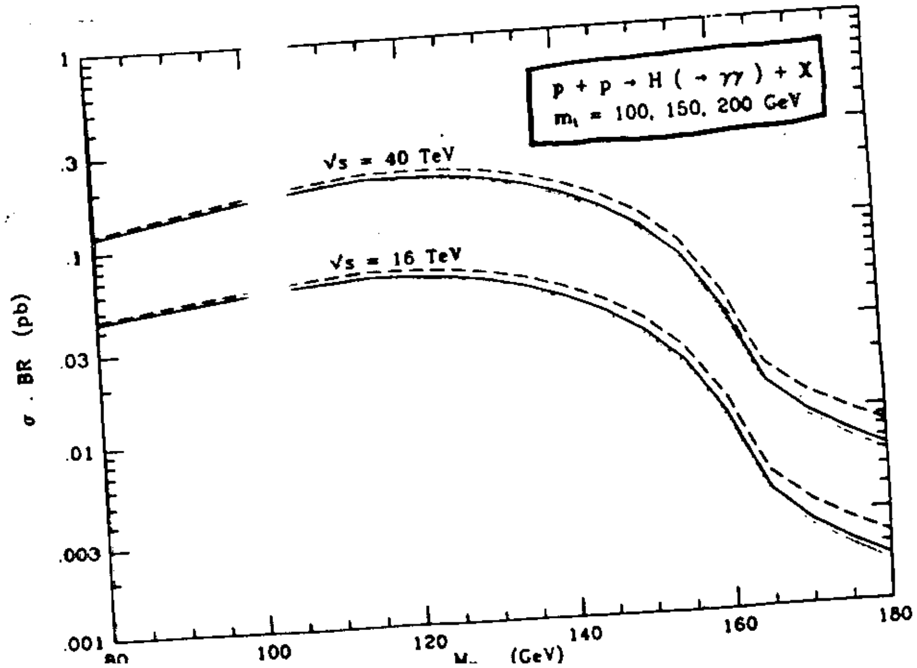


Fig. 7



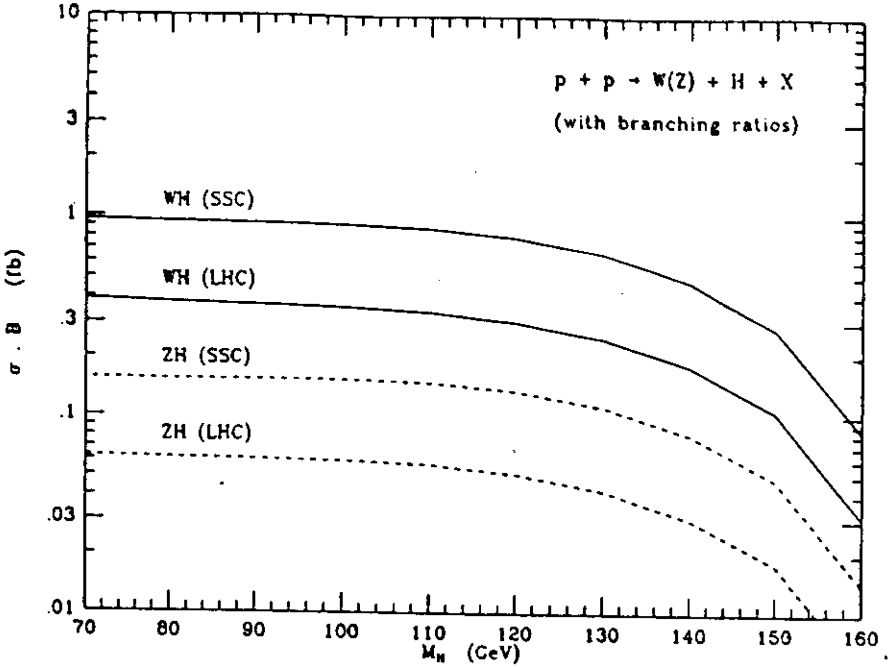


Fig. 9

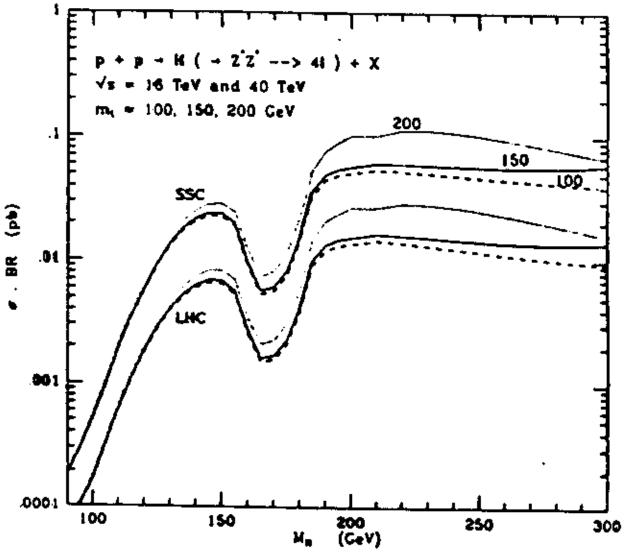


Fig. 10