INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

SMR.626 - 23
(Lect. IV)

SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

15 June - 31 July 1992

NON-PERTURBATIVE ELECTROWEAK THEORY

L. McLERRAN
Department of Physics
University of Minnesota
Minneapolis, MN
USA

Electroweak B+L Violation at High T and Making the Baryon Asymmetry at T~100 GeV

Outline of the Lectures:

I Anomalies and Level Crossing

II Topology, Vacua + Energy Barriers

- A. O(3) sigma model example
- B. Electroweak Theory
- C. Topology, Sphalerons + Instantons

III Transition Rates

- A. How to compute rates at
  - i) Low $T \ll M_W(T)$
  - ii) Intermediate $M_W(T) \ll T \ll M_W(T)/\alpha_W$
  - iii) High $T \gg M_W(T)/\alpha_W$
- B. Results of computations
- C. Apparent paradoxes
  - i) Making a fish at high T
  - ii) Contradictions with instantons

IV Baryogenesis + Cosmology

- A. Sakharov Conditions
- B. General Scenario
- C. Phase transitions in EW theory

V Explicit Examples

- A. B generation in the wall
- B. Scattering from the wall
- C. 2 Higgs doublet model

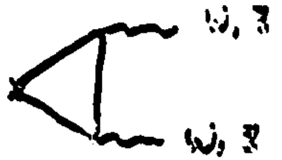
VI Dynamics of the Wall

- A. Shape of the wall
- B. Damping of the wall
- C. Hydrodynamic Instabilities

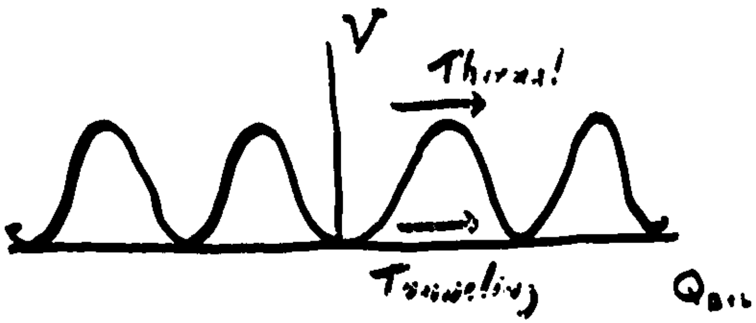
Brief Introduction:

Until 5 yrs. ago, was believed that
$\Delta B \neq 0$ dynamics only 'useful'
for $E \sim 10^{15}$ GeV $\sim E_{GUT}$

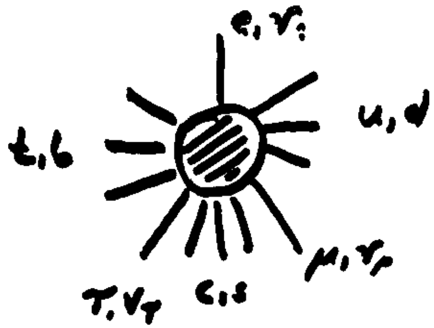
But, in EW theory $\exists$ anomaly

J^{μ}_{B+L} 

$\Rightarrow \quad \partial_\mu J^{\mu}_{B+L} \neq 0$



't Hooft computed low energy tunneling rate



EW theory knows about doublets,
but not about color, family
or lepton vs quark

Compute rate: Need WKB tunneling solution

Tunneling $\Rightarrow$ forbidden region

$$e^{iEt} \to e^{-Et}$$

Euclidean solutions of Yang-Mills equations $\Rightarrow$ 't Hooft-Belavin-Polyakov Instanton

Solution is known for EW theory

(Technical complication: must constrain size of instanton and in end integrate over all sizes)

Solution of form

$$A^a_\mu(x,\rho) \;,\quad \phi(x,\rho)$$

$$\begin{pmatrix} e(x,\rho) \\ \nu(x,\rho) \end{pmatrix} = \text{zero modes}$$

Can compute

$$\langle \psi_1 \cdots \psi_N \rangle \sim e^{-S}$$

S = WKB action

S from classical action, classical field is associated with $\sim 1/\alpha_W$ quanta

$$S \sim 1/\alpha_W$$

$$\Gamma/V \sim e^{-4\pi/\alpha_W} \sim 10^{-173}$$

$$\Gamma/V \sim 10^{-173}$$

Space-time volume of universe:

$$t \sim 10^{10}\,yr \sim 10^{17}\,sec$$
$$d \sim 10^{17}\,sec \times c \sim 10^{26}\,m \sim 10^{41}\,Fm \sim 10^{43}\;1/M_W$$

$$Vt \sim 10^{172}\;1/M_W^4 \;!!$$

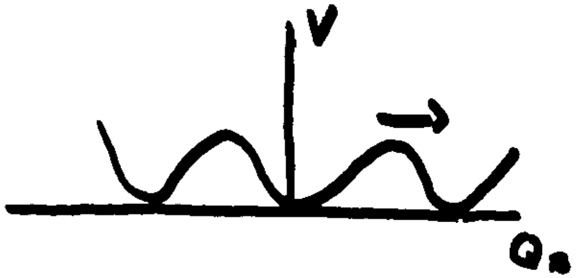
$$\#\,\text{protons}/V \sim 1\,m^{-3} \sim 10^{-54}\,M_W^{3}$$

Probability of single p decay $\sim 10^{-54}$; but need $\Delta B = 3 \Rightarrow$ never!

Even if rate was big, $\Delta B = 3$ would be hard to see experimentally (Big mass ~ GUT rate)

What about $T \neq 0$ or transitions over top of barrier?

Unsuppressed?



Early 'conjectures'
Linde; Polyakov; Dimopoulos + Susskind

Dropped because

1. t' Hoofts calculation
2. All $\Delta B \neq 0$ processes $e^{-4\pi/\alpha_W}$
3. No known path over top of barrier
4. Instantons at high T can be computed and [redacted]
   $S > 4\pi/\alpha_W$

Problem dropped until mid 80's when

Manton: Argued there is a finite action path over the top of the barrier

Manton + Klinkhamer: Computed height of barrier using sphaleron

Sphaleron: Classical, static solution of equations of motion which is unstable under a small perturbation

Kuzmin-Rubakov-Shaposhnikov:

Strong arguments that rate

$$\sim e^{-E_{sp}/T}$$

and $e^{-E_{sp}/T} \to 0$ as $T \to \infty$

Arnold-McLerran

Argued rate computable $M_W(T) \ll T \ll \frac{M_W(T)}{\alpha_W}$

Computed and showed

$$R_{\Delta B \neq 0} / R_{exp} \sim 10^9 \text{ for } T \approx 100 \text{ GeV}$$

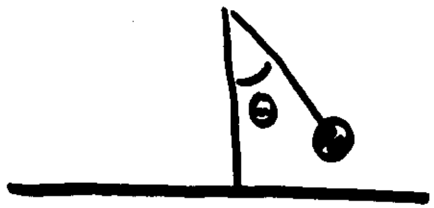
Resolved instanton 'paradoxes'

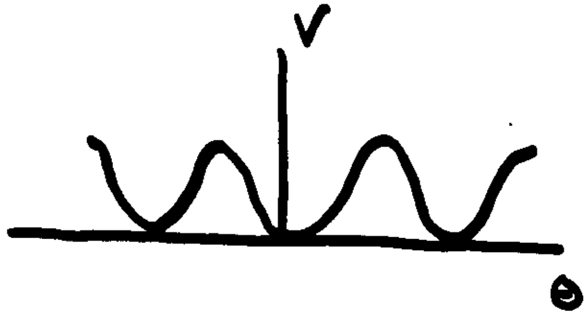
Grigoriev, Rubakov, Shaposhnikov

Convincing results from numerical simulations

An example to convince you that instantons are not relevant for high T barrier crossing

The Quantum Mechanical Pendulum





At high T, prob $e^{-E/T}$ to be in state with $E \approx T \gg V_{barrier}$ probability big that in state which freely crosses over top of barrier

To understand how finite T instantons work:

Review of finite T formalism

Remember

Hamiltonian → Path Integral

$$\lim_{t\to\infty} \langle out | e^{-itH} | in \rangle$$
$$\to \int [d\varphi]\, e^{i\int_{-\infty}^{\infty} dt\, \mathcal{L}(\varphi)}$$

Finite T:

$$\mathrm{Tr}\, e^{-\beta H} \to \int_{\varphi(\beta)=\varphi(0)} [d\varphi]\, e^{-\int_0^\beta dt\, \mathcal{L}(\varphi)}$$

Bosons: Periodic under $t \to t+\beta$
Fermions: $\psi(t+\beta) = -\psi(t)$ (harder to derive!)

$t \to it \Rightarrow$ Euclidean space

Vector fields: Analytic continuation $A^0 \to -iA^0$
$g^{\mu\nu} \to \delta^{\mu\nu}$, $\gamma^0 \to -i\gamma^0$

Euclidean space formulation of pendulum:

$$Q^{TOP} \equiv \int_0^\beta dt\, \dot{x}(t) = x(\beta) - x(0)$$

$$A^{INST} \sim e^{-S_{INS}}$$

$$S = \int_0^\beta dt\ \tfrac{1}{2}\dot{x}^2 + V(x)$$

$$\xrightarrow[T\to\infty]{} \int_0^\beta dt\ \tfrac{1}{2}\dot{x}^2$$

since in time $1/T$, x must change by multiple of 2π but V finite

$$x_{INST} = \frac{2\pi t}{\beta} = 2\pi t T$$

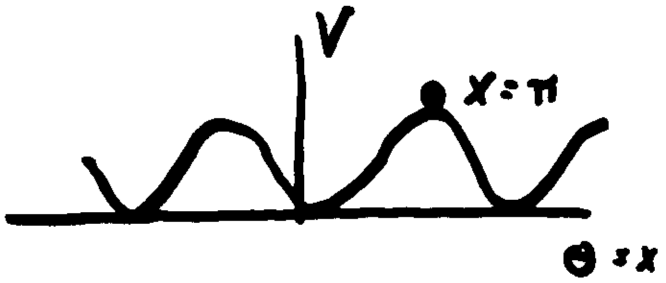
$$S = 2\pi^2 T \xrightarrow[T\to\infty]{} \infty$$

Instantons not relevant at high T.

Nevertheless

$$\text{Rate} \sim e^{-V_{sphaleron}/T} \sim e^{-E_{sp}/T} \to 1$$

as $T \to \infty$



Sphaleron:

$X = \pm\pi$

It is unstable

It is on a finite action, continuous path of deformations of X: $0 \le X \le 2\pi$

$$V_{sphaleron} = V(\pi)$$

Rate is big.

The U(1) Anomaly:

I will show that

$$\partial_\mu J^\mu_{B+L} \neq 0$$

in Electroweak Theory and that in Euclidean space is related to a topological charge

Later, will argue relation to simple fermion energy level crossing picture

We will consider the Euclidean path integral

$$\int [dA\, d\bar\Psi\, d\psi]\; e^{-S[\bar\Psi,\psi,A]}$$

$$[dA] = \prod_{x,\mu,a} dA^a_\mu(x) \quad \text{etc.}$$

$$S = \int d^4x\; \bar\Psi[-i\not{D} + M]\psi + \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu}$$

$$D = \partial - ig\, T\cdot A$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

Now, we use Fujikawa's trick:

The path integral must be invariant under a change of variables. Consider

$$\psi(x) \to e^{i\gamma_5 \alpha(x)} \psi(x)$$

$$\bar{\psi}(x) \to \bar{\psi}(x) e^{i\gamma_5 \alpha(x)}$$

(We will first derive axial U(1) anomaly & later argue case for B+L anomaly)

There will be two compensating changes:

Non-invariance of fermion measure:

$$\delta[d\bar{\psi} d\psi]$$

Non-invariance of action:

$$\delta S = \partial_\mu \alpha(x) \bar{\psi} \gamma^\mu \gamma_5 \psi + 2iM \alpha(x) \bar{\psi} \gamma_5 \psi$$

(α infinitesimal)

Consider $\delta[d\bar{\psi} d\psi]$

Recall that for Grassmann valued variables:

$$a_i^2 = 0 \;; \quad \{a_i, a_j\} = 0$$

$$\int da_i = 0 \qquad \int da_i \, a_i \neq 0$$

$\therefore$ Only non-zero N-dimensional Grassmann valued integral is

$$\int da_1 \cdots da_N \, a_1 \cdots a_N$$

Under $a_i' = \sum_j c_{ij} a_j$, integral must remain invariant. Note that

$$a_1' \cdots a_N' = \det C \; a_1 \cdots a_N$$

$\therefore$

$$[da'] = [\det C]^{-1} [da]$$

Now for an infinitesimal chiral rotation

$$C \simeq [1 + i\alpha(x)\gamma^5]\,\delta(x-y)$$

$$\ln \det{}^{-1} C = \exp \mathrm{tr} \ln C^{-1}$$

$$= \exp \mathrm{tr} \ln \frac{1}{1+i\alpha\gamma_5}$$

Evaluation of tr is UV singular.
Regulate in gauge invariant way

$$\not{D}\,\phi_N = \lambda_N \phi_N$$

$$\det C^{-1} = \exp -i \int d^4x\, \alpha(x) \sum_N \phi_N^\dagger(x) \gamma_5 \phi_N(x)$$

Sum over N is singular: Regulate

$$\sum_N \phi_N^\dagger(x) \gamma_5 \phi(x) \equiv \lim_{M\to\infty} \sum_N \phi_N^\dagger(x) \gamma_5 \phi_N(x) e^{-\lambda_N^2/M^2}$$

$$= \mathrm{tr}\, \gamma_5 e^{-\not{D}^2/M^2} = \lim_{\substack{M\to\infty \\ x\to y}} \int \frac{d^4k}{(2\pi)^4} e^{ik\cdot(x-y)}$$

$$\mathrm{tr}\, \gamma_5 \exp\{-(D^2 + 1/4\, g^2 [\gamma^\mu, \gamma^\nu] T\cdot F_{\mu\nu})/M^2\}$$

$$\ln \det{}^{-1} C = \lim_{\substack{M\to\infty \\ x\to y}} \int \frac{d^2k}{(2\pi)^2} e^{ik\cdot(x-y)}$$

$$\mathrm{tr}\, \gamma_5 \exp\{(-D^2 + \tfrac{1}{4} g^2 [\gamma^\mu, \gamma^\nu] T\cdot F_{\mu\nu})/M^2\}$$

In 1+1 dimensions $\gamma^0 = \sigma^1,\ \gamma^1 = \sigma^2,\ \gamma^5 = \sigma^3$

First non-vanishing term is

$$\sum_N \phi_N^\dagger \gamma_5 \phi_N = \lim_{M\to\infty} \tfrac{1}{4} g^2\, \mathrm{tr}\, \gamma_5 [\gamma^\mu, \gamma^\nu]\, T\cdot F_{\mu\nu}$$

$$/M^2 \int \frac{d^2k}{(2\pi)^2} e^{-k^2/M^2} = -g^2/4\pi\ \mathrm{tr}\, T\cdot \epsilon^{\mu\nu} F_{\mu\nu}$$

$$= -g^2/4\pi\ \epsilon\cdot F_{\mu\nu}$$

— — — — —

In 4-d

$$\sum_N \phi_N^\dagger \gamma_5 \phi_N = -g^2/32\pi^2\, F_{\mu\nu}^a\, F_a^{d\mu\nu}$$

$$F_{\mu\nu}^d = \tfrac{1}{2} \epsilon_{\mu\nu\lambda\sigma} F^{\lambda\sigma}$$

Plugging in to action path integral and requiring invariance ⇒

2-d

$$\partial_\mu \bar\psi \gamma^\mu \gamma_5 \psi - 2iM \bar\psi \gamma_5 \psi = \frac{-ig^2}{2\pi} \epsilon \cdot F$$

4-d

$$\partial_\mu \bar\psi \gamma^\mu \gamma_5 \psi - 2iM \bar\psi \gamma_5 \psi = \frac{-ig^2}{16\pi^2} F\tilde F^d$$

- - - - - - -

What happens in Electroweak Theory:

Make a B+L rotation
Left hand fields only get contribution from regulating trace
(None for right hand fields)

$$\partial_\mu J^\mu_{B+L} = \frac{g_w^2}{16\pi^2} N_F F\tilde F^d$$

(have ignored EM, set g_m or $\Theta_w = 0$)

Now

$$\frac{g^2}{32\pi^2} F F^d = \partial_\mu K^\mu$$

K^μ ≡ Chern-Simons current

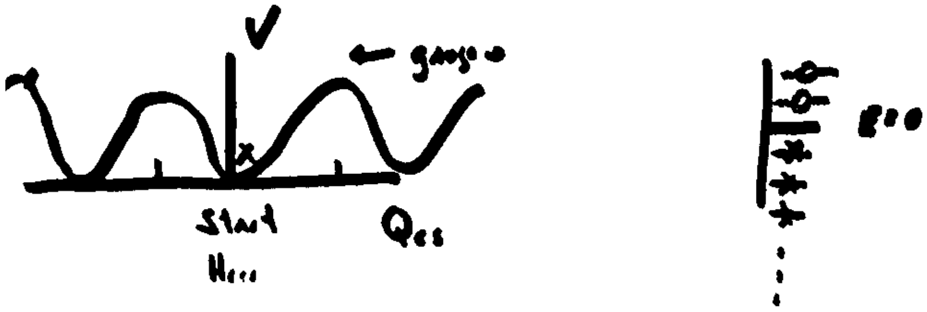
$$= \frac{g^2}{32\pi^2} \epsilon^{\mu\nu\lambda\kappa} \{ F^a_{\lambda\nu} W^a_\kappa - \tfrac{2}{3} g_w \epsilon^{abc} W^a_\nu W^b_\lambda W^c_\kappa \}$$

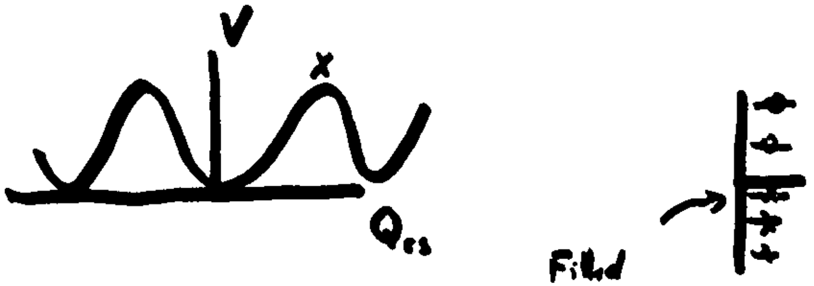
$\int d^4x\, \partial \cdot K$ ≡ topological charge, invariant under small gauge transformations

K^μ gauge dependent

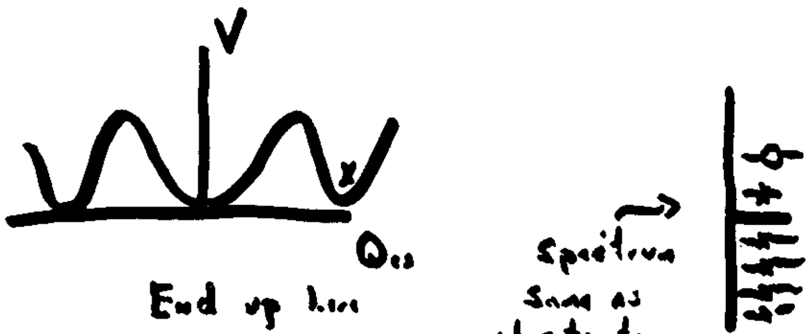
$J_{B+L} - 2N_F K$ conserved

Physical Interpretation of the Anomaly





Filled Level Crosses Zero



End up here

Spectrum Same as at start, but have 1 occupied pos energy state ⇒ particle production

An explicit example: 1+1 d electrodynamics

$$S = \int d^2x \left\{ \frac{1}{4} F^2 + \bar{\Psi} \tfrac{1}{i} \not{\partial} - e \not{A} \psi \right\}$$

Dirac equation is

$$\left\{ \frac{1}{i} \partial_0 + \left(\frac{1}{i} \gamma^5 \partial_1 - e \gamma^5 A_1 \right) \right\} \psi$$

Have used $A^0 = 0$ gauge

$\gamma^5 = \gamma^0 \gamma^1$ $\qquad \gamma^0 = i\sigma^1, \quad \gamma^1 = \sigma^2$

(i for Minkowski time)

$\gamma^5 = \sigma^3$

$$\therefore E\psi = \gamma^5 (k - eA_1) \psi$$

Now in a box

$$k = \frac{2\pi N}{L} + \frac{\pi}{L}$$

if we use anti-periodic spatial boundary condition

If in the box, we deform

$$A \to A + \frac{2\pi}{L} \Rightarrow \text{shift levels by 1 unit}$$

Gauge transform of A

$$A \to A + \nabla\Lambda \,, \quad \Lambda = \frac{2\pi x}{L}$$

Intermediate values not a gauge transform since

$$\psi' = e^{i\theta x/L}\,\psi \quad \text{periodic only}$$

if $\theta = 2\pi$

Notice that the topological charges

$$Q_{cs} \propto \epsilon_{\mu\nu} F^{\mu\nu} = \partial^0 A^1$$

$$\Delta Q_{cs} = \int dx \; A^1/_{2\pi} = 1$$

$$\delta Q_5 = 2\,\delta Q_{cs} = 2$$

Everything works here.

Index theorems exist for 3+1 d theory & prove level crossing picture

Addenda to Lecture I:

Uniqueness of anomaly:

Must specify regularization scheme

Must specify that gauged currents are conserved

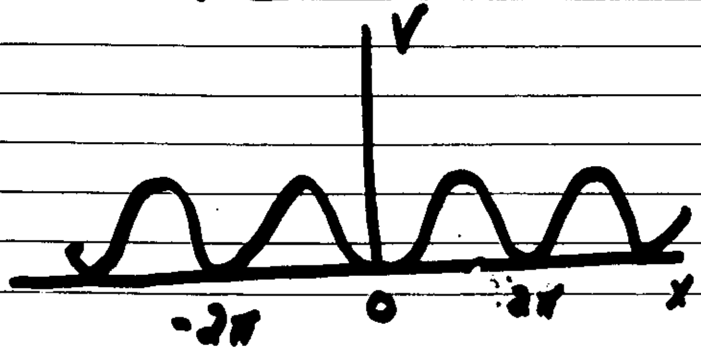
For axial anomaly, ⇒ require vector current is conserved ⇒

Fujikawa derivation agrees with standard Feynman diagram method

In general? Consequences?

- - - - - - - - - -

Relation between WKB tunneling + Instantons



We must solve Schrodinger Eqn.

$$\left\{\frac{p^2}{2M}+V\right\}\psi = E\psi$$

Let $\psi = e^{\chi}$

$$\left[\frac{p^2}{2M}\chi + \frac{1}{M}(p\chi)^2 + V\right] = E$$

$$(p\chi)^2 \simeq 2M(E-V)$$

$$\frac{d\chi}{dx} = i\sqrt{2M(E-V)} \qquad E>V$$

$$\frac{d\chi}{dx} = -\sqrt{2M(V-E)} \qquad E<V$$

$E=0$ ⇒

$$\frac{d\chi}{dx} = -\sqrt{2MV}$$

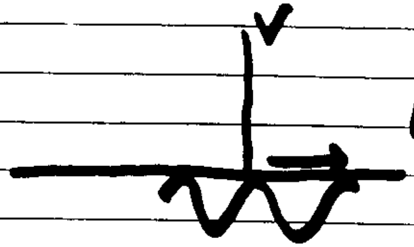
$$\psi(2\pi) = e^{-\int_0^{2\pi} dx\sqrt{2MV}}$$

On the other hand, consider

$$S = \int dt\, \frac{M}{2}\dot{x}^2 + V$$

$$M\ddot{x} = V'(x)$$

$V \to -V$ relative to ordinary mechanics



Look for soln

$$\frac{1}{2}M\dot{x}^2 = V(x) + E^{=0} \quad \text{(mult above by } \dot{x}\text{)}$$

$$\dot{x} = \sqrt{\frac{2V}{M}}$$

$$\psi = e^{-\int_0^{2\pi} dx\, \sqrt{2MV}} = e^{-\int_{-\infty}^{\infty} dt\, \frac{dx}{dt}\sqrt{2MV}}$$

$$= e^{-\int_{-\infty}^{\infty} dt\, 2V} = e^{-\int_{-\infty}^{\infty} dt\left[\frac{1}{2}M\dot{x}^2 + V\right]}$$

Lecture 2: Topology and Energy Barriers

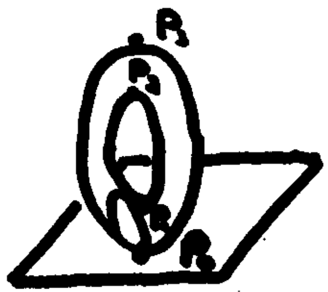
Manton
Klinkhamer & Manton

Theories with multiple minima:

Classify minima

Compute height of barriers (find the sphalerons)

An example: Torus



↑ Height

Existence of non-contractible loop $\Rightarrow \exists$ saddle points (need compactness too!!)

Compactness alone $\Rightarrow$ maxima and minima

O(3) Sigma Model in 1+1 Dimensions

Novikov et al Phys Rpt. 116, 103 (1984)

$$S = \frac{1}{2g^2}\int d^2x \, (\partial_\mu \hat{n} \cdot \partial^\mu \hat{n})$$

$$\hat{n}^2 = 1$$

Analogies to 3+1 d gauge theory:

- Scale invariant
- Asymptotically free
- Finite T: $M = g^2 T$
- In SUSY extension, J_5^μ anomalous
- Instantons

- - - - - - - - -

Comment on SUSY extension:

$$S = \frac{1}{2g^2}\int d^2x \left\{ \frac{1}{2}(\partial_\mu n_a)^2 + \frac{1}{2}\bar{\psi}_a \frac{1}{i}\not{\partial}\psi_a + \frac{1}{8}(\bar{\psi}\psi)^2 \right\}$$

$$n^2 = 1 \qquad n \cdot \psi = 0$$

$$J_5^\mu \equiv \epsilon^{ijk}\hat{n}_i \bar{\psi}_j \gamma^\mu \gamma_5 \psi_k$$

$$\partial_\mu J_5^\mu = \frac{ig^2}{\pi}\epsilon_{\mu\nu}\hat{n}\cdot(\partial_\mu \hat{n} \times \partial_\nu \hat{n})$$

$$Q = \frac{1}{8\pi}\int d^2x \, \epsilon_{\mu\nu}\hat{n}\cdot(\partial^\mu \hat{n} \times \partial^\nu \hat{n})$$

$$= \frac{1}{4\pi}\int d^2x \, \partial_\mu K^\mu$$

In representation where

$$\hat{n} = (\sin\Theta\cos\varphi, \sin\Theta\sin\varphi, \cos\Theta)$$

$$\Downarrow$$

$$K^\mu = \epsilon^{\mu\nu}\cos\Theta \, \partial_\nu \varphi \qquad \text{SKIP}$$

Example: $d^2x = d\Theta d\varphi$

$$0 \le \Theta \le \pi$$
$$0 \le \varphi \le 2n\pi \quad \Rightarrow Q = n$$

- - - - - - - - - - -

Instantons:

Most easily seen when define

$$z = x + iy$$

$$w = \frac{n_1 + i n_2}{1 + n_3}$$

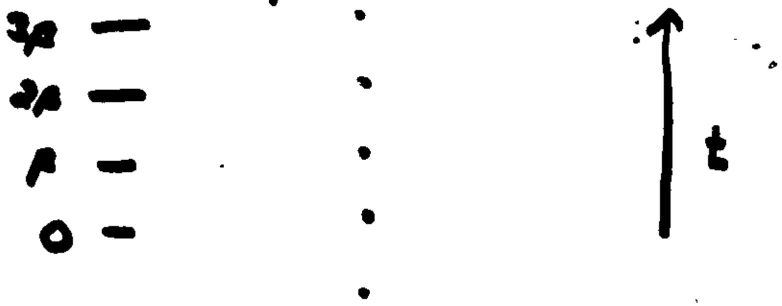
The action, +, and topological charge, −, are:

$$\binom{S}{Q} = \frac{4}{g^2}\int d^2x \frac{1}{(1+|w|^2)^2}\left\{\frac{\partial w}{\partial z}\frac{\partial \bar{w}}{\partial \bar{z}} \pm \frac{\partial w}{\partial \bar{z}}\frac{\partial \bar{w}}{\partial z}\right\}$$

Instanton soln $(Q=n)$

$$w_n = c \prod_{j=1}^{n} \frac{z-a_j}{z-b_j}$$

At finite T



$a_j = a + j\beta\,,\quad b_j = b + j\beta \quad (Q=1)$

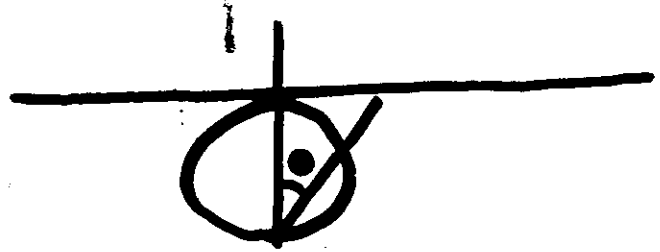
$$w = c\,\frac{\sinh \pi(z-a)/\beta}{\sinh \pi(z-b)/\beta}$$

$S_{inst} \sim 1/g^2$

Why $\exists$ instantons:

σ model: $O(3) \Rightarrow S_2$

2-d Euclidian space (regular at ∞)



$E_2 \to S_2 \Rightarrow$ Winding number: $S_2 \to S_2$

- - - - - - - -

Instantons: Tunneling to degenerate field configuration. What is barrier height?

Consider the mapping

$$\hat{n} = (\sin\mu \sin\theta,\ \sin^2\mu\cos\theta + \cos^2\mu,\ \sin\mu\cos\mu(\cos\theta - 1))$$

1. continuous, $\hat{n}^2 = 1$
2. fixed μ, θ is azimuthal angle of a circle

Example

$\hat{n} = (\sin\theta\cos\chi\,,\ \sin\theta\sin\chi\,,\ \cos\theta)$

$K^{\mu} = \epsilon^{\mu\nu}\cos\theta\,\partial_\nu\chi$

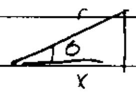
Let

$\chi(2\pi, r) = \chi(0, r) + 2n\pi$

$\cos\theta\,\big|_{r=0} = -1$

$\cos\theta\,\big|_{r=\infty} = +1$

$Q = \frac{1}{4\pi}\int$



$\frac{x}{r} = \cos\theta$

$l\cos\theta =$

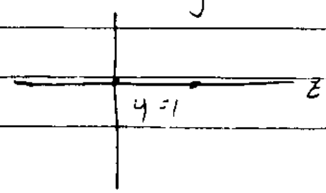
$y - 1 = z\tan\mu$

$y = 1 + z\tan\mu$

$\mu = 0 \Rightarrow y = 1$

$\mu = \pi/2 \quad z = 0$

$z = 0 \quad y = 1 \quad dy$



(6)

$\hat{n} = (\sin\mu\sin\theta,\ \sin^2\mu\cos\theta + \cos^2\mu,\ \sin\mu\cos\mu(\cos\theta - 1))$

3 . $\hat{n}(\mu, \theta=0) = \hat{n}(\mu, \theta=2\pi) = (0,1,0)$

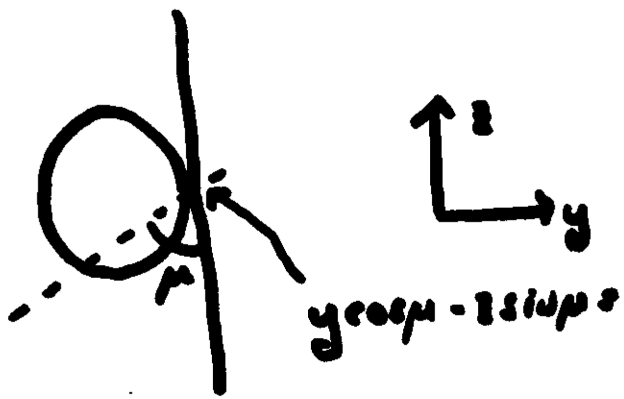
4 . $\hat{n}(\mu=0, \theta) = \hat{n}(\mu=\pi, \theta) = (0,1,0)$

5 . each point on S_2 occurs for at least one (μ, θ), and if $\hat{n}$ is not point $(0,1,0)$, $\mu(\hat{n})$ unique

6 . $\mu: 0 \to \pi,\ \theta: 0 \to 2\pi \Rightarrow Q_{top} = 1$

1-4 obvious

5: $\hat{n}(\theta)$ comes from intersection of plane with 2 sphere.



$\Delta Q = 1$

Let $\mu = \mu(t)$, $\Theta = \Theta(x)$

$\mu(-\infty) = 0 \qquad \Theta(-\infty) = 0$

$\mu(+\infty) = \pi \qquad \Theta(+\infty) = 2\pi$

Compute:

$$\frac{1}{8\pi} \hat{n}\cdot(\partial_\mu \hat{n} \times \partial_\nu \hat{n}) = \frac{1}{4\pi} \sin\mu (1-\cos\Theta)$$

$d\Theta \Rightarrow \delta Q = \frac{1}{2}(1-\cos\mu)$

Winds and $\mu = \frac{\pi}{2} \Rightarrow \delta Q = \frac{1}{2}$

What is sphaleron of this theory?

Problems:

- Scale invariance $\Rightarrow$ no static soln
- Colemans theorem $\Rightarrow$ ground state disordered.

$\Theta(x)$:

$$S = \frac{\sin^2\mu}{g^2} \int dx \left\{ \frac{1}{2}\Theta'^2 + \omega^2(1-\cos\Theta)\right\}$$

$\Theta_{sph} = 2\sin^{-1}\operatorname{sech}\omega x$

$E_{sp} = 2\omega/g^2$

- Is there another soln. with lower E_{sp}? small fluctuations
- $\Gamma/V \sim e^{-2\omega/g^2T} \sim e^{-2/\alpha_3}$

Prefactor? in principle, might vanish

Consider instead:

$$S = \frac{1}{g^2}\int d^2x \left\{ \frac{1}{2}(\partial \hat{n})^2 + \omega^2 (1-\hat{n}_3) \right\}$$

$\exists$ static solns. and $\mu = \pi/2 \Rightarrow$ maximum (to be minimized)

- - - - - - - -

High T and perturbation theory:

High T: $\int_0^\beta dt \sim \beta$

$$S \sim \frac{\omega\beta}{g^2}\int dx \left\{ \frac{1}{2}(\nabla n)^2 + (1-n_3)^2 \right\}$$

1-d coupling:

$$g'^2 = \frac{Tg^2}{\omega}$$

T large $\Rightarrow$ $T \gg \omega$

Weak coupling $\Rightarrow$ $T \ll \omega/g^2$

In electroweak theory $\alpha_3 = \frac{\alpha T}{M_W(T)}$

$M_W(T) \ll T \ll \underline{M_W(T)}$

Topology + Sphalerons in Electroweak Theory

↑ skip

3+1 Euclidian dimensions:

gauge transformation $U(x)$

$U(x) \xrightarrow{\text{continuous}} 1$

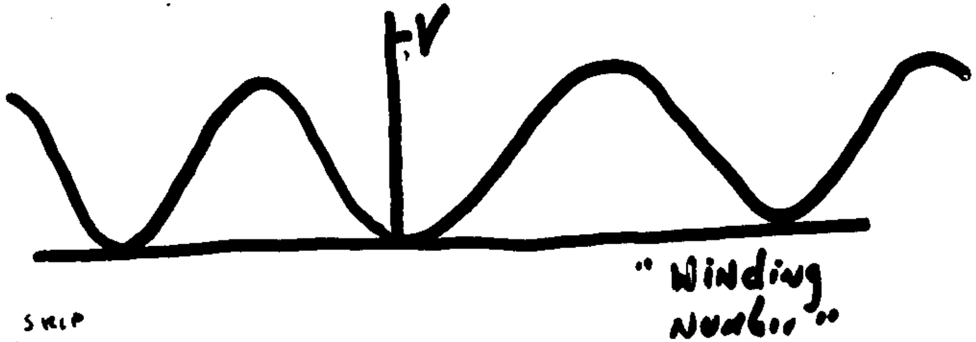
Brief comment: Only large gauge transforms can change $Q_{cs} \Rightarrow$ none continuously connected to identity $\Rightarrow$ expect ... : [illegible]

Some cannot:

Consider transforms $\lim_{|x|\to\infty} U(x) = 1$ with t fixed $\Rightarrow E_3 \to S_3$

On other hand manifold $SU(2)$ homeomorphic to S_3

$S_3 \to S_3$ winding number N



skip ↓

Topology ⇒ winding number ⇒ topological charge:

$$Q = \frac{g^2}{32\pi^2}\int d^4x\; F\tilde{F}^d$$

Chern-Symons charge K^0

$$\partial_\mu K^\mu = \frac{g^2}{32\pi^2} F\tilde{F}^d$$

- - - - - - - -

Electroweak instantons:

Exist in symmetric phase

$$\langle \phi \rangle = 0, \quad T > T_{CRIT}$$

We will, for simplicity, consider E-W theory with $\Theta_W = 0$ ⇒

$$S = \int d^4x \left\{ \frac{1}{4}F^2 + (D\phi)^\dagger (D\phi) + V(\phi) \right\}$$

$$V(\phi) = \lambda(\phi^\dagger\phi - v^2/2)^2$$

$$\phi_{VAC} = \frac{v}{\sqrt{2}}\, u_{-1/2}$$

We have ignored

- Photons $(\Theta_W = 0)$
- Fermions (except anomaly)

$$\frac{1}{\not{p} + M} = \frac{1}{\gamma^0 2\pi(j+\frac{1}{2})T + \vec{\gamma}\cdot\vec{p} + M} \sim \frac{1}{T}$$

- - - - - - - - -

Below T_c, ∃ field configurations with $Q \neq 0$

$$S \geq \int d^4x \frac{1}{4}F^2 \geq \int d^4x \frac{1}{4}|F\tilde{F}^d|$$

$$= S_{inst}$$

∴ such field configurations are suppressed as $e^{-S_{inst}}$

Energy Barriers and Topology:

Write

$$\Phi = \begin{pmatrix} \mathrm{Re}\,\phi_1 \\ \mathrm{Im}\,\phi_1 \\ \mathrm{Re}\,\phi_2 \\ \mathrm{Im}\,\phi_2 \end{pmatrix}$$

Let

$$\Phi(\mu,\theta,\varphi) = \begin{bmatrix} \sin\mu\sin\theta\cos\phi \\ \sin\mu\sin\theta\sin\phi \\ \sin^2\mu\cos\theta + \cos^2\mu \\ \sin\mu\cos\mu(\cos\theta - 1) \end{bmatrix}$$

Intersection of plane + sphere.

Can write

$$\left.\begin{aligned} \phi(\mu,\theta,\varphi) &= V u_{-1/2} \\ \tau\cdot A_\mu &= \frac{i}{g}(\partial_\mu V)V^{-1} \end{aligned}\right\} \begin{array}{l}\text{Now require} \\ \phi, A \to \\ \text{as } r\to\infty\end{array}$$

$$\Downarrow \qquad \Downarrow \qquad \Downarrow$$

$$\phi = (1-h(r))\begin{pmatrix} 0 \\ e^{i\mu}\cos\mu \end{pmatrix} + h(r)\, V u_{-1/2}$$

$$A = f(r)\,\frac{i}{g}(\partial V)V^{-1}$$

$$h(0) = \lim_{r\to 0} f(r)/r = 0, \quad h(\infty) = f(\infty) = 1$$

After piles of algebra, can prove

- $\mu \to \mu(t)$, $\mu(-\infty) = 0$, $\mu(\infty) = \pi \Rightarrow$
  $Q = 0 \rightarrow Q = 1$

  Arbitrary $\mu(\infty) \Rightarrow Q = Q(\mu)$,

  $$Q(\pi/2) = \frac{1}{2}$$

- At any μ, equations of motion consistent with assumed form

- $\mu = 0$, $\mu = \pi$, $E = 0$

- $\mu = \pi/2$:

$$E = \int d^3x \Big/ \frac{4}{g^2 r^2}\left(f'^2 + \frac{2}{r^2} f^2(1-f)^2\right) + \frac{v^2}{2}\left(h'^2 + \frac{2}{r^2} h^2(1-f)^2\right) + \frac{\lambda v^4}{4}(h^2-1)^2 \Big\}$$

A simpler form for the sphaleron:

Can gauge rotate.

Also $\exists$ global custodial $SU(2)_R$

A^μ unchanged

$\begin{pmatrix}\phi_1 \\ \phi_2\end{pmatrix}$ as a doublet

Take $SU_L(2)$: $\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$, $SU_R(2) = \begin{pmatrix}0 & -i\\ -i & 0\end{pmatrix}$

$$\left.\begin{aligned}\vec{A} &= 2v\,\frac{f(\xi)}{\xi}\,\hat{r}\times\vec{\tau}\\ \phi &= \sqrt{2}\,v\,h(\xi)\,\hat{r}\cdot\vec{\tau}\,u_{-1/2}\end{aligned}\right\}\text{DHN, Soni, Boguta}$$

$\xi = gvr$

Not $SU_L(2) + R$ invariant. Maximal invariance is $R + SU_R(2) + SU_L(2)$

$\Downarrow$

$\exists$ 3 translational zero modes

3 rotational "

Can show $\exists$ 1 unstable mode, all rest $E > 0$

$\boxed{\text{This is the electroweak sphaleron}}$

No known exact solutions:

Numerically:

$$E_{sp} = \frac{2M_W}{\alpha_W} A(\lambda/g^2)$$

$1.52 \leq A \leq 2.70$, $\lambda/g^2 = 1 \Rightarrow A = 2.07$

$$e^{-E_{sp}/kT} = e^{-1/\alpha_3(T)}$$

$\Rightarrow$ Larger rate

$$\Gamma/V \sim \underset{\tau_K ?}{K}\, e^{-1/\alpha_3(T)}$$ Kuzmin, Rubakov + Shaposhnikov

<u>Beware</u>: $\exists$ no sphaleron for $T > T_c$.

$R_{sp} \sim 1/M_W(T) \to \infty$ as $T \to T_c$

Potentially very dangerous!!

Another computation of Q_{ephal}

$$Q_{ep} = \int d^4x\ \partial\cdot K$$

$$= \int d^3x\ K^0$$

if $\int d^3x\ K^0\Big|_{t=-\infty} = 0$, and if $\lim_{r\to\infty} r^2\ \vec{\nabla}\cdot\vec{K} = 0$

Can make $\vec{\nabla}\cdot\vec{K}$ vanish rapidly at $r = \infty$: $U(r) = e^{i\Theta(r)\,\vec{\tau}\cdot\hat{r}}$

Let $\bar{\Theta}(r)$: $0 \le \bar{\Theta} \le \pi$ for $0 \le r \le \infty$

$\lim_{r\to\infty} \bar{\Theta}(r) \to \pi$ rapidly $\Rightarrow \vec{\nabla}\cdot\vec{K} \to 0.$

$$A_i^a = \frac{(1-2f)\cos\bar{\Theta} - 1}{g r^2}\,\epsilon_{iab}\, r^b$$

$$+ \frac{(1-2f)\sin\bar{\Theta}}{g r^3}\,(\delta_{ia} r^2 - r_i r_a)$$

$$+ \frac{1}{g}\,\frac{d\bar{\Theta}}{dr}\,\frac{r_i r_a}{r^2}$$

Compute $Q_{ep} = \frac{1}{2}$