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(Lect. II & III)

SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

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NEUTRINO PHYSICS

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Please note: These are preliminary notes intended for internal distribution only.

Models / Properties

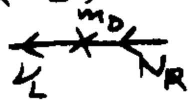
- Weyl Neutrino
  2 components: $\nu_L \underset{CP}{\leftrightarrow} \nu_R^c \equiv C\,\bar{\nu}_L^T$ $[i\gamma^2\gamma^0$ Weyl
- Active (SU_2-doublet), sterile (SU_2-singlet)
  [normal weak ints] [only via mixing]

$$\begin{pmatrix}\nu_e \\ e^-\end{pmatrix}_L \underset{CP}{\longleftrightarrow} \begin{pmatrix} e^+ \\ \nu_e^c \end{pmatrix}_R \qquad N_R \leftrightarrow N_L^c$$

- SM: $\nu_{eL}, \nu_{\mu L}, \nu_{\tau L}$ (active)
  - no mass w/o extra Higgs/Weyl ν's
  - L_e, L_μ, L_τ conserved

- Dirac Mass (conserves L)
  2 Weyl neutrinos $\quad \nu_L \underset{CP}{\leftrightarrow} \nu_R^c$
  $(N_R \neq \nu_R^c)$ $\quad N_R \underset{CP}{\leftrightarrow} N_L^c$

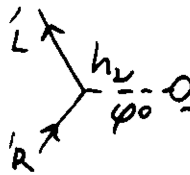
$$-\mathcal{L}_{Dirac} = m_D(\bar{\nu}_L N_R + \bar{N}_R \nu_L)$$
$$= m_D \bar{\nu}\nu$$
$$(\nu \equiv \nu_L + N_R)$$



4 components: $\nu_L, \nu_R^c, N_R, N_L^c$
$L = L_\nu + L_N$ conserved

Normal Dirac: ν_L = active, N_R = sterile

$\Delta I = \frac{1}{2} \Rightarrow$ Higgs doublet ν's



$v = \sqrt{2}\langle\varphi^0\rangle$
$= (\sqrt{2}G_F)^{-1/2}$
$= 246$ GeV

$m_D = h_\nu v$

$m_{\nu_e} < 10\,\text{eV} \Rightarrow h_{\nu_e} < 10^{-10}$

why is h so small?
- $h_\nu = 0$ at tree level?
  (no compelling models)

Weyl

$P_L \nu_L = \nu_L \qquad P_R \nu_R^c = \nu_R^c \qquad P_{L,R} = \frac{1 \mp \gamma_5}{2}$

- L, R have definite chirality (χ)
  $\chi \sim$ helicity $+ O(\frac{m}{E})$

Free Field:
$$\nu_L(x) = \sum_{\vec{p}} [b_L(\vec{p}) u_L(\vec{p}) e^{-ip\cdot x} + d_R^\dagger(\vec{p}) v_R(\vec{p}) e^{+ip\cdot x}]$$
$\sum_{\vec{p}} \to \int \frac{d^3p}{\sqrt{(2\pi)^3\, 2E_p}}$ $\qquad$ [no spin sum]

Dirac

$\nu = \nu_L + N_R \qquad P_L \nu = \nu_L$
$\qquad\qquad\qquad\quad P_R \nu = N_R$

Free Field:
$$\nu(x) = \sum_{\vec{p}} \sum_{s=L,R} [b_s(\vec{p}) u_s(\vec{p}) e^{-ip\cdot x} + d_s^\dagger(\vec{p}) v_s(\vec{p}) e^{+ip\cdot x}]$$

Majorana

$\nu = \nu_L + \nu_R^c \qquad P_L \nu = \nu_L$
$\qquad\qquad\qquad\quad P_R \nu = \nu_R^c$

Free Field:
$$\nu(x) = \sum_{\vec{p}} \sum_{s=L,R} [b_s(\vec{p}) u_s(\vec{p}) e^{-ip\cdot x} + b_s^\dagger(\vec{p}) v_s(\vec{p}) e^{+ip\cdot x}]$$
not $d_s^\dagger$

ZKM Dirac: Zeldovich, Konopinski, Mahmoud

ν_L, N_R both active

$N_{eR} = \nu^c_{\tau R} \Rightarrow L_e - L_\tau$ conserved

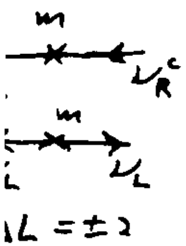
$\nu_{eL}, \nu^c_{eR}, \nu^c_{\tau R}, \nu_{\tau L}$

$N_{eR} = \nu^c_{\mu R} \Rightarrow L_e - L_\mu$ conserved

- need new Higgs
- limiting case of Majorana

Majorana Mass $(\Delta L = \pm 2)$

$N_R = \nu^c_R$

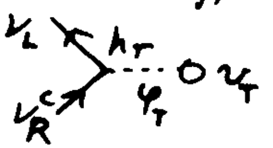


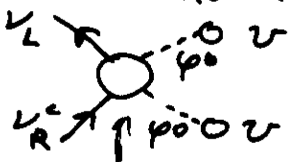
$$-\mathcal{L}_{majorana} = \frac{1}{2} m(\bar\nu_L \nu^c_R + \bar\nu^c_R \nu_L) = \frac{1}{2} m(\bar\nu_L C \bar\nu_L^T + H.c.) = \frac{1}{2} m \bar\nu\nu$$

$(\nu = \nu_L + \nu^c_R, \quad \nu = \nu^c = C\bar\nu^T)$

2 components: ν_L, ν^c_R

ctive: $\Delta I = 1 \Rightarrow$ Higgs Triplet or effective op.





$m \sim h_T v_T \qquad m \sim \frac{c v^2}{M}$

sterile: $\Delta I = 0 \Rightarrow$ bare mass, Higgs singlet, ...

- no distinction between Dirac & Majorana for $m \to 0$ (unless new interactions)
- both reduce to Weyl (N_R decouples)

irac ν: $\Delta L = 0$

lajorana ν: - $\Delta L = \pm 2$

- however, for $m_\nu \ll E_\nu$, approximate helicity conservation (of V, A interactions) plays role of L conservation

$\Rightarrow \Delta L \neq 0$ amplitude of $O\left(\frac{m_\nu}{E_\nu}\right)$

$\Rightarrow$ no difference between Dirac & Majorana for $m_\nu = 0$

example

$\pi^+ \to e^+ \nu_e \qquad \mathcal{L} \sim \frac{G_F}{\sqrt{2}} J^\mu_{had} \bar\nu \gamma_\mu (1-\gamma_5) e^-$

$\Rightarrow \nu_L = \frac{1-\gamma_5}{2}\nu$ produced ($x =$

$h = -1$: amplitude $= O(1)$

$h = +1$: amplitude $= O\left(\frac{m_\nu}{E_\nu}\right)$

$\nu_e p \to e^+ n \qquad \mathcal{L} \sim \frac{G_F}{\sqrt{2}} J^\mu_{had} \bar\nu \gamma_\mu (1-\gamma_5) e^-$

$= -\frac{G_F}{\sqrt{2}} J^\mu_{had} e^+ \gamma_\mu (1+\gamma_5) \nu$

$\nu^c_R = \frac{1+\gamma_5}{2}\nu$ absorbed ($x = +1$)

$\Rightarrow h = -1$: amplitude $= O\left(\frac{m_\nu}{E_\nu}\right)$

$h = +1$: amplitude $= O(1)$

Mixed Models

$$-\mathcal{L} = \frac{1}{2}\left(\bar{\nu}_L \ \ \bar{N}_L^c\right)\begin{pmatrix} m_T & m_D \\ m_D & m_S \end{pmatrix}\begin{pmatrix} \nu_R^c \\ N_R \end{pmatrix} + H.c.$$

m_T, m_S = Majorana masses $\quad \nu_L \leftrightarrow \nu_R^c$

m_D = Dirac mass $\quad N_L^c \leftrightarrow N_R$

- diagonalize 2×2 matrix $\Rightarrow$ 2 Majorana mass eigenstates $n_i = n_{iL} + n_{iR}^c$, $i=1,2$

$n_{iL} \leftrightarrow n_{iR}^c$

Special cases:

(a) $m_D = 0$ (Majorana)

$\Rightarrow n_{1L} = \nu_L, \ n_{2L} = N_L^c$

(b) $m_T = m_S = 0$ (Dirac)

degenerate Majorana ν's (mass m_D)

$$n_1 = \frac{1}{\sqrt{2}}(\nu_L + N_L^c) + \frac{1}{\sqrt{2}}(\nu_R^c + N_R)$$

$$n_2 = \frac{1}{\sqrt{2}}(\nu_L - N_L^c) - \frac{1}{\sqrt{2}}(\nu_R^c - N_R)$$

- can define $\nu = \frac{1}{\sqrt{2}}(n_1 + n_2) = \nu_L + N_R$

$\nu^c = \frac{1}{\sqrt{2}}(n_1 - n_2) = N_L^c + \nu_R^c$

$$\Rightarrow -\mathcal{L} = \frac{1}{2} m_D\left[\bar{n}_{1L} n_{1R}^c + \bar{n}_{2L} n_{2R}^c\right] + H.c.$$

$= m_D \bar{\nu}\nu$ [conserved L]

$\Rightarrow$ Dirac ν is a pair of degenerate Majorana ν's n_1, n_2 with $45°$ mixing

(c) $m_T \ll m_D$, $m_S \ll m_D$ (Pseudo-Dirac)

2 almost degenerate ν's with small breaking of L

example: $m_S = 0$, $m_T \ll m_D \Rightarrow m_\pm \simeq m_D \pm m_T/2$

$\Rightarrow n_+ = n_1 + \frac{m_T}{4m_D} n_2$, $n_- = -\frac{m_T}{4m_D} n_1 + n_2$

(d) $m_T = 0$, $m_D \ll m_S$ (seesaw)

$\nu_{1L} \simeq \nu_L - \frac{m_D}{m_S} N_L^c$, $\quad m_1 \sim \frac{m_D^2}{m_S} \ll m_D$

[illegible]

Laboratory / Experimental Constraints

- Neutrino Counting
- Kinematic limits
- Neutrinoless Double Beta Decay ($\beta\beta_{0\nu}$)
- Neutrino Oscillations
- Incoherent Mixing with Heavy ν's
- Cosmology / Astrophysics

Neutrino Counting

$N_\nu \geq 2$ direct (ν_e, ν_μ)

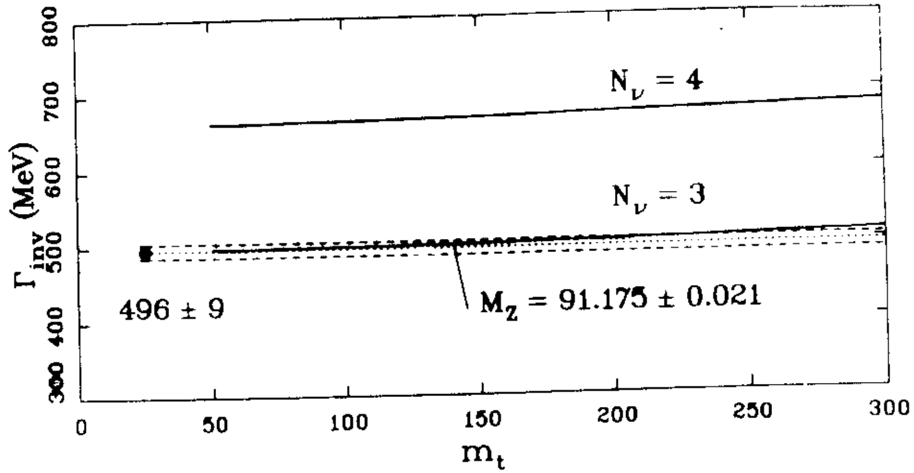
$N_\nu \geq 3$ interactions of τ

$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \begin{pmatrix} \nu_\mu \\ \mu^-_L \end{pmatrix} \tau^-_L$ Γ_τ suppressed

- Full strength decay
- $e^+e^- \to \tau^+\tau^-$ FB asym
- $\Gamma_{Z\to\tau^+\tau^-}$
- $\tau \to \nu\bar{\nu}\ell$ spectrum, rat

$\Rightarrow \left\{ \begin{matrix} \tau_L \text{ in doublet} \\ \tau_R \text{ in singlet} \end{matrix} \right\} \Rightarrow \nu_\tau$ exists

[also FCNC]



$N'_\nu = 3.04 \pm 0.04$ LEP, $N'_\nu = \dfrac{\Gamma_Z - \Gamma_{Z\to q\bar{q},\ell\bar{\ell}}}{\Gamma_{Z\to\nu_i\bar{\nu}_i}}$

doesn't include sterile

$N'_\nu = n_{\text{doublets}\atop m \ll 45} + \sum_i \underbrace{\varphi_i(m_{\nu_i})}_{\text{phase space for heavy doublets}}$

$+ \frac{1}{2} n_{\tilde{\nu}\,(m \ll 45)} + 2 n_{\text{majoron}} + \cdots$

$N''_\nu < 3.3$ nucleosynthesis

- only includes states with $m < O(30\text{ MeV})$
- includes singlet ν's with any signif. mixing

$\dfrac{N_{He^4}}{N_H} \longleftrightarrow \dfrac{N_n}{N_p} = e^{-(m_n - m_p)/T^*}$

T^* = decoupling of $\nu_e n \leftrightarrow e^- p$

when $\Gamma_{weak} \sim G_F^2 T^5 \sim H \sim \sqrt{G_N \rho}$

$\sim \sqrt{N''_\nu} \dfrac{T^2}{m_p}$

Primordial nucleosynthesis

- $\dfrac{N_n}{N_p} = e^{-(m_n - m_p)/T}$ as long as
  $\nu_e n \leftrightarrow e^- p$ remains in equilibrium $(T > T^*)$

- rate: $\Gamma \sim \underbrace{\langle \sigma v \rangle}_{(r_F^2 T^2)} \underbrace{n_{\nu, e}}_{T^3} \sim G_F^2 T^5$

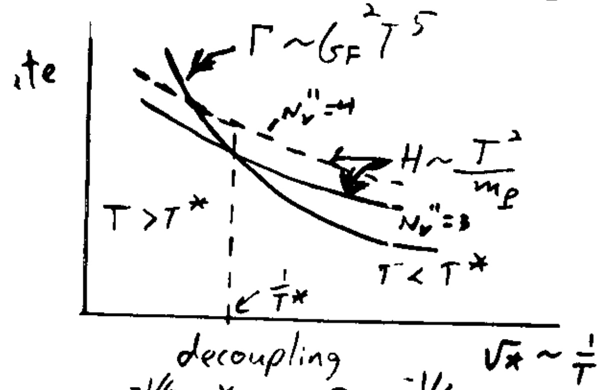
- equilibrium cannot be maintained when $\Gamma << H = \left[\dfrac{8\pi G_N \rho}{3}\right]^{1/2}$ = expansion rate

$H \sim 1.66\, g_*^{1/2} \dfrac{T^2}{m_P}$ $\qquad m_P = G_N^{-1/2}$ = Planck mass

$g_* = g_B + \frac{7}{8} g_F$

$g_B = 2 \quad (\gamma)$

$g_F = 4 + 2 N_\nu'' $
$\quad (e^+, e^-) \quad (\nu)$

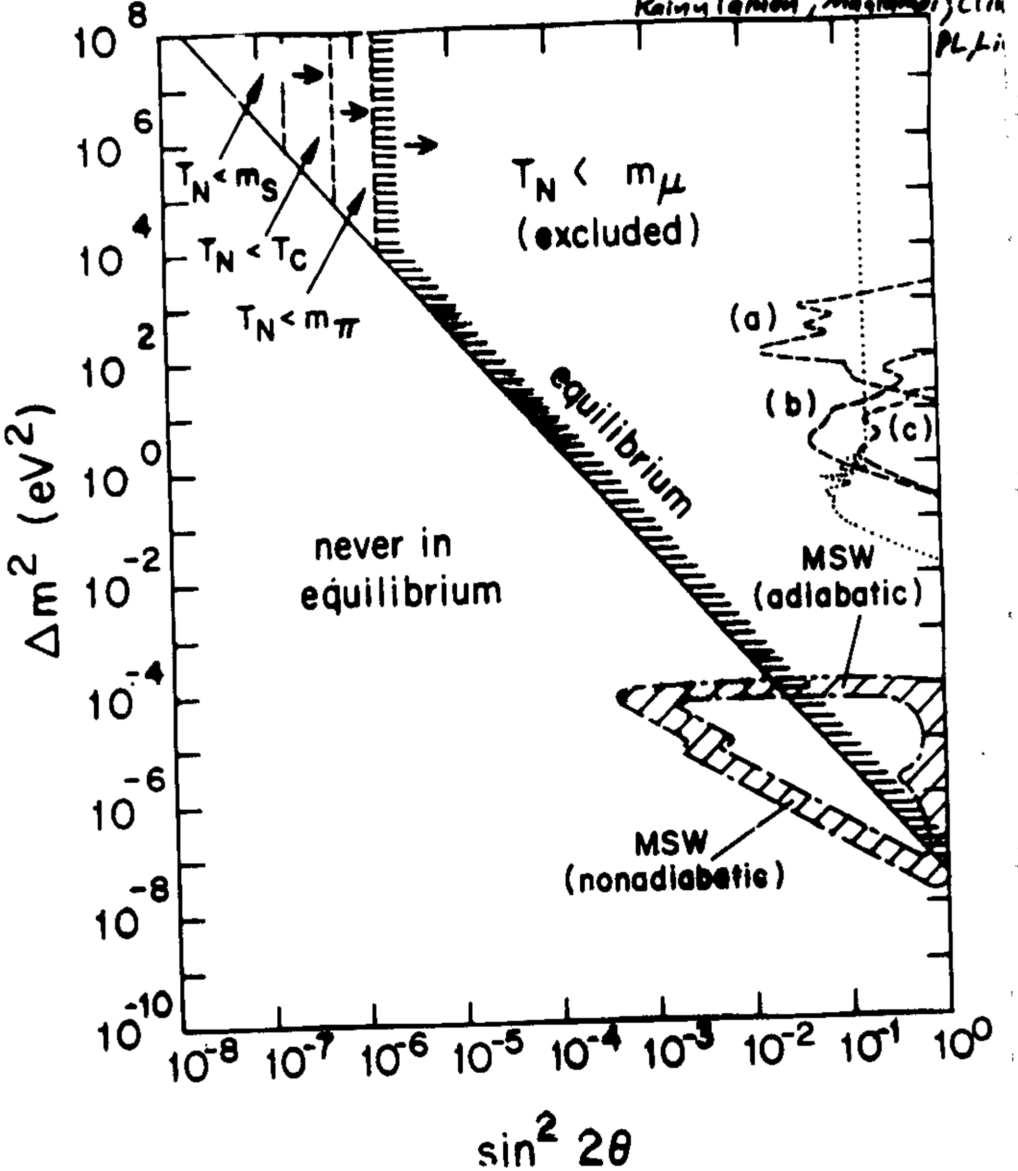


$g_*^{-1/6} T^* \sim \underbrace{\left((G_F^2 m_p)^{-1/3} \sim 3\ \text{meV} = O(m_n - m_p)\right)}$
- remarkable accident
- otherwise all H or all ^{4}He

$\Rightarrow \dfrac{N_n}{N_p} = e^{-(m_n - m_p)/T^*}$, $\qquad \dfrac{4 N_{He}}{N_H + 4 N_{He}} \sim \dfrac{2 N_n/N_p}{1 + N_n/N_p} \sim 24\%$

$\Rightarrow N_\nu'' < 3.3$

P.L.; Barbieri, Dolgov
Kainulainen; Thomson, McKellar; Enqvist, Kainulainen, Maalampi; Cline
PL Li



nucleosynthesis: $N_\nu'' < 3.3$

sterile (singlet) $\Rightarrow \Delta N_\nu = 1$

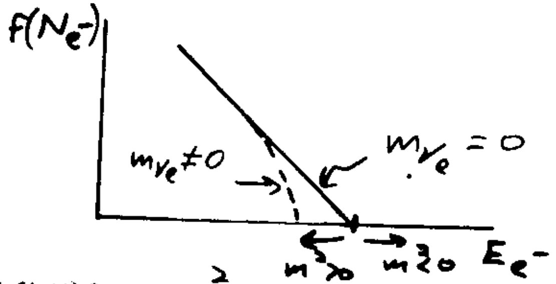
Kinematic limits

$^3He\, e^- \bar\nu_e$ spectrum

Mainz	$m_{\nu_e} < 7.2\ eV$	$m_\nu^2 = -39 \pm 34 \pm 15\ eV^2$
LANL	$m_{\nu_e} < 9.3\ eV$	$m_\nu^2 = -147 \pm 68 \pm 41\ eV^2$
INS-Tokyo	$m_{\nu_e} < 13\ eV$	$m_\nu^2 = -65 \pm 85 \pm 65\ eV^2$
Zurich	$m_{\nu_e} < 11.5\ eV$	$m_\nu^2 = -24 \pm 48 \pm 61\ eV^2$
Livermore	$m_{\nu_e} < 8\ eV$	$m_\nu^2 = -72 \pm 41 \pm 30\ eV^2$
SN1987A	$m_{\nu_e} < 0(20\ eV)$	
PSI†	$m_{\nu_\mu} < 270\ keV\, (90\% CL)$	$\pi \to \mu\nu$
ARGUS††	$m_{\nu_\tau} < 35\ MeV$	$\tau \to (5\pi)\nu_\tau$

ITEP: $m_{\nu_e} = 26 \pm 5\ eV$ excluded

$^3H \to {}^3He\, e^- \bar\nu_e$



- measure m_ν^2
- $m_\nu^2 < 0$ at 2.5σ
  $\Rightarrow$ Fluctuation?
  Common systematic?

SN1987A

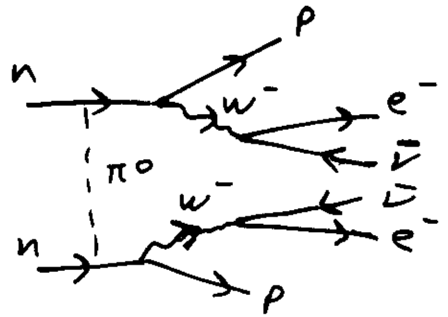
· For $m_{\nu_e} \neq 0$, expect high E_ν to arrive sooner ($\beta = 1 - \frac{m^2}{2E_\nu^2}$)

† original: $m_{\nu_\mu} < 250\ keV \xrightarrow[\text{new } m_{\pi^\pm}]{} 270\ keV$

†† new: $m_{\nu_\tau} < 31\ MeV$ | nucleosynthesis: only small window ...

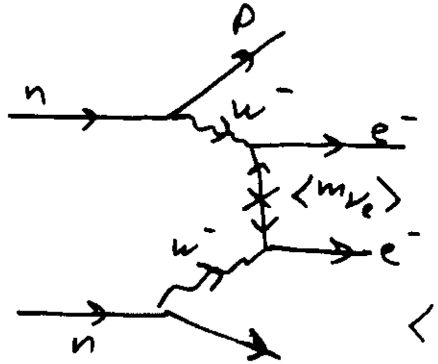
Double Beta Decay

$\beta\beta_{2\nu}$ $\quad (Z,N) \to (Z+2, N-2)\, e^- e^- \bar\nu \bar\nu$



$\Delta L = 0 \Rightarrow$ expected in SM
- calibrates calculation of matrix elements
- now observed

$\beta\beta_{0\nu}$ $\quad (Z,N) \to (Z+2, N-2)\, e^- e^-$



$\Delta L = 2$
- much larger phase space

$\langle m_{\nu_e} \rangle$ = effective Majorana mass

$$\langle m_{\nu_e} \rangle = \sum_i \xi_i U_{ei}^2 m_i F(m_i, A)$$

mass eigenstates of mass m_i

^{76}Ge: $T_{\beta\beta_{0\nu}} \gtrsim 10^{24}$ yr
$\Rightarrow \langle m_\nu \rangle < (2 \pm 1)\ eV$
Theory

$$\nu_e = \sum_i U_{ei} \nu_i$$

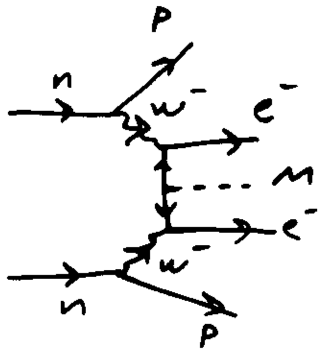
$\xi_i = CP\text{-parity} = \pm 1$, $\nu_{iL} = \xi_i C \bar\nu_{iR}^T$

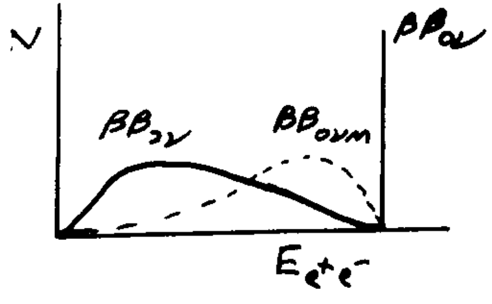
$$F(m_i, A) = \langle e^{-m_i r}/r \rangle / \langle 1/r \rangle \to \begin{cases} 1, & m_i \ll O(MeV) \\ c/m_i, & m_i \gg O(MeV) \end{cases}$$

- can have cancellations [e.g. Dirac]
- nuclear calc. cleaner than $\beta\beta_{0\nu}$

Other Contributions To $\beta\beta_{0\nu}$ D.O. Caldwell

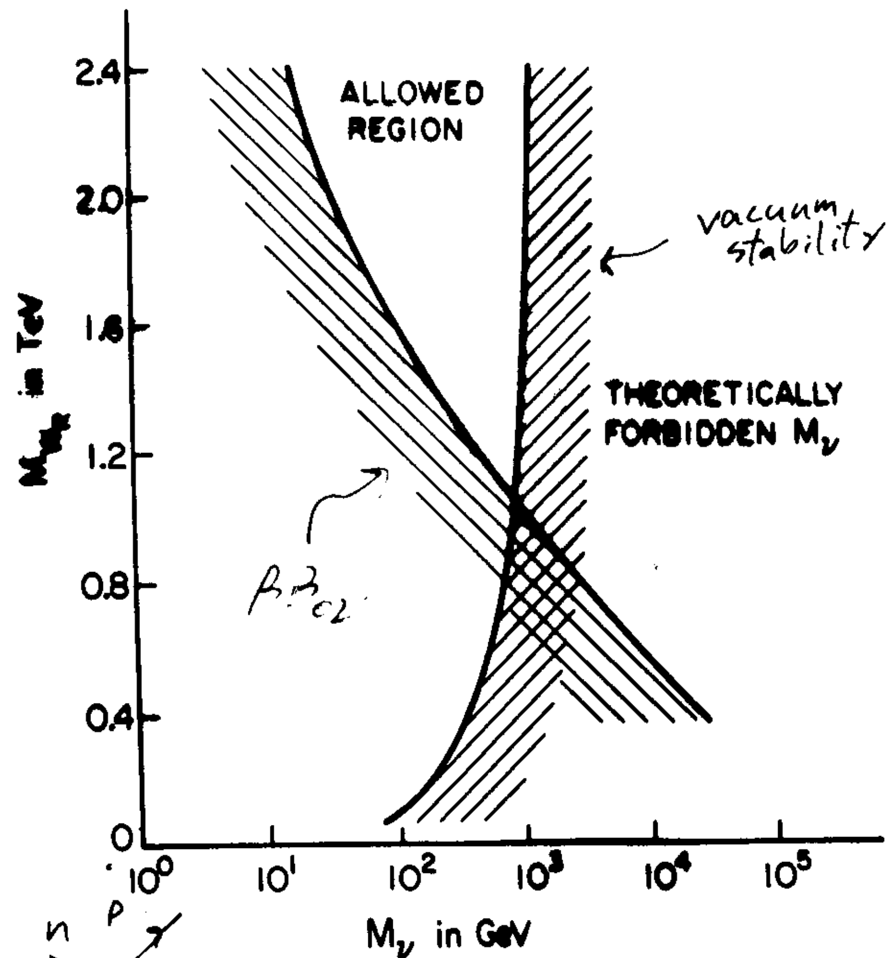
Majoron (Goldstone Boson) emission (models disfavored but hint in data (Mo, Irvine))



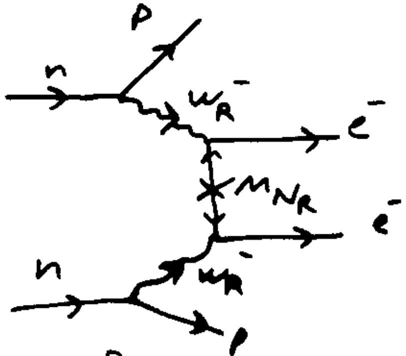


$SU_{2L} \times SU_{2R} \times U_1$

$+ \ |U_{Lij}| = |U_{Rij}|$

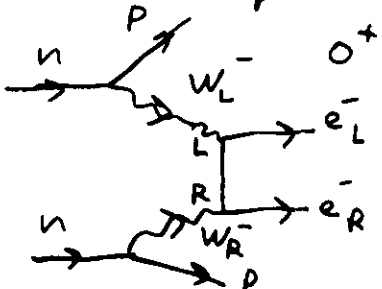


$SU_{2L} \times SU_{2R} \times U_1$ models

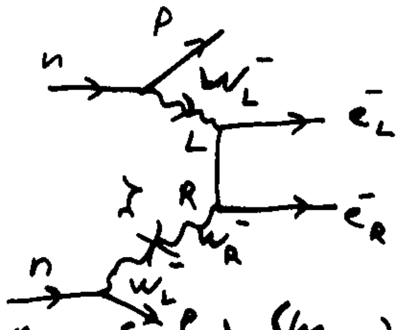


$g_R W_R^- \bar{e}_R \gamma^\mu N_R$

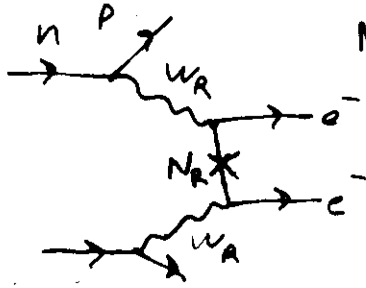
$\Rightarrow M_{N_R} > O(10-100)$ GeV For $M_{W_R} \gtrsim$ TeV



$0^+ \to 2^+$



Doi et al



$W_L^- \left(\nu_L \pm \Theta N_L^c\right) + W_R^- \left(-\Theta \nu_R^c + N_R\right) \Rightarrow \left\{ \begin{array}{l} \left(\frac{M_{W_L}}{M_{W_R}}\right)^2 \Theta < 10^{-6} \\ \lambda\Theta < 10^{-6} \end{array} \right.$

e_L^- e_R^-

expect $|\lambda| < \left(\frac{M_{W_L}}{M_{W_R}}\right)^2 < 10^{-3}$, $\Theta \sim \frac{m_D}{M_{N}} \lesssim 10^{-4} - 10^{-5}$

Mohapatra; Caldwell

Neutrino Oscillations

$|\nu_e\rangle = |\nu_1\rangle\cos\theta + |\nu_2\rangle \sin\theta$

$|\nu_\mu\rangle = -|\nu_1\rangle\sin\theta + |\nu_2\rangle\cos\theta$

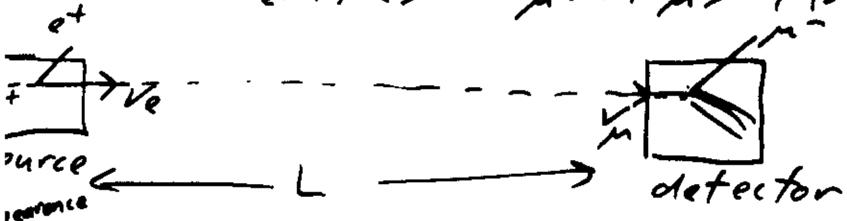
weak eigenstates [or ν_τ or N_L^c (sterile)] — mass eigenstates

requires both Dirac, Majorana masses

$|\nu(0)\rangle = |\nu_e\rangle$

$|\nu(t)\rangle = |\nu_1\rangle e^{-iE_1 t}\cos\theta + |\nu_2\rangle e^{-iE_2 t}\sin\theta$

$= \nu_e(t)|\nu_e\rangle + \nu_\mu(t)|\nu_\mu\rangle \neq |\nu_e\rangle$



actually, $\nu_\mu \to \nu_e$ in realistic exp.

disappearance $P(\nu_e\to\nu_e; L) = 1-\sin^2 2\theta \sin^2\frac{\Delta m^2 L}{4E}$

appearance $P(\nu_e\to\nu_\mu; L) = \sin^2 2\theta \sin^2\frac{\Delta m^2 L}{4E}$

$E \gg m_{1,2}$

L = distance

$\Delta m^2 = m_1^2 - m_2^2$

$\frac{\Delta m^2 L}{4E} \sim \frac{1.27\,\Delta m^2 L}{E}$ (eV^2, m, MeV)

Families: $\nu_a^0 = \sum_{i=1}^{F} V_{ai}\nu_i$

$\Rightarrow P(\nu_e\to\nu_a) = \sum_{i=1}^{F}|V_{ei}V_{ai}^*|^2$

$+ \mathrm{Re}\sum_{i\neq j} V_{ei}V_{ai}^* V_{ej}^* V_{ej}\, e^{-i(m_i^2-m_j^2)L/2E}$

Neutrino Oscillations

$\nu_e = \nu_1\cos\theta + \nu_2\sin\theta$

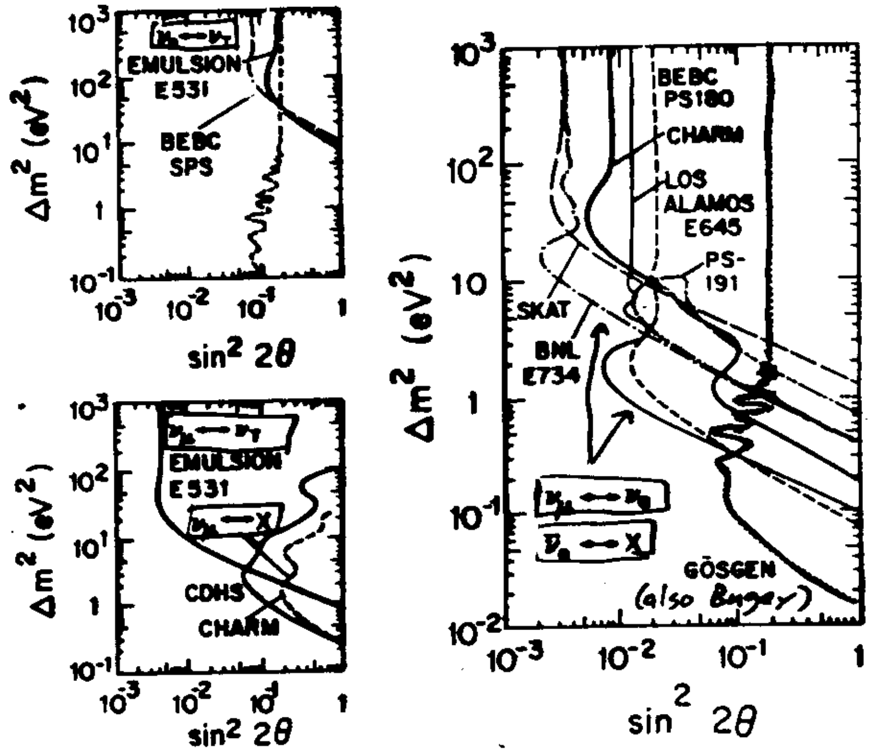
$\nu_x = -\nu_1\sin\theta + \nu_2\cos\theta$

weak eigenstates, mass eigenstates

- Can generalize to many states
- $\nu_x = \nu_\mu, \nu_\tau, \nu_s, \ldots$

$P(\nu_e\to\nu_x) = \sin^2 2\theta\, \sin^2\frac{\Delta m^2 L}{4E_\nu}$

$\Delta m^2 = m_1^2 - m_2^2$



Neutrino Oscillations

- Lab: no evidence
  - new $\nu_\mu \to \nu_\tau$ planned } small $\sin^2 2\theta$
    For CERN, FNAL, LANL
  - $\nu_\mu \to \nu_x$ (LANL) { small Δm^2, large $\sin^2 2\theta$
  - $\nu_e \to \nu_\tau$: would like $\sin^2 2\theta \sim .034$ (17 keV); present: .74 (hard)
- Solar
  - New GALLEX
  - MSW still preferred over cool sun

$\Delta m^2 \sim 6\times10^{-6}\,eV^2,\ \sin^2 2\theta \sim 7\times10^{-3}$

or $\Delta m^2 \sim 8\times10^{-6}\,eV^2,\ \sin^2 2\theta \sim 0.6$

- atmospheric ν_μ/ν_e deficit
  [ratio clean theoretically]

$$r = \frac{(\mu/e)_{data}}{(\mu/e)_{Theory}} \sim \begin{cases} 0.65 \pm .09, & \text{Kamiokande} \\ 0.64 \pm .15, & \text{IMB (single ring)} \end{cases}$$

$\rightarrow \begin{cases} \Delta m^2 \sim (10^{-3} - 1)\,eV^2 \\ \text{large mixing} \end{cases}$

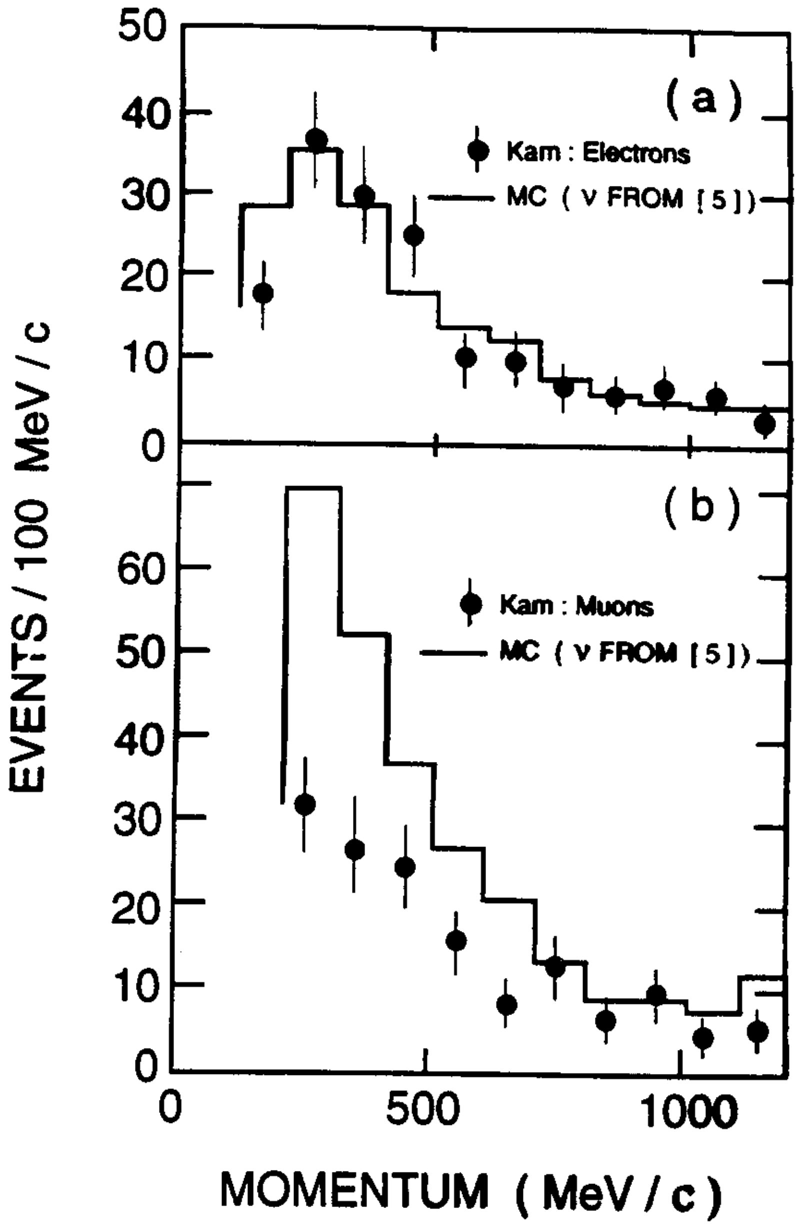


Fig. 2a,b

I-0108

D. O. Caldwell

Incoherent Mixing with Heavy ν

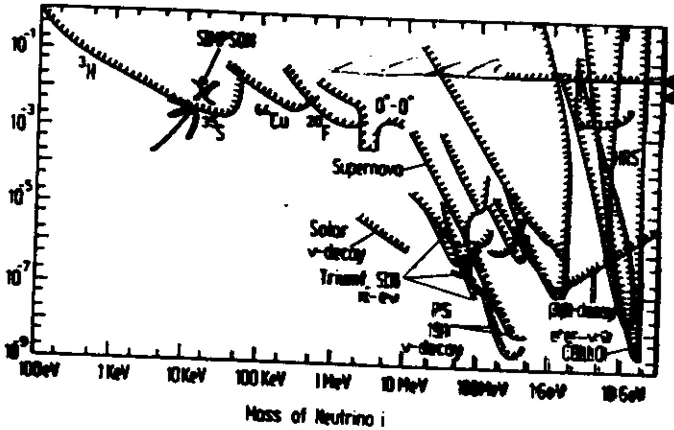
$\nu_e \sim \nu_1 + U_{ei} \nu_i$

$\nu_\mu \sim \nu_2 + U_{\mu i} \nu_i$

— universality, β decay (17 keV?), π, K decay; $\nu_h \to \nu_\ell e^+ e^-$ (beam dumps, νN, e^+e^-), ...

17 keV?

1% mixing



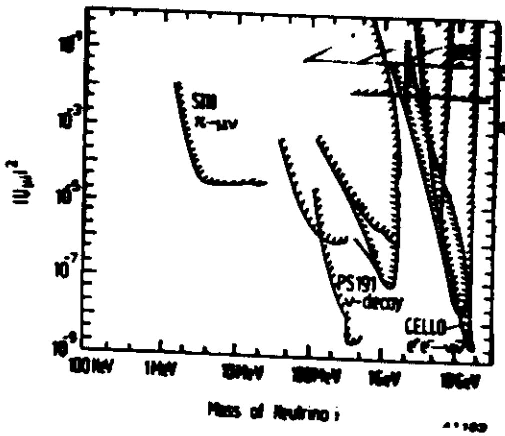
.08

.03

universality with (without) fine-tuned cancellations

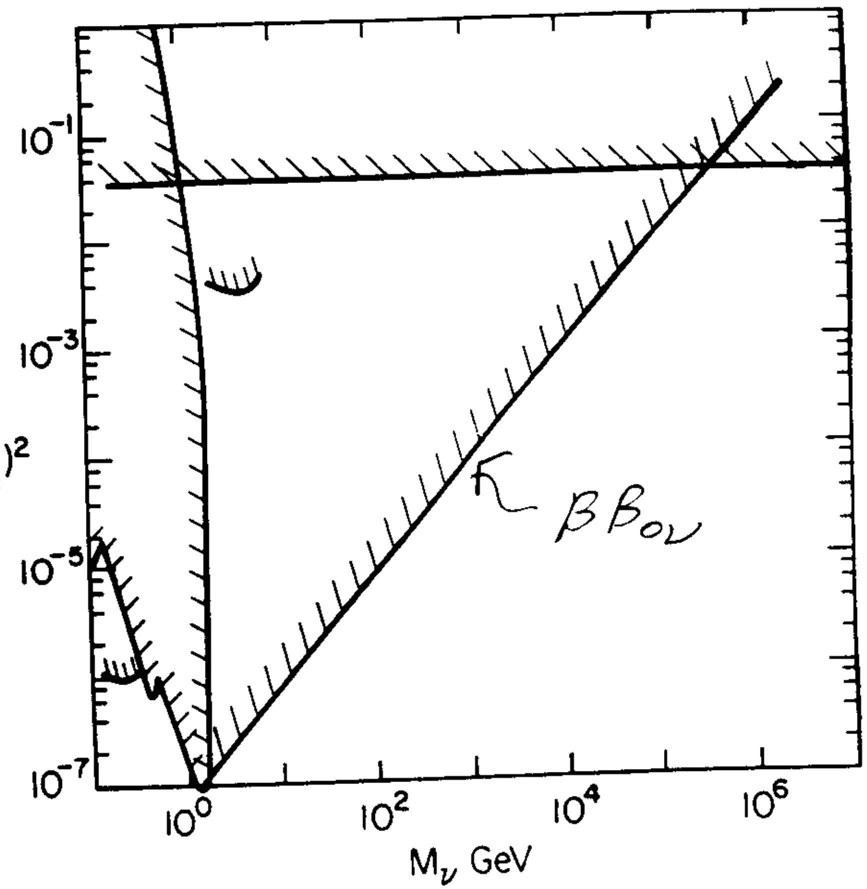
PL, London

LEP: $M_N < 40$ GeV excluded for doublets

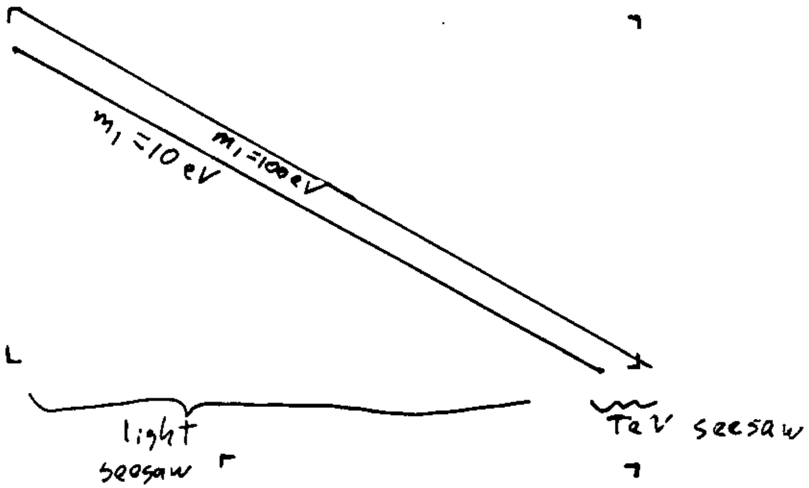


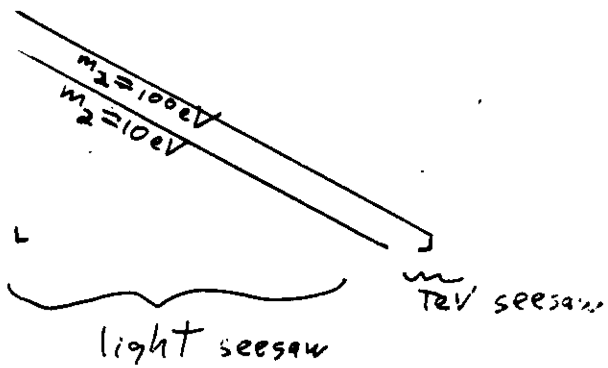
.05

.0024



Caldwell

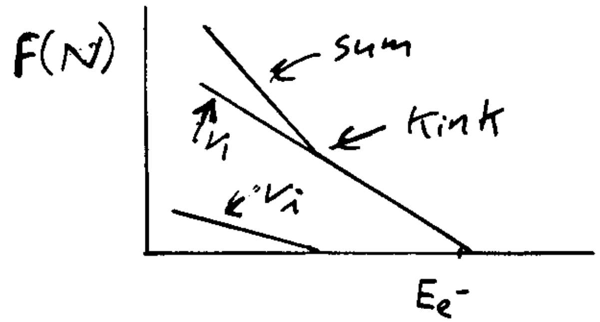


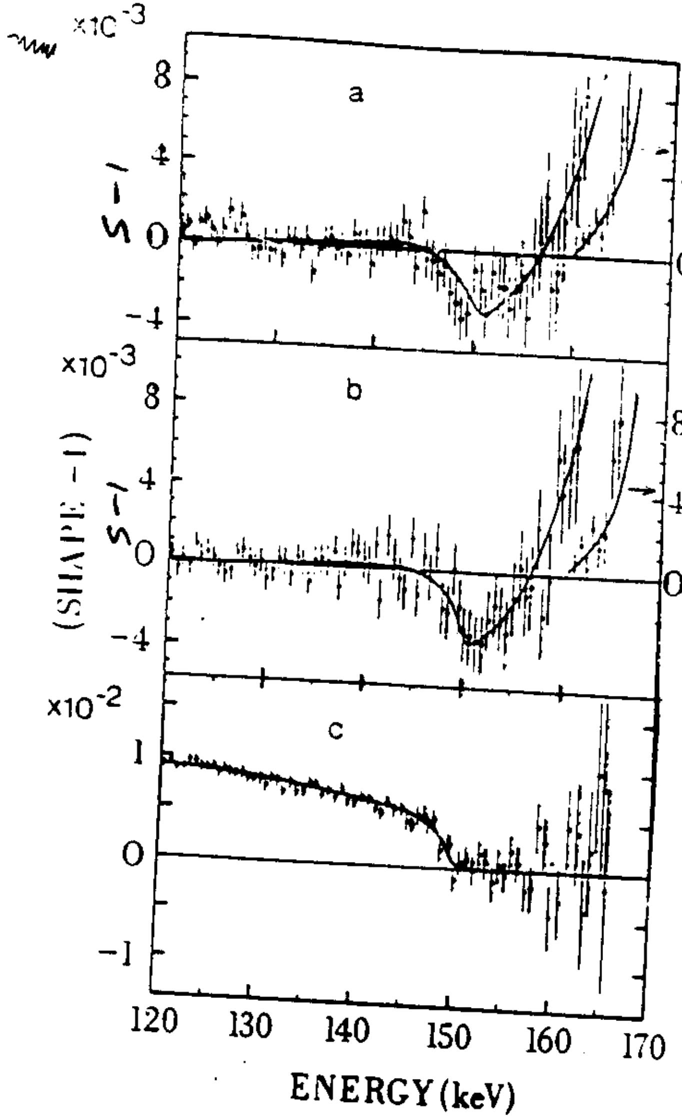


17 KeV

$$\nu_e \sim \nu_1 + U_{ei}\,\nu_i$$

- $\sum_{m_i \sim 17\,KeV} |U_{ei}|^2 \sim 0.01$ (solid state detectors)
- no effect (magnetic spectrometers)

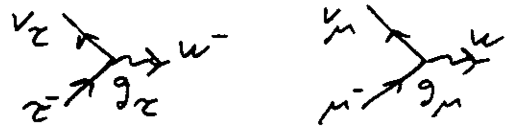




$$S = \frac{N_{obs}}{N_{Theory}}$$

Hime, Jelley
Oxford
^{35}S

The τ problem



$SU_2 \times U_1$ (universality): $\frac{g_\tau}{g_\mu} = 1$

data $[\Gamma_\tau,\ B(\tau \to \mu\nu\bar{\nu}),\ m_\tau]$:

$$\frac{g_\tau}{g_\mu} = \begin{cases} .975 \pm .010, & 3/92 \\ .985 \pm .010 & \text{present} \end{cases}$$

[Now only $1\frac{1}{2}\sigma \Rightarrow$ may be fluctuation

$$\Gamma_\tau = \frac{\Gamma_{\tau \to \mu\nu\bar{\nu}}}{B(\tau \to \mu\nu\bar{\nu})} = \frac{C\, G_F^2\, m_\tau^5}{B(\tau \to \mu\nu\bar{\nu})} \left(\frac{g_\tau}{g_\mu}\right)^2$$

3/92: world average: $m_\tau = 1784 \pm 3$ MeV

present (prelim) $m_\tau = \begin{cases} 1776.3 \pm 2.4 \pm 1.4, & \text{ARGUS} \\ 1776.9 \pm 0.4 \pm 0.3, & \text{Beijing} \end{cases}$

- if discrepancy is real, may suggest

$$\nu_\tau \sim \cos\theta\, \nu_3 + \sin\theta\, \nu_H,$$
$$m_{\nu_H} > m_\tau$$

$$\Rightarrow \Gamma_\tau \propto \cos^2\theta \Rightarrow \frac{g_\tau}{g_\mu} \sim \cos\theta \sim 1 - \frac{\theta^2}{2}$$

$$\Rightarrow \theta^2 \sim \begin{cases} .05 \pm .02, & 3/92 \\ .03 \pm .02, & \text{present} \end{cases}$$

if ν_H = active $\Rightarrow m_{\nu_H} > 40$ GeV (LEP)

if ν_H = sterile $\Rightarrow N_\nu' = 2 + \cos^4\theta \sim \begin{cases} 2.90 \pm .04, & 3/92 \\ 2.94 \pm .04, & \text{present} \end{cases}$

LEP: $N_\nu' = 3.04 \pm .04$

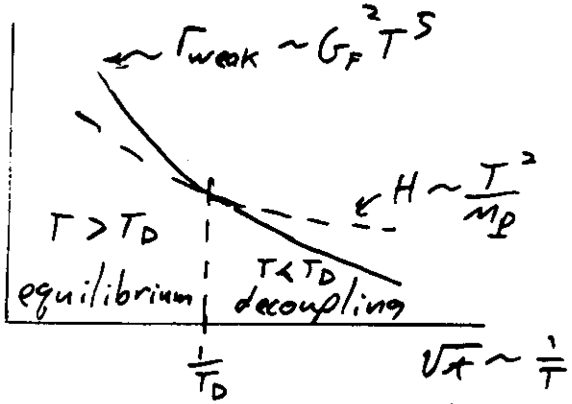
Cosmological limits

active ν: produce by $e^+e^- \overset{Z}{\leftrightarrow} \nu\bar{\nu}$, etc.

rate: $\Gamma_{weak} \propto \langle \sigma v \rangle n_T \sim G_F^2 T^5$

$\left[G_F^2 T^2 \right.$ ↑ Target density $\sim T^3$

expansion rate: $H = \left[\frac{8\pi G_N \rho}{3}\right]^{\frac{1}{2}} \sim \frac{T^2}{M_P}$ ← Planck mass $G_N^{-1/2} \sim 10^{19}$ GeV



$T_D \sim (G_F^2 M_P)^{-1/3} \sim 3$ MeV, $t_D \sim 1$ sec

at present: $T_{\nu 0} = \left(\frac{4}{11}\right)^{1/3} T_{\gamma 0} = \left(\frac{4}{11}\right)^{1/3} 2.7K \sim 1.9K$

↑ photon reheating $e^+e^- \to \gamma\gamma$ at $\sim 1/3$ MeV

$\Rightarrow n_{\nu 0} \sim T_{\nu 0}^3 \sim \frac{50}{cm^3} g_\nu \sim 300/cm^3$

[Clustered locally by $\sim 10^6$ if $m = O(10\,eV)$]

- also sterile if mixing ($\theta > 10^{-3}$)
- Detection of relic ν's ?

- if massive, $\rho_\nu = \underbrace{\sum_i}_{\substack{\text{light, stable, active} \\ +\text{(sterile with mixing)}}} m_{\nu_i} n_{\nu_i}$

present density $\Rightarrow \sum_i m_{\nu_i} < 28$ eV [HDM for $m \gtrsim 1$ eV

[or $m \gtrsim$ GeV, Lee-Weinberg]

[large + small scale structure suggests HDM + CDM]

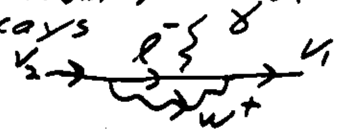
Unstable neutrinos

- still require $m_\nu < 28\,eV \left[\frac{t_0}{\tau_\nu}\right]^{\frac{1}{2}}$

From present energy density ($t_0 \sim 10^{10}$ yr = age) [redshifting]

[stronger limits from growth of structu

- normal weak decays



$\nu_2 \to \nu_1 \gamma$

$\nu_2 \to \nu_1 e^+e^-$

- too slow; excluded by energy density, CBR, SN, nucleosynthesi $\Rightarrow m_{\nu_\mu, \nu_\tau} < 28$ eV unless new decay mech

- Fast, invisible decays/annihilations

$\nu_2 \to \nu_1 \nu_1 \bar{\nu}_1$ - too slow

$\nu_2 \to \nu_1 F$ - Familon (G.B. of Family sy
 - constrained by $\tau \to \mu$

$\nu_2 \to \bar{\nu}_1 M$ - Majoron (GB of $\cancel{L}$
 - hard to implemen

$\nu\bar{\nu} \to MM$ - no viable models

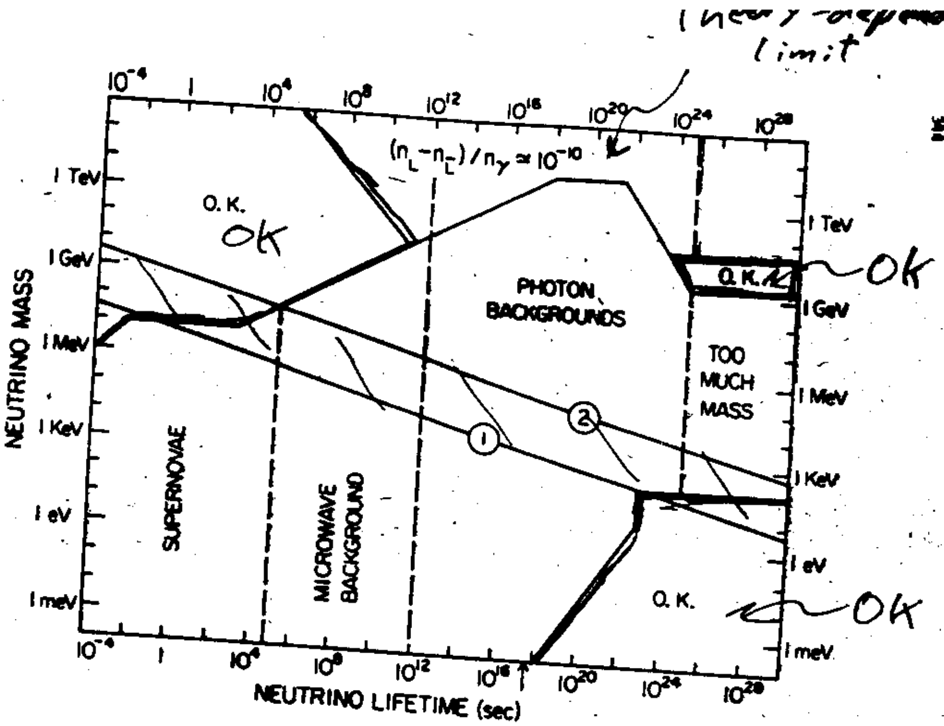


FIGURE [illegible] - Approximate astrophysical and cosmological restrictions on the mass and lifetime of a hypothetical neutrino. The arrow indicates the present age of the universe. The assumptions and origin of the constraints are explained in the text. The allowed regions are labeled "O.K."

Typical range for $\nu_H \to e^+ e^- \nu_L$, $\nu_H \to \nu_L \gamma$:

(1) $\tau = 10^4 \left(\frac{1\,\mathrm{MeV}}{m_\nu}\right)^5$ s

(2) $\tau = 10^{14} \left(\frac{1\,\mathrm{MeV}}{m_\nu}\right)^5$ s

Turner

"Theoretical papers on Neutrino Mass are an excellent candidate for the dark matter of the Universe."

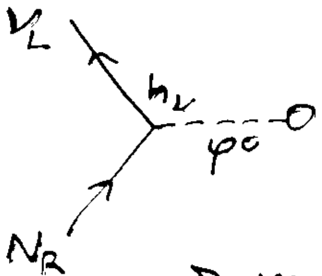
Ray Davis, Jr.

Models of Neutrino Mass

Normal Dirac: ν_L = active
N_R = sterile $\neq \nu_R^c$

$$-\mathcal{L}_{Dirac} = m_D(\bar\nu_L N_R + \bar N_R \nu_L)$$

$\Delta L = 0$
$\Delta I = \frac{1}{2} \Rightarrow$ ordinary Higgs doublet



$v = \sqrt{2}\langle\varphi^0\rangle$
$= (\sqrt{2}\, G_F)^{-1/2} \simeq 246$ GeV

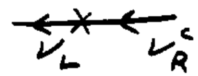
$\Rightarrow m_D = h_\nu v$
$m_{\nu_e} < 10$ eV $\Rightarrow h_{\nu_e} < 10^{-10}$

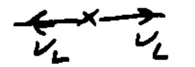
- why is h_ν so small?
(but why is $\frac{m_e}{m_t} < 10^{-5}$)
- $h_\nu = 0$ at Tree level?
(no compelling models)

- ZKM Dirac — Zeldovich, Konopinski, Mahmoud
- ν_L, N_R both active
- e.g. $N_{eR} = \nu_{\tau R}^c \Rightarrow L_e - L_\tau$ conserved
- limiting case of Majorana
- need new Higgs ($\Delta I = 1$)

- Majorana Mass ($\Delta L = \pm 2$)
$N_R = \nu_R^c$

$$-\mathcal{L}_{Majorana} = \frac{1}{2} m(\bar\nu_L \nu_R^c + \bar\nu_R^c \nu_L)$$
$$= \frac{1}{2} m(\bar\nu_L C \bar\nu_L^T + H.c.)$$
$$= \frac{1}{2} m \bar\nu \nu$$



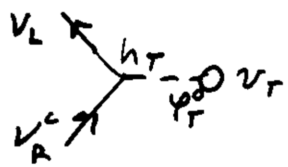


$\Delta L = \pm 2$

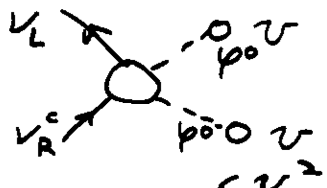
$(\nu \equiv \nu_L + \nu_R^c = \nu^c = C\bar\nu^T)$
2 components: ν_L, ν_R^c

sterile: $\Delta I = 0 \Rightarrow$ bare mass or Higgs singlet

active: $\Delta I = 1 \Rightarrow$ Higgs triplet or effective operator

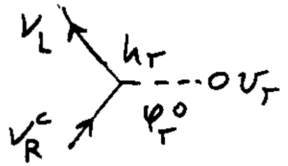


$m \sim h_T v_T$



$m \sim \frac{C v^2}{M}$

- elementary triplet (Gelmini-Roncadelli)



$$\vec{\varphi}_T = (\varphi_T^0, \varphi_T^-, \varphi_T^{--})$$

$$-\mathcal{L} = \tfrac{1}{2} h_T (\bar\nu_L \; \bar e_L) \, \vec\tau \cdot \vec\varphi_T \begin{pmatrix} e_R^c \\ -\nu_R^c \end{pmatrix}$$

$$= \tfrac{1}{2} h_T (\bar\nu_L \; \bar e_L) \begin{pmatrix} \varphi_T^- & \sqrt{2}\varphi_T^0 \\ \sqrt{2}\varphi_T^{--} & -\varphi_T^- \end{pmatrix} \begin{pmatrix} e_R^c \\ -\nu_R^c \end{pmatrix}$$

$m = h_T v_T$
$= \sqrt{2} h_T \langle \varphi_T^0 \rangle$

- most versions: spontaneous $\cancel{L}$

$\Rightarrow$ triplet majoron, $M_T \sim \mathrm{Im}\, \varphi_T^0$

$\Rightarrow v_T = \frac{m}{h_T} < (2-10)$ KeV stellar energy loss

$\Rightarrow$ hierarchy problem

- but $Z \to M_T \chi_T \Rightarrow \Delta N_\nu' = 2$

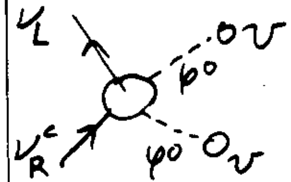
χ_T: $\mathrm{Re}\,\varphi_T^0$, $m_{\chi_T} = O(v_T)$

$\Rightarrow$ excluded by LEP ($N_\nu' = 3.04 \pm .04$)

- all epi-Higgs* variants with M_T excluded by N_ν', N_ν'', $K^+ \to \ell^+ \nu M_T$, $\nu_s \to \nu_s$

*epi-Higgs: arbitrary, complicated Higgs sector invented by theorists (from epicycles)

- effective operator



$$-\mathcal{L} = \frac{c}{M} \bar\nu_L \nu_R^c \varphi\varphi$$

triplet

M: new physics scale

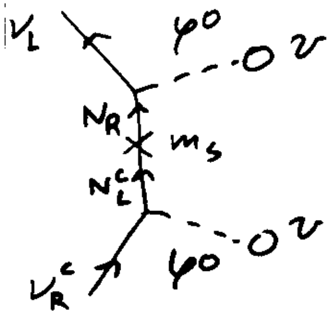
$\Rightarrow m_\nu \sim \frac{c v^2}{M} \ll v$, for $c = O(1)$, $v \ll M$

e.g., $c = 1$, $M = M_P = 10^{19}$ GeV

$\Rightarrow m_\nu \sim 10^{-5}$ eV

- seesaw realization

Gell-Mann, Ramond, Slansky, Yanagida



$$-\mathcal{L} = \tfrac{1}{2} (\bar\nu_L \; \bar N_L^c) \begin{pmatrix} m_T & m_D \\ m_D & m_S \end{pmatrix} \begin{pmatrix} \nu_R^c \\ N_R \end{pmatrix}$$

$$\nu_L = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_L \qquad N_R = \begin{pmatrix} N_e \\ N_\mu \\ N_\tau \end{pmatrix}_R$$

(active) (sterile)

3×3: $m_T = m_T^T$ { - elementary triplet
- assume $m_T = 0$ (can be dangerous)

3×3 $m_S = m_S^T$ { - bare mass or Higgs singlet

3×3 m_D { - Higgs doublet

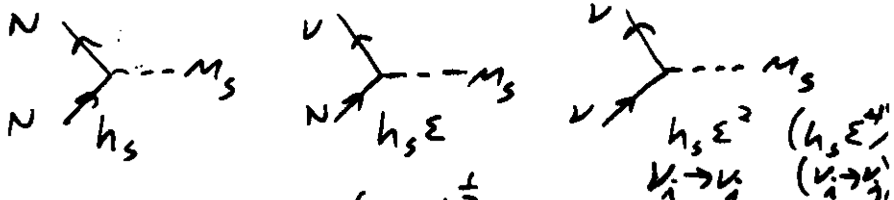
- assume eigenvalues of $m_S \gg m_D$

$\Rightarrow$ 3 heavy: m_S $[N + O(\varepsilon)\nu]$

3 light: $m_\nu = m_D m_S^{-1} m_D^T$ $[\nu + O(\varepsilon) N]$

$\varepsilon = O\left(\frac{m_D}{m_S}\right)$

- hundreds of versions
  - scale of M_S
    - $M_X \sim 10^{16}$ GeV (GUT) $(\to m_\nu \sim 10^{-13}, 10^{-8}, 10^{-3}$ eV$)$
    - $M_R \sim 10^{12}$ GeV (int. breaking, axion, non-renorm. op.) $(\to m_\nu \sim 10^{-9}, 10^{-4}, 10$ eV$)$
    - 1 TeV $(\to 1, 10^{4}, 10^{9}$ eV$)$
  - hierarchy of M_S eigenvalues (linear, quadratic, loop-induced)
  - Form of m_D $\begin{cases} m_D \sim m_u, & \text{(GUT)} \\ m_D = O(m_u), & \text{string-inspired} \\ m_D = O(m_e) & \text{—} \end{cases}$
  - variations: Dirac, extended, SUSY (gauginos), incomplete, ...
  - Global or Gauged L
    - global $\Rightarrow$ singlet majoron M_S



$$\varepsilon \sim \frac{m_D}{M_S} \sim \left(\frac{m_\nu}{M_S}\right)^{\frac{1}{2}} \ll 1$$

- no fast $\nu_i \to \nu_j M_S$ decay in simplest versions $O(h_s^2 \varepsilon^8)$

- 3 viable models inspired by coupling constant unification

- "old" SUSY-GUTs (large Higgs reps, eg 126 of SO_{10})
  - often, $m_D = m_u$, $m_e = m_d$ Bludman, Kennedy, PL

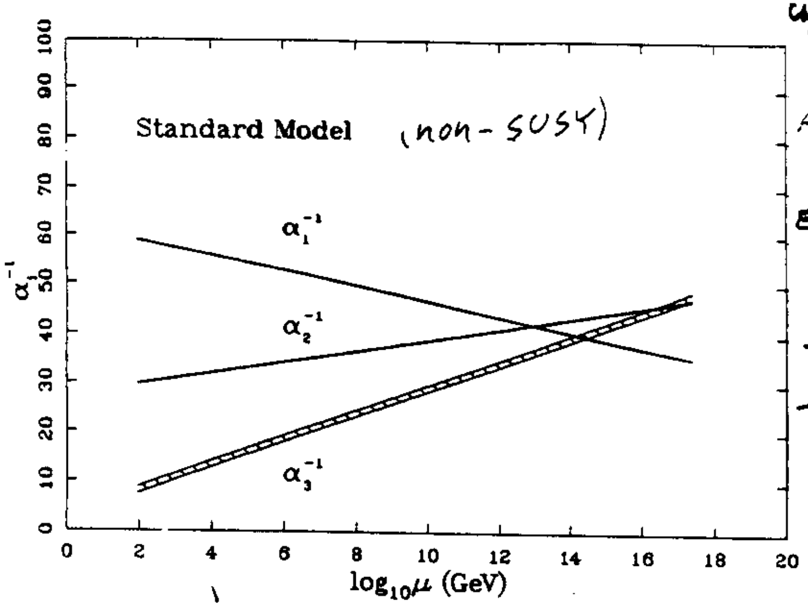
$$\Rightarrow m_{\nu_i} \sim c_i \frac{m_{u_i}^2}{M_{S_i}} \qquad c_i \sim .05 - .1 \text{ (rad corr)}$$

$$M_{S_i} \sim (10^{-2} - 1) M_X \qquad M_X \sim 10^{16} \text{ GeV (unification)}$$

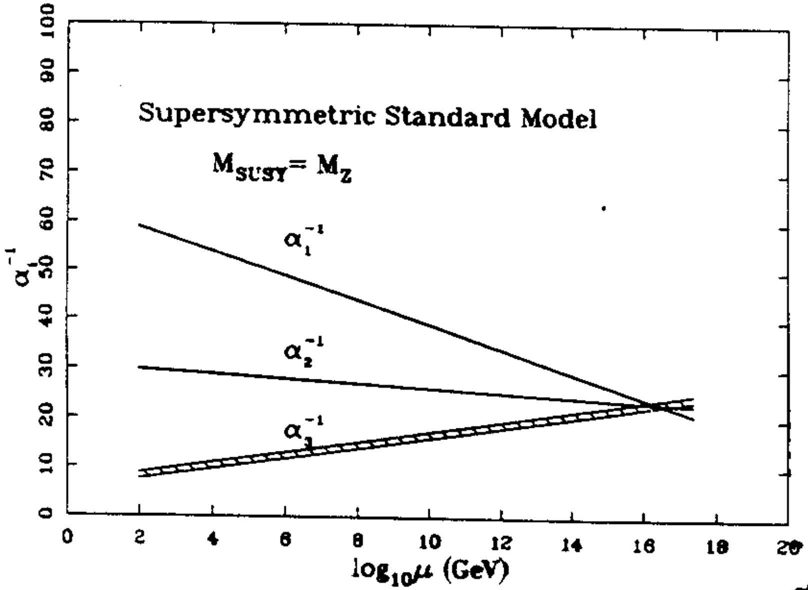
$$\Rightarrow m_{\nu_e} \lesssim 10^{-11} \text{ eV}$$
$$m_{\nu_\mu} \sim (10^{-8} - 10^{-6}) \text{ eV}$$
$$m_{\nu_\tau} \sim (10^{-4} - 1) \text{ eV}$$
$$V_{lepton} \sim V_{CKM}$$

but, also:
$$\frac{m_d}{m_s} = \frac{m_e}{m_\mu}$$
$(\frac{1}{20})$ $(\frac{1}{200})$

- solar $\nu_e \to \nu_\tau$ in disfavored small angle region



updated From PL, '90 Amaldi, de Boer, Furstenau
Ellis, Kelley, Nanopoulos; Langacker
SU_5, SO_{10}, ... etc, PL
- also excluded by proton decay



similar for $M_{SUSY} = 1$ TeV

Proton decay: $d=6$, suppressed; $d=5$, dangerous

inputs

$\alpha^{-1}(M_Z) = 127.9 \pm 0.2$

$\sin^2\hat\theta_W(M_Z) = 0.2325 \pm 0.0007$

$\alpha_s(M_Z) = 0.124 \pm 0.010$

K, L

- string-inspired models (or "new" SUSY-GUTs with small Higgs reps)
- m_{s_i} not generated at tree-level
- effective ops may be left over from gravity

$$-L_{eff} = \frac{c}{M_c} \bar N_L^c N_R \Phi \Phi$$

$M_c \sim 10^{18}$ GeV = compactification scale

Φ = 10-plet of SO_{10}

$\langle \Phi \rangle \sim M_X \sim 10^{16}$ GeV = unification scale

$$\Rightarrow M_s \sim c \frac{|\langle\Phi\rangle|^2}{M_c} \sim 10^{-2} c\, |\langle\Phi\rangle|$$

example: large radius Calabi-Yau:

$$c \sim e^{-R^2/\alpha'}, \quad \frac{M_X}{M_c} \sim \frac{1}{\sqrt{2R^2/\alpha'}}$$

R = radius, α' = string tension

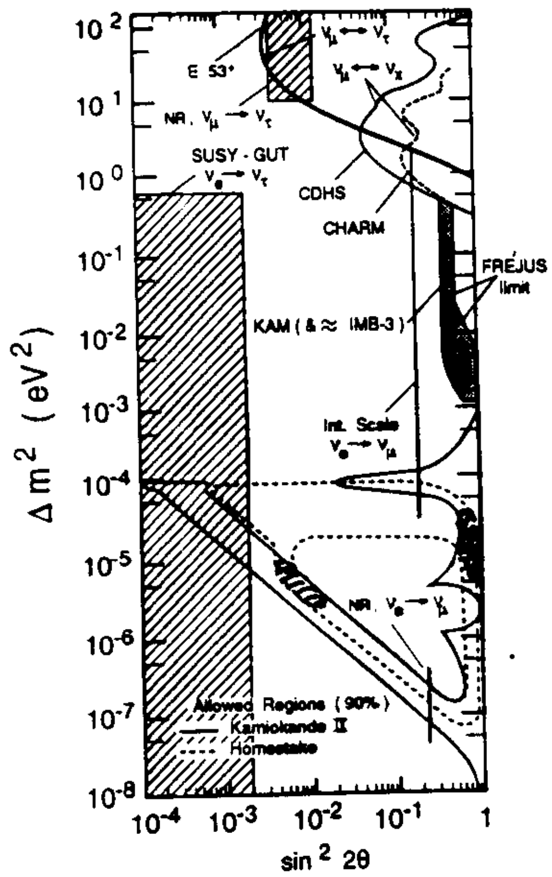
$\Rightarrow M_s \sim 10^{11}$ GeV

$$\Rightarrow \begin{matrix} m_{\nu_e} \sim 10^{-7} \text{ eV} \\ m_{\nu_\mu} \sim 10^{-3} \text{ eV} \\ m_{\nu_\tau} \sim 10 \text{ eV} \end{matrix} \quad \begin{matrix} \} \ \nu_e \to \nu_\mu \text{ in Sun} \\ \} \ \text{HDM} \end{matrix}$$

- no V_{lepton} prediction

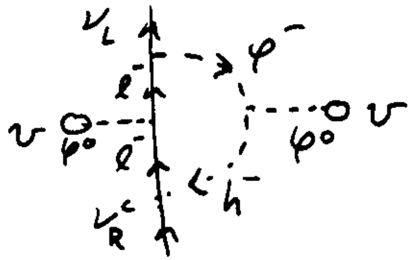
- intermediate-scale ordinary GUTs
  - similar to string-inspired for int. scale $M_R \sim 10^{11}$ GeV (simplest version: 10^{10})

  Typically, $V_{lepton} \sim V_{CKM}$



Radiative Generation of ν masses

- explain smallness of m_ν by higher order
- but, models very complicated (epi-Higgs)



Zee

h^- = charged SU_2-singlet with L-violating couplings

- often Pseudo-Dirac e.g. $L_e - L_\mu + L_\tau \simeq$ conserved

- major motivation: large m_ν but small $\langle m_{\nu e} \rangle$

- recently: applied to 17 keV
  Barbieri, Hall,
  Babu, Mohapatra, Rothstein
  ⋮

Dynamical Symmetry Breaking

Giudice, Raby
Appelquist, Terning

Composite Fermions