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SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

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PRECISION TESTS OF THE ELECTROWEAK MODEL

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Please note: These are preliminary notes intended for internal distribution only.

Precision Tests of Electroweak Theories–II

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Lecture 2

ICTP, Trieste, July 16, 1992

CERN-PPE/91-232
20 Dec 1991

EUROPEAN ORGANISATION FOR NUCLEAR RESEARCH

Electroweak Parameters of the Z^0 Resonance and the Standard Model

The LEP Collaborations:
ALEPH, DELPHI, L3 and OPAL [1]

Abstract

The four LEP experiments have each performed precision measurements of Z^0 parameters. A method is described for combining the results of the four experiments, which takes into account the experimental and theoretical systematic errors and their correlations. We apply this method to the 1989 and 1990 LEP data, corresponding to approximately 650,000 Z^0 decays into hadrons and charged leptons, to obtain precision values for the Z^0 parameters. We use these results to test the Standard Model and to constrain its parameters.

(Submitted to Physics Letters B)

[1] Lists of authors can be found in references 1 to 4.

Parameter	Average Value	χ^2
M_Z (GeV)	91.175 ± 0.021	3.4
Γ_Z (GeV)	2.487 ± 0.010	2.0
σ_h^0 (nb)	41.33 ± 0.23	2.3
R_e	20.91 ± 0.22	
R_μ	20.80 ± 0.18	
R_τ	21.02 ± 0.23	
Γ_e (MeV)	83.20 ± 0.55	1.0
Γ_μ (MeV)	83.35 ± 0.86	5.1
Γ_τ (MeV)	82.76 ± 1.02	0.3
$Br(Z^0 \to e^+e^-)$ (%)	3.345 ± 0.020	
$Br(Z^0 \to \mu^+\mu^-)$ (%)	3.351 ± 0.034	
$Br(Z^0 \to \tau^+\tau^-)$ (%)	3.328 ± 0.040	

Table 3. Average LEP Line shape parameters. Values for χ^2 of the weighted average are quoted for those parameters given directly by the four experiments.

$$R_\ell \equiv \frac{\Gamma(Z^0 \to hadrons)}{\Gamma(Z^0 \to \ell^+\ell^-)}, \quad \ell = e, \mu, \tau$$

$$\Gamma_{inv} = \Gamma_Z - \Gamma_{had} - 3\Gamma_\ell$$

$$\Gamma_{inv}/\Gamma_\ell = \sqrt{\frac{12\pi R_\ell}{M_Z^2 \sigma_h^0}} - R_\ell - 3$$

Parameter	Average Value	χ^2
R_l	20.89 ± 0.13	
Γ_{had} (GeV)	1.740 ± 0.012	
Γ_l (MeV)	83.24 ± 0.42	0.4
$Br(Z^0 \to hadrons)$ (%)	69.93 ± 0.31	
$Br(Z^0 \to l^+l^-)$ (%)	3.347 ± 0.013	
A^0_{FB}	0.0138 ± 0.0049	
$g^2_{V_l}$	$(1.16 \pm 0.41) \times 10^{-3}$	2.9
$g^2_{A_l}$	0.2493 ± 0.0013	0.8
$g^2_{V_l}/g^2_{A_l}$	0.0047 ± 0.0017	
Γ_{inv} (MeV)	498 ± 8	
Γ_{inv}/Γ_l	5.985 ± 0.085	

$$N_\nu = 3.00 \pm 0.05$$

Table 4. Average LEP Z^0 parameters assuming lepton universality. Values for χ^2 of the weighted average are quoted for those parameters given directly by the four experiments.

− A^0_{FB} defined using Lepton Universality

$$A^0_{FB} \equiv 3\,\frac{g^2_{V_\ell}\, g^2_{A_\ell}}{(g^2_{V_\ell} + g^2_{A_\ell})^2}$$

Based on Combined LEP data (Dec. '91)

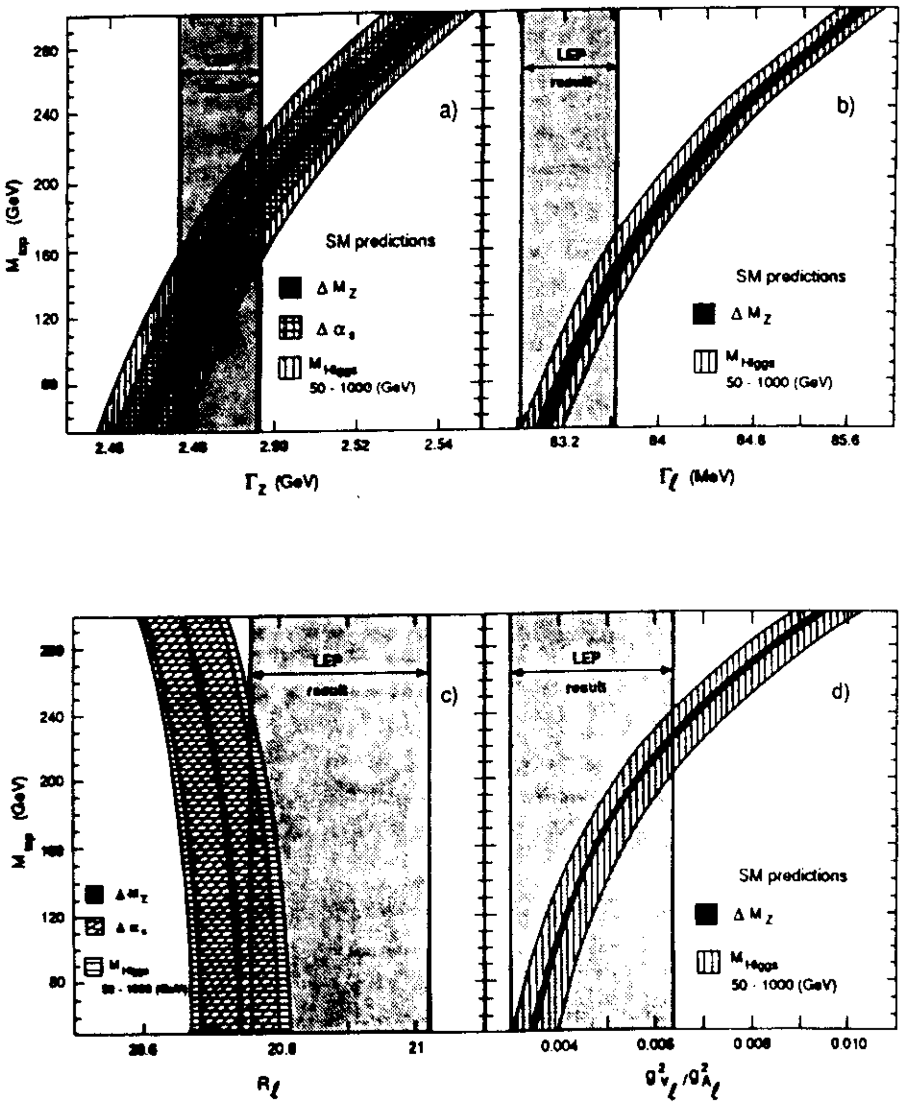


Figure 1: Z parameters as a function of M_t together with Standard Model prediction. a) Γ_Z, b) Γ_l, c) R_l, d) $g^2_{V_l}/g^2_{A_l}$.

LEP Collaborations
CERN-PPE/91-232
(Dec. '91)

	α_s unconstrained	α_s constrained	α_s unconstrained + Collider and ν data	α_s constrained + Collider and ν data
M_t (GeV)	$94^{+63\ +23}_{-\dots\ -24}$	$124^{+40\ +21}_{-56\ -21}$	$124^{+28\ +16}_{-31\ -18}$	$132^{+27\ +18}_{-31\ -19}$
α_s	$0.141 \pm 0.017 \pm 0.002$	0.123 ± 0.007	0.138 ± 0.015	0.123 ± 0.007
χ^2/ d.o.f.	0.3/2	2.2/3	1.5/5	3.0/6
$\sin^2\theta_{eff}^{lept}$	-	$0.2337 \pm 0.0014\ ^{+0.0001}_{-0.0004}$	$0.2337 \pm 0.0010\ ^{+0.0003}_{-0.0004}$	$0.2335 \pm 0.0009\ ^{+0.0001}_{-0.0004}$
$\sin^2\theta_W \equiv 1 - M_W^2/M_Z^2$	-	$0.2299^{+0.0067}_{-0.0048}$	0.2299 ± 0.0033	0.2290 ± 0.0033
M_W (GeV)	-	$80.01^{+0.27}_{-0.37}$	80.01 ± 0.19	80.06 ± 0.19

Table 7. Results of fits to LEP and other data for M_t and α_s. In the first and third columns α_s is unconstrained whereas, in the others it is constrained to the value 0.118 ± 0.008. In the third and fourth columns data are included from the $p\bar{p}$ experiments UA2 [11]: $M_W/M_Z = 0.8813 \pm 0.0041$, and CDF [10]: $M_W = 79.91 \pm 0.39$ GeV and from the neutrino experiments, CDHS [12] and CHARM [13]: $\sin^2\theta_W = 0.2300 \pm 0.0064$. The first error is experimental while the second error corresponds to 50 GeV $< M_H <$ 1000 GeV. The dots for the negative error of M_t indicate that it reaches the bound for open top production, which is not implemented in the parametrisations used.

$$\boxed{m_t = \left(149^{+21}_{-27} \pm 16\right)\ \text{GeV}}$$

(50 GeV $< m_H$ $<$ 1000 GeV)

$\Rightarrow$ $m_t < 191$ GeV @ 90 % C.L.

Langacker ('92

$$\boxed{m_t = 155 \pm 30\ \text{GeV}}$$

Altarelli (Moriond '92)

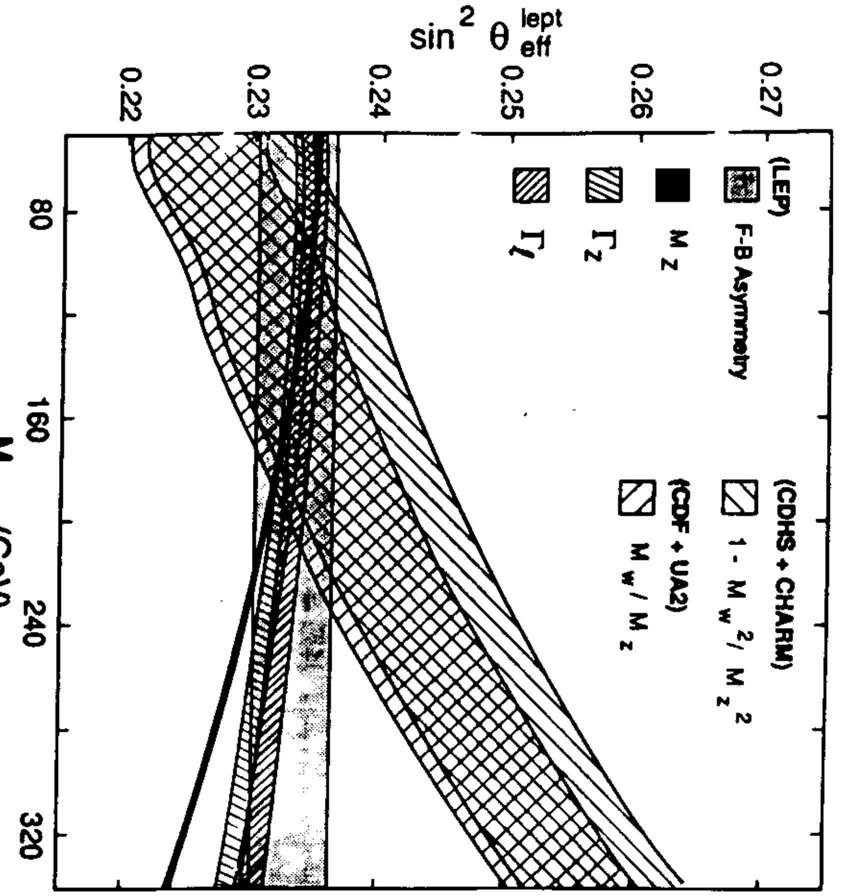


Figure 2: Constraints on $\sin^2\theta_{eff}^{lept}$ versus M_t from different measurements corresponding to 1σ limits

$$\sin^2\theta_{eff}^{lept} \equiv \frac{1}{4}\left(1 - g_{V\ell}/g_{A\ell}\right)$$

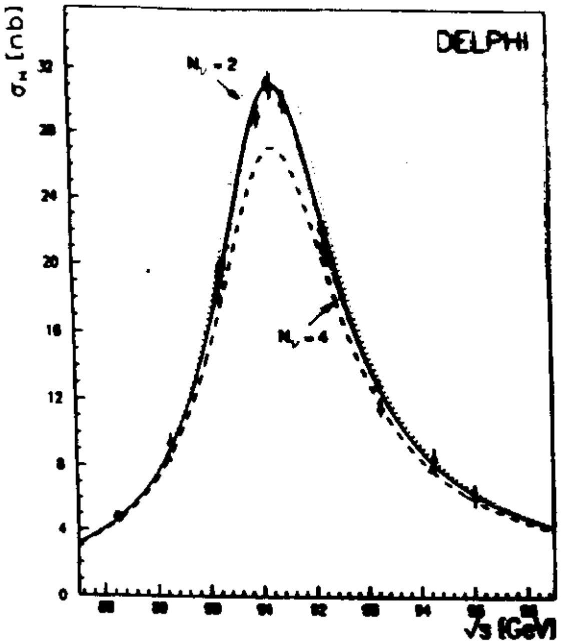


Figure 16: Measurement of the Z line shape, and comparison with the MSM expectation for N_ν = 2, 3, and 4 (DELPHI)

error, a significant improvement of N_ν is expected from further running. Ultimately, the number of neutrino families will be measured to ± 0.03 ($\pm 1\%$).

Figure 16 shows the most prominent achievement in the first year of data taking at LEP: the precise mapping of the Z line shape as measured with hadronic events. The data are quite consistent with $N_\nu = 3$, whereas $N_\nu = 2$ and $N_\nu = 4$ are clearly ruled out. The data shown in Fig. 16 are from the DELPHI experiment. The analogous data from the ALEPH, L3, and OPAL experiments look of course equally convincing.

2.6 Forward–backward asymmetry of leptons

With the exception of L3, who studied electrons and muons only, the LEP experiments presented results on the forward–backward asymmetry of electrons, muons, and taus, for all energies which have been employed during the scan of the Z resonance.

On the peak, the leptonic forward–backward asymmetry is given in the improved Born approximation by

$$A_f^{FB} = \frac{3}{4} \mathcal{A}_e \mathcal{A}_f ,$$

with

$$\mathcal{A}_f = 2 \frac{\bar{g}_{Vf} \bar{g}_{Af}}{\bar{g}_{Vf}^2 + \bar{g}_{Af}^2} .$$

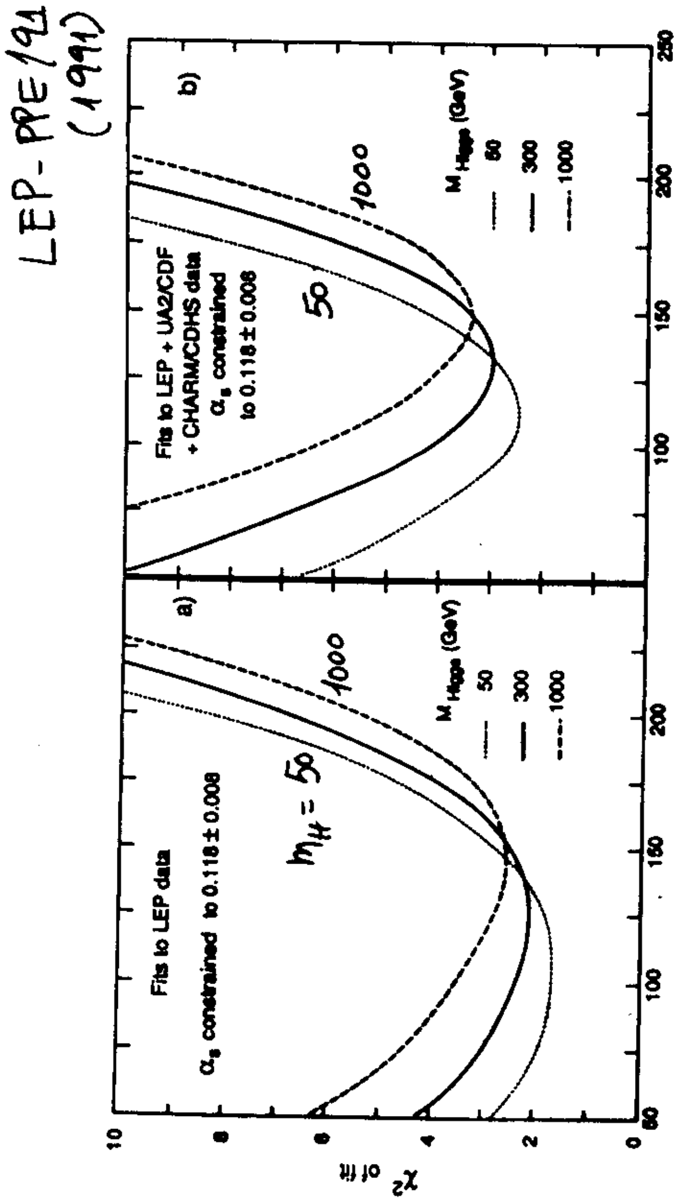


Figure 8: χ^2 as function of M_t from a fit to a) LEP data with 3 degrees of freedom, b) LEP data and $p\bar{p}$-collider and ν data for 5 degrees of freedom.

A Compilation of $\sin^2\theta_W(m_Z)_{\overline{MS}}$

$(m_H = 100\ \text{GeV})$

$R_\nu \equiv \dfrac{\sigma(\nu_\mu N \to \nu_\mu X)}{\sigma(\nu_\mu N \to \mu X)} \Rightarrow 0.230 \pm 0.003 \pm 0.005 + 0.0008\left(\dfrac{m_t}{m_W}\right)^2$

$\overset{(-)}{\nu}_\mu p$ Elastic Scattering $\Rightarrow 0.2[illegible] \pm 0.033$

$\dfrac{\sigma(\nu_\mu e \to \nu_\mu e)}{\sigma(\bar\nu_\mu e \to \bar\nu_\mu e)} \Rightarrow 0.2325 \pm 0.010 \pm 0.0006$

$A_{FB}\,(LEP) \Rightarrow 0.228 \pm 0.003$

$A_{LR}\,(\tau\ \text{Pol.})_{ALEPH} \Rightarrow 0.231 \pm 0.007$

All LEP data $\Rightarrow 0.2322 \pm 0.00[illegible]$

World average: $\boxed{\sin^2\theta_W(m_Z)_{\overline{MS}} = 0.2326} \pm 0.00[illegible]$

$m_t = 130$ GeV, $m_H = 100$ GeV, $m_Z = 91.[illegible]$ GeV

Marciano)

(Langacker)

α_s-determination @ LEP + SLC

T. Hebbeker
Aachen Report '92
(to appear in Phys. Reports)

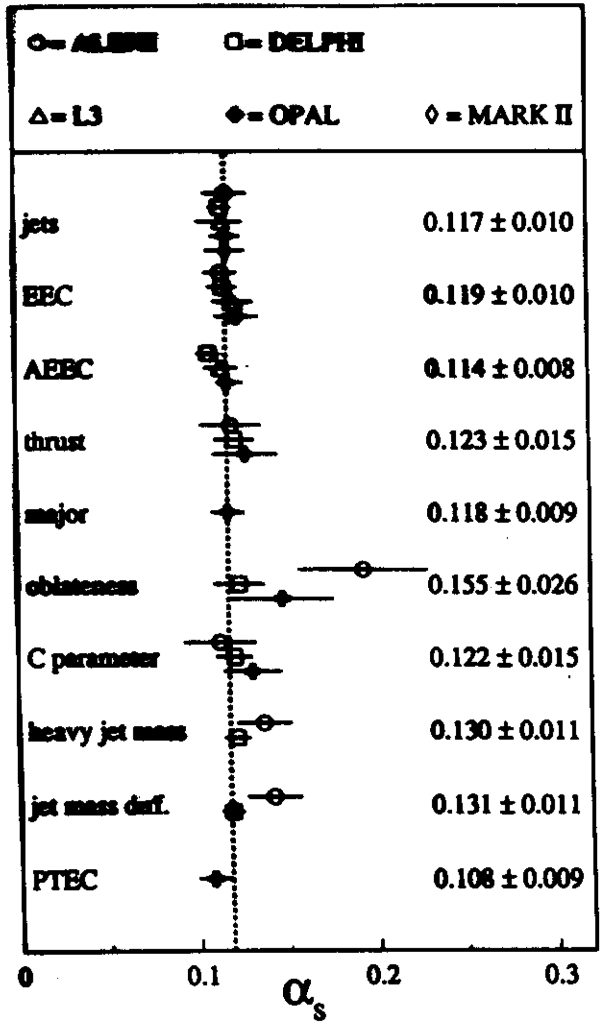
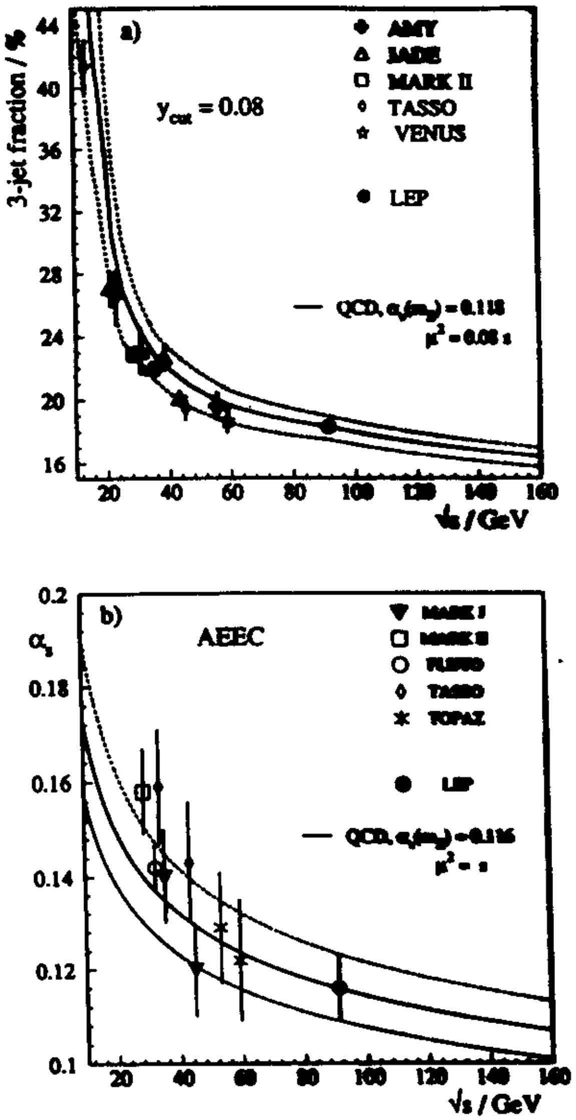


Figure 4.7: α_s from event topology

(T. Hebekker)



a) Energy dependence of the 3-jet fraction, b) energy dependence of α_s from AEEC

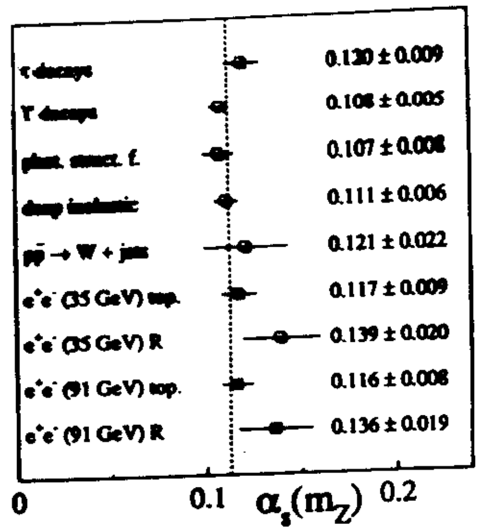


Figure 4.12: α_s values at the scale $\mu = m_Z$

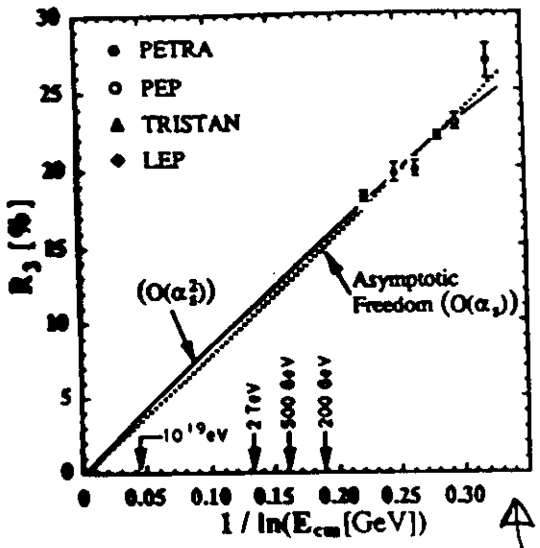


Figure 4.14: 3-jet fractions, from S. Bethke

$$\boxed{\alpha_s(m_Z) = 0.12 \pm 0.01} \quad \text{(Altarelli Moriond '92)}$$

$$\Rightarrow \quad \Lambda^{(4)}_{\overline{MS}} = 0.362^{+0.202}_{-0.149} \text{ GeV}$$

Appendix A

Λ and $\alpha_S(m_Z)$

α_s	$\Lambda^{(5)}$	$\Lambda^{(4)}$	$\Lambda^{(3)}$		α_s	$\Lambda^{(5)}$	$\Lambda^{(4)}$	$\Lambda^{(3)}$
0.091	0.031	0.054	0.078		0.121	0.267	0.380	0.453
0.092	0.034	0.059	[illegible]		0.122	0.281	0.398	0.472
0.093	0.038	0.065	[illegible]		0.123	0.296	0.417	0.492
0.094	0.041	0.070	[illegible]		0.124	0.312	0.437	0.513
0.095	0.045	0.076	0.106		0.125	0.328	0.457	0.533
0.096	0.049	0.082	0.114		0.126	0.344	0.477	0.554
0.097	0.054	0.090	0.123		0.127	0.361	0.498	0.576
0.098	0.058	0.096	0.131		0.128	0.379	0.520	[illegible]
0.099	0.063	0.103	0.140		0.129	0.397	0.542	[illegible]
0.100	0.068	0.111	0.149		0.130	0.416	0.565	0.644
0.101	0.074	0.120	0.160		0.131	0.436	0.589	[illegible]
0.102	0.080	0.128	0.170		0.132	0.456	0.613	[illegible]
0.103	0.086	0.137	0.181		0.133	0.476	0.637	0.715
0.104	0.092	0.146	0.191		0.134	0.498	0.664	0.741
0.105	0.099	0.156	0.203		0.135	0.519	[illegible]	[illegible]
0.106	0.107	0.167	0.216		0.136	0.542	0.715	0.791
0.107	0.114	0.177	0.228		0.137	[illegible]	0.742	0.817
0.108	0.122	0.188	0.240		0.138	[illegible]	0.769	0.843
0.109	0.131	0.200	0.255		0.139	[illegible]	0.798	[illegible]
0.110	0.140	0.213	0.269		0.140	[illegible]	0.826	[illegible]
0.111	0.149	0.225	0.283		0.141	[illegible]	0.856	[illegible]
0.112	0.159	0.238	0.298		0.142	[illegible]	[illegible]	[illegible]
0.113	0.169	0.252	0.313		0.143	[illegible]	0.916	[illegible]
0.114	0.179	0.265	0.328		0.144	[illegible]	0.947	[illegible]
0.115	0.190	0.280	0.344		0.145	[illegible]	0.978	[illegible]
0.116	0.202	0.296	0.362		0.146	[illegible]	1.009	[illegible]
0.117	0.214	0.312	0.379		0.147	[illegible]	1.043	[illegible]
0.118	0.226	0.327	0.396		0.148	[illegible]	1.076	1.120
0.119	0.239	0.344	0.414		0.149	[illegible]	1.109	1.148
0.120	0.253	0.362	0.434		0.150	[illegible]	1.143	1.176

Table A.1: Relation between $\alpha_s(m_Z)$ and $\Lambda^{(N_f)}$ according to formula (2.19)

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Determination of CG Coefficients of QCD + alternative models

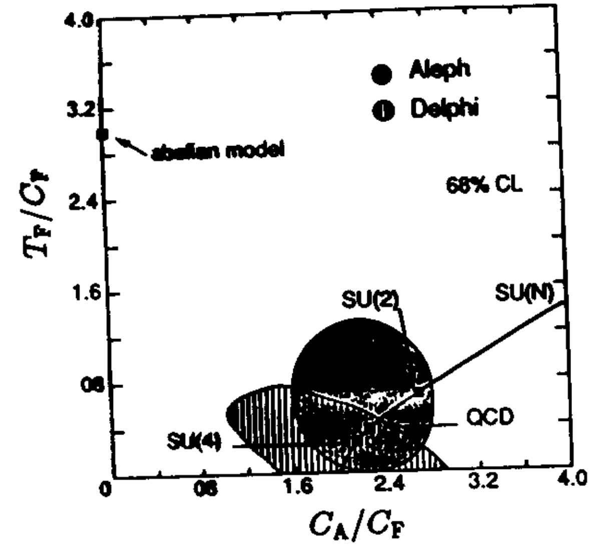

Figure 5.8: QCD color factors as measured by ALEPH and DELPHI

	C_A/C_F $g \to gg$	T_F/C_F $g \to q\bar{q}$
LEP	2.0 ± 0.3	0.3 ± 0.2
QCD	2.25	0.375
abelian	0	3

Table 5.2: Color factor ratios

Test of the Non-abelian Character of QCD

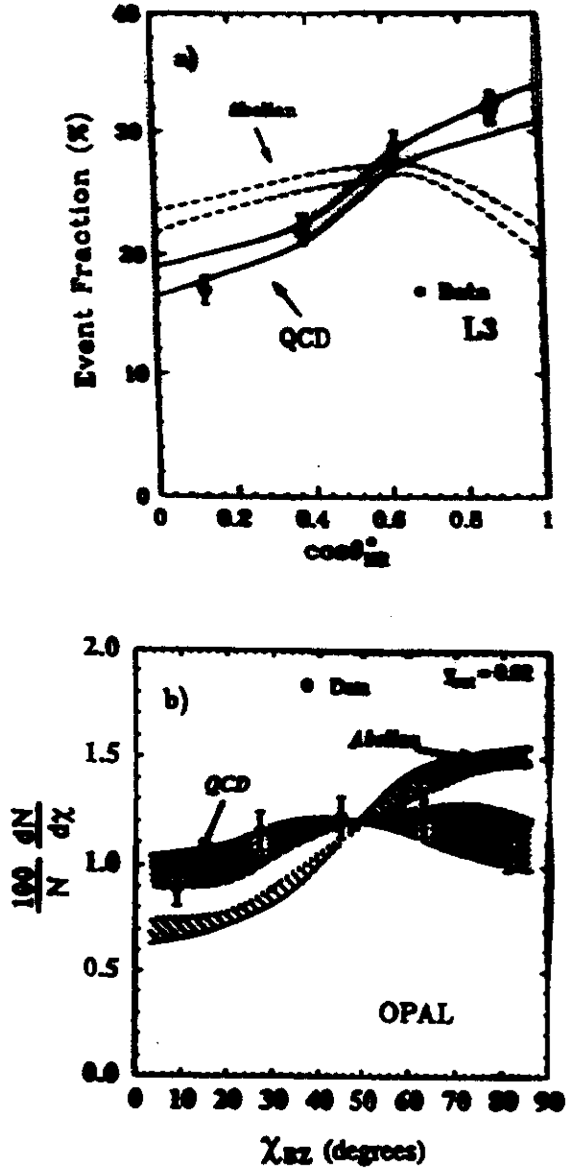

Figure 5.7: a) Nachtmann-Reiter angle measured by L3, b) Bengtsson-Zerwas angle a… by OPAL

Extrapolation to High Energies

Langacker, J. Ellis et al., Amaldi et al., Marciano, Sirlin ....

Input:

$$\alpha^{-1}(m_Z) = 127.9 \pm 0.2$$

$$\sin^2\theta_W(m_Z)_{\overline{MS}} = 0.2326 \pm 0.0008$$

$$\alpha_S(m_Z) = 0.118 \pm 0.007$$

$$\Rightarrow \alpha_1^{-1}(m_Z) = \frac{3}{5}\alpha^{-1}(m_Z)\cos^2\theta_W(m_Z)_{\overline{MS}} = 58.89 \pm 0.11$$

$$\alpha_2^{-1}(m_Z) = \alpha^{-1}(m_Z)\sin^2\theta_W(m_Z)_{\overline{MS}} = 29.75 \pm 0.11$$

$$\alpha_3^{-1}(m_Z) = \alpha_S^{-1}(m_Z) = 8.47 \pm 0.5c$$

- Extrapolation to high energies done by RGE's
- GUTS $\Rightarrow$ Unification of α_i^{-1} at a common GUT-Scale, m_X
- Extrapolation depends mainly on the Low-en SM Particles and SUSY Partners

Amaldi et al.,
Langacker

- Simple GUT Models ($SU(5)$, $SO(10)$, E_6)

Characterized by breaking in a single step
$$M_X \rightarrow M_Z$$

No unification for a single M_X

$\Rightarrow$ Simple GUT models are disfavoured!

Simple GUT models also in

— Failure of simple GUT models in predicting $\sin^2\theta_W$ (Marciano, Sirlin; Langacker)

$$\sin^2\theta_W(m_Z)_{\overline{MS}} = 0.208 \pm 0.004(N_H - 1) + 0.006 \ln\left(\frac{400\ \text{MeV}}{\Lambda_{\overline{MS}}(n_f=4)}\right)$$

$$\Rightarrow \boxed{\sin^2\theta_W(m_Z)_{\overline{MS}} \simeq 0.21 - 0.215} \quad (0.2326 \pm 0.0008)$$

- SUSY-GUT Models (MSSM)

$$M_X \rightarrow M_{SUSY}$$

$\Rightarrow$ Unification of α_i^{-1} for a single M_X!

— Hard to determine M_{SUSY}

in agreement with

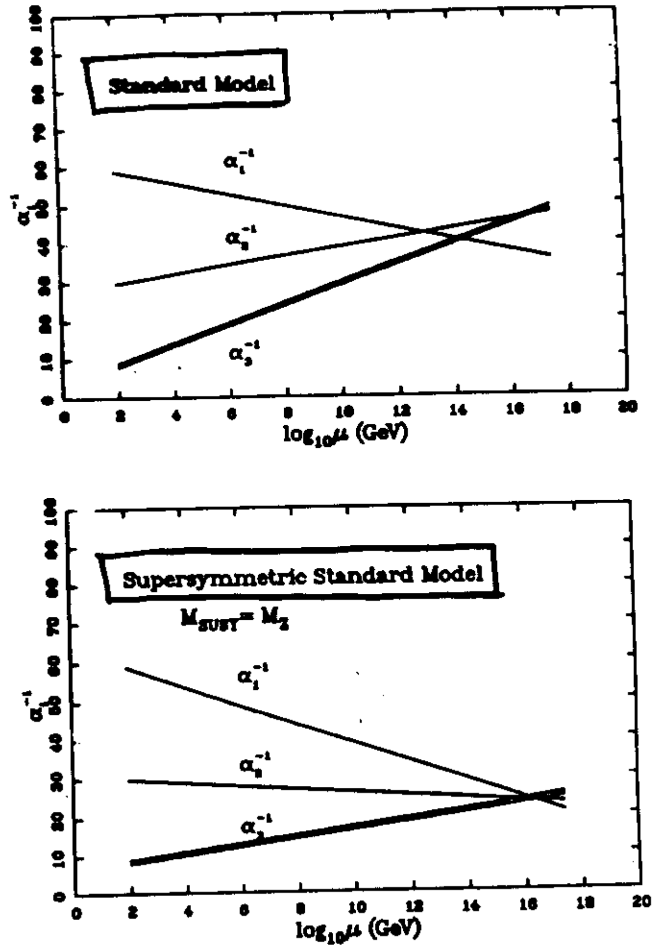

Figure 8: Running coupling in (a) the standard model and (b) in the minimal supersymmetric extension of the standard model (MSSM) with two Higgs doublets for $M_{SUSY} = M_Z$. The corresponding figure for $M_{SUSY} = 1$ TeV is almost identical. It is seen that the couplings unify at $\simeq 10^{16}$ GeV in the MSSM.

Langacker '91

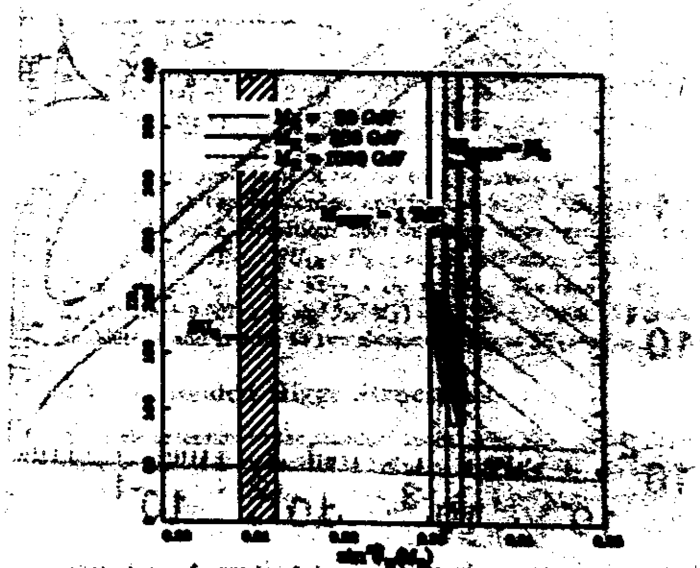

Figure 9: 90% CL regions in $\sin^2\hat{\theta}_W(M_Z)$ vs m_t for M_H = 50, 250, and 1000 GeV, compared with the predictions of ordinary and SUSY-GUTs.

General Analysis of Oblique Corr.

$\overline{MS}$ Scheme

$$\hat{S}_W^2 \equiv \sin^2\hat{\theta}_W(m_Z)$$

$$\hat{C}_W^2 = 1 - \hat{S}^2$$

$$A \equiv \left(\frac{\pi\alpha}{\sqrt{2}\,G_\mu}\right)^{1/2}$$

- $\mu = m_Z$
- $\overline{MS}$ Scheme for Counter terms

henceforth ~~drop~~ $\hat{S}_W = S_W$ etc

Relations

- $$\boxed{S_W^2 C_W^2 = \frac{A^2}{m_Z^2(1-\Delta r_W)}}$$

$$(\Delta r_W)_{\overline{MS}} = \mathrm{Re}\left[\frac{\Pi_{ZZ}(m_Z^2)}{m_Z^2} - \frac{\Pi_{WW}(0)}{m_W^2} - \frac{2\delta e}{e}\right]_{\overline{MS}}$$

$$+ \text{(box, vertex) Contribution}$$

- $$\boxed{m_W^2 = \frac{A^2}{S_W^2(1-\Delta\hat{r}_W)}}$$

$$(\hat{\Delta} r_W)_{\overline{MS}} = \mathrm{Re}\left[\frac{\Pi_{WW}(m_W^2) - \Pi_{WW}(0)}{m_W^2} - \frac{2\delta e}{e}\right]_{\overline{MS}}$$

- Partial Widths (Improved Born App.)

$$\boxed{\Gamma(Z^0 \to f\bar{f}) = N_c^f \frac{\alpha(m_Z)}{s_W^2 c_W^2} \frac{m_Z \bar{\rho}_f(m_Z^2)}{48} \times \left[1 + (1 - 4|Q_f|\bar{\kappa}_f s_W^2)^2\right]}$$

$$\bar{\rho}_f(q^2) = 1 + \frac{\bar{\Pi}_{ZZ}(q^2)}{q^2 - m_Z^2}\Big|_{\overline{MS}} + \dots$$

$$\bar{\kappa}_f(q^2) = 1 - \frac{c}{s} \frac{\bar{\Pi}_{\gamma Z}(q^2)}{q^2}\Big|_{\overline{MS}}$$

where

$$\bar{\Pi}_{ZZ}(q^2) = \left[\Pi_{ZZ}(q^2) - \mathrm{Re}\,\Pi_{ZZ}(m_Z^2) - i\frac{q^2}{m_Z^2} \mathrm{Im}\,\Pi_{ZZ}(m_Z^2)\right]_{\overline{MS}}$$

$$\bar{\Pi}_{\gamma Z}(q^2) = \left[\Pi_{\gamma Z}(q^2) - \Pi_{\gamma Z}(0)\right]_{\overline{MS}}$$

[$\bar{\Pi}_{\gamma Z}(q^2)$ is like $\hat{\Pi}_{\gamma\gamma}(q^2)$ ... to δe earlier]

- Equivalent Form for $\Gamma(Z^0 \to \ell\bar{\ell})$ + A_{FB}^{ℓ}

Leptonic Width

$$\Gamma(Z^0 \to \ell\bar{\ell}) = \frac{G_\mu m_Z^3}{6\pi\sqrt{2}} (\bar{v}_\ell^2 + \bar{a}_\ell^2)$$

FB Asymmetry

$$A_{FB}^{\ell} = 3 \frac{\bar{v}_e \bar{a}_e}{\bar{v}_e^2 + \bar{a}_e^2} \frac{\bar{v}_\ell \bar{a}_\ell}{\bar{v}_\ell^2 + \bar{a}_\ell^2}$$

with

$$\bar{v}_\ell = -\frac{1}{2}\sqrt{\rho_{eff.}}\,(1 - 4\sin^2\bar{\theta}_W)$$

$$\bar{a}_\ell = -\frac{1}{2}\sqrt{\rho_{eff.}}$$

$$\boxed{\rho_{eff.} = \frac{\alpha(m_Z)\pi}{s_W^2 c_W^2} \frac{\bar{\rho}_\ell(m_Z^2)}{\sqrt{2}\,G_\mu m_Z^2}}$$

$$\boxed{\sin^2\bar{\theta}_W = \bar{\kappa}_\ell s_W^2}$$

- Weak Charge [Coherent Atomic Scat $e + Cs \to e + Cs$]

$$\boxed{Q_W(Z, A) = \rho_{PV}\left[2Z - A - 4Z\,X_{PV}(0) \dots\right]}$$

$$\rho_{..} = 1 + \Pi_{WW}(0) - \Pi_{ZZ}(0) + \dots$$

$$\boxed{\rho_{pv} = 0.9799 + \frac{\alpha(m_Z)}{\pi} F(m_t, m_H, m_W)}$$

$$F(m_t, m_H, m_W) = \frac{3}{8 s_W^2} \frac{m_t^2}{m_W^2} + \frac{3\xi}{8 s_W^2}\left[\frac{\ln(c_W^2/\xi)}{c_W^2 - \xi} + \frac{1}{c_W^2}\frac{\ln \xi}{1-\xi} \right]$$

$$\xi = m_H^2 / m_Z^2$$

$$\Rightarrow \rho_{pv}(m_t = 130 \text{ GeV},\ m_H = 100 \text{ GeV}) = 0.9849$$

$$x_{pv} = 1.003 \pm 0.0025$$

$$\Rightarrow \boxed{Q_W(^{133}_{55}Cs) = -73.20 \pm 0.13}$$

$m_t = 130$ GeV
$m_H = 100$ GeV
$m_Z = 91.17$ GeV

Expt.

$$\boxed{Q_W(^{133}_{55}Cs) = -71.04 \pm 1.58 \pm 0.88}$$

Noecker

Electroweak Parameters

$$M_{W^\pm} = 80.14 \pm 0.31 \text{ GeV}$$

$$M_{Z^0} = 91.175 \pm 0.021 \text{ GeV}$$

$$\Rightarrow \boxed{\delta(M_W - M_Z) = 0.32 \text{ GeV}}$$

$$\Gamma(Z^0 \to \text{hadrons}) = 1.740 \pm 0.009 \text{ GeV}$$

$$\Gamma(Z^0 \to b\bar{b}) = 0.385 \pm 0.023 \text{ GeV}$$

$$\Rightarrow \boxed{\frac{\delta\Gamma(Z)}{\Gamma(Z)} = 0.06}$$

$$\sin^2\bar{\theta}_W = 0.2319 \pm 0.0028$$

$$\rho_{eff} = 0.998 \pm 0.0028$$

$$Q_W(^{133}_{55}Cs) = -71.04 \pm 1.58 \pm 0.88$$

$$\Rightarrow \boxed{\frac{\Delta Q_W}{Q_W} \simeq 0.025}$$

Peskin, Takeuchi
SLAC-PUB-5618

Combining (3.8) with (2.10) and (2.12), we find

$$\frac{m_W^2}{m_Z^2} - c_0^2$$
$$= s_0^2 - s_W^2$$
$$= -\left\{ \frac{e^2 c^2}{s^2(c^2-s^2) m_Z^2} \left[\Pi_{33}(m_Z^2) - 2s^2\Pi_{3Q}(m_Z^2) - \frac{s^2}{c^2}\Pi_{11}(0) - \frac{c^2-s^2}{c^2}\Pi_{11}(m_W^2) \right] + \frac{e^2 s^2 c^2}{c^2-s^2}\left[\Pi'_{QQ}(m_Z^2) - \Pi'_{QQ}(0) \right] \right\},$$

$$s_*^2(q^2) - s_0^2$$
$$= \left\{ \frac{e^2}{c^2-s^2}\left[\frac{\Pi_{33}(m_Z^2) - 2s^2\Pi_{3Q}(m_Z^2) - \Pi_{11}(0)}{m_Z^2} - (c^2-s^2)\frac{\Pi_{3Q}(q^2)}{q^2} \right] + \frac{e^2 s^2}{c^2-s^2}\left[s^2\Pi'_{QQ}(m_Z^2) - c^2\Pi'_{QQ}(0) + (c^2-s^2)\Pi'_{QQ}(q^2) \right] \right\}. \tag{3.9}$$

The remaining starred functions are defined as deviations from 1, and so the formulae of Section 2 may be evaluated directly. From (2.26) and (2.18), we have

$$\boxed{\begin{aligned} \rho_*(0) - 1 &= \frac{e^2}{s^2c^2m_Z^2}\left[\Pi_{11}(0) - \Pi_{33}(0)\right], \\ Z_{Z*}(q^2) - 1 &= \frac{e^2}{s^2c^2}\left[\frac{d}{dq^2}\left(\Pi_{33} - 2s^2\Pi_{3Q} + s^4\Pi_{QQ}\right)\Big|_{q^2=m_Z^2} - (c^2-s^2)\Pi'_{3Q}(q^2) - s^4\Pi'_{QQ}(q^2) \right], \\ Z_{W*}(q^2) - 1 &= \frac{e^2}{s^2}\left[\frac{d}{dq^2}\Pi_{11}\Big|_{q^2=m_W^2} - \Pi'_{11}(q^2) \right]. \end{aligned}} \tag{3.10}$$

We note again that (3.9) and (3.10) present only the oblique corrections to the various starred parameters. The calculation of the full standard model corrections is much more involved and cannot be expressed in such simple formulae. But it is remarkable that the entire influence of new physics, to the extent that it is purely oblique, follows from the relatively simple relations (3.9) and (3.10) and the use of the starred functions to renormalize the tree-level formulae. This point has, of course, been known for a long time; for example, it is the major result of ref. 6.

If the physics included in the vacuum polarization diagrams is associated with new heavy particles of mass much larger than m_Z, the vacuum polarization amplitudes will have rapidly convergent Taylor series expansions in q^2. Then, it is natural to expand the various Π's in q^2 and to neglect terms of order q^4 and above. This gives

$$\boxed{\begin{aligned} \Pi_{QQ}(q^2) &\approx q^2\Pi'_{QQ}(0), \\ \Pi_{3Q}(q^2) &\approx q^2\Pi'_{3Q}(0), \\ \Pi_{33}(q^2) &\approx \Pi_{33}(0) + q^2\Pi'_{33}(0), \\ \Pi_{11}(q^2) &\approx \Pi_{11}(0) + q^2\Pi'_{11}(0). \end{aligned}} \tag{3.11}$$

This approximation should only induce a relative error of (m_Z^2/m_T^2), where m_T is the scale where the new physics resides.

When we insert the approximate formulae (3.11) into (3.9) and (3.10), three linear combinations of the six Taylor series coefficients must cancel out, since these equations are differences of radiative corrections which fix α, G_F, and m_Z. These subtractions remove the ultraviolet divergences of perturbation theory, and so the three combinations which remain must be differences of coefficients with cancelling ultraviolet divergences. It is natural to define these three ultraviolet-finite combinations of Taylor series coefficients as our [illegible] parameters:[13,14]

$$\boxed{\begin{aligned} \alpha S &\equiv 4e^2\left[\Pi'_{33}(0) - \Pi'_{3Q}(0)\right], \\ \alpha T &\equiv \frac{e^2}{s^2c^2m_Z^2}\left[\Pi_{11}(0) - \Pi_{33}(0)\right], \\ \alpha U &\equiv 4e^2\left[\Pi'_{11}(0) - \Pi'_{33}(0)\right]. \end{aligned}} \tag{3.12}$$

Several different notations for these parameters appear in the literature; we review these notations and their interrelations in Appendix C.

We have already noted that, when the approximation (3.11) is inserted into (3.9) and (3.10), these formulae reduce to linear functions of S, T, and U. In fact, the relations are quite simple:

c_0, s_0 defined by $\sin 2\theta_0 = \left[\frac{\pi \alpha_{*,0}(m_Z^2)}{\sqrt{2}G_F m_Z^2}\right]^{1/2}$

$\alpha_{*,0}(m_Z^2) = \frac{\alpha}{1-\Delta\alpha} \approx 1/128$

$$\boxed{\begin{aligned} \frac{m_W^2}{m_Z^2} - c_0^2 &= \frac{\alpha c^2}{c^2-s^2}\left[-\frac{1}{2}S + c^2T + \frac{c^2-s^2}{4s^2}U\right], \\ s_*^2(q^2) - s_0^2 &= \frac{\alpha}{c^2-s^2}\left[\frac{1}{4}S - s^2c^2T\right], \\ \rho_*(0) - 1 &= \alpha T, \\ Z_{Z*}(q^2) - 1 &= \frac{\alpha}{4s^2c^2}S, \\ Z_{W*}(q^2) - 1 &= \frac{\alpha}{4s^2}(S+U). \end{aligned}} \tag{3.13}$$

The error involved in the reduction from the original perturbative formulae to (3.13) is of order $\alpha(m_Z/m_T)^2$, where m_T is the scale of new physics. Subject to

bservable	Measured Value	Reference	Standard Model
$_W/m_Z$	0.8791 ± 0.0034	51	0.8787
$_Z$ [GeV]	2.487 ± 0.009	23	2.484
	0.2977 ± 0.0042	32	0.3001
	0.0317 ± 0.0034	32	0.0302
$_Z$	20.94 ± 0.12	23	20.78
$^2(m_Z^2)$	0.2317 ± 0.0030	23	0.2337
$^b_{FB}$	0.135 ± 0.031	23	0.0848
$_\tau$	-0.152 ± 0.045	53	-0.1297
$_W(^{133}_{55}\mathrm{Cs})$	-71.04 ± 1.81	54	-73.31

Table 1

$\gamma \sim\bullet\sim \gamma = i\, e^2\, \Pi_{QQ}\, g^{\mu\nu} + \cdots$

$Z \sim\bullet\sim \gamma = i\, \frac{e^2}{cs}\, (\Pi_{3Q} - s^2\Pi_{QQ})\, g^{\mu\nu} + \cdots$

$Z \sim\bullet\sim Z = i\, \frac{e^2}{c^2s^2}\, (\Pi_{33} - 2s^2\Pi_{3Q} + s^4\Pi_{QQ})\, g^{\mu\nu} + \cdots$

$W \sim\bullet\sim W = i\, \frac{e^2}{s^2}\, \Pi_{11}\, g^{\mu\nu} + \cdots$

Fig. 1

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APPENDIX A

The following formulae represent the dependence of various observables on the starred functions. For comparison with experiment, vertex and box corrections that are not included in the starred functions must be added to these expressions though they generally turn out to be small. An exception is $\Gamma_{b\bar{b}}$ which receives an important vertex correction involving the t-quark. QCD corrections are included in N_{quark} and k_A.

Z^0 widths:

$$\Gamma_Z = Z_{Z*}\frac{\alpha_* m_Z}{6s_*^2c_*^2}\sum_f (I_{3f} - s_*^2 Q_f)^2 N_f\Big|_{q^2=m_Z^2}$$
$$= 3\Gamma_{\nu\bar{\nu}} + 3\Gamma_{\ell^+\ell^-} + \Gamma_{had}$$
$$\Gamma_{\nu\bar{\nu}} = Z_{Z*}\frac{\alpha_* m_Z}{24 s_*^2 c_*^2}\Big|_{q^2=m_Z^2}$$
$$\Gamma_{\ell^+\ell^-} = Z_{Z*}\frac{\alpha_* m_Z}{6s_*^2c_*^2}\left[\left(-\frac{1}{2}+s_*^2\right)^2 + (s_*^2)^2\right]\Big|_{q^2=m_Z^2}$$
$$\Gamma_{u\bar{u}} = \Gamma_{c\bar{c}} = Z_{Z*}\frac{\alpha_* m_Z}{6s_*^2c_*^2}\left[\left(\frac{1}{2}-\frac{2}{3}s_*^2\right)^2 + \left(-\frac{2}{3}s_*^2\right)^2\right]N_{quark}\Big|_{q^2=m_Z^2}$$
$$\Gamma_{d\bar{d}} = \Gamma_{s\bar{s}} = \Gamma_{b\bar{b}} = Z_{Z*}\frac{\alpha_* m_Z}{6s_*^2c_*^2}\left[\left(-\frac{1}{2}+\frac{1}{3}s_*^2\right)^2 + \left(\frac{1}{3}s_*^2\right)^2\right]N_{quark}\Big|_{q^2=m_Z^2}$$
$$\Gamma_{had} = 2\Gamma_{u\bar{u}} + 3\Gamma_{d\bar{d}}$$

Asymmetries at the Z^0 pole:

$$A_{LR} = \left[\frac{(g_{L*}^e)^2 - (g_{R*}^e)^2}{(g_{L*}^e)^2 + (g_{R*}^e)^2}\right]$$
$$= \frac{\left[-\frac{1}{2}+s_*^2(q^2)\right]^2 - \left[s_*^2(q^2)\right]^2}{\left[-\frac{1}{2}+s_*^2(q^2)\right]^2 + \left[s_*^2(q^2)\right]^2} = \frac{2[1-4s_*^2(q^2)]}{1+[1-4s_*^2(q^2)]^2}$$

$$A_{FB}^b = \frac{3}{4}A_{LR}\left[\frac{(g_{L*}^b)^2 - (g_{R*}^b)^2}{(g_{L*}^b)^2 + (g_{R*}^b)^2}\right]\left(1 - k_A\frac{\alpha_s}{\pi}\right)$$
$$= \frac{3}{4}A_{LR}\frac{\left[-\frac{1}{2}+\frac{1}{3}s_*^2(q^2)\right]^2 - \left[\frac{1}{3}s_*^2(q^2)\right]^2}{\left[-\frac{1}{2}+\frac{1}{3}s_*^2(q^2)\right]^2 + \left[\frac{1}{3}s_*^2(q^2)\right]^2}\left(1 - k_A\frac{\alpha_s}{\pi}\right)$$
$$A_{FB}^\ell = \frac{3}{4}[A_{LR}]^2$$
$$P_\tau = -\left[\frac{(g_{L*}^\tau)^2 - (g_{R*}^\tau)^2}{(g_{L*}^\tau)^2 + (g_{R*}^\tau)^2}\right] = -A_{LR}$$

Deep Inelastic Neutrino Scattering:

$$g_L^2 = \rho_*(0)^2\left[(g_{L*}^u)^2 + (g_{L*}^d)^2\right] = \rho_*(0)^2\left[\frac{1}{2} - s_*^2(0) + \frac{5}{9}s_*^4(0)\right]$$
$$g_R^2 = \rho_*(0)^2\left[(g_{R*}^u)^2 + (g_{R*}^d)^2\right] = \rho_*(0)^2\left[\frac{5}{9}s_*^4(0)\right]$$
$$R_\nu = g_L^2 + r g_R^2 = \rho_*(0)^2\left[\frac{1}{2} - s_*^2(0) + \frac{5}{9}(1+r)s_*^4(0)\right]$$
$$R_{\bar{\nu}} = g_L^2 + \frac{g_R^2}{r} = \rho_*(0)^2\left[\frac{1}{2} - s_*^2(0) + \frac{5}{9}\left(1+\frac{1}{r}\right)s_*^4(0)\right]$$

Atomic Parity Violation:

$$C_{1u} = 2\rho_*(0)[g_{L*}^e - g_{R*}^e][g_{L*}^u + g_{R*}^u] = \rho_*(0)\left[-\frac{1}{2} + \frac{4}{3}s_*^2(0)\right]$$
$$C_{1d} = 2\rho_*(0)[g_{L*}^e - g_{R*}^e][g_{L*}^d + g_{R*}^d] = \rho_*(0)\left[\frac{1}{2} - \frac{2}{3}s_*^2(0)\right]$$
$$Q_W(Z,N) = -2[(2Z+N)C_{1u} + (Z+2N)C_{1d}] = -\rho_*(0)\left[N - (1-4s_*^2(0))Z\right]$$

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APPENDIX B

The following numbers are evaluated with $m_t = 150GeV$, $m_H = 1000GeV$, $e^2 = 4\pi\alpha_{*,0}(m_Z^2) = 4\pi/129$, $s^2 = \sin^2\theta_w|Z = 0.23$, and $\alpha_s = 0.12$. The constant terms on the right hand sides are the Standard Model predictions including oblique and direct corrections, and QED and QCD corrections. They are dependent on the values of m_t and m_H while the coefficients of S, T, and U are not. For example, the standard model predictions for $\Gamma_{d\bar{d}}$ and $\Gamma_{b\bar{b}}$ differ due to the fact that $\Gamma_{b\bar{b}}$ receives an m_t dependent correction from the vertex diagrams containing the t-quark. However, the coefficients for S and T are the same because the oblique corrections are common. To evaluate R_ν and $R_{\bar{\nu}}$, we have used the CDHS[65] values of r. Other experiments should use their own measured values of r together with the formulae for g_L^2, g_R^2 below.

$$\frac{m_W}{m_Z} = 0.8787 - [3.15\times10^{-3}]S + [4.86\times10^{-3}]T + [3.70\times10^{-3}]U$$
$$\Gamma_Z = 2.484 - [9.58\times10^{-3}]S + [2.615\times10^{-2}]T \quad (\text{GeV})$$
$$\Gamma_{\ell^+\ell^-} = 0.0835 - [1.91\times10^{-4}]S + [7.83\times10^{-4}]T \quad (\text{GeV})$$
$$\Gamma_{u\bar{u}} = 0.2962 - [1.92\times10^{-3}]S + [3.67\times10^{-3}]T \quad (\text{GeV})$$
$$\Gamma_{d\bar{d}} = 0.3823 - [1.72\times10^{-3}]S + [4.20\times10^{-3}]T \quad (\text{GeV})$$
$$\Gamma_{b\bar{b}} = 0.3779 - [1.72\times10^{-3}]S + [4.20\times10^{-3}]T \quad (\text{GeV})$$
$$\Gamma_{had} = 1.7348 - [9.00\times10^{-3}]S + [1.993\times10^{-2}]T \quad (\text{GeV})$$
$$R_Z = \Gamma_{had}/\Gamma_{\ell^+\ell^-} = 20.78 - [5.99\times10^{-2}]S + [4.24\times10^{-2}]T$$
$$s_*^2(m_Z^2) = 0.2337 + [3.59\times10^{-3}]S - [2.54\times10^{-3}]T$$
$$A_{LR} = -P_\tau = 0.1297 - [2.82\times10^{-2}]S + [2.00\times10^{-2}]T$$
$$A_{FB}^b = 0.0848 - [1.97\times10^{-2}]S + [1.40\times10^{-2}]T$$
$$A_{FB}^\ell = 0.0126 - [6.72\times10^{-3}]S + [4.76\times10^{-3}]T$$
$$g_L^2 = 0.3001 - [2.67\times10^{-3}]S + [6.53\times10^{-3}]T$$
$$g_R^2 = 0.0302 + [9.17\times10^{-4}]S - [1.94\times10^{-4}]T$$
$$R_\nu = 0.3126 - [2.32\times10^{-3}]S + [6.46\times10^{-3}]T \qquad (r = 0.383)$$
$$R_{\bar{\nu}} = 0.3821 - [2.77\times10^{-3}]S + [6.03\times10^{-3}]T \qquad (r = 0.371)$$
$$Q_W(^{133}_{55}Cs) = -73.31 - 0.790S - 0.011T$$

$m_t = 150$ GeV
$m_H = 1000$ GeV
$\alpha_s = 0.12$

APPENDIX C

In this appendix, we clarify the relation of the parameters of oblique corrections defined in Section 3 to those of other authors.

First of all, we would like to clarify precisely how our formalism is a specialization of the formalism of Kennedy and Lynn.[12] As a part of their analysis, Kennedy and Lynn defined a running Fermi constant $G_{F*}(q^2)$ and a running ρ-parameter $\rho_*(q^2)$ as:

$$\frac{1}{4\sqrt{2}G_{F*}(q^2)} = \frac{v^2}{4} + [\Pi_{11}(q^2) - \Pi_{3Q}(q^2)],$$
$$\frac{1}{\rho_*(q^2)} = 1 - 4\sqrt{2}G_{F*}(q^2)\,[\Pi_{11}(q^2) - \Pi_{33}(q^2)]. \qquad (C.1)$$

These functions enable us to write

$$\frac{Z_{Z*}}{q^2 - M_{Z*}^2} = \frac{1}{q^2 - \dfrac{e_*^2}{s_*^2c_*^2}\dfrac{1}{4\sqrt{2}G_{F*}\rho_*}},$$
$$\frac{Z_{W*}}{q^2 - M_{W*}^2} = \frac{1}{q^2 - \dfrac{e_*^2}{s_*^2}\dfrac{1}{4\sqrt{2}G_{F*}}}.$$

Therefore, in the original version of the Kennedy and Lynn formalism, the effects of oblique corrections were summarized into just four starred functions: $e_*^2(q^2)$, $s_*^2(q^2)$, $G_{F*}(q^2)$, and $\rho_*(q^2)$.

Kennedy and Lynn further define:

$$\Delta_\rho(q^2) \equiv \Pi_{11}(q^2) - \Pi_{33}(q^2),$$
$$\Delta_1(q^2) \equiv -[\Pi_{11}(q^2) - \Pi_{11}(0) - \Pi_{3Q}(q^2)], \qquad (C.2)$$
$$\Delta_3(q^2) \equiv -[\Pi_{33}(q^2) - \Pi_{33}(0) - \Pi_{3Q}(q^2)]$$

These Δ's determine the running of $\rho_*(q^2)$, $G_{F*}(q^2)$, and $G_{F*}(q^2)\rho_*(q^2)$ in the following way:

$$\frac{1}{\rho_*(q^2)} = 1 - 4\sqrt{2}G_{F*}(q^2)\Delta_\rho(q^2),$$
$$\frac{1}{4\sqrt{2}G_{F*}(q^2)} = \frac{1}{4\sqrt{2}G_F} - \Delta_1(q^2), \qquad (C.3)$$
$$\frac{1}{4\sqrt{2}G_{F*}(q^2)\rho_*(q^2)} = \frac{1}{4\sqrt{2}G_F\rho_*(0)} - \Delta_3(q^2).$$

- One can also express (m_t, m_H) dependence by $\delta(S), \delta(T), \delta(U)$

<u>Higgs</u>

$$\delta S \simeq \frac{1}{12\pi} \ln\left(\frac{m_H^2}{m_{H,ref}^2}\right)$$

$$\delta T = \frac{3}{16\pi c_W^2} \ln\left(\frac{m_H^2}{m_{H,ref}^2}\right)$$

$$\delta U = 0$$

<u>Top</u>

$$\delta S \approx \frac{1}{6\pi} \ln\left(\frac{m_t^2}{m_{t,ref}^2}\right)$$

$$\delta T \approx \frac{3}{16\pi s_W^2 c_W^2}\left(\frac{m_t^2 - m_{t,ref}^2}{m_Z^2}\right)$$

$$\delta U \approx \frac{1}{2\pi} \ln\left(m_t^2 / m_{t,ref}^2\right)$$

In general $\delta U \ll (\delta S, \delta T)$

Relations between ε_i + (S, T, U) Variables

$$\left(1 - \frac{m_W^2}{m_Z^2}\right)\frac{m_W^2}{m_Z^2} = \frac{\pi\alpha(m_Z)}{\sqrt{2} G_F m_Z^2 (1-\Delta r_W)}$$

Here $\alpha(m_Z) = \alpha/(1-\Delta\alpha)$

$$(1-\Delta r)_W = (1-\Delta\alpha)(1-\Delta r_W).$$

$$\boxed{g_A = -\frac{\sqrt{\rho}}{2} = -\frac{1}{2}\left(1 + \frac{\Delta\rho}{2}\right) \text{ and } \frac{g_V}{g_A} = 1 - 4\bar{s}_W^2 = 1 - 4(1 + \Delta k')\, s_0^2}$$

$\Rightarrow$ <u>$\Delta\rho, \Delta r, \Delta k'$</u>

Altarelli, Barbieri, Jadach

$$\boxed{\begin{aligned} \varepsilon_1 &= \Delta\rho \\ \varepsilon_2 &= c_0^2 \Delta\rho + \frac{s_0^2 \Delta r_W}{(c_0^2 - s_0^2)} - 2 s_0^2 \Delta k' \\ \varepsilon_3 &= c_0^2 \Delta\rho + (c_0^2 - s_0^2)\Delta k' \end{aligned}}$$

- s_0, c_0 defined by $s_0^2 c_0^2 = \dfrac{\pi\alpha(m_Z)}{\sqrt{2} G_F m_Z^2}$ with $\alpha(m_Z) = 1/128.$
  $c_0^2 = 0.2314$
  fn $m_Z = 91.175$ GeV

$$\boxed{\begin{aligned} \Delta\varepsilon_1 &= \alpha T, \\ \Delta\varepsilon_2 &= -\alpha U/(4 s_0^2) \\ \Delta\varepsilon_3 &= \alpha S/(4 s_0^2), \end{aligned}}$$

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(Moriond '92)

Values of the asymmetries at $\sqrt{s} = m_Z$.

J. Nash
(Moriond '92)

$$\boxed{\begin{aligned} A_{FB}^{l} &= (1.74 \pm 0.38)\, 10^{-2}. \\ A_{pol}^{\tau} &= 0.140 \pm 0.024 \\ A_{FB}^{b} &= 0.094 \pm 0.014 \text{ (corrected for } B\bar{B} \text{ mixing)} \end{aligned}}$$

Given the present values of m_W/m_Z, Γ_l and A_{FB}^{l} we obtain:

Altarelli
(Moriond '92)

$$\varepsilon_1 = \Delta\rho = (0.15 \pm 0.41)\, 10^{-2}$$
$$\varepsilon_2 = (-0.71 \pm 0.83)\, 10^{-2}$$
$$\varepsilon_3 = (-0.02 \pm 0.56)\, 10^{-2}$$

Essentially one only needs to assume charged lepton universality in order to utilize the information on A_{pol}^{τ}.

$+ A_{Pol.}^{\tau} \Longrightarrow$

$$\varepsilon_1 = \Delta\rho = (0.16 \pm 0.41)\, 10^{-2}$$
$$\varepsilon_2 = (-0.72 \pm 0.82)\, 10^{-2}$$
$$\varepsilon_3 = (-0.01 \pm 0.51)\, 10^{-2}$$

QCD effect in A_{FB}^{b}: $(A_{FB}^{b})_{meas} = (A_{FB}^{b})_{SM}(1 - 0.79\, \alpha_s(m_Z)/\pi) \simeq 0.97\, (A_{FB}^{b})_{SM}$

$+ A_{FB}^{b} \Longrightarrow$

$$\varepsilon_1 = \Delta\rho = (0.16 \pm 0.40)\, 10^{-2}$$
$$\varepsilon_2 = (-0.73 \pm 0.82)\, 10^{-2}$$
$$\varepsilon_3 = (0.02 \pm 0.48)\, 10^{-2}$$

ALL LEP
+ Low Energy
data

$$\varepsilon_1 = \Delta\rho = (0.16 \pm 0.32)\, 10^{-2}$$
$$\varepsilon_2 = (-0.72 \pm 0.7\ldots)\, 10^{-2}$$

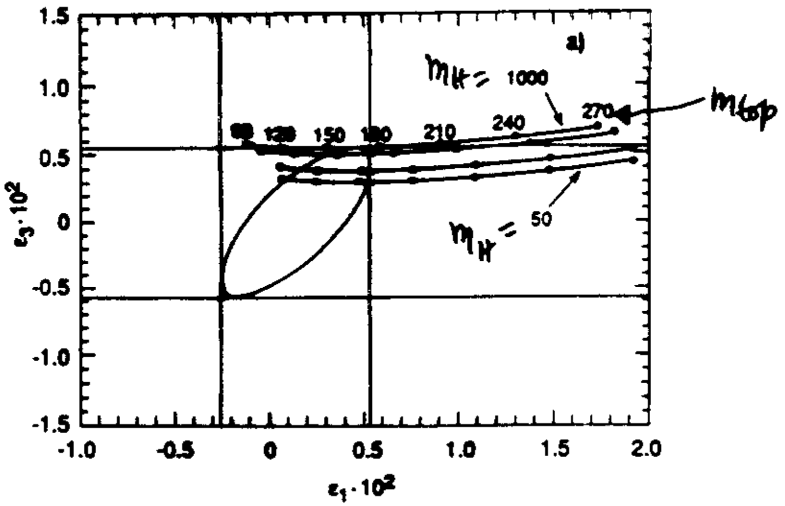

Figure 3a

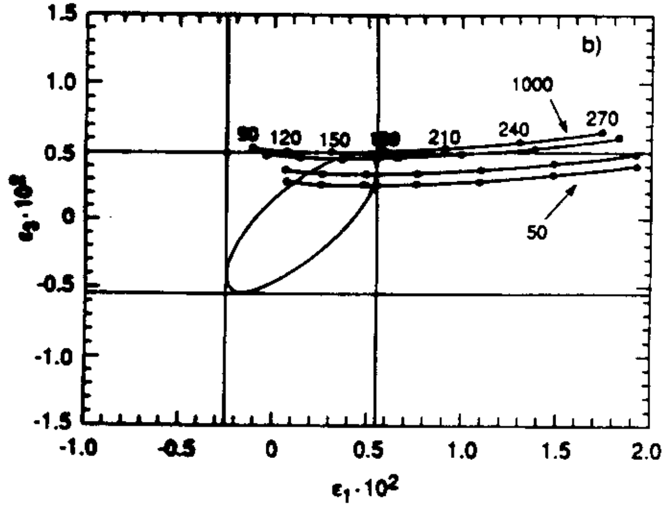

Figure 3b

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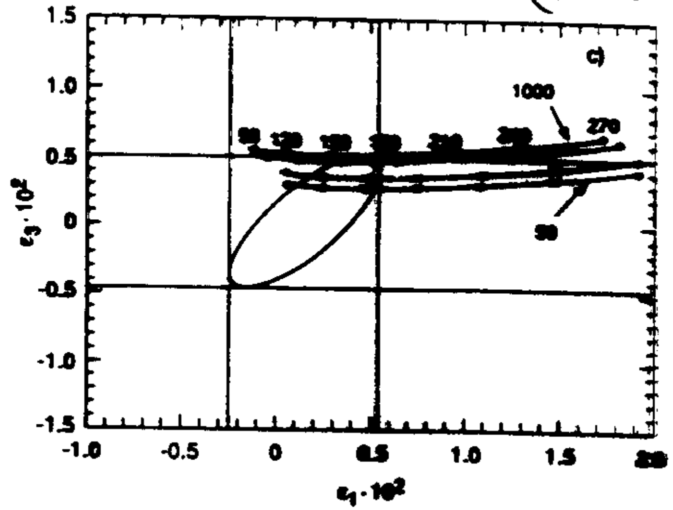


Figure 3c

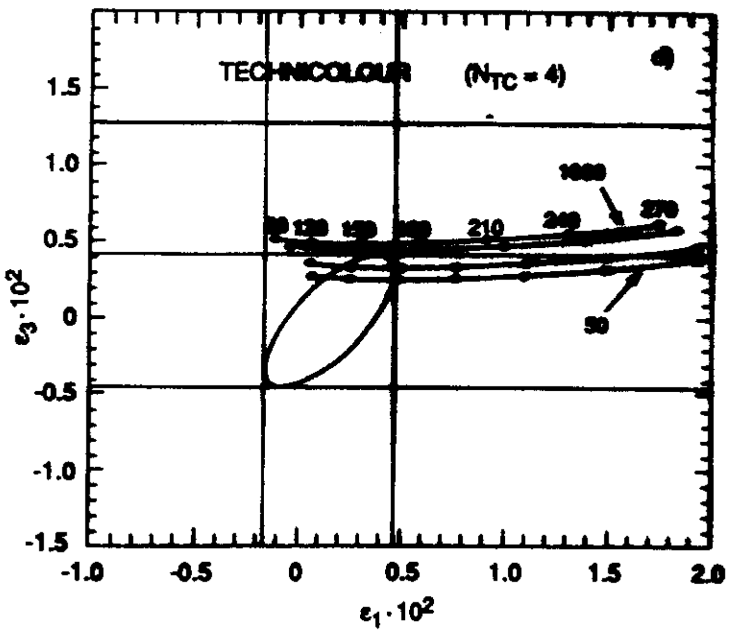
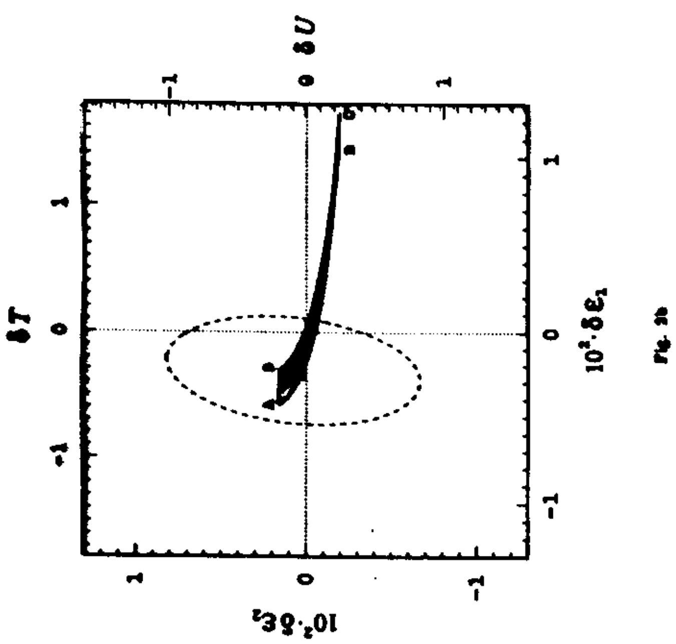
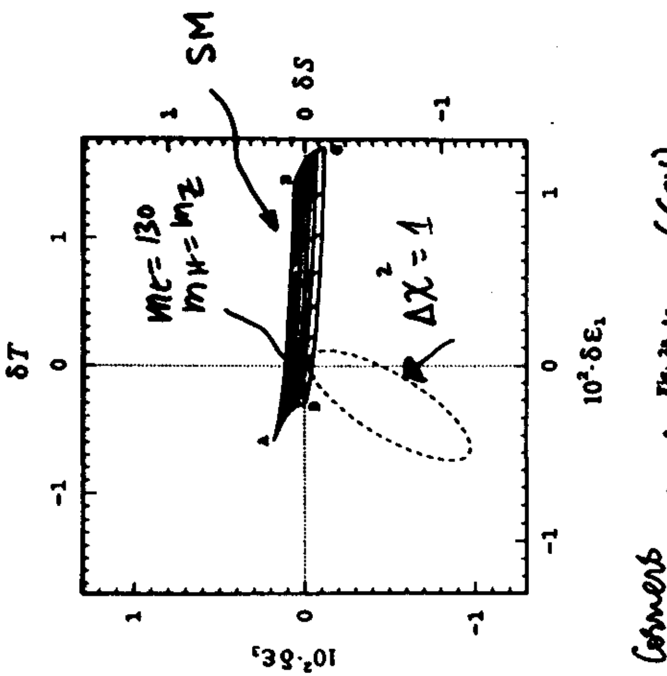


Figure 3d

J. Ellis, G. Fogli, E. Lisi
CERN-TH. 96/92

$\varepsilon_i\ (\equiv (S, T, U))$ analysis





1 Corners

	m_H (GeV)	m_t (GeV)
A =	1000	90
B =	1000	250
C =	50	250
D =	50	90

(Ellis, Fogli, Lisi)

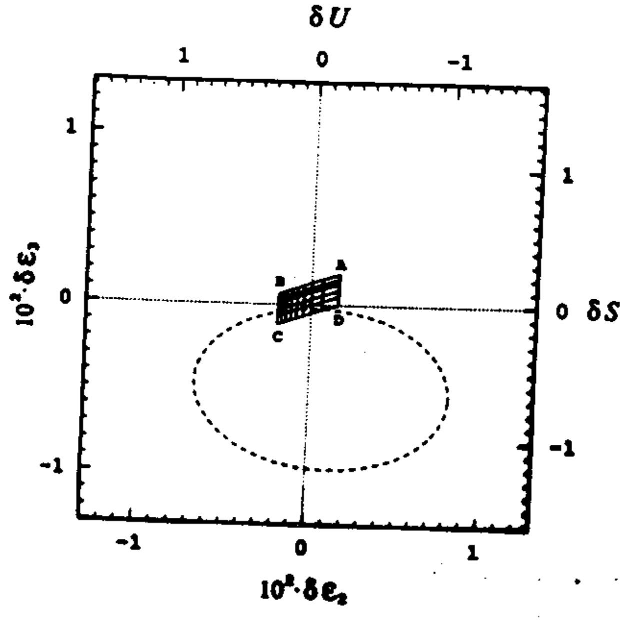


Fig. 2c

Fig. 2 — Projections of Fig. 1 onto (a) the $(\delta\epsilon_1, \delta\epsilon_3)$ or $(\delta S, \delta T)$ plane, (b) the $(\delta\epsilon_1, \delta\epsilon_2)$ or $(\delta T, \delta U)$ plane, and (c) the $(\delta\epsilon_2, \delta\epsilon_3)$ or $(\delta S, \delta U)$ plane. Again, the reference choices $(m_t, M_H) = (130\ \text{GeV}, M_Z)$ have been made. The Standard Model surface of Fig. 1 is bounded by the extreme points A, B, C, D, whose reference values in terms of (m_t, M_H) are (masses in GeV): $A = (90, 1000)$, $B = (250, 1000)$, $C = (250, 50)$, $D = (90, 50)$. The intermediate points correspond to $m_t = 110, 130, \ldots, 210, 230$ GeV and $M_H = M_Z, 100, 300, 500$ GeV.

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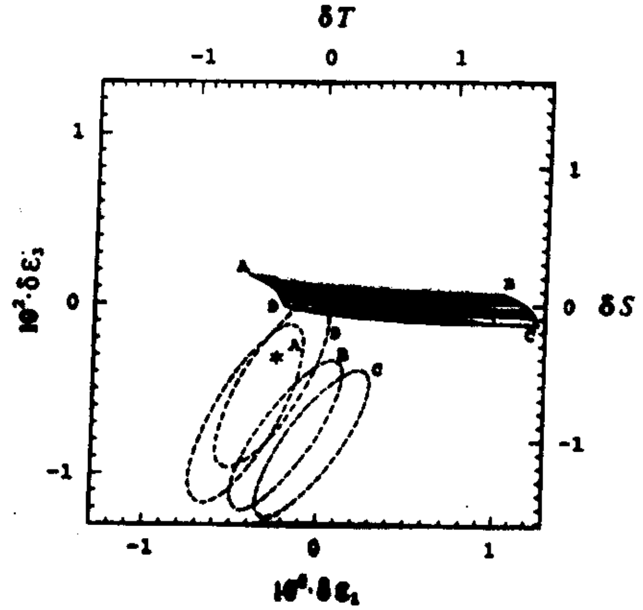

Fig. 3 — As for Fig. 2a, but for the different choices of the reference values of (m_t, M_H) corresponding to the points A, B, C, D of fig. 2. In each case the origin has been shifted to the Standard Model $(m_t, M_H) = (130\ \text{GeV}, M_Z)$ point. The bounds on the $\delta\epsilon_i|_{\text{new}}$ for each of the cases A, B, C, D are given by the differences between the corresponding ellipses and Standard Model points. The star (*) marks the reference point used in ref. [12] to define $(\delta\epsilon_1, \delta\epsilon_3)$.

Deviations from SM

(J. Ellis, Fogli, Lisi

$\epsilon_1 = \alpha T = -(0.23 \pm 0.32) \times 10^{-2}$

$\epsilon_2 = -\alpha U / 4 s_W^2 = +(0.06 \pm 0.74) 10^{-2}$

$\epsilon_3 = -(0.50 \pm 0.47) \times 10^{-2}$

$= \dfrac{\alpha S}{4 s_W^2}$ $\left(\begin{array}{c}\text{Notation} \\ s_W^2 = s_0^2 = 0.2319 \\ c_W^2 = c_0^2\end{array}\right)$

lculated for the ref.

lue $(m_t, m_H) = (130, M_Z)$

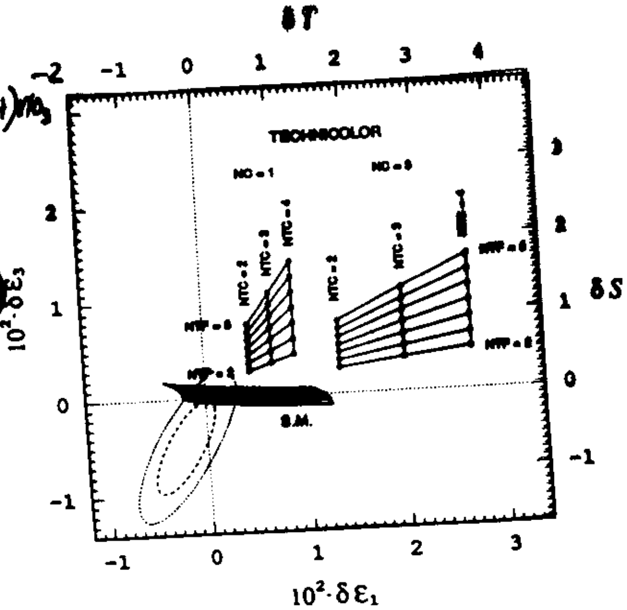

Fig. 4 — The predictions of various technicolour models [14], [15], compared with the $\Delta\chi^2 = 1, 2.7$ ellipses and the Standard Model values taken from Fig. 2a, for the reference choices $(m_t, M_H) = (130\ \text{GeV}, M_Z)$.

ontributions in TC Models

$\delta S_{TC} \simeq 0.05\, N_{TF} N_{TC} + 0.127$

> 0 (data favours $S \lesssim 0$)

$\delta T_{TC} \simeq N_C N_{TC} \dfrac{1}{12\pi s_W^2 c_W^2} \left(\dfrac{m_t}{m_Z}\right)^2 \simeq 0.15\, N_C N_{TC} \left(\dfrac{m_t^2}{m_Z^2}\right)$

(based on Peskin, Takeuchi)

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TC Models less favoured by d.L

Extended Gauge Models

L-R Symmetric Models

Pati, Salam; Mohapatra, Pati; Senjanovic; ......

$$SU(2)_L \otimes SU(2)_R \otimes U(1)$$

$$\boxed{\mathcal{L}_{NC} = ig\, J^{\mu}_{3L} W_{3L,\mu} + i\, g/\lambda\, J^{\mu}_{3R} W_{3R,\mu} + ig' J^{\mu}_{B-L} B}$$

$$J^{\mu}_{3L,3R} = \bar{\psi}\gamma^{\mu} T_{3L,3R}\psi$$

$$J^{\mu}_{B-L} = \bar{\psi}\gamma^{\mu}\frac{B-L}{2}\psi$$

$$B-L = Q - T_{3L} - T_{3R}$$

$$(\lambda = g_L/g_R\ ,\ g \equiv g_L)$$

Electromagnetic Field

$$A = \sin\theta\, W_L^3 + \lambda\sin\theta\, W_R^3 + yB$$

$$y = (\cos^2\theta - \lambda^2\sin^2\theta)^{1/2}\ ;\ g\sin\theta = g'y = e$$

$$\begin{pmatrix} A \\ Z_{0R} \\ Z_{0L} \end{pmatrix} = \begin{pmatrix} \sin\theta & \lambda\sin\theta & y \\ 0 & \frac{y}{\cos\theta} & -\lambda\tan\theta \\ \cos\theta & -\lambda\sin\theta\tan\theta & -y\tan\theta \end{pmatrix}\begin{pmatrix} W_L^3 \\ W_R^3 \\ B \end{pmatrix}$$

$$\Rightarrow \mathcal{L}_{NC} = ieQA_{\mu}J^{\mu,em} + \frac{ieZ_{0L,\mu}}{\sin\theta\cos\theta}(J^{\mu}_{3L} - \sin^2\theta J^{\mu}_{em}) + ig' Z_{0R}(y J^{\mu}_{3R} - \lambda^2\sin^2\theta J^{\mu}_{B-L})$$

L-R Symmetric Models

$$g_L = g_R \Rightarrow \lambda = 1\ ;\ y = (\cos^2\theta - \sin^2\theta)^{1/2}$$

$$\Rightarrow$$

$$\boxed{\begin{aligned}\mathcal{L}_{NC} = ieQJ^{em}_{\mu}A^{\mu} &+ \frac{ie\, Z^{\mu}_{0L}}{\sin\theta\cos\theta}(J_{\mu,3L} - \sin^2\theta J_{\mu}^{em}) \\ &+ \frac{ie}{(\cos^2\theta - \sin^2\theta)^{1/2}} Z^{\mu}_{0R}\left((\cos^2\theta - \sin^2\theta)J_{\mu,3R} - \sin^2\theta J_{\mu,B-L}\right)\end{aligned}}$$

- Z^{μ}_{0L}, Z^{μ}_{0R} are Current Eigenstates
- Mass Eigenstates $\equiv Z, Z'$

$$\begin{pmatrix} Z \\ Z' \end{pmatrix} = \begin{pmatrix} \cos\xi_0 & \sin\xi_0 \\ -\sin\xi_0 & \cos\xi_0 \end{pmatrix}\begin{pmatrix} Z_{0L} \\ Z_{0R} \end{pmatrix}$$

- ignoring mixing in $W^{\pm}_{L,R}$

$$\Rightarrow \boxed{\frac{m_W^2}{m_Z^2\cos^2\theta} = \rho_0} \Rightarrow 2 \text{ new Parameters } (m_{Z'}, \xi_0)$$

$$\rho_0 = 1 + \Delta\rho_M + \Delta\rho_{SB}$$

with $\Delta\rho_M = \left[(m^2_{Z'}/m^2_Z)^2 - 1\right]\sin^2\xi_0 > 0$

G. Alt[illegible]

CERN-TH-[illegible]

Limits on ξ_0 and $m_{Z'}$ from LEP

$$\Gamma(Z \to f\bar{f}) = \frac{G_F m_Z^3}{6\pi\sqrt{2}} \rho N_c \left[\tilde{v}_f^2 + \tilde{a}_f^2\right]$$

$$A_{FB} = 3 \frac{(\tilde{v}^e \tilde{a}^e)}{(\tilde{v}^e)^2 + (\tilde{a}^e)^2} \frac{(\tilde{v}^f \tilde{a}^f)}{(\tilde{v}^f)^2 + (\tilde{a}^f)^2}$$

$$\rho = 1 + \Delta\rho_M + \Delta\rho_{SB} + \Delta\rho_t$$

$$\Delta\rho_t = \frac{3 G_F m_t^2}{8\pi^2\sqrt{2}} + \cdots$$

$$\Delta\rho_{SB} = \frac{\alpha}{16\pi s_W^2} \frac{11}{3} \left(\ln m_H^2/m_t^2 - 5/6\right) + \cdots$$

$$\tilde{v}^f = \cos\xi_0 \, v_S^f + \sin\xi_0 \, v_N^f$$

$$\tilde{a}^f = \cos\xi_0 \, a_S^f + \sin\xi_0 \, a_N^f$$

$$v_S^f = T_{3L}^f - 2\tilde{s}_W^2 Q_f$$

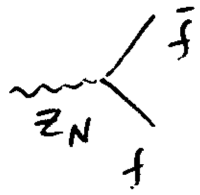
$$a_S^f = -T_{3L}^f$$

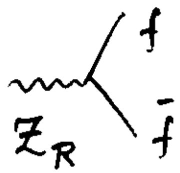
with $\tilde{s}_W^2 \tilde{c}_W^2 = \dfrac{\pi\alpha(m_Z)}{\sqrt{2} G_F m_Z^2 \rho}$

$$\alpha(m_Z) = \frac{\alpha}{\ldots} = 1/128.8$$

Table 1

Vector and axial couplings of fermions to the unmixed new vector boson Z_N in the case of extra U(1) and LR models. For left-right models, we have put in the text $\lambda = g_L/g_R = 1$. As a consequence $y = \sqrt{\cos 2\theta}$.





Extra U(1) Models
$v_N^u = 0,$
$a_N^u = -\frac{2}{3}\sin\theta\cos\theta_2,$
$v_N^d = \frac{1}{2}\sin\theta(\cos\theta_2 + \sqrt{\frac{5}{3}}\sin\theta_2)$
$a_N^d = \sin\theta(-\frac{1}{6}\cos\theta_2 + \frac{1}{2}\sqrt{\frac{5}{3}}\sin\theta_2),$
$v_N^e = -\frac{1}{2}\sin\theta(\cos\theta_2 + \sqrt{\frac{5}{3}}\sin\theta_2),$
$a_N^e = a_N^d,$
$v_N^\nu = -\sin\theta(\frac{1}{6}\cos\theta_2 + \frac{1}{2}\sqrt{\frac{5}{3}}\sin\theta_2)$
$a_N^\nu = -v_N^\nu.$
Left-Right Models
$v_N^f = \left[\frac{\cos^2\theta}{\lambda y}(T_{3R} - 2\lambda^2\sin^2\theta Q_{em}) + \frac{\lambda\sin^2\theta}{y}(T_{3L} - 2\sin^2\theta Q_{em}\right],$
$a_N^f = \left[\frac{\cos^2\theta}{\lambda y}T_{3R} - \frac{\lambda\sin^2\theta}{y}T_{3L}\right]$

$$\tilde{S}_W^2 = S_W^2 - \frac{S_W^2 C_W^2}{(C_W^2 - S_W^2)} \Delta\rho_M$$

- [For $\Gamma(Z^0 \to b\bar{b})$ + $A_{FB}(b)$

$1 + \Delta\rho_t \longrightarrow 1 - \frac{1}{3}\Delta\rho_t$

$\tilde{S}_W^2 \longrightarrow \tilde{S}_W^2 (1 + \frac{2}{3}\Delta\rho_t)$]

$\Rightarrow$ Shifts in $\Gamma(Z^0 \to f\bar{f})$ + $a_{FB}(f)$, ...]

e.g.

$$\boxed{\delta\Gamma_f = \frac{G_F m_Z^3}{24\pi\sqrt{2}} (A_f \Delta\rho_M + B_f \xi_0)}$$

$A_f = A_f(v_s^f, a_s^f)$

$B_f = B_f(v_N^f, a_N^f)$

$\Rightarrow$ 2 additional parameters

$\Delta\rho_M$, ξ_0

- LEP data $\Rightarrow$ $\boxed{\begin{matrix}\Delta\rho_M < 0.018 \\ \xi_0 < 0.01\end{matrix}}$

- Minimal L-R Sym. models
 250 (800) GeV

Extra $U(1)_{Y'}$ embedded in High. Group

Theories with Extra U(1) — Langacker, Luo; Langacker; Altarelli et al.

$SO(10) \to SU(5) \times U(1)_\chi$

$E_6 \to SO(10) \times U(1)_\psi$

$E_6 \to SU(3) \times SU(2) \times U(1) \times U(1)_\eta$

$$\mathcal{L}_{NC} = i g W_{3L} J_{3L} + i g' B J_Y + i g'_{\hat{Y}} Z_0 J_{\hat{Y}}$$

$$J_{\hat{Y}} = J_{Y'} \cos\theta_2 - J_{Y''} \sin\theta_2$$

$\Rightarrow$ v_N^f and a_N^f functions of θ_2

LEP data $\Rightarrow$ Constraints on ξ_0, $\Delta\rho_M$ as a function of θ_2.

$\Rightarrow$ $\boxed{\begin{matrix}\Delta\rho_M < 0.01 \\ \xi_0 < 0.02\end{matrix}}$ for all values of θ_2

G. Altarelli
(Moriond '92)

$M_t = M_H = 100\,GeV$
$\alpha_s = 0.12$

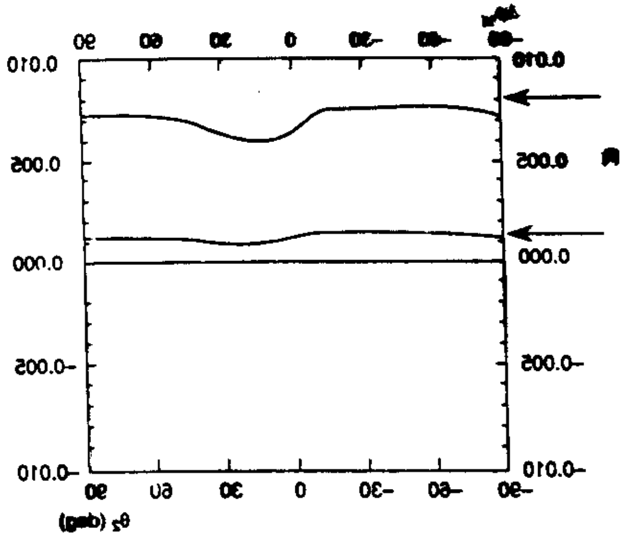


Figure 1

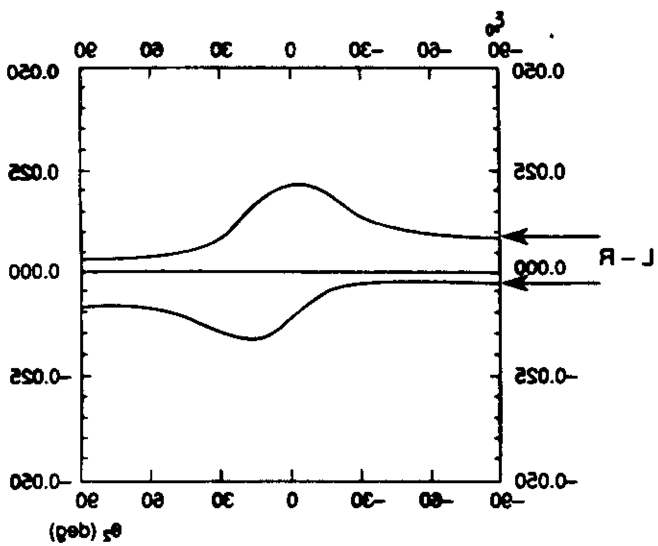


Figure 2

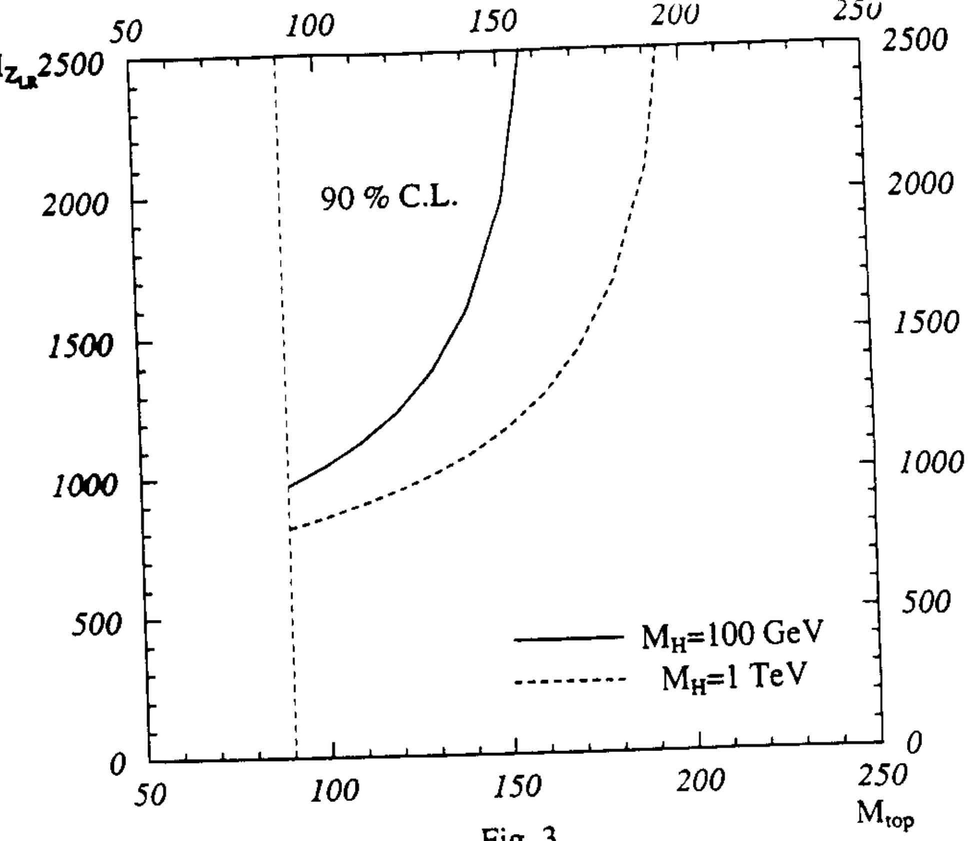


Fig. 3

G. Altarelli et al.
CERN-TH-6051/91

Langacker

	Z_χ	Z_ψ	Z_η	Z_{LR}	Z'
ρ_0 = free	322(343)	158(172)	181(194)	389(419)	756(824)
$\rho_0 = 1$	321(340)	160(174)	182(195)	389(421)	779(855)
$\sigma = 0$	553(592)	585(624)	211(221)	857(914)	-
$\sigma = 1$	553(592)	163(178)	483(531)	857(914)	-
$\sigma = 5$	553(592)	716(785)	773(842)	857(914)	-
$\sigma = \infty$	553(592)	912(1000)	923(1004)	857(914)	-
15-plet	273	250	266	299	335
16-plet	248	246	236	281	-
27-plet	224	160	182	-	-

Table 9: Lower limits on the mass M_2 of an extra Z boson for the $Z_\chi, Z_\psi, Z_\eta, Z_{LR}$, and Z' models. The indirect limits are for the ρ_0 =free, $\rho_0 = 1$, and for minimal Higgs models with $\sigma = 0, 1, 5, \infty$. Both the 95% CL and 90% (in parentheses) CL limits are given. In all cases, $m_t > 89$ GeV and $\sin^2\hat{\theta}_W(M_Z)$ are free parameters. The direct limits are 95% CL based on the CDF result [50], assuming $B(Z_2 \to e^+e^-)$ for models with three 15-plets, 16-plets, and 27-plets of fermions light enough for the Z_2 to decay into. The 27-plet limits for the Z_χ and Z_η assume that the $Z_2 \to t\bar{t}$ channel is closed. From [12].

	Z_χ	Z_ψ	Z_η	Z_{LR}	Z'
ρ_0 = free, θ	0.0012 (50)	0.0040(58)	−0.021(12)	0.0018(41)	−0.0038(29)
θ_{min}	−0.0070	−0.0060	−0.038	−0.0048	−0.0067
θ_{max}	+0.0094	+0.012	+0.002	+0.0079	+0.0020
$\rho_0 = 1, \theta$	0.0019(44)	0.0047(47)	−0.021(11)	0.0024(32)	−0.0036(32)
θ_{min}	−0.0048	−0.0025	−0.038	−0.0025	−0.0086
θ_{max}	+0.0097	+0.013	−0.002	+0.0083	+0.0005

Table 10: Best fit values and 95% CL upper (θ_{max}) and lower (θ_{min}) limits on the mixing angle θ for the ρ_0 = free and $\rho_0 = 1$ models. The numbers in parentheses are the uncertainties in the best fit values. From [12].

	Z_χ	Z_ψ	Z_η	Z_{LR}	Z'	$SU_2 \times U_1$
ρ_0 = free	300(282)	307(290)	294(275)	309(292)	307(290)	310(294)
$\rho_0 = 1$	182(174)	182(172)	167(156)	181(172)	183(175)	182(174)

Table 11: 95 (90)% CL upper limits on m_t for the $Z_\chi, Z_\psi, Z_\eta, Z_{LR}, Z'$ models for the cases ρ_0 = free and $\rho_0 = 1$, compared to the limits obtained assuming $SU_2 \times U_1$. The direct CDF limit $m_t > 89$ GeV is incorporated in the analysis. The limits in the minimal Higgs models are similar to the $\rho_0 = 1$ case. From [12].