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REVIEW OF LATTICE THEORIES

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Please note: These are preliminary notes intended for internal distribution only.

Claudio Rebbi - Boston University

Review of Lattice Theories

Lattice gauge theories (LGT) realize the dreams of a quantum field theorist

$$\langle O \rangle = Z^{-1} \int DA\, D\bar{\psi}\, D\psi\; O(A,\bar{\psi},\psi)\, e^{-S(A,\bar{\psi},\psi)}$$

but the r.h.s. is a purely formal expression, which must be given a meaning through the process of regularization. This is normally carried out in the context of perturbation theory, but then, we are taught, the perturbative expansion does not converge.

In L.G.T. the r.h.s. is well defined, it is a number.

$\int DA$ is not well defined, unless one fixes a gauge. But then the emergence of Gribov copies may invalidate the standard gauge fixing procedure.

In L.G.T. $\int DA$, replaced by $\int \prod_{x,\mu} dU_x^\mu$ ($U_x^\mu = e^{igA_x^\mu a}$), is a well defined integral over a group manifold. There is no need to fix a gauge.

$$\int D\bar{\psi}\, D\psi\; e^{-\bar{\psi}(D(A)+m)\psi} = \mathrm{Det}\,(D(A)+m)$$

which is again a formal expression that must be regularized. In L.G.T. $\mathrm{Det}\,(D(A)+m)$ as well as the matrix element of the propagator $(D(A)+m)^{-1}$ are well defined and calculable.

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Of course ....

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PHYSICS GOALS OF THE QCD TERAFLOP PROJECT

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Lattice calculations for QCD

Key words/facts

$$S = S(U_x^\mu, \bar\psi_x, \psi_x) = S_G(U_x^\mu, g) + S_F(U_x^\mu, \bar\psi_x, \psi_x)$$

g bare coupling constant ≈ 1
normally used

$$\beta = \frac{6}{g^2} \qquad (5 \le \beta \le 7)$$

$$S_F(U_x^\mu, \bar\psi_x, \psi_x) = \sum_{x,y} \bar\psi_x \left(D_{xy}(U) + m\delta_{xy} \right) \psi_y$$

↳ Wilson fermions
staggered fermions

$$\int \prod_{x\mu} dU_x^\mu \prod_x (d\bar\psi_x d\psi_x)\, e^{-S_G - \bar\psi (D(U)+m)\psi} =$$

$$= \int \prod_{x\mu} dU_x^\mu \; e^{-S_G(U)} \det(D(U)+m)$$

neglect (approx $\underset{= \text{const}}{\det} = 1$) → "quenched QCD"

keep → "QCD with dynamical fermions"

Key words/facts cont'd

lattice spacing a

renormalization $\quad a = a(g) = a(\beta)$

quenched QCD

$\beta \simeq 6 \qquad a \approx 0.1\,\mathrm{fm}\ (2\,\mathrm{GeV})^{-1}$

$\beta \simeq 6.4 \quad a \approx 0.05\,\mathrm{fm}\ (4\,\mathrm{GeV})^{-1}$

QCD with dynamical fermions

$\beta = 5.6 \quad m_q a = 0.01 \iff \beta = 6 \quad m_q = \infty$ (quenched)

($m_q a = 0.01 \quad a^{-1} \approx 2\,\mathrm{GeV} \Rightarrow m_q \approx 20\,\mathrm{MeV}$

(but, with lattice regularization

($20\,\mathrm{MeV} \approx \frac{1}{2} m_s$)

Typical lattice sizes

$N_s^3 \times N_t = 16^3 \times 32 \qquad (1.6^3 \times 3.2\ \mathrm{fm}^4)$

$24^3 \times 48$

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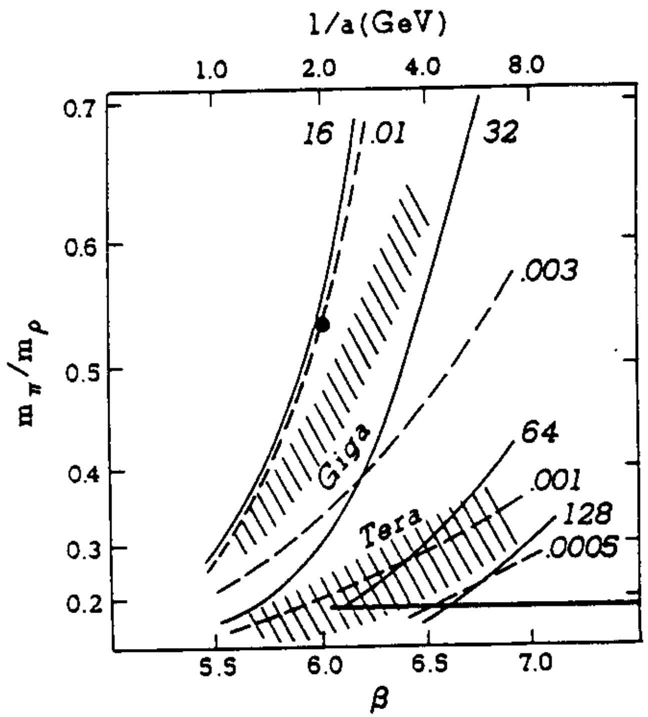
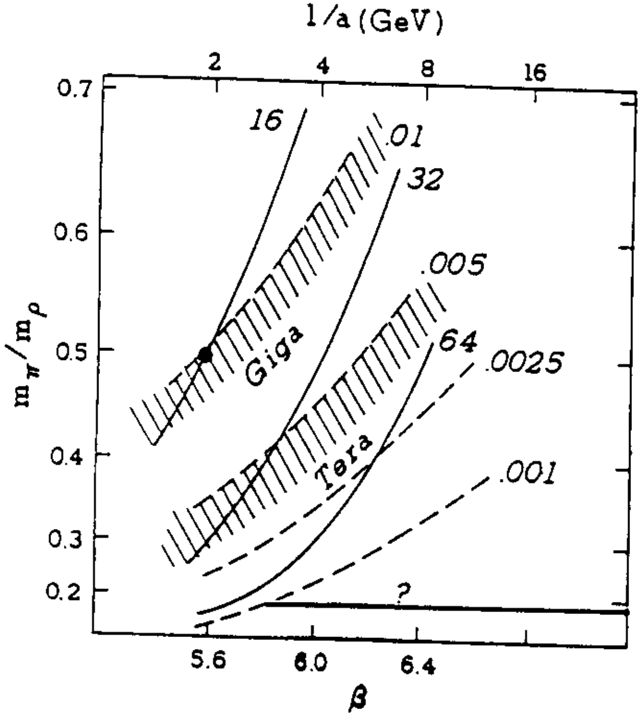


Fig. 2.1. Estimate of required operating parameters and computational effort for the quenched calculation assuming current algorithms and a lattice of 3–4 pion Compton wavelengths on a side. Bar at lower right: target scaling range at the physical value of m_π/m_ρ. Black dot: rough locus of current calculations. Cross hatched regions: range of parameters accessible at one Gigaflop and one Teraflop. Dashed lines: approximate value of quark mass m_q needed for the calculation. Solid line: approximate size of lattice needed.



g. 2.2. The same as Fig. 2.1, but for full QCD with 4 flavors. For the two flavor estimate add 2 to the value of β.

Spectrum calculations

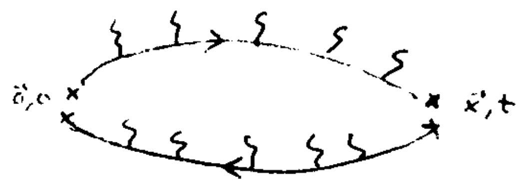
Masses are obtained from rate of decay of correlation functions in Euclidean space-time

$$\mathcal{O}(t) = \Big\langle \sum_{\vec{x}} \big(\bar{\psi}(\vec{x},t)\,\Gamma\,\psi(\vec{x},t)\big) \quad \bar{\psi}(\vec{0},0)\,\Gamma\,\psi(\vec{0},0) \Big\rangle$$

$$= \sum_{\text{mesons}} \langle \emptyset |\ \bar{\psi}(0)\,\Gamma\,\psi(0)\ | \text{meson}, \vec{p}=0\rangle\, e^{-m_{\text{meson}}t} \langle \text{meson}, \vec{p}=0 | \bar{\psi}\,\Gamma\,\psi | \emptyset$$

Calculate $\langle \mathcal{O} \rangle$ from

$$(D(U) + m)^{-1}_{0,x}$$



configuration per configuration, and then sum over configurations ($\Rightarrow \int \mathcal{D}U_x^{\mu}$)

quenched

with dynamical quarks

Spectrum calculations, cont'd

similarly for baryons, glueballs

(source $\mathrm{Tr}\, U_x^{\mu\nu}$ made of U_x^{μ}) etc..

In practice calculations are more sophisticated and make use of extended sources to enhance

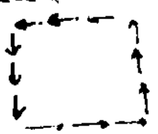
$$\langle \emptyset \,|\, \text{source} \,|\, \text{phys} \rangle$$

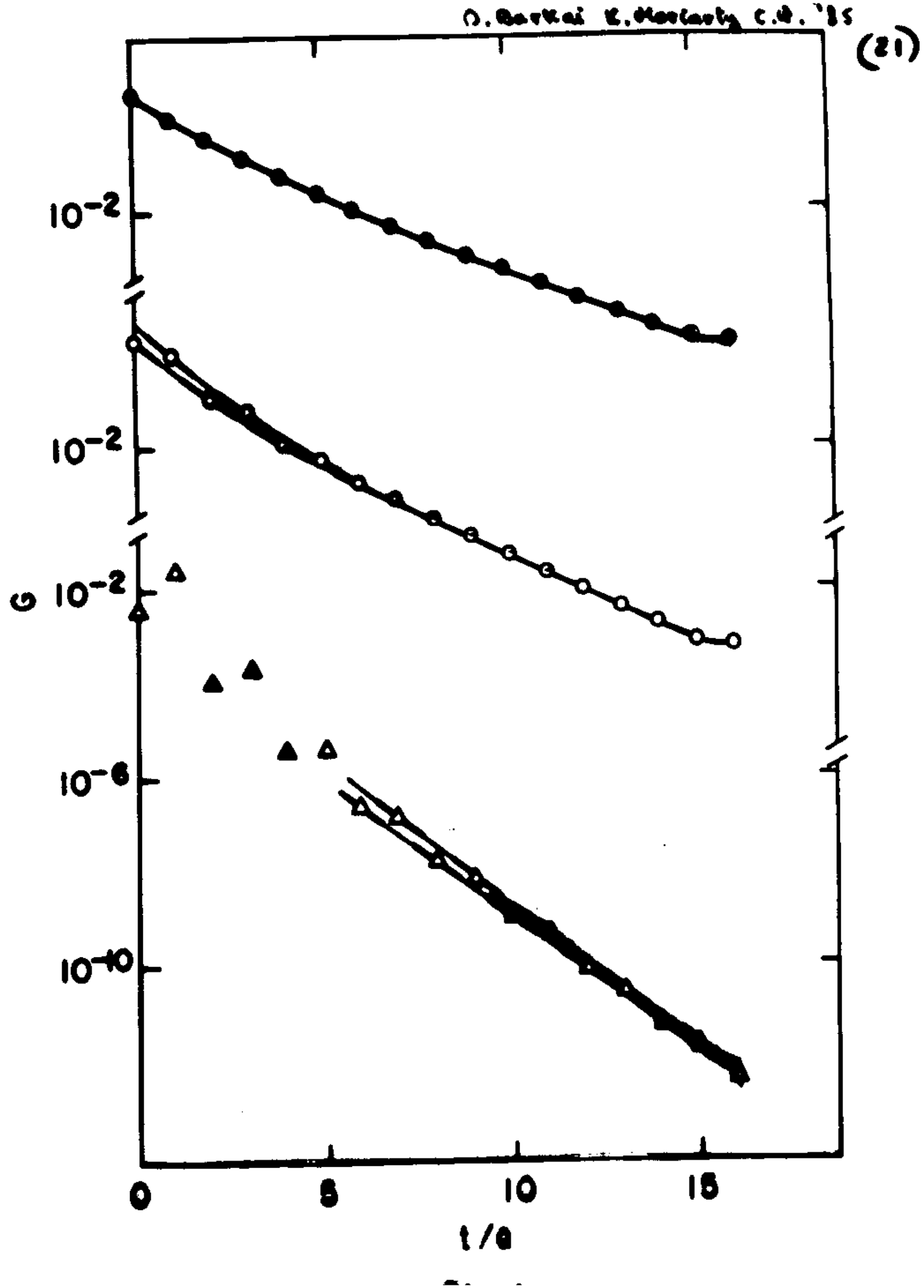
e.g.

$$\bar\Phi \Gamma \Phi \quad \text{where} \quad \Phi = \sum_{\vec x,\, t=0} \underbrace{(D+m)^{-1}_{\vec x, \vec 0}}_{\text{propagator over the time-slice } t=0} \psi_{\vec x}$$

wall sources (fix the gauge at $t=0$, use then a uniform quark source over the whole time slice)

spread-out glueball sources:

use  after a few iterations of $\uparrow \Rightarrow \uparrow + \ldots$



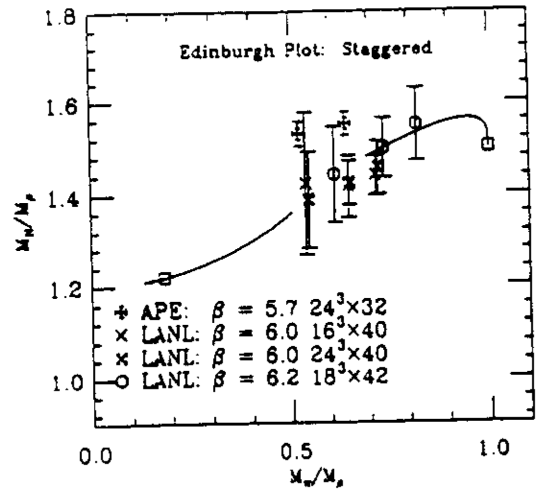


Fig. 3.1. Quenched QCD mass ratios "Edinburgh Plot" for Staggered fermions.

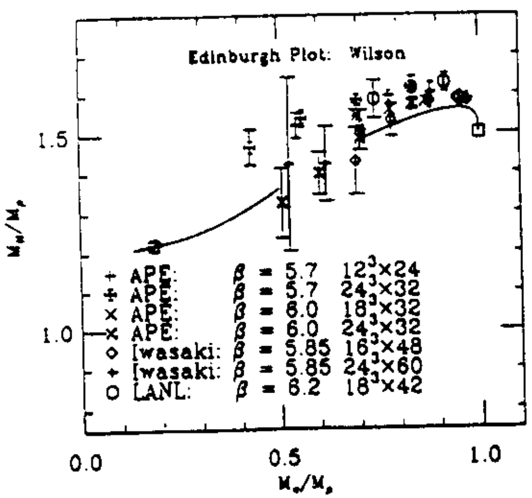


Fig. 3.2. Quenched QCD mass ratios "Edinburgh Plot" for Wilson fermions.

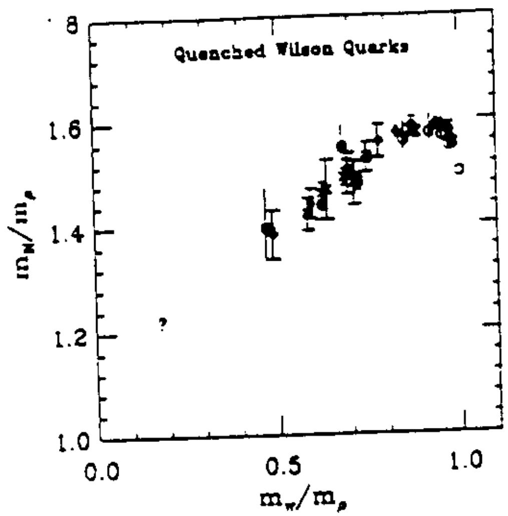


FIGURE 1

Edinburgh plot for quenched Wilson quarks. The octagon at the right is the heavy quark limit, and the question mark at the left is the physical point. A question mark is used to remind us that we don't really know what the quenched approximation should produce. Here the squares are HEMCGC results [22] at $6/g^2 = 5.85$ and 5.95, the diamonds APE results [7,23] at $6/g^2 = 6.0$ and 6.3, the crosses Iwasaki *et al.* [5] at $6/g^2 = 5.85$ and 6.0, and the octagons GF11 [4] at $6/g^2 = 5.7$.

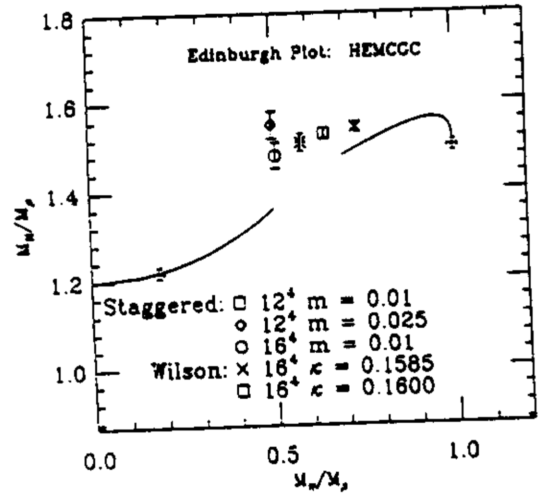


Fig. 3.3. Full QCD mass ratios "Edinburgh Plot".

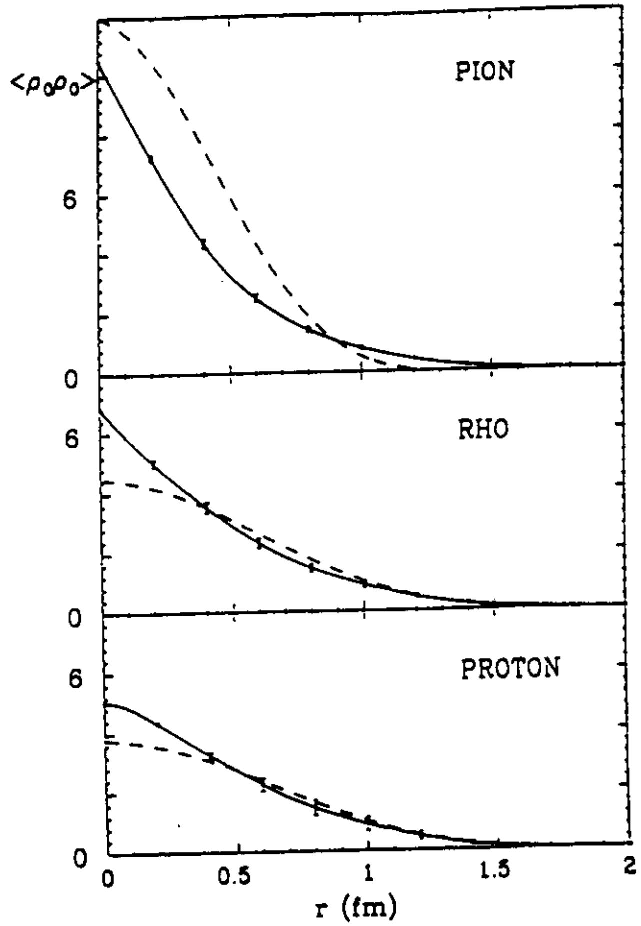


Fig. 7.3. Spatial dependence of density-density correlation functions for up and down quarks in the pion, rho, and proton. The error bars denote the lattice results, and are joined by a solid line to guide the eye. The dashed curves show the corresponding correlation function in the bag model. Al correlation functions are normalized to a volume integral of unity.

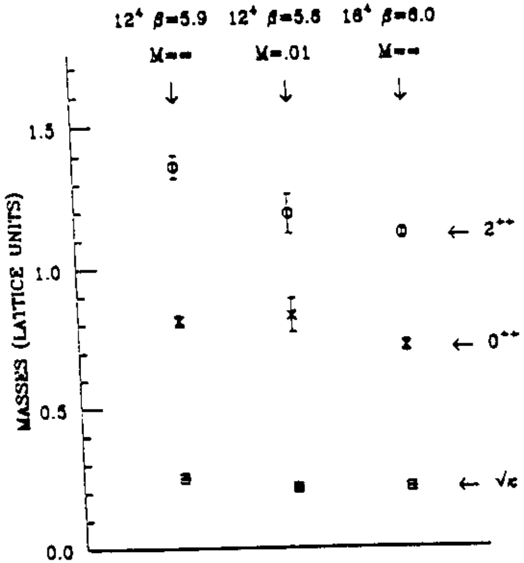
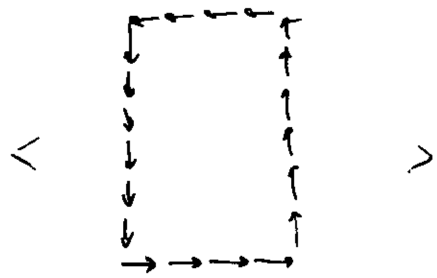
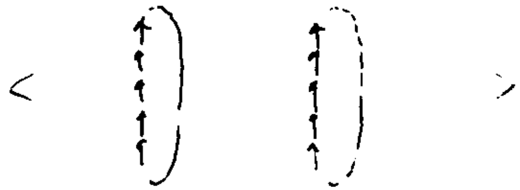


Fig. 4.8. Comparison of the glueball spectra with and without dynamical fermions.

Static potentials and other observables related to the physics of heavy quarks

$V(r)$ from 

or 

Lattice calculations for heavy quark physics provide very accurate determination of α_{QCD}

(?. Lepage . [illegible] !!!!)

$\alpha^{(4)}_{\overline{MS}}$ (5 GeV)	β	source
0.174 ± 0.012	5.7-6.1	ψ (Fermilab)
0.171 ± 0.012	6.0	Υ (NRQCD gr.)
0.169 ± 0.016	5.7	Υ (")
0.173 ± 0.012	5.7	ψ (")
0.216 ± 0.021	—	extrapol. from LEP

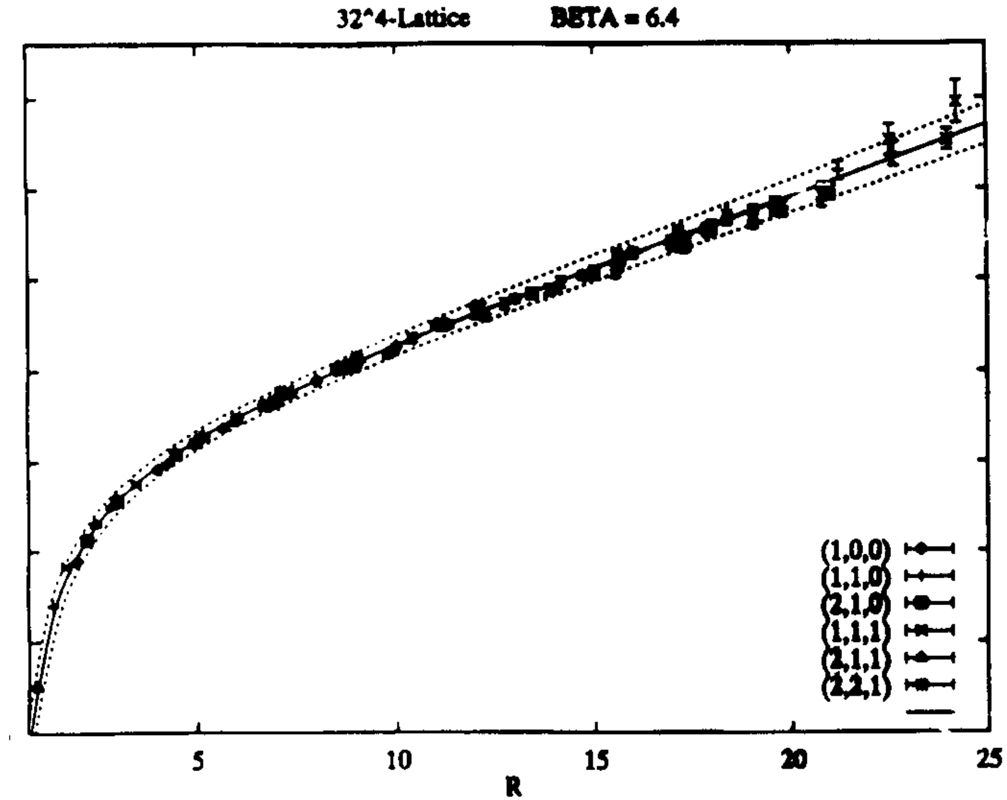


Figure 3: The potential $V(R)$ for the 32^4 lattice at $\beta = 6.4$. The various off-axis entries are indicated by different symbols. The error bars refer to statistical errors only, while the dashed error band incorporates both statistical and systematic errors.

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Static Quark-Antiquark Potential: Scaling Behaviour and Finite Size Effects in $SU(3)$ Lattice Gauge Theory[1]

Finite temperature QCD

Use

$$Z = \mathrm{Tr}_{phys}\, e^{-\frac{1}{kT} H} = \int \mathcal{D}A\,\mathcal{D}\bar{\psi}\,\mathcal{D}\psi\; e^{-\int_0^{1/kT} dt\, \mathcal{L}}$$

to relate finite temperature QFT to lattice periodic in Euclidean time (antiperiodic for fermionic variables) and extent

$$\Delta t = N_t a = \frac{1}{kT}$$

(set $k=1$)

$$T = \frac{1}{N_t a} \qquad \left(T = \frac{1}{N_t a(\beta)} \right)$$

(T increases for decreasing N_t and for increasing β)

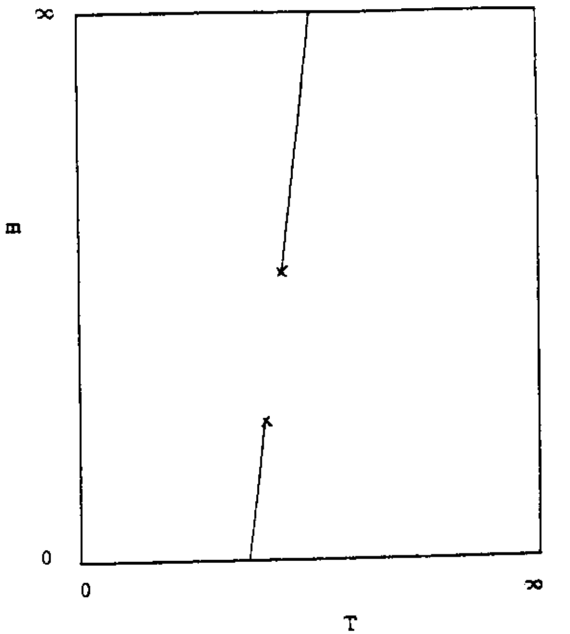
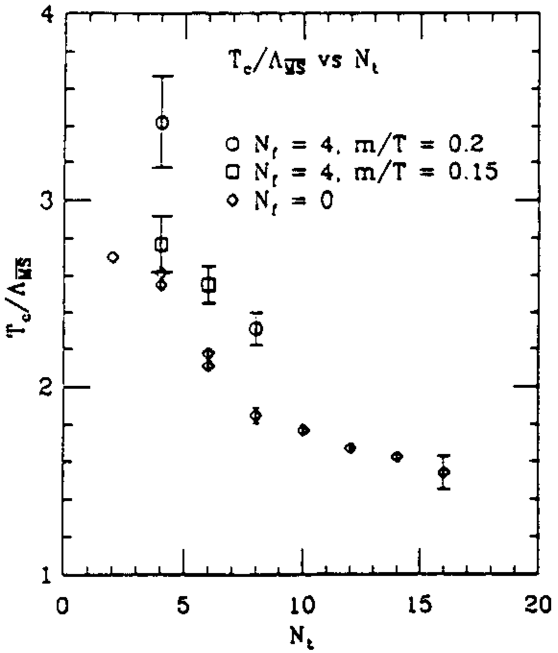


Fig. 8.1. Schematic phase diagram for QCD in quark mass m and temperature T.

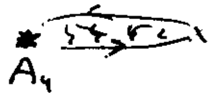


Matrix element calculations

Use again Green's functions with 2 or 3 operators to create the particle states and the currents

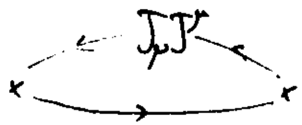
e.g.

$$\langle \bar\Psi \gamma_4 \gamma_5 \Psi \quad \bar\Psi \gamma_5 \Psi \rangle \text{ for } f_\pi$$



$$\langle s\gamma_5 \bar d \quad (\bar s \gamma_\mu (1-\gamma_5) d)(\bar s \gamma_\mu (1-\gamma_5) d) \quad s \gamma_5 \bar d \rangle$$

for B_K



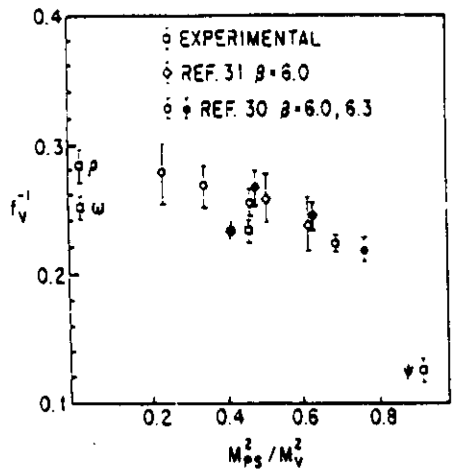


Fig. 2. Vector meson decay constants as a function of the meson mass.

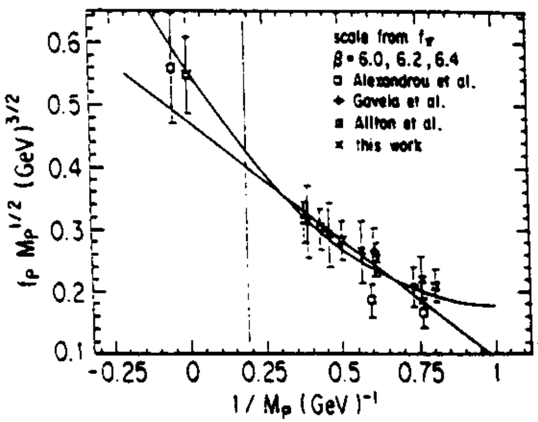


Fig. 3. The quantity $f_{M_P} M_P^{1/2}$ is reported as a function of the inverse pseudoscalar mass. The results of several calculations at $\beta = 6.0$, 6.2 and 6.4, with fully propagating quarks and in the static limit, are shown. The curves refer to linear a quadratic fits in $1/M_p$ to the points. The vertical line identify the point corresponding to the B-meson.