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(Lect. IV)

SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

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NEUTRINO PHYSICS

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Please note: These are preliminary notes intended for internal distribution only

Solar Neutrinos

		†SSM
Homestake (Cl) ^{8}B, ^{7}Be (Time dependence)	2.1 ± 0.3 SNU [6σ] 10^{-36}/s-atom	8.0 ± 1.0
Kamiokande ($\nu e \to \nu e$) (KII + 220 days KIII) ^{8}B	[0.49 ± 0.08]% [3σ]	$1 \pm .14$
GALLEX (Ga) pp, ^{7}Be, ^{8}B	83 ± 20 SNU [2σ] $\pm 19 \pm 8$	132^{+7}_{-6} minimal Flux: >80
SAGE (Ga) - not used in Fits	pub 12/91: $20^{+15}_{-20} \pm 30$ 6/92: higher	

- all errors $\pm 1\sigma$

- †SSM: Bahcall, Pinsonneault:
  helium diffusion; new S_{17};
  good agreement with other
  calculations, helioseismology, etc

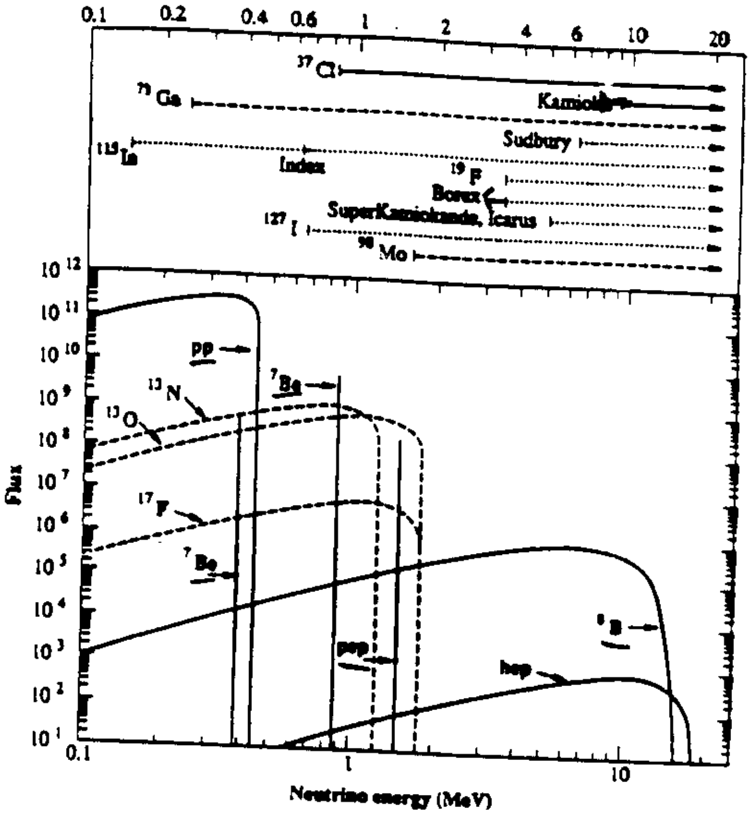
SSM uncertainties (1000 SSM's, Bahcall, Ulrich)

$\Rightarrow T_c = 1 \pm .0057$ + nuclear
(production, detection) uncertainties
- excluded

Table II. Reactions.

Reaction	Number	Terminations (%)	Neutrino Energy (MeV)
$p + p \to {}^{2}H + e^{+} + \nu_e$	1a	99.75	≤ 0.420
or			
$p + e + p \to {}^{2}H + \nu_e$ (PEP)	1b	0.5	1.442
${}^{2}H + p \to {}^{3}He + \gamma$	2	100	
${}^{3}He + p \to {}^{4}He + e^{+} + \nu_e$ (hep)	3	0.00002	18.77
or			
${}^{3}He + {}^{3}He \to \alpha + 2p$	4	85	
or			
${}^{3}He + {}^{4}He \to {}^{7}Be + \gamma$	5	15	
${}^{7}Be + e^{-} \to {}^{7}Li + \nu_e$	6	15	0.861 (90%) 0.383 (10%)
${}^{7}Li + p \to 2\alpha$	7		
or			
${}^{7}Be + p \to {}^{8}B + \gamma$	8	0.02	
${}^{8}B \to {}^{8}Be^{*} + e^{+} + \nu_e$	9		<15
${}^{8}Be^{*} \to 2\alpha$	10		

	Ga (SNU)	Cl (SNU)	Kam
pp	70.8	—	
pep	3.0	0.2	—
hep	0.06	0.03	Trace.
^{7}Be	34.3	1.1	—
^{8}B	14.0	6.1	100%
^{13}N	3.8	0.1	
^{15}O	6.1	[illegible]	—



igure 2 : Solar neutrino energy spectrum (adapted from [1]). Neutrino fluxes from ontinuum sources are in $cm^{-2}s^{-1}MeV^{-1}$. Line fluxes are in $cm^{-2}s^{-1}$. The insert above ives the sensitivity interval of the different detectors above the threshold. Full lines : xisting detectors. Dashed lines : detectors in installation. Dotted lines : projects.

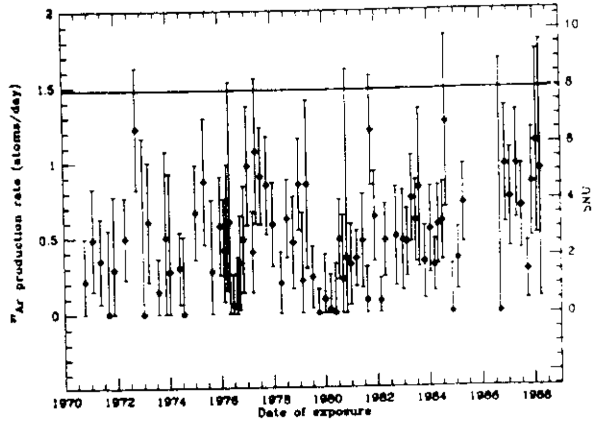


Figure 10.5 Observational results from the chlorine solar neutrino experiment For details see Davis (1968), Davis, Harmer, and Hoffman (1968), Davis, Harmer, and Neeley (1969), Davis, Evans, Radeka, and Rogers (1972), Bahcall and Davis (1976), Davis, Evans, and Cleveland (1978), Davis (1978), Cleveland, Davis, and Rowley (1984), Rowley, Cleveland, and Davis (1985), Davis (1987), and Section 10.6. This figure contains data on more recent runs, 83-99, discussed in Section 10.6 and generously made available by R. Davis, Jr. and B. Cleveland. The line at 7.9 SNU across the top of the figure represents the prediction of the standard model

Figure 10.5 shows the results for each of the runs. The vertical error bars indicate 1σ errors. The combined production rate for the 61 runs that is inferred with the aid of the maximum likelihood analysis is

$$\text{Production rate} = 0.462 \pm 0.040 \ ^{37}\text{Ar atoms day}^{-1}. \qquad (10.7)$$

Subtracting the small background rate given in Eq. (10.5) and converting to solar neutrino units using Eq. (10.3), the capture rate in the tank is estimated to be

$$\text{Capture rate} = (2.05 \pm 0.3)\ \text{SNU}. \qquad (10.8)$$

From J. Bahcall,

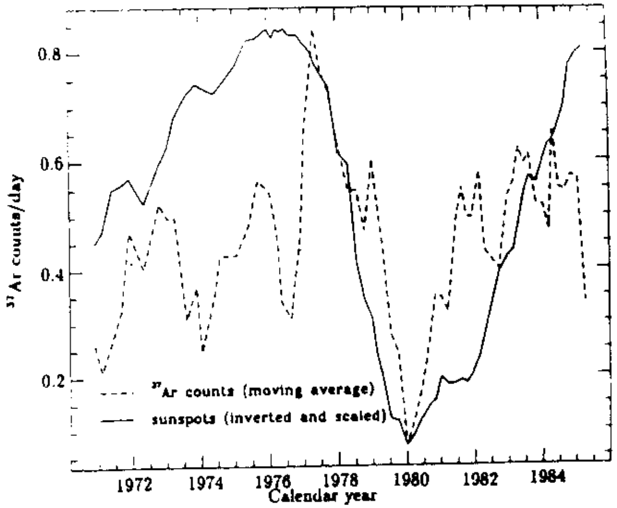


Figure 10.7 SNUs versus spots The figure shows the average monthly sunspot number (solid curve) versus the moving averaged SNU rate (dashed curve) as a function of calendar year. The scale of the ordinate is arbitrary for sunspots, which are scaled to the same peak to peak range, and inverted (small sunspot number at the top).

discusses a possible theoretical interpretation of the suggested correlation in terms of a large neutrino magnetic moment.

Figure 10.7 shows both the observed sunspot number and the five point moving average of the neutrino capture rate [in captures per day, which is linearly related to SNUs via Eq. (10.3)]. The neutrino capture rates were smoothed, following Davis (1987), because the individual measurements constitute a rather noisy data set. The scale of the ordinate for sunspots is arbitrary. For convenience in visualizing the results, Figure 10.7 has the same peak to peak range for spots and ^{37}Ar captures per day. Also for convenience, the sunspot number is plotted so that it decreases from the bottom to the top of Figure 10.7, while the capture rate is plotted in the more conventional way with larger values at the top of the figure.

From J. Bahcall, Neutrino

- SSM excluded ($T_c = 1 \pm .0057$)

- non-standard sm's: T_c = free
  WIMPS, B_{core}, rotation, ...
  - excluded by spectrum (Cl/Kam)
  Bludman, Hata, Kennedy, PL

- neutrino decay: almost excluded
  by SN1987A + reactor
  (possible ν-component loophole)

- electromagnetic moments (Time dependence)
  - not seen by Kam
  - need huge μ_ν

- vacuum and "just so" oscillations
  - need $\sin^2 2\theta \sim 1$
  - (Cl/Kam) Fails except at node ("just so")

- matter-enhanced (MSW) oscillations
  $\nu_e \to \nu_\mu, \nu_\tau$
  $\nu_e \to \nu_s$ (sterile) BHKL

- MSW + non-standard SM
  $\Rightarrow T_c = 1.03^{+.03}_{-.05}$ (90% CL)
  BHKL

- Future experiments

Non-Standard solar models (cool sun)

1000 SSMs: Bahcall, Ulrich

$\varphi(pp) \sim T_c^{-1.2}$, $\varphi(^7Be) \sim T_c^{8}$, $\varphi(^8B) \sim T_c^{18}$

$$R = \frac{Rate}{(Rate)_{SSM}}$$

σ production [correlated between exps.

$$R_{Cl} = .28 \pm .04 = \underbrace{(1 \pm .033)}_{\text{detector cr. sec.}} \big[.775\,(1\pm.10)\,T_c^{18} + .150\,(1 \pm .036)\,T_c^{8} + \text{small Term}$$

$$R_{K\,II+III} = .49 \pm .08 = (1 \pm .10)\,T_c^{18}$$

$$R_{Ga} = .63 \pm .15 = (1 \pm .04)\big[.538\,(1 \pm .0022)\,T_c^{-1.2} + .271\,(1 \pm .036)\,T_c^{8} + .105\,(1 \pm .10)\,T_c^{18} + \text{small}\big]$$

K+Cl: $T_c = .92 \pm .01$, $\chi^2 = 13.78$

$\Rightarrow$ excluded at 99.99%

K+Cl+Ga: $T_c = .92 \pm .01$, $\chi^2 = 15.46$, 99.99%

K+Ga: $T_c = .96 \pm .01$, $\chi^2 = 2.64$, 89%

weaker hypothesis:

K+Cl: $\varphi(^7Be) \sim T_c^{n_{Be}}$, $\varphi(^8B) \sim T_c^{n_B}$

– for $n_{Be} \leq n_B$

$\Rightarrow \chi^2 = 10.21$ [99.9%]

$\Rightarrow$ Cool sun very unlikely

Table I: T_c Fit for Kamiokande II+III, Homestake, and GALLEX

	$T_c \pm \Delta T_c$	χ^2	C.L.
Kam	0.961 ± 0.010	0	—
Cl	0.901 ± 0.015	0	—
GALLEX	0.860 ± 0.042	1.31	—
Kam+Cl	0.921 ± 0.013	13.78	>99.99
Kam+GALLEX	0.960 ± 0.010	2.64	89.26
Cl+GALLEX	0.902 ± 0.013	1.49	77.56
Kam+Cl+GALLEX	0.920 ± 0.012	15.46	99.99

Table II: Best Fit of Kamiokande II+III, Homestake, and GALLEX

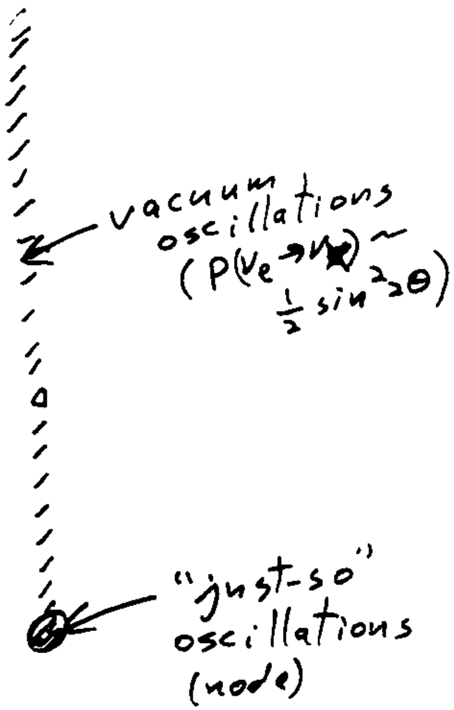
	$T_c = 1$		T_c=free	
	Non-adiabatic	Large-mixing	Non-adiabatic	Large-mixing
$\sin^2 2\theta$	6.7×10^{-3}	0.73	7.7×10^{-3}	0.31
$\Delta m^2(\mathrm{eV}^2)$	6.1×10^{-6}	1.1×10^{-5}	8.9×10^{-6}	1.1×10^{-5}
T_c	1 ± 0.0057	1 ± 0.0057	$1.03^{+[illegible]}_{-[illegible]}$	$1.05^{+[illegible]}_{-[illegible]}$
χ^2	0.56	3.52	0	0

Table III: Best Fit for Oscillations into Sterile Neutrino

	$T_c = 1$		T_c=free	
	Non-adiabatic	Large-mixing	Non-adiabatic	Large-mixing
$\sin^2 2\theta$	9.7×10^{-3}	0.83	7.5×10^{-3}	0.15
$\Delta m^2(\mathrm{eV}^2)$	3.8×10^{-6}	4.8×10^{-6}	4.0×10^{-6}	3.5×10^{-6}
T_c	1 ± 0.0067	1 ± 0.0067	$0.99^{+[illegible]}_{-[illegible]}$	$1.15^{+0.005}_{-0.18}$
χ^2	3.17	8.95	3.13	4.91

vacuum oscillations

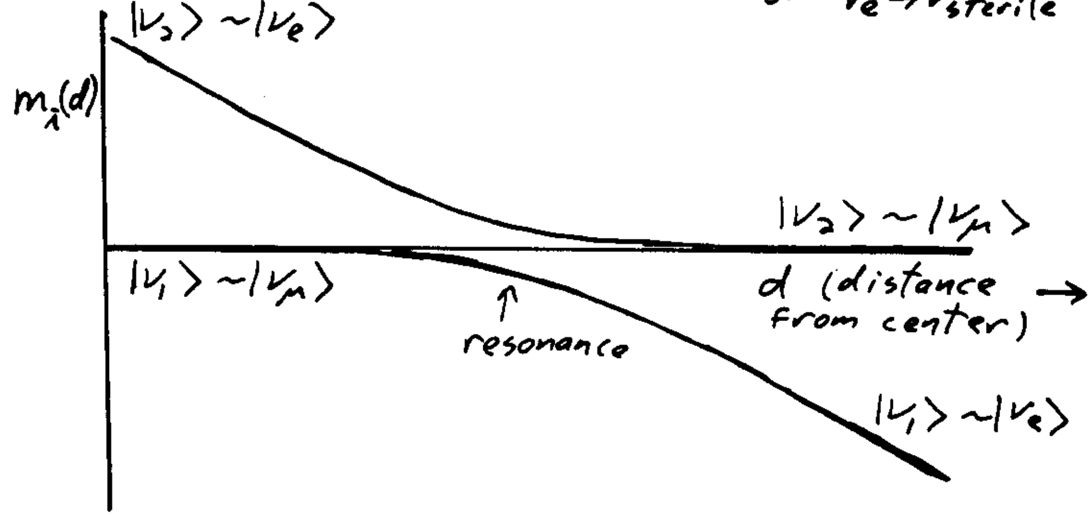
$$P(\nu_e \to \nu_x) \sim \sin^2 2\theta \, \sin^2 \frac{\Delta m^2 L}{4E} \xrightarrow[L \gg \frac{E}{\Delta m^2}]{} \frac{1}{2} \sin^2 2\theta$$



Mikheyev - Smirnov - Wolfenstein (MSW) resonant conversion in matter

- tiny vacuum mixing may be amplified by interaction with matter
- ν_e effective mass $\Rightarrow \nu_e^{eff} - \nu_\mu$ degeneracy $\Rightarrow$ 45° effective mixing

[or $\nu_e \to \nu_\tau$ or $\nu_e \to \nu_{sterile}$



Vacuum:

$$|\nu_e\rangle = |\nu_1\rangle \cos\theta_v + |\nu_2\rangle \sin\theta_v$$

$$|\nu_\mu\rangle = -|\nu_1\rangle \sin\theta_v + |\nu_2\rangle \cos\theta_v$$

$$|\nu(0)\rangle = |\nu_e\rangle$$

$$\Rightarrow |\nu(t)\rangle = \nu_e(t)\,|\nu_e\rangle + \nu_\mu(t)\,|\nu_\mu\rangle$$

$$\Rightarrow P_{\nu_e \to \nu_\mu} = \sin^2 2\theta_v \, \sin^2 \underbrace{\frac{\Delta m^2 L}{4E}}_{\pi L / l_\nu}$$

$l_\nu = \frac{4\pi E}{|\Delta m^2|}$ = oscillation length

$$i\frac{d}{dt}\begin{pmatrix}\nu_e(t)\\ \nu_\mu(t)\end{pmatrix} = M\begin{pmatrix}\nu_e(t)\\ \nu_\mu(t)\end{pmatrix}$$

matter term; charged current $\nu_e e^- \to \nu_e e^-$

$$M = 2\pi\begin{pmatrix}\frac{\Delta m^2}{2E}\cos 2\theta_\nu + \sqrt{2}G_F N_e & -\frac{\Delta m^2}{4E}\sin 2\theta_\nu \\ -\frac{\Delta m^2}{4E}\sin 2\theta_\nu & 0\end{pmatrix} + c\mathbb{1}$$

doesn't affect oscillatio

$$\Delta m^2 = m_1^2 - m_2^2$$

$$\Rightarrow |\nu_e\rangle = |\nu_1^m\rangle\cos\theta_m + |\nu_2^m\rangle\sin\theta_m$$

$$|\nu_\mu\rangle = -|\nu_1^m\rangle\sin\theta_m + |\nu_2^m\rangle\cos\theta_m$$

matter eigenstates

$$\sin^2 2\theta_m = \frac{\sin^2 2\theta_\nu}{\left[1 \pm \frac{2 l_\nu}{l_0}\cos 2\theta_\nu + \frac{l_\nu^2}{l_0^2}\right]}$$

sign of Δm^2

$\nu \to \bar{\nu}$: $\pm \to \mp$

$$l_m = \frac{l_\nu}{\left[1 \pm \frac{2 l_\nu}{l_0}\cos 2\theta_\nu + \frac{l_\nu^2}{l_0^2}\right]^{1/2}}$$

assume θ small

$l_\nu = \frac{4\pi E}{|\Delta m^2|}$ = vacuum osc. length

$\frac{2\pi}{l_0} \equiv \sqrt{2}G_F N_e$

l_m = matter osc. length: $P_{\nu_e\to\nu_\mu} = \sin^2 2\theta_m \sin^2\left(\frac{\pi L}{l_m}\right)$

resonance: $M_{11} = 0 \Rightarrow \frac{l_\nu}{l_0} = \cos 2\theta_\nu$

$(\Delta m^2 < 0 \Rightarrow m_{\nu_e} < m_{\nu_\mu})$

$\Rightarrow \theta_m = 45° \gg \theta_\nu$, $l_m = \frac{l_\nu}{\sin 2\theta_\nu} \gg l_\nu$

resonance: $\frac{l_\nu}{l_0} = \frac{2E}{|\Delta m^2|}\sqrt{2}G_F N_e = \cos 2\theta_\nu$

- but $N_e < N_e^{core}$

$\Rightarrow E(\text{MeV}) > 10^5 \cos 2\theta_\nu |\Delta m^2(\text{eV}^2)|$

For resonanc in sun

adiabatic condition

$$\frac{\Delta(l_\nu/l_0)}{\frac{d}{dr}(l_\nu/l_0)} = \frac{\tan 2\theta_\nu}{\frac{1}{N_e}\frac{dN_e}{dr}} > l_m^{res} = \frac{l_\nu}{\sin 2\theta_\nu}$$

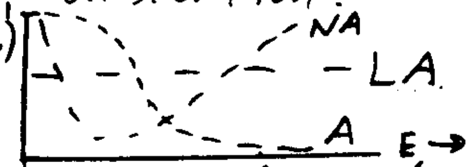
$\Rightarrow E(\text{MeV}) < 2\times 10^8 \sin 2\theta_\nu \tan 2\theta_\nu |\Delta m^2(\text{e}$

- MSW triangle:
  - 3 solutions
  - GUT, interm. seesaw
  - day/night
  - SN 1987A

- $\nu_e \to \nu_{sterile}$ $[N_e \to N_e - \frac{1}{2}N_n$
  - nucleosynthesis limits]
  - no neutral current in Kamio

- spectral distortion:

$P(\nu_e\to\nu_e)$



- no obvious t dependence
(magnetic tubes insufficient)

- Parke Formula (non-adiabatic)

$P_{jump} = e^{-x}$, $x = \pi h \sin^2\theta \frac{\Delta m^2}{E}$

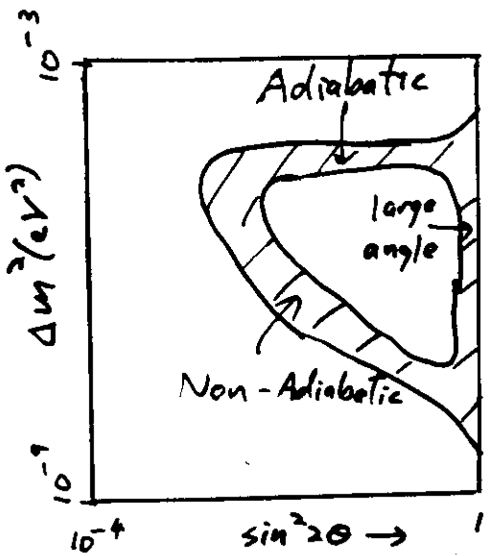
$h = \left|-\frac{1}{n_e}\frac{dn_e}{dr}\right|^{-1}$ = scale heigh

MSW analysis

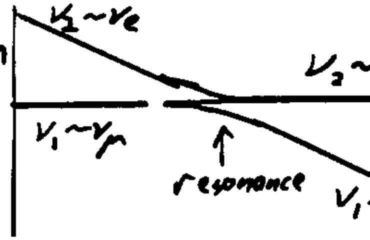
Mikheyev, Smirnov, Wolfenstein

- matter enhanced oscillations from coherent $\nu_e e^-$ scattering

$$i\frac{d}{dt}\begin{pmatrix}\nu_e(t)\\ \nu_\mu(t)\end{pmatrix} = \begin{pmatrix}\frac{\Delta m^2}{2E}\cos 2\theta + \sqrt{2}G_F n_e & -\frac{\Delta m^2}{4E}\sin 2\theta \\ -\frac{\Delta m^2}{4E}\sin 2\theta & 0\end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}$$



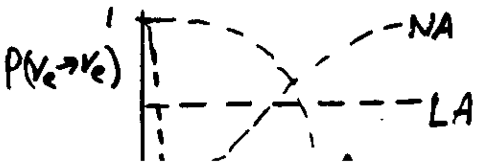
MSW Triangle of fixed survival prob. for a given E



Adiabatic (A) – level crossing: Full conversion if resonance $\Rightarrow$ high energy suppressed (like cool Sun)

Nonadiabatic (NA) – Finite jump probability $\Rightarrow$ low energy suppressed

large angle (LA) – extension of vacuum oscill. in $\sin^2\frac{\Delta m^2 L}{4E} \rightarrow \frac{1}{2}$ limit $\Rightarrow$ equal suppression of all E



MSW + SSM

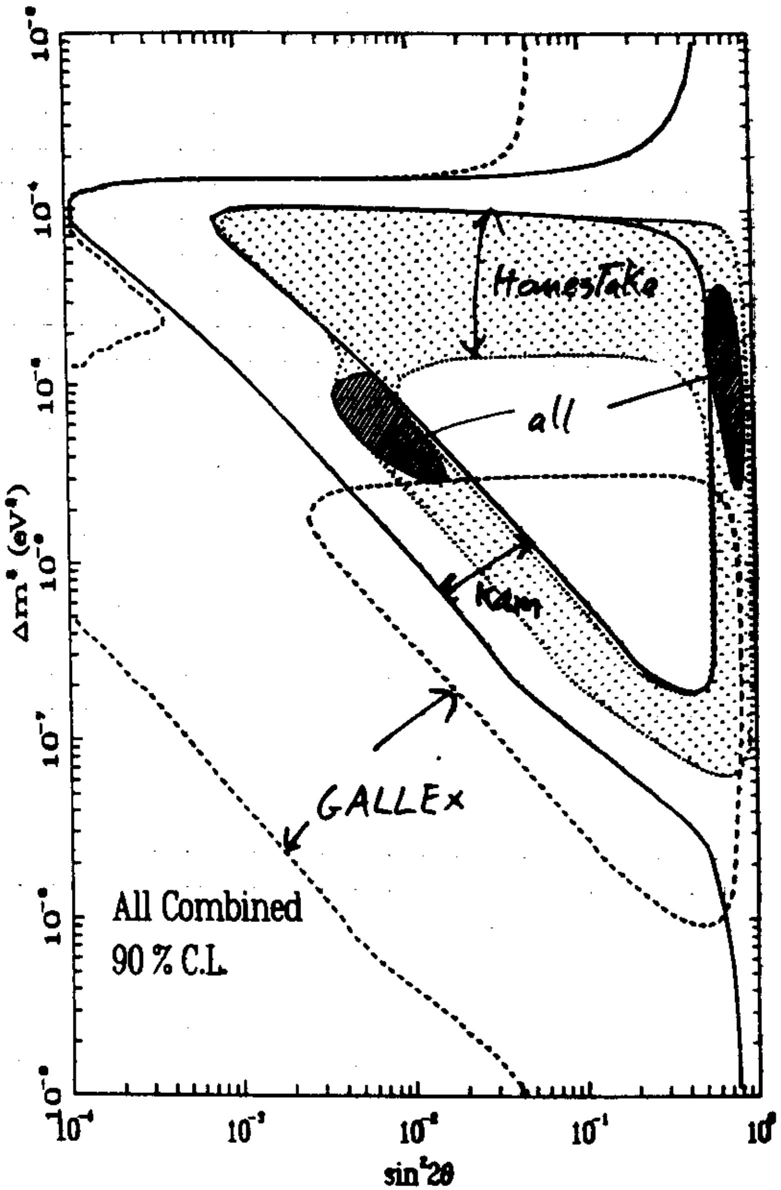
- use Bahcall & Pinsonneault SSM for production distribution $p(r)$ for each flux component, and $n_e(r)$ (and $n_n(r)$ for $\nu_e \rightarrow \nu_s$) for MSW
- Theory errors from
  - SSM
    - (a) $T_c = 1.0 \pm .005 \Rightarrow$ (opacities)
    - b) production cross sections
    - c) detection cross sections
  - MSW uncertainties (small)
    - d) $p(r)$, $n_e(r)$
- Kam. detector resolutions, threshold, effic. and $\nu_\mu e \rightarrow \nu_\mu e$ included (not day-night)

Two solutions (Homestake, Kamiokande, GALL...

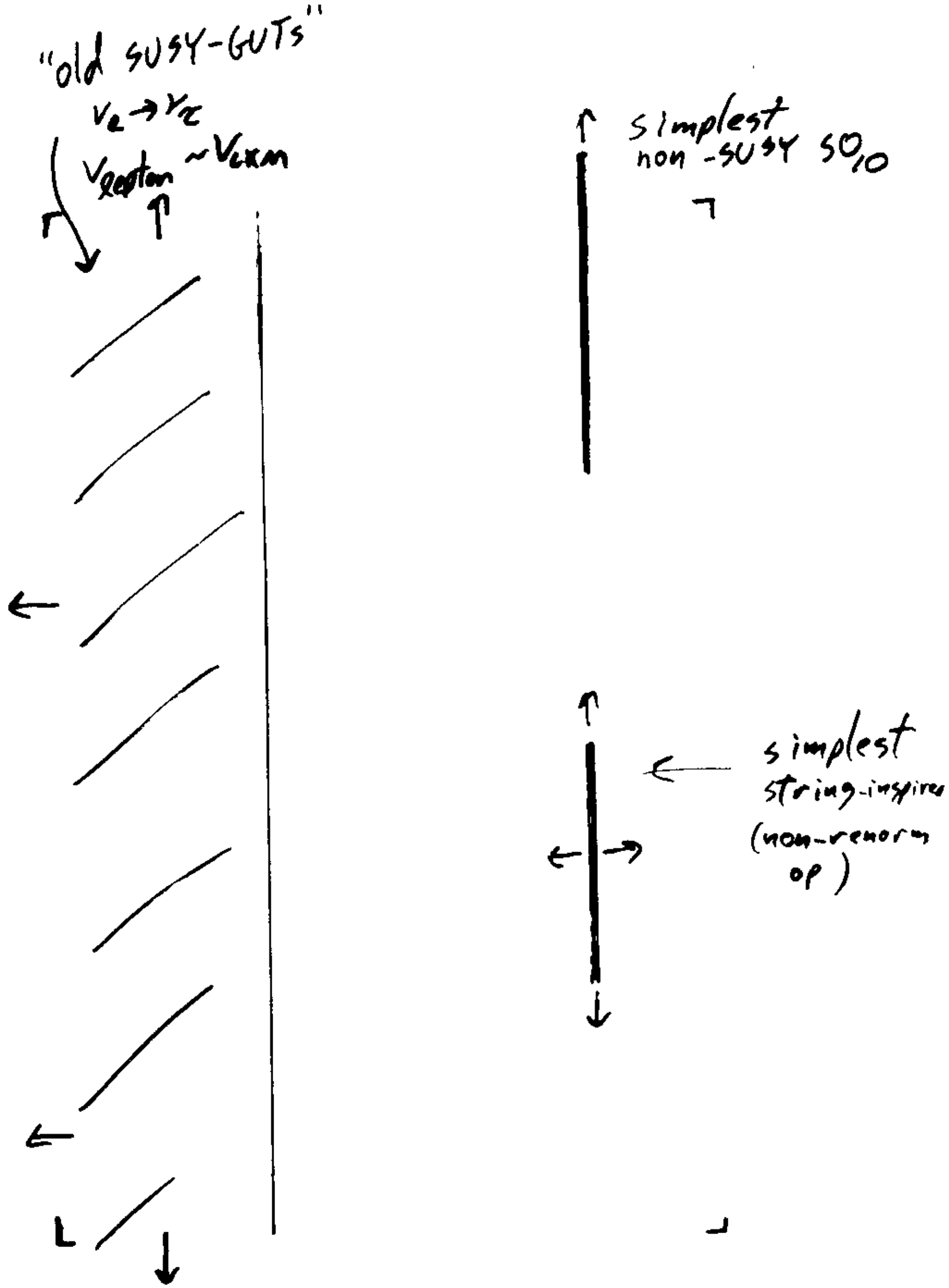
	Non-Adiabatic	large angle
$\sin^2 2\theta$	6.7×10^{-3}	0.73
$\Delta m^2 (eV^2)$	6.1×10^{-6}	1.1×10^{-5}
χ^2	0.56 – good fit	3.5 [Cl/K... – poor fit – within 90% CL for 2df

- General range of Δm^2 consistent with GUT-motivated models
- $V_{lepton} \neq V_{CKM}$
- $\nu_e \rightarrow \nu_s$ (motivated by 17 keV):
  - $n_e \rightarrow n_e - \frac{1}{2}n_n$
  - no $\nu_e \rightarrow \nu_e$

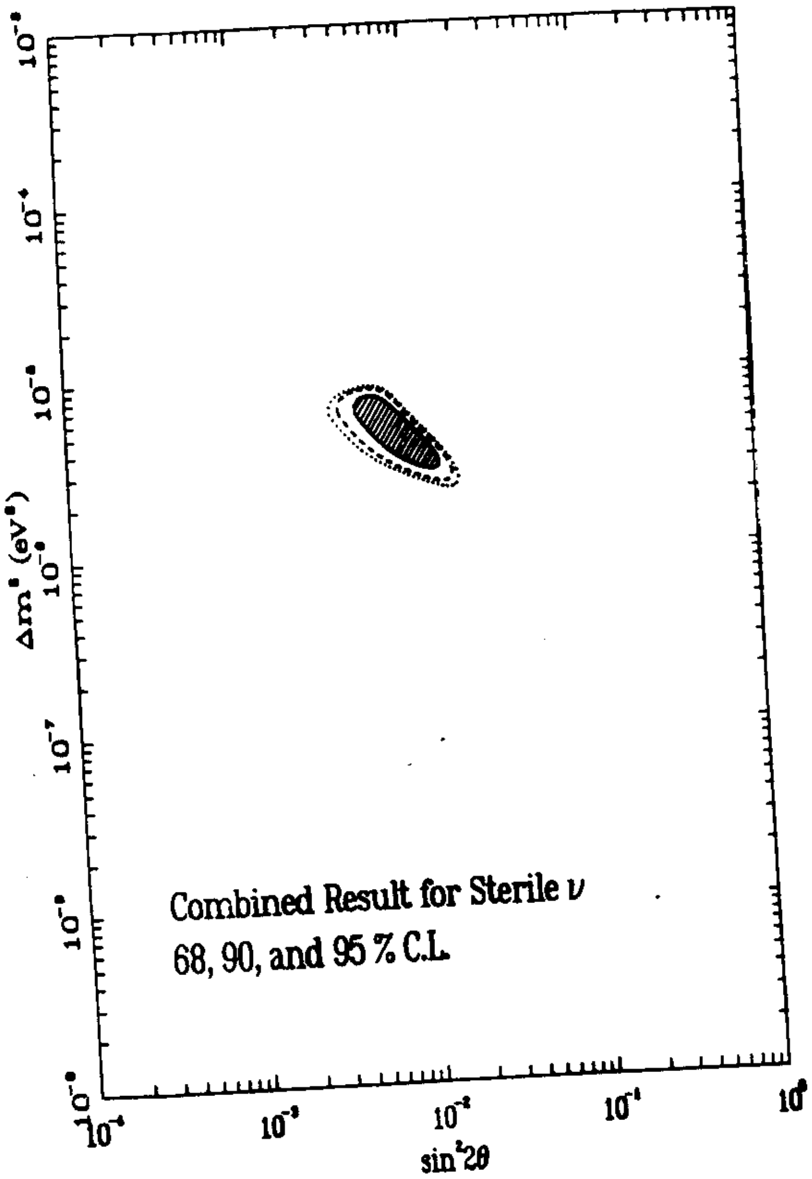
  non-adiabatic $\chi^2 = 3.2$, 0... large angle: ...



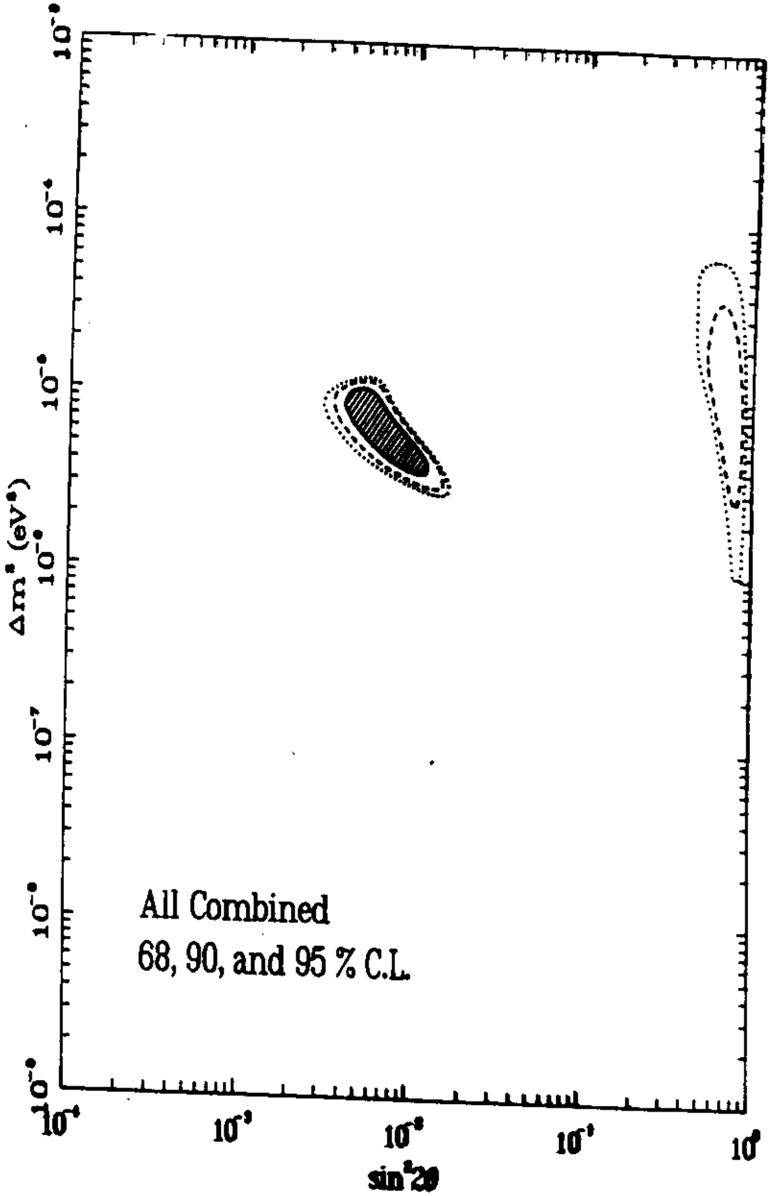
Homestake
all
Kam
GALLEx
All Combined
90 % C.L.
Δm² (eV²)
sin²2θ



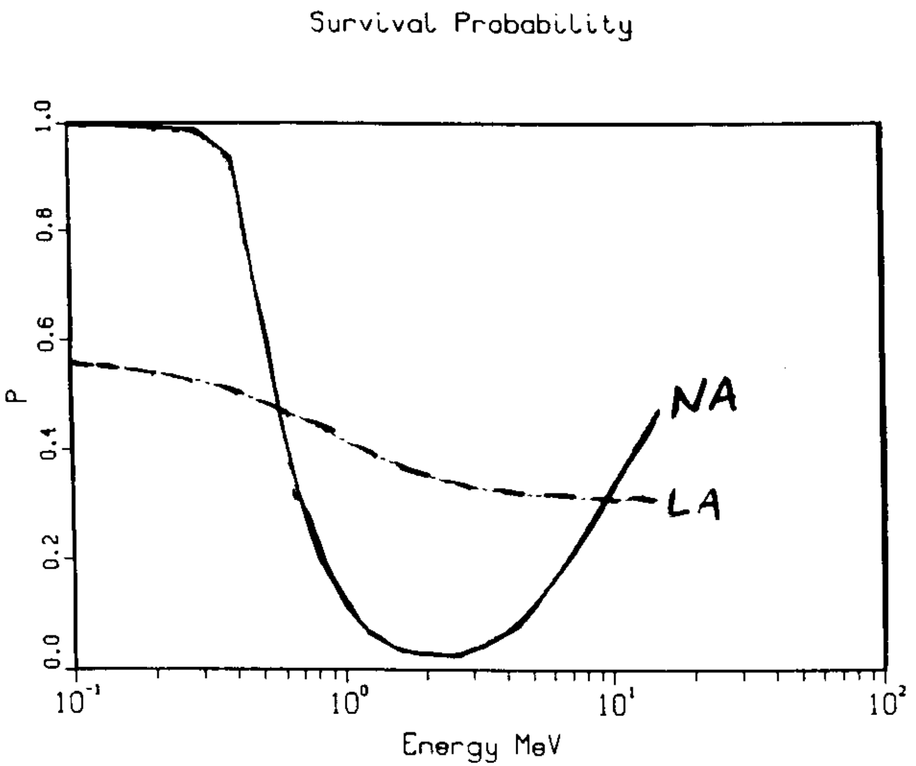
"old SUSY-GUTs"
νe → ντ
Vlepton ~ VCKM
simplest
non-SUSY SO10
simplest
string-inspired
(non-renorm
op)



Combined Result for Sterile ν
68, 90, and 95 % C.L.
Δm² (eV²)
sin²2θ



All Combined
68, 90, and 95 % C.L.
Δm² (eV²)
sin²2θ



MSW + non-standard solar model

- allow T_c = free
- same nuclear physics unc. as SSM case
- 3 parameters: T_c, $\sin^2 2\Theta$, Δm^2

$\Rightarrow$ $T_c = 1.03^{+.03}_{-.05}$ [90% CL]

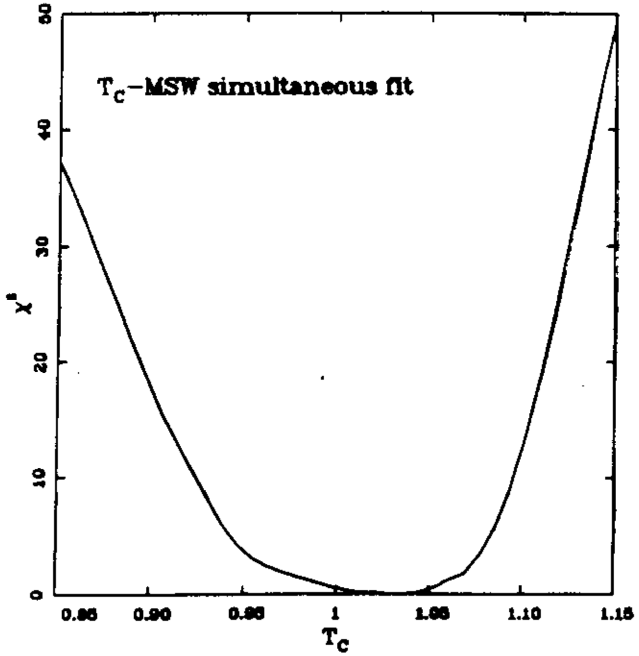
- T_c "measured" by solar ν exps, even allowing MSW
- consistent with SSM prediction $T_c = 1 \pm .0057$

- MSW parameter range broadened

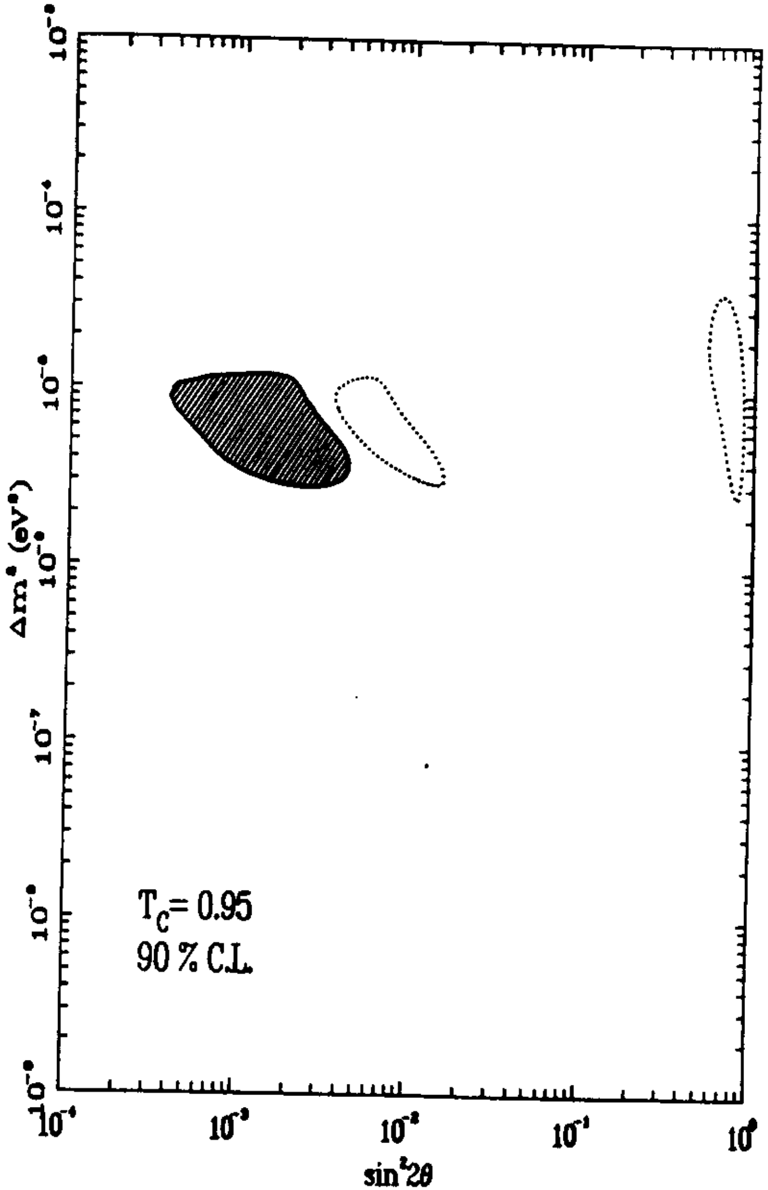
low $T_c \Rightarrow$ $\begin{cases} \text{smaller } \sin^2 2\Theta \text{ for NA} \\ \text{LA disappears} \end{cases}$

high $T_c \Rightarrow$ move to inside of MSW Triangle

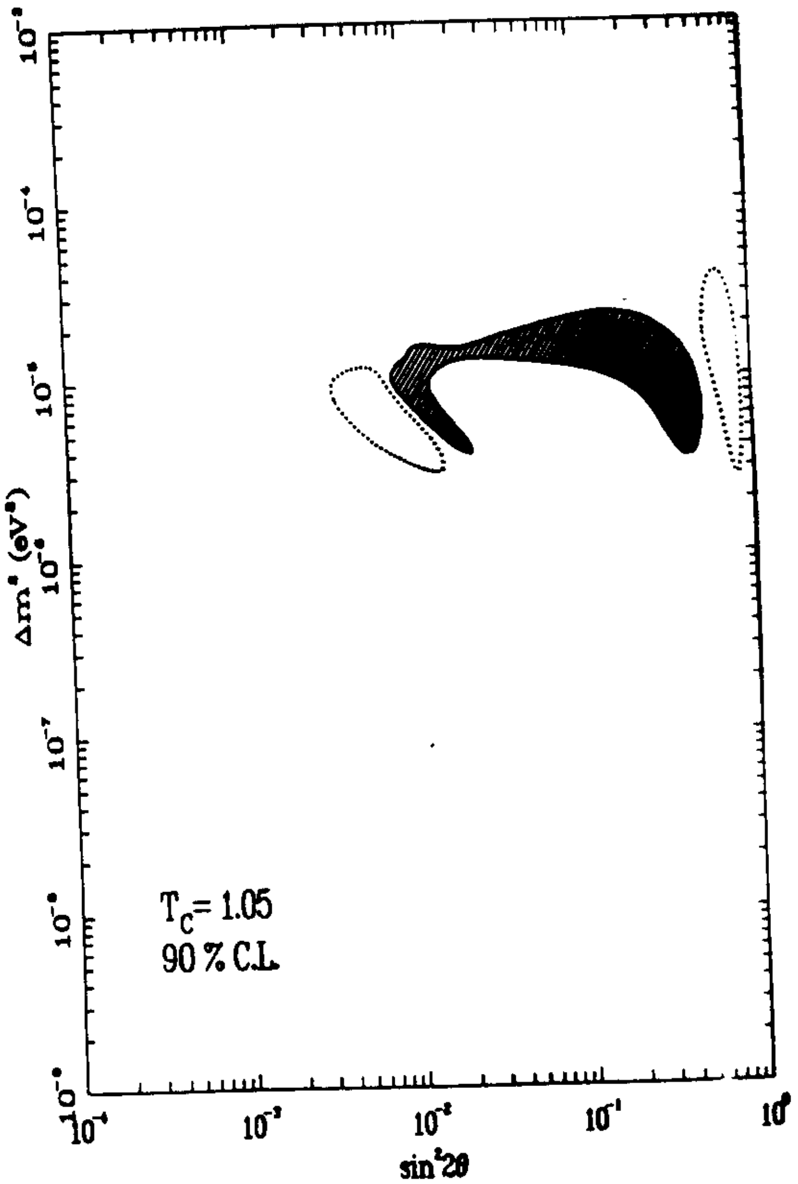
(two regions merge)



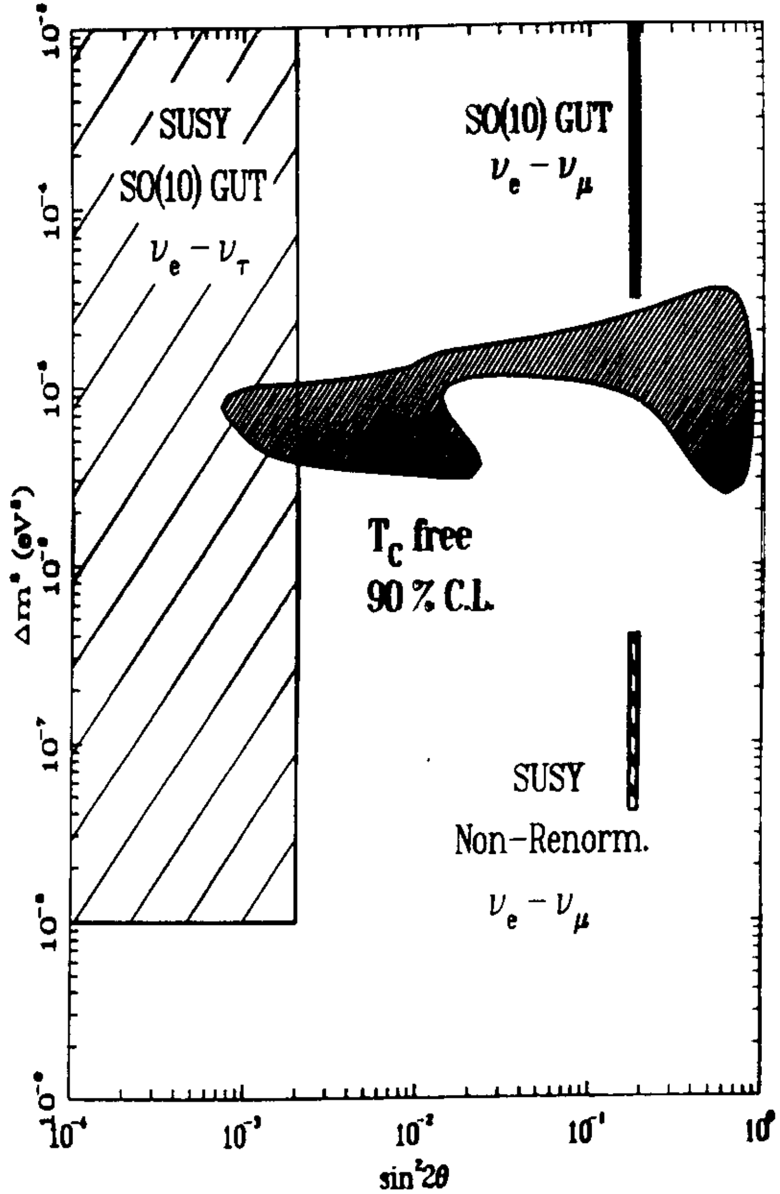
T$_C$–MSW simultaneous fit
χ^2
T$_C$
0.85 0.90 0.95 1 1.05 1.10 1.15
0 10 20 30 40 50



Δm^2 (eV^2)
$\sin^2 2\theta$
T$_C$= 0.95
90 % C.L.



$T_C = 1.05$
90 % C.L.
Δm^2 (eV^2)
$\sin^2 2\theta$



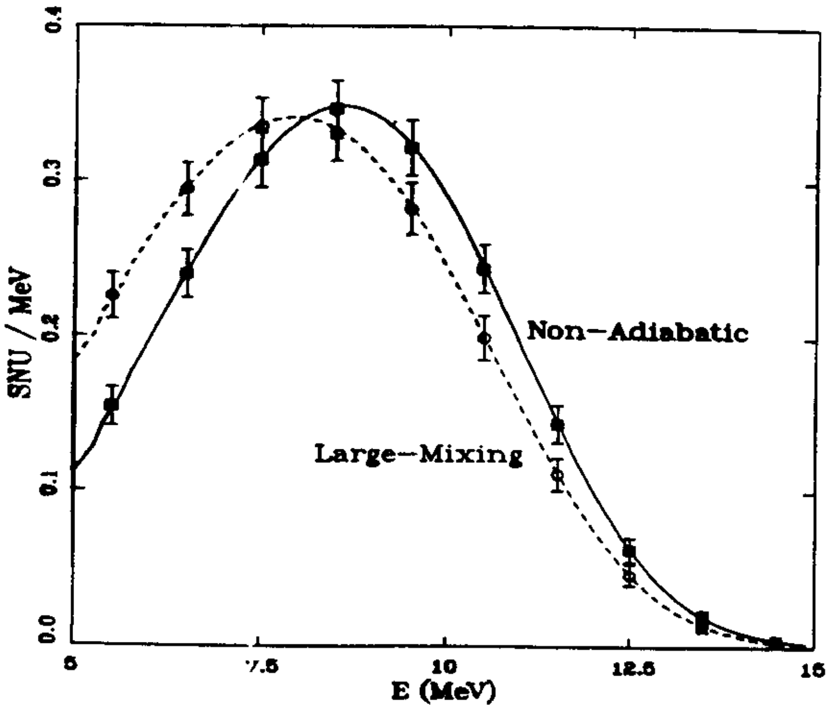
SUSY
SO(10) GUT
$\nu_e - \nu_\tau$
SO(10) GUT
$\nu_e - \nu_\mu$
T_C free
90 % C.L.
SUSY
Non-Renorm.
$\nu_e - \nu_\mu$
Δm^2 (eV^2)
$\sin^2 2\theta$

SNO CC spectrum, 2yr running

[CC $\nu_e D \to e^- pp$

NC $(\nu_e, \nu_\mu, \nu_\tau) D \to \nu n p$ $[n + {}^{35}Cl \to \gamma]$

also $\nu e^- \to \nu e^-$]



SNO can establish N-A MSW by spectral distortion

[also NC for $\nu_e \to \nu_\mu, \nu_\tau$]

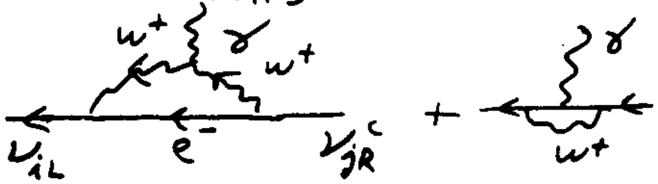
Large-mixing $\simeq$ cool sun

Table IV: Predicted Rates for Future Detectors

	Rate / SSM	
	Non-adiabatic	Large-mixing
SNO (Charged Current)	0.15 - 0.5	0.15 - 0.3
SNO ($\nu - e$ scattering)	0.3 - 0.6	0.3 - 0.45
Super-Kamiokande	0.3 - 0.6	0.3 - 0.45
Borexino ($^7Be\nu - e$)	0.15 - 0.6	0.3 - 0.6

Electromagnetic Moments

Majorana:



(or new physics)

$\Rightarrow \mu_{ij}\ \bar{\nu}_{iL}\sigma^{\mu\nu}\nu_{jR}^c F_{\mu\nu} + H.C.$ [also electric dipole

- Can show $\mu_{ij} = -\mu_{ji} \Rightarrow$ transition moment only for Majorana
- diagonal and transition for Dirac (diagonal = transition between Majorana components)

Solar Neutrinos:

- Time dependence (anti-correlation with sun-spots) claimed by Homestake

Suggests $\nu_{iL} \to \nu_{jR}^c$ [preferred: MSW resonance

or $\nu_L \to \nu_R$ [MSW suppressing

in sun

- not confirmed by Kamiokande

Could reconcile with:

...kata, ...unakawa; ..., Mohapatra, Rothstein; ...hmedov; ...zuki, Mori, Numata, Oyama; ...kugita, Yanagida; ..., Mori, Oyama, Suzuki

(a) Combined flavor + magnetic effects $\Rightarrow$ low energy more affected

(b) $\nu_{eL} \to \nu_{\mu R}^c \to \nu_{eR}^c$] interacts in Kam. [but $E_e > 7.5$ MeV $\Rightarrow$ fails – Rothstein

(c) $\nu_{eL} \to \nu_{eR}$] interaction in Kam via μ or new int [but $N_\nu'' = 4$]

- Okun et al: need $\mu_\nu \sim (3-10)\times 10^{-11}\mu_B$ for solar ν's

- standard model for Dirac ν_e

Lee, Shrock, Fujikawa

$$\mu_\nu = \frac{3G_F m_\nu m_e}{4\pi^2\sqrt{2}}\mu_B \sim 3\times 10^{-19}\left(\frac{m_\nu}{1\,eV}\right)\mu_B$$

Limits:

lab: $\mu_{\bar{\nu}_e} < 4\times 10^{-10}\mu_B$

$\mu_{\nu_\mu} < 9.5\times 10^{-10}\mu_B$

Red giants (Raffelt)

$\mu_\nu < \begin{cases} 2\times 10^{-12}\mu_B & \text{Dirac transition} \\ 3\times 10^{-12}\mu_B & \text{Majorana, Dirac diagonal} \end{cases}$

stellar cooling $(\gamma \to \nu\bar{\nu})$ $\mu_\nu < 1.1\times 10^{-11}\mu_B$ Fukugita, Yazaki; Raffelt, Dearborn

nucleosynthesis $\mu_\nu < 0.5\times 10^{-10}\mu_B$

SN1987A $\mu_\nu < (10^{-13} - 10^{-12})\mu_B$] Dirac only

Matter suppression

$$\frac{d}{dt}\begin{pmatrix}\nu_{eL}(t)\\ \nu_{eR}(t)\end{pmatrix} = 2\pi \begin{pmatrix} \sqrt{2}\,G_F\,(N_e - \underbrace{\tfrac{1}{2}N_n}_{\text{neutral current}}) & \mu B \\ \mu B & 0 \end{pmatrix}\begin{pmatrix}\nu_{eL}(t)\\ \nu_{eR}(t)\end{pmatrix}$$

+Kmeder

im, Marciano

[like MSW, only $m_1 = m_2$, $N_e \to N_e - \frac{1}{2}N_n$,

$-\frac{\Delta m^2}{4E}\sin 2\theta_v \to \mu B$]

in vacuum, $N_e = N_n = 0 \Rightarrow \nu_{eL}, \nu_{eR}$ degenerate

$$\Rightarrow P_{\nu_{eL}\to\nu_{eR}}(t) = \sin^2(\mu B t)$$

- in matter (constant density)

$$P_{\nu_{eL}\to\nu_{eR}}(t) = \frac{(2\mu B)^2}{a_{\nu_e}^2 + (2\mu B)^2}\sin^2\left[\left(a_{\nu_e}^2 + (2\mu B)^2\right)^{\frac{1}{2}}\frac{t}{2}\right]$$

$a_{\nu_e} = \sqrt{2}\,G_F\left(N_e - \frac{1}{2}N_n\right)$

- effect suppressed unless $2\mu B \gtrsim a_{\nu_e}$
- marginally satisfied for $\begin{cases}\mu \sim (3-10)\times 10^{-11}\mu_B \\ B \sim 10^3\,G\end{cases}$

Alternative: transition moment (Dirac or Majorana)

Dirac: $\mathcal{L}_{eff} \sim \mu_{ij} F_{\alpha\beta}\,\bar{\psi}_{iL}\,\sigma^{\alpha\beta}\psi_{jR} + h.c.$

$i = j \Rightarrow$ moment

$i \neq j \Rightarrow$ transition moment

$\nu_{eL} \to \underset{\uparrow}{\nu_{\mu R}}$ or $\nu_{eL} \to \underset{\uparrow}{\nu_{\tau R}}$

sterile, SN1987A limits still valid

Majorana:

$$\mathcal{L}_{eff} \sim \mu_{ij} F_{\alpha\beta}\,\bar{\psi}_{iL}\,\sigma^{\alpha\beta}\psi^c_{jR} + h.c.$$

$$\psi^c_{jR} = C\left(\overline{\psi_{jL}}\right)^T \quad \text{(vanishes for } i = j\text{)}$$

$\Rightarrow \nu_{eL} \to \underbrace{\nu^c_{\mu R} \text{ or } \nu^c_{\tau R}}_{\text{normal weak interactions}}$

$\Rightarrow$ no SN1987A (or nucl.) bounds

- can have resonant enhancement

$$\frac{d}{dt}\begin{pmatrix}\nu_{eL}\\ \nu_R\end{pmatrix} = 2\pi\begin{pmatrix}\frac{\Delta m^2}{2E} + \sqrt{2}\,G_F\left(N_e - \frac{1}{2}N_n\right) & \mu_{12} B \\ \mu_{12} B & c\end{pmatrix}\begin{pmatrix}\nu_{eL}\\ \nu_R\end{pmatrix}$$

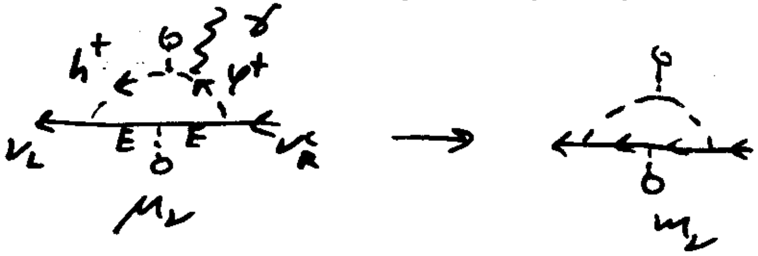
$c = 0$, Dirac

$c = \sqrt{2}\,G_F N_n/2$, Majorana

- resonance for $\frac{\Delta m^2}{2E} = -\sqrt{2}\,G_F\left(N_e - \frac{1}{2}N_n\right)$
- similar to MSW, only $-\frac{\Delta m^2 \tan 2\theta_v}{2\mu E}$

Models for enhanced μ_ν [epi-Higgs]

- $\nu_R^c \to \nu_L \Rightarrow$ charged Higgs, $SU_{2L} \times SU_{2R} \times U_1$, SUSY, mirror Fermions, ...
- most models: removing γ generates too large m_ν (Fine-tuning needed)

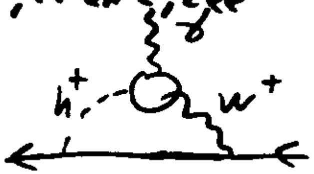


Voloshin symmetry [see Vysotsky reviews]

- SU_ν symmetry between $\nu_{eL}, \nu_{\mu R}^c$
  - mass term: SU_ν triplet
  - μ_ν: SU_ν singlet

  $\Rightarrow m_\nu$ suppressed by SU_ν-breaking (cancellation of diagrams)

Barr, Freire, Zee



- induced $W h \gamma$ vertex
- no corresponding mass term

17 KeV

$\nu_e \sim \nu_1 + \theta \nu_3$

$m_3 \sim 17$ KeV

$\theta^2 \sim .0085$

Caldwell, PL

$\nu_3 \neq \nu_\mu$ (oscillations)

$\nu_3 \neq \nu_{\text{Fourth Family}}$ [LEP: $N_\nu' = 3.04 \pm .04$]

$\nu_3 \neq \nu_s$ [$N_\nu'' < 3.3$]

$\Rightarrow \nu_3 \simeq \nu_\tau$

- ν_τ probably not Dirac with sterile partner: SN1987A pulse length [new conflicting calcs: Fuller et al, Burrows et al]
  - new interactions to trap would violate $N_\nu'' < 3.3$

    loopholes:
    - resonance (Cline)
    - leptoquark (Babu et al)
  - Fast decays harder to arrange than Majorana

    $[\nu_\tau \to \nu_e F \Rightarrow \tau \to e f]$
- ν_τ not pseudo-Dirac with sterile partner unless $\Delta m < 10^{-7} - 10^{-11}$ eV [$N_\nu'' < 3.3$]

Scenarios/Conclusions

- several hints of m_ν:
  Theory, solor, atm. $\frac{\nu_\mu}{\nu_e}$, 17 KeV, HDM

- Solar
  - MSW strongly Favored over cool Sun
  - SSM+MSW: - nonadiabatic & large angle solutions
    - Δm^2 consistent with simple GUT/string seesaws, but $V_{lepton} \neq V_{CKM}$
  - nonstandard SM + MSW:
    - $T_c = 1.03^{+.03}_{-.05}$
    - broader $\sin^2 2\Theta$ range
  - ν_τ could be cosm. relevant ($\Rightarrow \nu_e \leftrightarrow \nu_\tau$ osc.)

- Other possibilities need more complicated models

- atmospheric ν_μ/ν_e deficit
  - $\nu_\mu \to \nu_\tau$ (or $\nu_\mu \to \nu_e$) with large mixing

  Scenario: $\begin{cases} \nu_e \to \nu_\mu, \text{ in sun} \\ \nu_\mu \to \nu_\tau, \text{ atmospheric} \\ \nu_\tau \text{ not cosm. relevant} \end{cases}$

17 KeV - $\nu_{17} \sim \nu_\tau$
- $\nu_e \to \nu_\tau$ observable
- Two solns: (Majorana) $\begin{cases} m_{\nu_\mu} \sim m_{\nu_\tau}, \ L_e - L_\mu + L_\tau \text{ conserved} \\ m_{\nu_\mu} > m_{\nu_\tau}, \ \nu_\mu \to \nu_\tau \text{ observable} \end{cases}$

[Dirac option if SN1987A or N_ν'' constraints relaxed; conflicting calcs?]

- for $m_{\nu_\tau} - m_{\nu_\mu} \sim 10^{-7}$ eV, could have atm. osc. also
- solar MSW: need $\nu_e \to \nu_s$ ad hoc light sterile
- little room for HDM
  [Caldwell loophole: $m_{\nu_e} \sim m_{\nu_s} \sim 3$ eV
  $m_{\nu_s} - m_{\nu_e} \sim 10^{-6}$ eV
  $\sin^2 2\Theta \sim 1$
  [solar, N_ν'' marginal]]

- Lab osc. exps ($\nu_e \to \nu_\tau$, $\nu_\mu \to \nu_\tau$, $\nu_\mu \to \nu_x$) extremely important

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SMR.626 - 23
(Lect. IV)

SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

15 June - 31 July 1992

NON-PERTURBATIVE ELECTROWEAK THEORY

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Electroweak B+L Violation at High T and Making the Baryon Asymmetry at $T \sim 100$ GeV

Outline of the Lectures:

I Anomalies and Level Crossing

II Topology, Vacua + Energy Barriers

A. O(3) sigma model example
B. Electroweak Theory
C. Topology, Sphalerons + Instantons

III Transition Rates

A. How to compute rates at
  i) Low $T \ll M_W(T)$
  ii) Intermediate $M_W(T) \ll T \ll M_W(T)/\alpha_W$
  iii) High $T \gg M_W(T)/\alpha_W$

B. Results of computations
C. Apparent paradoxes
  i) Making a fish at high T
  ii) Contradictions with instantons

IV Baryogenesis + Cosmology

A. Sakharov Conditions
B. General Scenario
C. Phase transitions in EW theory

V Explicit Examples

A. B generation in the wall
B. Scattering from the wall
C. 2 Higg's doublet model

VI Dynamics of the Wall

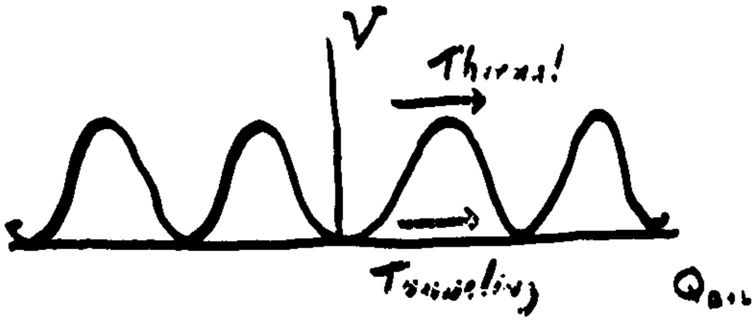
A. Shape of the wall
B. Damping of the wall
C. Hydrodynamic Instabilities

Brief Introduction:

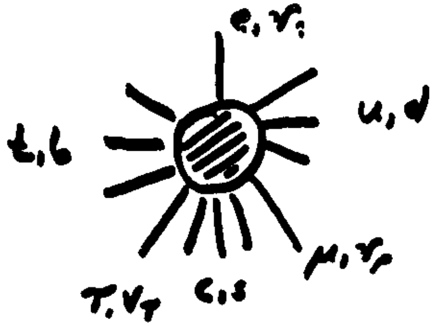
Until 5 yrs. ago, was believed that
$\Delta B \neq 0$ dynamics only 'useful'
for $E \sim 10^{15}\,\text{GeV} \sim E_{GUT}$

But, in EW theory $\exists$ anomaly

$$\Rightarrow \quad \partial_\mu J^\mu_{B+L} \neq 0$$



't Hooft computed low energy tunneling rate



EW theory knows about doublets,
but not about color, family
or lepton vs quark

Compute rate: Need WKB tunneling solution

Tunneling ⇒ forbidden region

$$e^{iEt} \rightarrow e^{-Et}$$

Euclidean solutions of Yang-Mills equations ⇒ 't Hooft-Belavin-Polyakov Instanton

Solution is known for EW theory

(Technical complication: must constrain size of instanton and in end integrate over all sizes)

Solution of form

$A^a_\mu(x,\rho)$, $\phi(x,\rho)$

$\begin{pmatrix} e(x,\rho) \\ \nu(x,\rho) \end{pmatrix}$ = zero modes

Can compute

$\langle \psi_1 \cdots \psi_N \rangle \sim e^{-S}$

S = WKB action

S from classical action, classical field is associated with $\sim 1/\alpha_W$ quanta

$S \sim 1/\alpha_W$

$\Gamma/V \sim e^{-4\pi/\alpha_W} \sim 10^{-173}$

$\Gamma/V \sim 10^{-173}$

Space-time volume of universe:

$t \sim 10^{10}\,yr \sim 10^{17}\,sec$

$d \sim 10^{17}\,sec \cdot c \sim 10^{26}\,m \sim 10^{41}\,Fm \sim 10^{43}\ 1/M_W$

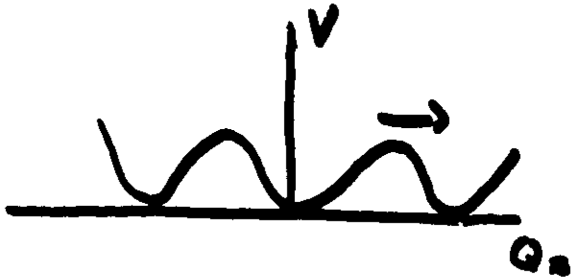
$Vt \sim 10^{173}\ 1/M_W^4$!!

$\#\,\text{protons}/V \sim 1\,m^{-3} \sim 10^{-54}\ M_W^{3}$

Probability of single p decay $\sim 10^{-54}$; but need $\Delta B = 3 \Rightarrow$ Never!

Even if rate was big, $\Delta B = 3$ would be hard to see experimentally (Big mass $\sim$ GUT rate)

What about $T \neq 0$ or transitions over top of barrier?
Unsuppressed?



Early 'conjectures'
Linde; Polyakov; Dimopoulos + Susskind

Dropped because

1. 't Hoofts calculation
2. All $\Delta B \neq 0$ processes $e^{-4\pi/\alpha_w}$
3. No known path over top of barrier
4. Instantons at high T can be computed and $S > 4\pi/\alpha_w$

Problem dropped until mid 80's when

Manton: Argued there is a finite action path over the top of the barrier

Manton + Klinkhammer: Computed height of barrier using sphaleron

Sphaleron: Classical, static solution of equations of motion which is unstable under a small perturbation

9

Kuzmin - Rubakov - Shaposhnikov:

Strong arguments that rate

$$\sim e^{-E_{sp}/T}$$

and $e^{-E_{sp}/T} \to 0$ as $T \to \infty$

Arnold - McLerran

Argued rate computable $M_W(T) \ll T \ll \frac{M_W(T)}{\alpha_W}$

Computed and showed

$R_{\Delta B \neq 0} / R_{exp} \sim 10^9$ for $T \approx 100$ GeV

Resolved instanton 'paradoxes'

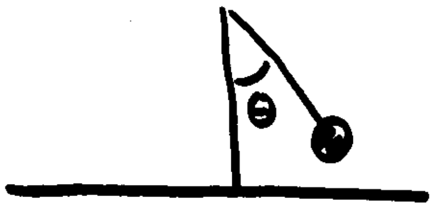
Grigoriev, Rubakov, Shaposhnikov

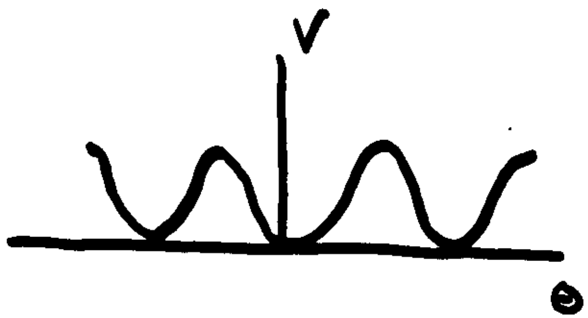
Convincing results from numerical simulations

10

An example to convince you that instantons are not relevant for high T barrier crossing

The Quantum Mechanical Pendulum





At high T, prob $e^{-E/T}$ to be in state with $E \geq T \gg V_{barrier}$

probability big that in state which freely crosses over top of barrier

To understand how finite T instantons work:

Review of finite T formalism

Remember

Hamiltonian $\Rightarrow$ Path Integral

$$\lim_{t\to\infty} \langle out| e^{-itH} |in\rangle$$

$$\to \int [d\psi]\, e^{i\int_{-\infty}^{\infty} dt\, \mathcal{L}(\psi)}$$

Finite T:

$$Tr\, e^{-\beta H} \to \int_{\psi(\beta)=\psi(0)} [d\psi]\, e^{-\int_0^\beta dt\, \mathcal{L}(\psi)}$$

Bosons: Periodic under $t \to t+\beta$

Fermions: $\psi(t+\beta) = -\psi(t)$ (harder to derive!)

$t \to it \Rightarrow$ Euclidean space

Vector fields: Analytic continuation $A^0 \to -i A^0$

$g^{\mu\nu} \to \delta^{\mu\nu}$, $\gamma^0 \to -i\gamma^0$

Euclidean space formulation of pendulum:

$$Q^{TOP} \equiv \int_0^\beta dt\, \dot{X}(t) = X(\beta) - X(0)$$

$$A^{INST} \sim e^{-S_{INS}}$$

$$S = \int_0^\beta dt\ \tfrac{1}{2}\dot{X}^2 + V(X)$$

$$\xrightarrow[T\to\infty]{} \int_0^\beta dt\, \tfrac{1}{2}\dot{X}^2$$

since in time $1/T$, X must change by multiple of 2π but V finite

$$X_{INST} = \frac{2\pi t}{\beta} = 2\pi t T$$

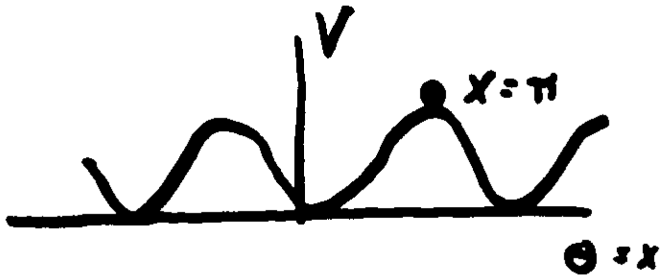
$$S = 2\pi^2 T \xrightarrow[T\to\infty]{} \infty$$

Instantons not relevant at high T.

Nevertheless

$$\text{Rate} \sim e^{-V_{barrier}/T} \sim e^{-E_{sp}/T} \to 1$$

as $T \to \infty$



Sphaleron:

$\chi = \pm \pi$

It is unstable

It is on a finite action, continuous path of deformations of χ: $0 \le \chi \le 2\pi$

$$V_{barrier} = V(\pi)$$

Rate is big.

The U(1) Anomaly:

I will show that

$$\partial_\mu J^\mu_{B+L} \neq 0$$

in Electroweak Theory and that in Euclidean space is related to a topological charge

Later, will argue relation to simple fermion energy level crossing picture

We will consider the Euclidean path integral

$$\int [dA \, d\bar{\Psi} \, d\psi] \, e^{-S[\bar{\Psi},\psi,A]}$$

$[dA] = \prod_{x,\mu,a} dA^\mu_a(x)$ etc.

$$S = \int d^4x \, \bar{\Psi}[-i\not{D} + M]\psi + \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu}$$

$$D = \partial - ig\,\tau \cdot A$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

Now, we use Fujikawa's trick:

The path integral must be invariant under a change of variables. Consider

$$\psi(x) \to e^{i\gamma_5\alpha(x)}\psi(x)$$

$$\bar{\psi}(x) \to \bar{\psi}(x)\, e^{i\gamma_5\alpha(x)}$$

(We will first derive axial U(1) anomaly & later argue case for B+L anomaly)

There will be two compensating changes:

Non-invariance of fermion measure:

$$\delta[d\bar{\psi}d\psi]$$

Non-invariance of action:

$$\delta S = \partial_\mu\alpha(x)\,\bar{\psi}\gamma^\mu\gamma_5\psi$$
$$+\,2iM\alpha(x)\,\bar{\psi}\gamma_5\psi$$

(α infinitesimal)

Consider $\delta[d\bar{\psi}d\psi]$

Recall that for Grassman valued variables:

$$a_i^2 = 0\ ; \quad \{a_i, a_j\} = 0$$

$$\int da_i = 0 \qquad \int da_i\, a_i \neq 0$$

$\therefore$ Only non-zero N-dimensional Grassman valued integral is

$$\int da_1 \cdots da_N\; a_1 \cdots a_N$$

Under $a_i' = \sum_j C_{ij} a_j$, integral must remain invariant. Note that

$$a_1' \cdots a_N' = \det C\;\, a_1 \cdots a_N$$

$\therefore$

$$[da'] = [\det C]^{-1}\,[da]$$

Now for an infinitesimal chiral rotation

$$C \simeq [1 + i\alpha(x)\gamma^5]\,\delta(x-y)$$

$$\ln \det{}^{-1} C = \exp \operatorname{tr} \ln C^{-1}$$

$$= \exp \operatorname{tr} \ln \frac{1}{1+i\alpha\gamma_5}$$

Evaluation of tr is UV singular.
Regulate in gauge invariant way

$$\not{D}\,\phi_N = \lambda_N \phi_N$$

$$\det C^{-1} = \exp -i \int d^4x\, \alpha(x) \sum_N \phi_N^\dagger(x)\,\gamma_5\,\phi_N(x)$$

Sum over N is singular: Regulate

$$\sum_N \phi_N^\dagger(x)\,\gamma_5\,\phi_N(x) = \lim_{M\to\infty} \sum_N \phi_N^\dagger(x)\,\gamma_5\,\phi_N(x)\, e^{-\lambda_N^2/M^2}$$

$$= \operatorname{tr} \gamma_5\, e^{-\not{D}^2/M^2} = \lim_{\substack{M\to\infty \\ x\to y}} \int \frac{d^4k}{(2\pi)^4}\, e^{ik\cdot(x-y)}$$

$$\operatorname{tr} \gamma_5 \exp\{-(D^2 + \tfrac{1}{4} g\,[\gamma^\mu,\gamma^\nu]\,\tau\cdot F_{\mu\nu})/M^2\}$$

$$\ln \det{}^{-1} C = \lim_{\substack{M\to\infty \\ x\to y}} \int \frac{d^dk}{(2\pi)^d}\, e^{ik\cdot(x-y)}$$

$$\operatorname{tr} \gamma_5 \exp\{(-D^2 + \tfrac{1}{4} g\,[\gamma^\mu,\gamma^\nu]\,\tau\cdot F_{\mu\nu})/M^2\}$$

In 1+1 dimensions $\gamma^0 = \sigma^1,\ \gamma^1 = \sigma^2,\ \gamma^5 = \sigma^3$

First non-vanishing term is

$$\sum_N \phi_N^\dagger\,\gamma_5\,\phi_N = \lim_{M\to\infty} \tfrac{1}{4} g\, \operatorname{tr} \gamma_5\, [\gamma^\mu,\gamma^\nu]\,\tau\cdot F_{\mu\nu}$$

$$/M^2 \int \frac{d^2k}{(2\pi)^2}\, e^{-k^2/M^2} = -g/4\pi\, \operatorname{tr} \tau\cdot \epsilon^{\mu\nu} F_{\mu\nu}$$

$$= -g/4\pi\ \epsilon\cdot F_{u(1)}$$

— — — — — —

In 4-d

$$\sum_N \phi_N^\dagger\,\gamma_5\,\phi_N = -g^2/32\pi^2\, F^a_{\mu\nu}\, \tilde{F}_a^{\mu\nu}$$

$$\tilde{F}^a_{\mu\nu} = \tfrac{1}{2}\epsilon_{\mu\nu\lambda\sigma} F^{\lambda\sigma}$$

Plugging in to action path integral and requiring invariance ⇒

2-d

$$\partial_\mu \bar{\psi}\gamma^\mu\gamma_5\psi - 2iM\bar{\psi}\gamma_5\psi = -\frac{ig^2}{2\pi}\epsilon \cdot F$$

4-d

$$\partial_\mu \bar{\psi}\gamma^\mu\gamma_5\psi - 2iM\bar{\psi}\gamma_5\psi = \frac{-ig^2}{16\pi^2} F\tilde{F}$$

- - - - - - - -

What happens in Electroweak Theory:

Make a B+L rotation

Left hand fields only get contribution from regulating trace

(None for right hand fields)

$$\partial_\mu J^\mu_{B+L} = \frac{g_w^2}{16\pi^2} N_f F\tilde{F}$$

(have ignored EM, set g_m or $\theta_w = 0$)

Now

$$\frac{g^2}{32\pi^2} F\tilde{F} = \partial_\mu K^\mu$$

K^μ ≡ Chern-Symons current

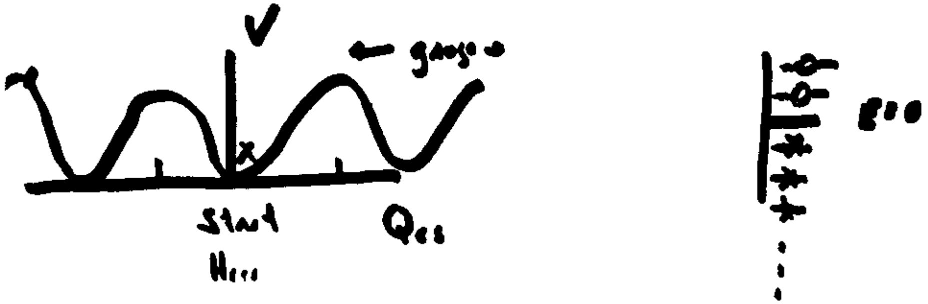
$$= \frac{g^2}{32\pi^2}\epsilon^{\mu\nu\lambda\kappa}\left\{F^a_{\lambda\nu}W^a_\kappa - \frac{2}{3}g_w\epsilon^{abc} W^a_\nu W^b_\lambda W^c_\kappa\right\}$$

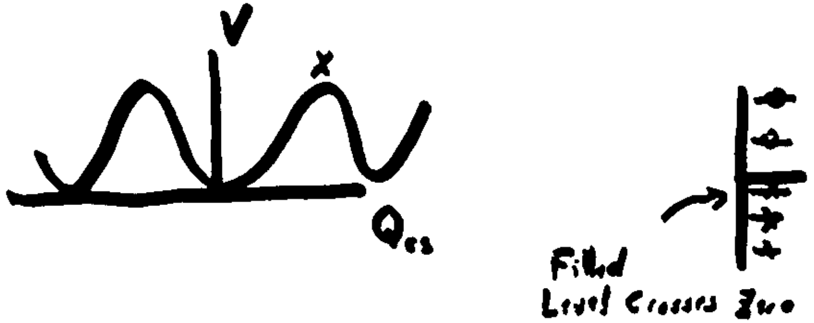
$\int d^4x\, \partial\cdot K$ ≡ topological charge, invariant under small gauge transformations

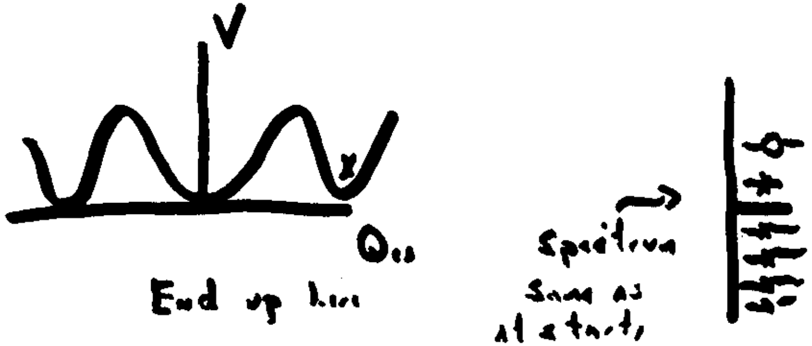
K^μ gauge dependent

$J_{B+L} - 2N_f K$ conserved

Physical Interpretation of the Anomaly







but have 1 occupied pos energy state ⇒ particle production

An explicit example: 1+1 d electrodynamics

$$S = \int d^2x \left\{ \frac{1}{4} F^2 + \bar{\Psi} \frac{1}{i} \not{\partial} - e \not{A} \psi \right\}$$

Dirac equation is

$$\left\{ \frac{1}{i} \partial_0 + \left(\frac{1}{i} \gamma^5 \partial_1 - e \gamma^5 A_1 \right) \right\} \psi$$

Have used $A^0 = 0$ gauge

$\gamma^5 = \gamma^0 \gamma^1$

$\gamma^0 = i\sigma^1$, $\gamma^1 = \sigma^2$

(i for Minkowski time)

$\gamma^5 = \sigma^3$

$$\therefore E\psi = \gamma^5 (k - eA_1) \psi$$

Now in a box

$$k = \frac{2\pi N}{L} + \frac{\pi}{L}$$

if we use anti-periodic spatial boundary conditions

If in the box, we deform

$$A \to A + \frac{2\pi}{L} \Rightarrow \text{shift levels by 1 unit}$$

Gauge transform of A

$$A \to A + \nabla\Lambda \,, \quad \Lambda = \frac{2\pi x}{L}$$

Intermediate values not a gauge transform since

$$\psi' = e^{i\theta x/L}\psi \quad \text{periodic only}$$

if $\theta = 2\pi$

Notice that the topological charges

Q_{cs} is $\epsilon_{\mu\nu} F^{\mu\nu} = \partial^0 A^1$

$$\Delta Q_{cs} = \int dx \, A^1/2\pi = 1$$

$$\delta Q_5 = 2\,\delta Q_{cs} = 2$$

Everything works here.

Index theorems exist for 3+1 d theory & prove level crossing picture

Addenda to Lecture I:

Uniqueness of anomaly:

Must specify regularization scheme

Must specify that gauged currents are conserved

For axial anomaly, ⇒ require vector current is conserved ⇒

Fujikawa derivation agrees with standard Feynman diagram method

In general? Consequences?

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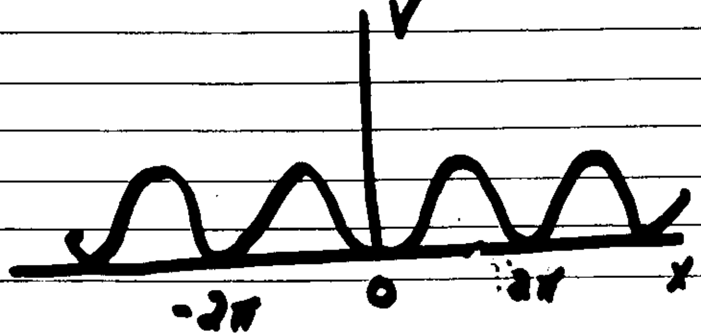
Relation between WKB tunneling & Instantons



We must solve Schrodinger Eqn.

$$\left\{\frac{p^2}{2M}+V\right\}\psi = E\psi$$

Let $\psi = e^{\chi}$

$$\left[\frac{p^2}{2M}\chi + \frac{1}{M}(p\chi)^2 + V\right] = E$$

$$(p\chi)^2 \simeq 2M(E-V)$$

$$\frac{d\chi}{dx} = i\sqrt{2M(E-V)} \qquad E>V$$

$$\frac{d\chi}{dx} = -\sqrt{2M(V-E)} \qquad E<V$$

$E=0$ ⇒

$$\frac{d\chi}{dx} = -\sqrt{2MV}$$

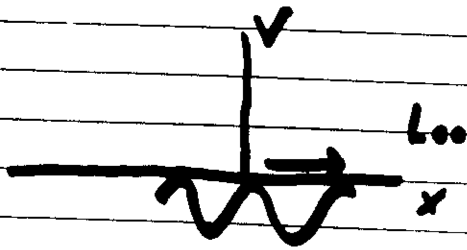
$$\psi(2\pi) = e^{-\int_0^{2\pi} dx\,\sqrt{2MV}}$$

On the other hand, consider

$$S = \int dt \, \frac{M}{2}\dot{x}^2 + V$$

$$M\ddot{x} = V'(x)$$

$V \to -V$ relative to ordinary mechanics



Look for soln

$$\frac{1}{2}M\dot{x}^2 = V(x) + E^{=0} \quad \text{(mult above by } \dot{x}\text{)}$$

$$\dot{x} = \sqrt{\frac{2V}{M}}$$

$$\psi = e^{-\int_0^{2\pi} dx \sqrt{2MV}} = e^{-\int_{-\infty}^{\infty} dt \frac{dx}{dt}\sqrt{2MV}}$$

$$= e^{-\int_{-\infty}^{\infty} dt \, 2V} = e^{-\int_{-\infty}^{\infty} dt \left[\frac{1}{2}M\dot{x}^2 + V\right]}$$

Lecture 2: Topology and Energy Barriers

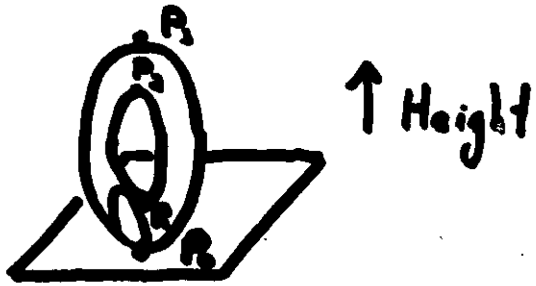
Manton
Klinkhamer
& Manton

Theories with multiple minima:

Classify minima

Compute height of barriers
(find the sphalerons)

An example: Torus



Existence of non-contractible loop $\Rightarrow \exists$ saddle points
(need compactness too!)

Compactness alone $\Rightarrow$ maxima and minima

O(3) Sigma Model in 1+1 Dimensions

Novikov et al. Phys. Rpt. 116, 103 (1984)

$$S = \frac{1}{2g^2}\int d^2x\,(\partial_\mu \hat{n}\cdot\partial^\mu \hat{n})$$

$$\hat{n}^2 = 1$$

Analogies to 3+1 d gauge theory:

- Scale invariant
- Asymptotically free
- Finite T: $M = g^2 T$
- In SUSY extension, J_5^μ anomalous
- Instantons

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Comment on SUSY extension:

$$S = \frac{1}{2g^2}\int d^2x\,\left\{\tfrac{1}{2}(\partial_\mu n_a)^2 + \tfrac{1}{2}\bar{\psi}_a\, \tfrac{1}{i}\not{\partial}\,\psi_a + \tfrac{1}{8}(\bar{\psi}\psi)^2\right\}$$

$$n^2 = 1 \qquad n\cdot\psi = 0$$

$$J_5^\mu = \epsilon^{ijk}\,\hat{n}_i\,\bar{\psi}_j\,\gamma^\mu\gamma_5\,\psi_k$$

$$\partial_\mu J_5^\mu = \frac{i g^2}{\pi}\,\epsilon_{\mu\nu}\,\hat{n}\cdot(\partial_\mu\hat{n}\times\partial_\nu\hat{n})$$

$$Q = \frac{1}{8\pi}\int d^2x\,\epsilon_{\mu\nu}\,\hat{n}\cdot(\partial^\mu\hat{n}\times\partial^\nu\hat{n})$$

$$= \frac{1}{4\pi}\int d^2x\,\partial_\mu K^\mu$$

In representation where

$$\hat{n} = (\sin\Theta\cos\varphi,\ \sin\Theta\sin\varphi,\ \cos\Theta)$$

$$\Downarrow$$

$$K^\mu = \epsilon^{\mu\nu}\cos\Theta\,\partial_\nu\varphi$$

SKIP

Example: $d^2x = d\Theta\, d\varphi$

$$0 \le \Theta \le \pi \qquad \Rightarrow Q = n$$
$$0 \le \varphi \le 2n\pi$$

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Instantons:

Most easily seen when define

$$z = x + iy$$

$$w = \frac{n_1 + i n_2}{1 + n_3}$$

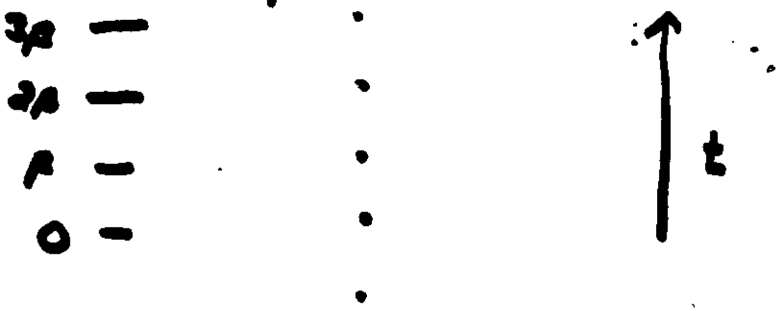
The action, +, and topological charge, −, are:

$$\binom{S}{Q} = \frac{4}{g^2}\int d^2x \frac{1}{(1+|w|^2)^2}\left\{\frac{\partial w}{\partial z}\frac{\partial \bar{w}}{\partial \bar{z}} \pm \frac{\partial w}{\partial \bar{z}}\frac{\partial \bar{w}}{\partial z}\right.$$

Instanton soln $(Q=n)$

$$w_n = c\prod_{j=1}^{n}\frac{z-a_j}{z-b_j}$$

At finite T



$a_j = a + j\beta,\quad b_j = b + j\beta \quad (Q=1)$

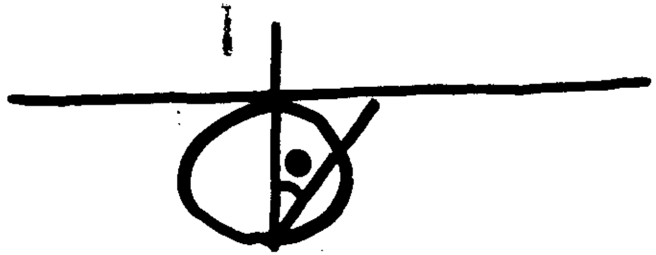
$$w = c\,\frac{\sinh \pi(z-a)/\beta}{\sinh \pi(z-b)/\beta}$$

$S_{inst} \sim 1/g^2$

Why $\exists$ instantons:

σ model: $O(3) \Rightarrow S_2$

2-d Euclidian space (regular at ∞)



$E_2 \to S_2 \Rightarrow$ Winding number: $S_2 \to S_2$

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Instantons: Tunneling to degenerate field configuration. What is barrier height?

Consider the mapping

$$\hat{n} = (\sin\mu \sin\theta,\ \sin^2\mu\cos\theta + \cos^2\mu,\ \sin\mu\cos\mu(\cos\theta - 1))$$

1. continuous, $\hat{n}^2 = 1$
2. fixed μ, θ is azimuthal angle of a circle

Example

$\hat{n} = (\sin\theta \cos\chi,\ \sin\theta \sin\chi,\ \cos\theta)$

$K^{\mu} = \epsilon^{\mu\nu} \cos\theta\, \partial_\nu \chi$

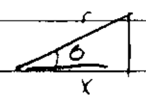
Let

$\chi(2\pi, r) = \chi(0, r) + 2n\pi$

$\cos\theta\,|_{r=0} = -1$

$\cos\theta\,|_{r=\infty} = +1$

$Q = \frac{1}{4\pi}\int$



$\frac{x}{r} = \cos\theta$

$l \cos\theta =$

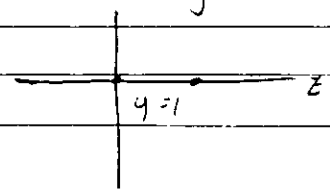
$y - 1 = z \tan\mu$

$y = 1 + z \tan\mu$

$\mu = 0 \Rightarrow y = 1$

$\mu = \pi/2 \quad z = 0$

$z = 0 \quad y = 1 \qquad dy \quad dy$



(6)

$\hat{n} = (\sin\mu \sin\theta,\ \sin^2\mu \cos\theta + \cos^2\mu,\ \sin\mu\cos\mu(\cos\theta - 1))$

3. $\hat{n}(\mu, \theta = 0) = \hat{n}(\mu, \theta = 2\pi) = (0, 1, 0)$

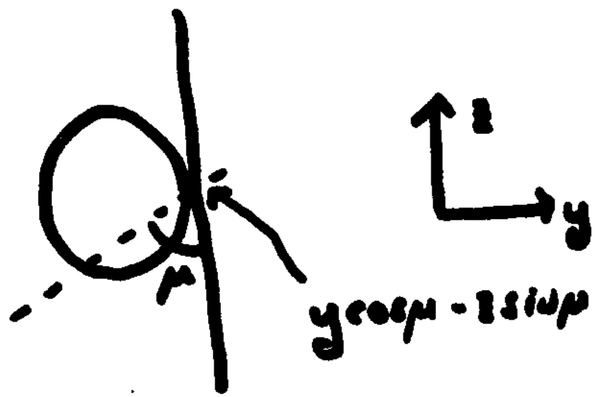
4. $\hat{n}(\mu = 0, \theta) = \hat{n}(\mu = \pi, \theta) = (0, 1, 0)$

5. each point on S_2 occurs for at least one (μ, θ), and if $\hat{n}$ is not point $(0,1,0)$, $\mu(\hat{n})$ unique

6. $\mu: 0 \to \pi,\ \theta: 0 \to 2\pi \Rightarrow Q_{top} = 1$

1-4 obvious

5: $\hat{n}(\theta)$ comes from intersection of plane with 2 sphere.



$y \cos\mu - z \sin\mu = \cos\mu$

G: $\Delta Q = 1$

Let $\mu = \mu(t)$, $\Theta = \Theta(x)$

$\mu(-\infty) = 0$ $\quad\quad$ $\Theta(-\infty) = 0$

$\mu(+\infty) = \pi$ $\quad\quad$ $\Theta(+\infty) = 2\pi$

Compute:

$\frac{1}{8\pi} \hat{n} \cdot (\partial_\mu \hat{n} \times \partial_\nu \hat{n}) = \frac{1}{4\pi} \sin\mu (1-\cos\Theta)$

$d\Theta \Rightarrow \quad \delta Q = \frac{1}{2}(1-\cos\mu)$

<u>Winds</u> and $\mu = \frac{\pi}{2} \Rightarrow \delta Q = \frac{1}{2}$

What is sphaleron of this theory?

Problems:

- Scale invariance ⇒ no static soln
- Colemans theorem ⇒ ground state disordered.

$\Theta(x)$:

$$S = \frac{\sin^2\mu}{g^2} \int dx \left\{ \frac{1}{2}\Theta'^2 + \omega^2(1-\cos\Theta) \right\}$$

$$\Theta_{sph} = 2 \sin^{-1} \operatorname{sech} \omega x$$

$$E_{sp} = 2\omega / g^2$$

- Is there another soln. with lower E_{sp}? small fluctuations
- $\Gamma/V \sim e^{-2\omega/g^2 T} \sim e^{-2/\alpha_3}$

Prefactor? in principle, might vanish

Consider instead:

$$S = \frac{1}{g^2}\int d^2x \left\{ \frac{1}{2}(\partial \vec{n})^2 + \omega^2 (1 - \vec{n}_3) \right\}$$

$\exists$ static solns. and $\mu = \pi/2 \Rightarrow$ maximum (to be minimized)

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High T and perturbation theory:

High T: $\int_0^\beta dt \sim \beta$

$$S \sim \frac{\omega\beta}{g^2}\int dx \left\{ \frac{1}{2}(\nabla n)^2 + (1 - n_3)^2 \right\}$$

1-d coupling:

$$g_1^2 = \frac{T g^2}{\omega}$$

T large $\Rightarrow$ $T \gg \omega$

Weak coupling $\Rightarrow$ $T \ll \omega/g^2$

In electroweak theory $\alpha_3 = \frac{\alpha T}{M_W(T)}$

$M_W(T) \ll T \ll \frac{M_W(T)}{\ldots}$

Topology & Sphalerons in Electroweak Theory

↑ skip

3+1 Euclidian dimensions:

gauge transformation $U(x)$

$U(x) \xrightarrow{\text{continuous}} 1$

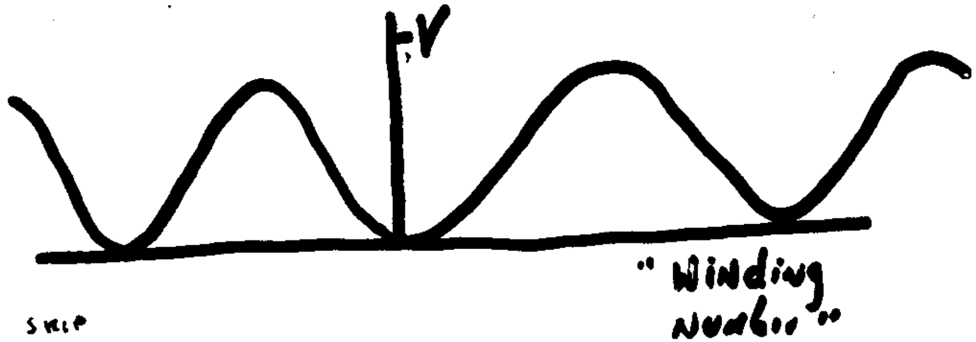
Brief comment: Only large gauge transforms can change $Q_{CS} \Rightarrow$ not continuously connected to identity $\Rightarrow$ expect ... : [illegible]

Some cannot:

Consider transforms $\lim_{|x|\to\infty} U(x) = 1$ with t fixed $\Rightarrow E_3 \to S_3$

On other hand manifold $SU(2)$ homeomorphic to S_3

$S_3 \to S_3$ winding number N



skip ↓

Topology ⇒ winding number ⇒ topological charge:

$$Q = \frac{g^2}{32\pi^2} \int d^4x \; F F^d$$

Chern-Symons charge K^0

$$\partial_\mu K^\mu = \frac{g^2}{32\pi^2} F F^d$$

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Electroweak instantons:

Exist in symmetric phase

$$\langle \phi \rangle = 0, \quad T > T_{CRIT}$$

We will, for simplicity, consider E-W theory with $\Theta_W = 0$ ⇒

$$S = \int d^4x \left\{ \tfrac{1}{4} F^2 + (D\phi)^\dagger (D\phi) + V(\phi) \right\}$$

$$V(\phi) = \lambda \left(\phi^\dagger \phi - v^2/2 \right)^2$$

$$\phi_{vac} = \frac{v}{\sqrt{2}} \, u_{-1/2}$$

We have ignored

- Photons $(\Theta_W = 0)$
- Fermions (except anomaly)

$$\frac{1}{\not{p} + M} = \frac{1}{\gamma^0 2\pi (j + \frac{1}{2}) T + \vec{\gamma} \cdot \vec{p} + M} \sim \frac{1}{T}$$

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Below T_c, ∃ field configurations with $Q \neq 0$

$$S \geq \int d^4x \, \tfrac{1}{4} F^2 \geq \int d^4x \, \tfrac{1}{4} |F F^d| = S_{inst}$$

∴ such field configurations are suppressed as $e^{-S_{inst}}$

Energy Barriers and Topology:

Write

$$\Phi = \begin{pmatrix} \mathrm{Re}\,\phi_1 \\ \mathrm{Im}\,\phi_1 \\ \mathrm{Re}\,\phi_2 \\ \mathrm{Im}\,\phi_2 \end{pmatrix}$$

Let

$$\Phi(\mu,\theta,\varphi) = \begin{bmatrix} \sin\mu \sin\theta \cos\phi \\ \sin\mu \sin\theta \sin\phi \\ \sin^2\mu \cos\theta + \cos^2\mu \\ \sin\mu \cos\mu (\cos\theta - 1) \end{bmatrix}$$

Intersection of plane + sphere.

Can write

$$\left.\begin{aligned} \phi(\mu,\theta,\varphi) &= V u_{-1/2} \\ \tau \cdot A_\mu &= \frac{i}{g}(\partial_\mu V)V^{-1} \end{aligned}\right\} \text{Now require } \phi, A \to \text{ as } r \to \infty$$

$$\Downarrow \quad \Downarrow \quad \Downarrow$$

$$\phi = (1-h(r))\begin{pmatrix} 0 \\ e^{-i\mu}\cos\mu \end{pmatrix} + h(r) V u_{-1/2}$$

$$A = f(r)\,\frac{i}{g}(\partial V)V^{-1}$$

$$h(0) = \lim_{r\to 0} f(r)/r = 0, \quad h(\infty) = f(\infty) = 1$$

After piles of algebra, can prove

- $\mu \to \mu(t)$, $\mu(-\infty) = 0$, $\mu(\infty) = \pi \Rightarrow$
  $Q = 0 \to Q = 1$
  Arbitrary $\mu(\infty) \Rightarrow Q = Q(\mu)$.
  $$Q(\pi/2) = \frac{1}{2}$$
- At any μ, equations of motion consistent with assumed form
- $\mu = 0$, $\mu = \pi$, $E = 0$
- $\mu = \pi/2$:

$$E = \int d^3x \Big\{ \frac{4}{g^2 r^2}\left(f'^2 + \frac{2}{r^2} f^2(1-f)^2\right) + \frac{v^2}{2}\left(h'^2 + \frac{2}{r^2} h^2(1-f)^2\right) + \frac{\lambda v^4}{4}(h^2-1)^2 \Big\}$$

A simpler form for the sphaleron:

Can gauge rotate.

Also $\exists$ global custodial $SU(2)_R$

A^μ unchanged

$\begin{pmatrix}\phi_1 \\ \phi_2\end{pmatrix}$ as a doublet

Take $SU_L(2)$: $\begin{pmatrix}0 & 1 \\ 1 & 0\end{pmatrix}$, $SU_R(2) = \begin{pmatrix}0 & -i \\ -i & 0\end{pmatrix}$

$$\left.\begin{aligned} \vec{A} &= 2v \frac{f(\xi)}{\xi} \hat{r} \times \vec{\tau} \\ \phi &= \sqrt{2}\, v\, h(\xi)\, \hat{r} \cdot \vec{\tau}\, u_{-1/2} \end{aligned}\right\} \text{DHN, Soni, Boguta}$$

$\xi = g v r$

__Not__ $SU_L(2) + R$ invariant. Maximal invariance is $R + SU_R(2) + SU_L(2)$

$\Downarrow$

$\exists$ 3 translational zero modes
3 rotational "

Can show $\exists$ 1 unstable mode, all rest $E > 0$

$\boxed{\text{This is the electroweak sphaleron}}$

No known exact solutions:

Numerically:

$$E_{sp} = \frac{2 M_W}{g \alpha} A(\lambda/g^2)$$

$1.52 \leq A \leq 2.70$, $\lambda/g^2 = 1 \Rightarrow A = 2.07$

$$e^{-E_{sp}/\alpha T} = e^{-1/\alpha_2(T)}$$

$\Rightarrow$ Larger rate

$$\Gamma/V \sim \underset{\tau_K ?}{K}\, e^{-1/\alpha_2(T)}$$ Kuzmin, Rubakov, & Shaposhnikov

__Beware__: $\exists$ no sphaleron for $T > T_c$.

$R_{sp} \sim 1/M_W(T) \rightarrow \infty$ as $T \rightarrow T_c$

Potentially very dangerous!!

Another computation of Q_{ephal}

$$Q_{sp} = \int d^4x\, \partial\cdot K$$

$$= \int d^3x\, K^0$$

if $\int d^3x\, K^0\Big|_{t=-\infty} = 0$, and if $\lim_{r\to\infty} r^2\, \vec{\nabla}\cdot\vec{K} = 0$

Can make $\vec{\nabla}\cdot\vec{K}$ vanish rapidly at $r = \infty$: $U(r) = e^{i\Theta(r)\,\vec{\tau}\cdot\hat{r}}$

Let $\bar{\Theta}(r)$: $0 \le \bar{\Theta} \le \pi$ for $0 \le r \le \infty$

$\lim_{r\to\infty} \bar{\Theta}(r) \to \pi$ rapidly $\Rightarrow \vec{\nabla}\cdot\vec{K} \to 0$.

$$A_i^a = \frac{(1-2f)\cos\bar{\Theta} - 1}{g r^2}\, \epsilon_{rab} r^b$$

$$+ \frac{(1-2f)\sin\bar{\Theta}}{g r^3}\,(\delta_{ia} r^2 - r_i r_a)$$

$$+ \frac{1}{g}\,\frac{d\bar{\Theta}}{dr}\,\frac{r_i r_a}{r^2}$$

Compute $Q_{sp} = \frac{1}{2}$