INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

SMR.626 - 25
Lect. I & II

SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

15 June - 31 July 1992

# BARYON AND LEPTON NUMBER VIOLATION IN THE ELECTROWEAK MODEL

V. RUBAKOV
Institute for Nuclear Research
Academy of Sciences of Russia
Moscow
Russian Federation

# INSTANTONS

BELAVIN, POLYAKOV, TYUPKIN, SCHWARZ '75

ERPRETATION IN TERMS VACUUM STRUCTURE – 't Hooft '76
Jackiw, Rebbi '76
Callan, Dashen, Gross '76

G-Mills-Higgs 't Hooft '76
Affleck '81

non-conservation 't Hooft '76

• – INSTANTON IN ABELIAN HIGGS MODEL IN (1+1) DIMENSIONS ≡ ABRIKOSOV-NIELSEN-OLESEN VORTEX

ELECTRODYNAMICS WITH HIGGS FIELD IN (1+1) DIM.

$$S_M = \int d^2x \left(-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu\varphi|^2 - V(\varphi)\right)$$

BREAKS U(1) SYMMETRY

otate to euclidean time $t \to -ix^0$

$A_0 \to iA_0$ Sometimes important to remember

$$e^{iS_M} \to e^{-S_E}$$

$$S_E \equiv S = \int d^2x \left[\tfrac{1}{4}F_{\mu\nu}F_{\mu\nu} + |D_\mu\varphi|^2 + V(\varphi)\right]$$

$g^{00} = g^{11} = 1$ ; $\mu,\nu = 0,1$; $D_\mu\varphi = (\partial_\mu - igA_\mu)\varphi$



INSTANTON ≡ (TOPOLOGICALLY NON-TRIVIAL) SOLUTION TO CLASSICAL EUCLIDEAN FIELD EQUATIONS WITH FINITE ACTION.

• INFINITY: $F_{\mu\nu} = 0 \Rightarrow A_\mu = \frac{1}{g}\partial_\mu\alpha(x)$

$V(\varphi) = 0 \Rightarrow |\varphi| = v$ Higgs v.e.v.

$D_\mu\varphi = 0 \Rightarrow \varphi = v e^{i\alpha(x)}$

CHOOSE THE GAUGE $\alpha(x) = \alpha(\theta)$

POLAR ANGLE IN SPACE-TIME

SINGLE-VALUEDNESS OF $\varphi$

$\Downarrow$

$$\alpha(\theta = 2\pi) = \alpha(\theta = 0)$$

Mapping of $S^1$ (SPATIAL INFINITY) $\to S^1$ (HIGGS CONFIGUR. SPACE)

NON-TRIVIAL MAPPING (WINDING NUMBER = 1)

$\alpha(\theta) = \theta$ (up to trivial gauge transformations)

$$\text{WINDING NUMBER} = \frac{g}{2\pi}\oint_{r\to\infty} A_\mu\, dx^\mu = \frac{g}{4\pi}\int \varepsilon_{\mu\nu}F_{\mu\nu}\, d^2x$$

GAUGE INVARIANT QUANTITY

NB: INSTANTON DESCRIBES TUNNELING BETWEEN NEIGHBOURING VACUA:

GAUGE TRANSFORM TO $A_0 = 0$

$$1 = \frac{g}{4\pi}\int \varepsilon_{\mu\nu} F_{\mu\nu} = \underbrace{\frac{g}{2\pi}\left[\int dx^1 A_1\Big|_{t=+\infty} - \int dx^1 A_1\Big|_{t=-\infty}\right]}$$

CHANGE IN CHERN-SIMONS NUMBER OF VACUUM

— o —

ACTUAL FIELD AT FINITE $r$:

$$A_\mu = \frac{1}{g} A(r)\, \partial_\mu \alpha(x)$$

$$\varphi = v\, B(r)\, e^{i\alpha(x)}$$

$\alpha(x) = \theta$; $A(r)$ AND $B(r)$ TO BE FOUND BY SOLVING CLASSICAL FIELD EQS.

<u>AT LARGE $r$:</u>

$$A(r) = 1 + c\,\frac{e^{-m_V r}}{\sqrt{r}} + \mathcal{O}\left(\frac{g^2}{\lambda}\right)$$

$$B(r) = 1 + c'\,\frac{e^{-m_H r}}{\sqrt{r}} + \mathcal{O}\left(\frac{g^2}{\lambda}\right)$$

at $\lambda \gg g^2$

<u>AT SMALL $r$:</u> $A(0) = B(0) = 0$ (NO SINGULARITY)

$A(x)$ AND $\varphi(x)$ DECAY SLOWLY, BUT GAUGE-INVARIANT QUANTITIES LIKE $F_{\mu\nu}$ DECAY FASTLY.

$$A_\mu \to A_\mu - \frac{1}{g}\partial_\mu \alpha = A'_\mu$$

$$\varphi \to e^{-i\alpha}\varphi = \varphi'$$

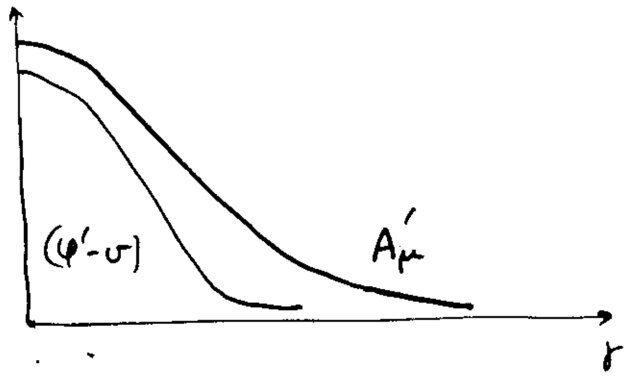
[ $\alpha(x) = \theta$ IS SINGULAR AT $r = 0$ ]

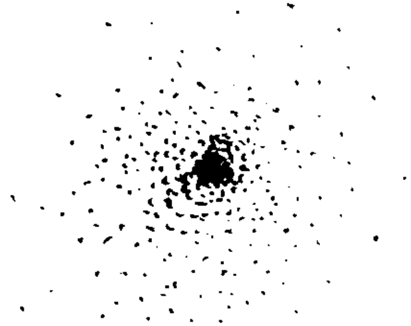
$A'_\mu$ IS SINGULAR AT $r=0$, BUT THIS IS PURE GAUGE SINGULARITY ($F_{\mu\nu}$ IS REGULAR)

LARGE $r$:

$$A'_\mu(x) \propto \frac{c}{\sqrt{r}} e^{-m_V r}$$

$$\varphi'(x) - v \propto \frac{c'}{\sqrt{r}} e^{-m_H r}$$



## INSTANTON IN PURE YANG-MILLS THEORY IN (3+1) DIMENSIONS

$$S_{Y.M.}\ (\text{EUCLIDEAN}) = \int d^4x\ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu}$$

$a = 1, 2, 3$ for $SU(2)$ gauge group to be considered hereafter

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\,\varepsilon^{abc} A^b_\mu A^c_\nu$$

SCALE INVARIANCE: ACTION IS INVARIANT UNDER TRANSFORMATION

$$A_\mu(x) \to \lambda A_\mu(\lambda x)$$

hus, If $A_\mu(x)$ is a solution to classical field equations, then $\lambda A_\mu(\lambda x)$ is also a solution $\Rightarrow$ expect a one parameter set of solutions (in fact, number of parameters is larger than one - see below).

Topology of the instanton.

Finite action $\Rightarrow$ $A^a_\mu(x)$ is pure gauge at infinity

In matrix notations $A_\mu = ig \frac{\tau^a}{2} A^a_\mu$,

$\ldots = \Omega\,\partial\,\Omega^{-1}$ where $\Omega(n^\mu)$ is gauge

$\underline{\Omega(n^\mu) \in SU(2)}$ at every $n^\mu$

($n^\mu$ = unit radius-vector in 4-dim's)

$\Omega$ defines mapping

$$S^3_{(\text{infinity of 4d spacetime})} \to S^3_{(SU(2)\text{-group})}$$

These mappings are characterized of integer winding number $q$.

$$\boxed{q = \frac{g^2}{32\pi^2} \int F^a_{\mu\nu} \tilde{F}^a_{\mu\nu}\, d^4x}$$

$$\tilde{F}^a_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\lambda\rho} F^a_{\lambda\rho}$$

This is because $F_{\mu\nu}\tilde{F}_{\mu\nu} = \partial_\mu K_\mu$, and the integral $\int d^4x F\tilde{F} \to \int_{|x|\to\infty} d\sigma^\mu K_\mu$;

for pure gauge $A_\mu$

$$\int d\sigma^\mu K_\mu \propto q$$

SEARCH FOR FINITE ACTION SOLUTIONS TO FIELD EQUATIONS WITH

$$q \propto \int F\tilde{F} \neq 0.$$

BPST TRICK: WE WANT TO minimize the action.

ow,

$$\int \left(F^a_{\mu\nu} - \tilde{F}^a_{\mu\nu}\right)\left(F^a_{\mu\nu} - \tilde{F}^a_{\mu\nu}\right) d^4x \geq 0$$

Equality iff $F^a_{\mu\nu} = \tilde{F}^a_{\mu\nu}$

$$\Downarrow$$

$$\int F^{a\,2}_{\mu\nu} - 2\int F^a_{\mu\nu}\cdot\tilde{F}^a_{\mu\nu} + \int \tilde{F}^a_{\mu\nu}\tilde{F}^a_{\mu\nu} \geq 0$$

$$= \int F^a_{\mu\nu} F^a_{\mu\nu}$$

$$\Downarrow$$

$$S = \frac{1}{4}\int F^a_{\mu\nu}F^a_{\mu\nu} \geq \frac{1}{4}\int F^a_{\mu\nu}\tilde{F}^a_{\mu\nu} = \frac{8\pi^2}{g^2}q$$

$$S \geq \frac{8\pi^2}{g^2} q$$

Equality iff

order diff. equ. $\boxed{F^a_{\mu\nu} = \tilde{F}^a_{\mu\nu}}$ Solve this self-duali[ty] equation instead of field equ

ld equ. is automatically satisfied

$$D_\mu F_{\mu\nu} = D_\mu \tilde{F}_{\mu\nu} \equiv 0 \qquad \text{(Bianchi identity)}$$

NB: $q<0$ - anti-instantons; obey $F = -\tilde{F}$.

One instanton solution:

$$A^a_\mu(x) = \frac{2}{g}\,\frac{\bar{\eta}_{\mu\nu a}x_\nu}{x^2+1}$$

where $\bar{\eta}_{ija} = \varepsilon_{ija}$; $\bar{\eta}_{0ia} = \delta_{ia}$ and $\eta_{\mu\nu a}$ is antisymmetric in $\mu\nu$

($\eta_{\mu\nu a} \equiv$ 't Hooft symbols in physics literature)

More one-instanton solutions:

One can: i) displace the instanton $\Rightarrow x^\mu \to x^\mu - x_0^\mu$

arbitrary numbers

ii) rotate in internal space

$$A^a_\mu \to U^{ab}A^b_\mu$$

arbitrary $O(3)$ matrix ($SU(2)$ in adjoint rep.)

iii) make scale transformation:

$$A_\mu(x) = \lambda A_\mu(\lambda x)$$

iv) Make $O(4)$-rotations $x^\mu \to \Lambda^\mu{}_\nu x^\nu$

GENERAL SOLUTION

$$A^a_\mu(x) = \frac{2}{g} U^{ab} \frac{\bar{\eta}_{\mu\nu b}(x^\nu - x_0^\nu)}{(x-x_0)^2 + \rho^2}$$

Number of parameters:

$$3\,(U^{ab}) + 4\,(x_0^\mu) + 1\,(\rho) = 8$$

SLOW DECAY AT INFINITY:

$$A_\mu = \Omega\,\partial_\mu \Omega^{-1} \propto \frac{1}{|x|} \quad \text{at} \quad |x| \to \infty$$

$$\Omega(n^\mu) = \sigma^\mu n^\mu \qquad (\text{TOPOLOGY}: S^3 \to S^3)$$

$$\sigma^0 = 1, \quad \sigma^i = i\tau^i$$

MAKE GAUGE TRANSFORMATION $\Omega^{-1}(n^\mu)$ EVERYWHERE IN 4D SPACE-TIME

$\Downarrow$

PURE GAUGE SINGULARITY AT $x = 0$ (for $x_0 = 0$) BUT RAPID DECREASE AS $|x| \to \infty$

$$\boxed{A^a_\mu = \frac{2\rho^2}{g} U^{ab} \frac{\bar{\eta}_{\mu\nu b}(x^\nu - x_0^\nu)}{(x-x_0)^2[(x-x_0)^2+\rho^2]}}$$

Instanton parameters: $x_0^\mu$, $\rho$ and 3 IN $U^{ab}$

ACTION: $S_{INST} = \dfrac{8\pi^2}{g^2}$



TO SUMMARIZE:

INSTANTON IS LOCAL MINIMUM OF ACTION IN SECTOR $q = 1$;

$$A_\mu \propto \frac{1}{g}, \quad S = \frac{8\pi^2}{g^2}, \quad A \propto \frac{1}{|x|^3} \text{ AS } |x| \to \infty$$

DIPOLE IN 4D

8-parameter family parametrized by position $x_0^\mu$, size $\rho$ and orientation in internal space.

INSTANTONS INDUCE $\not{B}$ IN EW THEORY (McLerran's lectures)

FORGET ABOUT FERMIONS UNTIL THE END OF THESE LECTURES:

- EVEN PURE BOSONIC THEORIES ARE COMPICATED ENOUGH
- FERMIONS ARGUED TO PLAY SUBDOMINANT ROLE IN INSTANTON TRANSITIONS (Espinosa '91)

## "INSTANTONS" IN SU(2) YANG-MILLS-HIGGS

(SU(2) SECTOR OF ELECTROWEAK THEORY)

LIDEAN ACTION:

$$S = \int \left[ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + (D_\mu \varphi)^+ (D_\mu \varphi) + V(\varphi) \right] d^4x$$

$$D_\mu \varphi = \left( \partial_\mu - i g \frac{\tau^a}{2} \right) \varphi \quad ; \quad V(\varphi) = \frac{\lambda}{2} \left( \varphi^+ \varphi - \frac{v}{\sqrt{2}} \right)^2$$

$\varphi$ is in fundamental (doublet) rep. of SU(2).

symmetry is spontaneously broken altogether:

LL THREE VECTOR BOSONS AQUIRE EQUAL MASSES

$$m_W = g v$$

IGGS DOUBLET IS

$$\varphi = \begin{pmatrix} \chi_1 + i \chi_2 \\ v + \eta + i \chi_3 \end{pmatrix}$$

$\chi_1, \chi_2, \chi_3$ ARE EATEN UP BY VECTOR FIELDS

O MAKE MASSIVE VECTOR BOSONS

IS THE HIGGS boson, $m_\eta = \sqrt{\lambda}\, v = m_H$

W THEORY IN THE LIMIT $\sin \theta_{\dots} \to 0$ ]

THERE ARE NO INSTANTONS IN THIS THEORY

BUT THAT REALLY DOES NOT MATTER.

• THERE ARE NO INSTANTONS (FINITE ACTION solutions to euclidean field equations).

SCALING ARGUMENT (a la' Derrick '64)

Suppose THERE WERE SUCH A SOLUTION $A(x)$, $\varphi(x)$

THIS WOULD BE AN EXTREMUM OF EUCLIDEAN ACTION

CONSIDER A NEIGHBOURING (IN CONFIGURATION SPACE) CONFIGURATION

$$A'_\mu = \lambda A_\mu(\lambda x), \quad \varphi'(x) = \varphi(\lambda x), \quad \lambda = 1 + \epsilon, \ \epsilon \text{ small}$$

ACTION FOR NEW CONFIGURATION

$$S[A', \varphi'] = \int \frac{1}{\lambda^4} d^4x \left[ \lambda^4 \cdot \frac{1}{4} F^2_{\mu\nu}(A) + \lambda^2 |D_\mu \varphi|^2 + V(\varphi) \right]$$

old fields

$$= \int F^2 + \frac{1}{\lambda^2} \int |D\varphi|^2 + \frac{1}{\lambda^4} \int V(\varphi)$$

ACTION IS NOT STATIONARY AT $\lambda = 1$

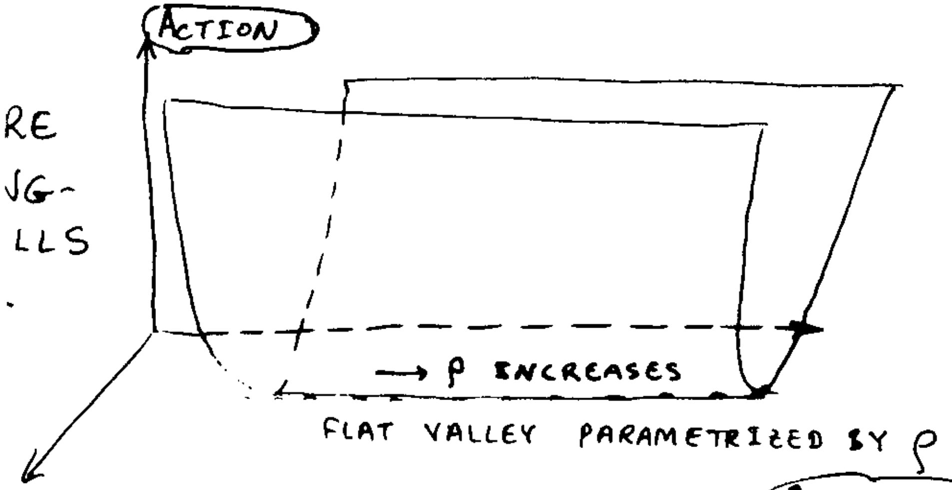
$$\left. \frac{\partial S}{\partial \lambda} \right|_{\lambda = 1} = -2 \int |D\varphi|^2 - 4 \int V(\varphi) < 0$$

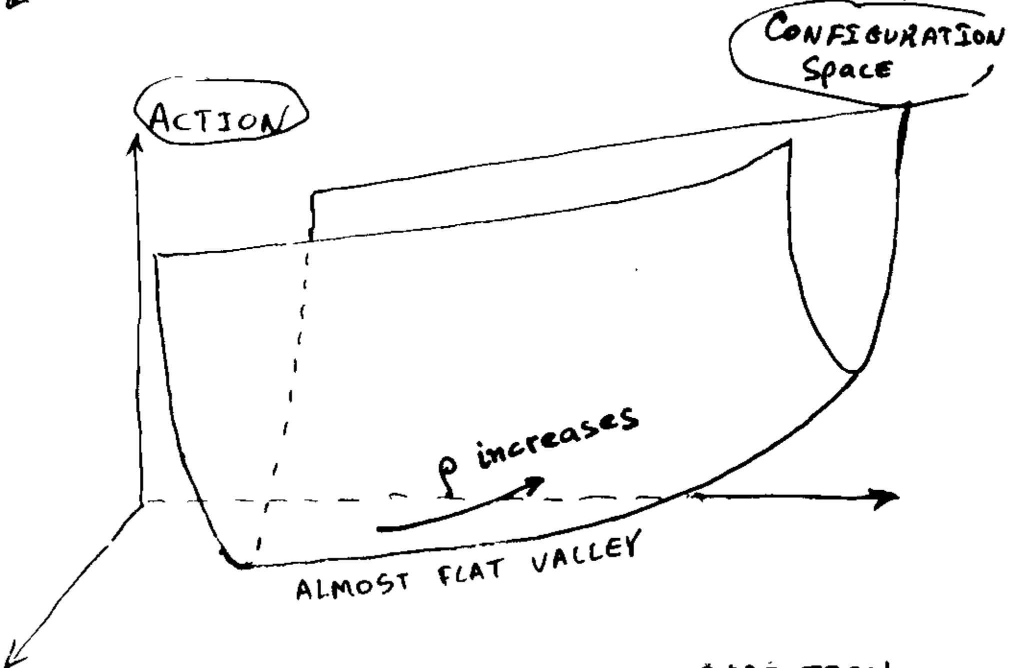
$$\Downarrow$$

• THAT REALLY DOES NOT MATTER.

THERE EXIST FINITE ACTION CONFIGURATIONS THAT ARE ALMOST STATIONARY POINTS OF THE ACTION

RE
NG-
LLS




ALMOST FLAT: CURVATURE IN ρ DIRECTION
[illegible] BY $\rho^2$ COMPARED TO ORTHOGONAL

CONSIDER NOW PATH INTEGRAL (EUCLIDEAN)

PURE YANG-MILLS:

$$\int [dA]\, e^{-S[A]}$$

$A_{inst}(\rho)$ IS A MINIMUM FOR EVERY $\rho$

(FORGET ABOUT $x_0$ AND $U^{ab}$ for a moment)

SADDLE-POINT APPROXIMATION:

$$\int [dA]\, e^{-S[A]} = \int d\rho\, e^{-S_{inst}} \int [d(\Delta A)]\, e^{-\Delta A\cdot M\cdot \Delta A}$$

integral over fluctuations orthogonal to $\rho$-direction (flat)

YANG-MILLS-HIGGS

INTEGRATE OVER FLUCTUATIONS ORTHOGONAL TO $\rho$-DIRECTION IN GAUSSIAN APPROXIMATION (CURVATURE IS LARGE) BUT KEEP INTEGRAL OVER $\rho$ NON-GAUSSIAN

$$\int [dA][d\varphi]\, e^{-S[A,\varphi]} = \int d\rho\, e^{-S_{inst}(\rho)} \int_{\substack{\text{ORTHOGONAL}\\ \text{FLUCTUATIONS}}}$$

integral goes along the bottom of the valley.

one should have this "ALMOST SADDLE POINT" FORMULA lethod to separate $\rho$-variable:

$$\int [dA\, d\varphi]\, e^{-S[A,\varphi]} =$$

$$= \int d\rho \int [dA\, d\varphi]\, e^{-S[A,\varphi]}\, \delta(B(A) - \rho)$$

where $B(A)$ is some functional that is

- Not invariant under rescaling $A \to \lambda A(\lambda x)$
- Is equal to $\rho$ for pure Yang-Mills instanton

or example, $B(A) \propto \left( \int F^3 d^4x \right)^{-1/2}$

$$\Downarrow$$

minimize action under condition
$$B(A) = \rho$$
$$\Downarrow$$

Eqn's become $\frac{\delta S}{\delta A} = \Lambda \frac{\delta B}{\delta A}$

$\Lambda$ — Lagrange multiplier

ey now have solutions.

other words, minimum of the action
der the condition $B(A) = \rho$ now exists.

Orders of various terms in action

eading: $\int d^4x F_{\mu\nu}^2 \propto \frac{1}{g^2}$

b-leading: $\int |D_\mu \varphi|^2 d^4x \propto \rho^2 v^2$ (because $\varphi \propto v$) $v =$ v.e.v. of $\varphi$

ib-ib-leading: $\int V(\varphi)\, d^4x \propto \lambda \rho^4 v^4$ $\left( V = \lambda \left( \varphi^+\varphi - \frac{v}{\sqrt{2}} \right)^2 \right.$

If $\rho^2 v^2 \ll \frac{1}{g^2}, \frac{1}{\lambda}$ orders are different

To minimize the action, one first minimizes leading term

$$\Downarrow$$

$$A_\mu^a(x) = A_\mu^a(x,\rho) \Big|_{\text{inst. of pure Yang-Mills}} \Rightarrow \frac{1}{4}\int F_{\mu\nu}^2 d^4x = \frac{8\pi^2}{g^2}$$

Then one minimizes subleading term (correction to leading term is irrelevant to this order), at given $A_\mu^a$:

$$\varphi_{inst} = v \left[ \frac{(x - x_0)^2}{(x-x_0)^2 + \rho^2} \right]^{1/2} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

(singular gauge, where $A_{inst} \sim \frac{1}{|x|^3}$)

$\Downarrow$ $\left( |D \varphi|^2 d^4 x \ldots \right.$

THUS, THE TOTAL ACTION

$$\boxed{S_{INST}(\rho) = \frac{8\pi^2}{g^2} + \pi^2 v^2 \rho^2} + O(g^2 v^4 \rho^4, \lambda v^4 \rho^4)$$

HE SHAPE OF THE FIELDS $A_{inst}, \varphi_{inst}$ EED NOT BE ONE GIVEN ABOVE IN THE EGION $|x| \gg \rho$ WHERE THE CONTRIBUTION NTO ACTION IS SMALL ANYWAY.

IN FACT, IN THAT REGION FIELDS DECAY EXPONENTIALLY.

INDEED, MODIFIED FIELD EQN IS

$$\frac{\delta S}{\delta A} = \Lambda \frac{\delta B}{\delta A}, \quad \frac{\delta S}{\delta \varphi} = 0$$

T LARGE $|x|$ FIELDS ARE SMALL

$$\Downarrow$$

FIELD EQS. BECOME LINEARIZED ($B \sim \int F^3 \Rightarrow \frac{\delta B}{\delta A} \sim F^2$ - SMALL)

$$\Downarrow$$

AT LARGE $|x|$ FIELDS OBEY FREE FIELD EQUATIONS AND BEHAVE LIKE

$$A_{inst} \propto c e^{-m_V |x|}$$

$$\varphi_{inst} \propto v + c' e^{-m_H |x|}$$



THE REGION

$$|x| \gg \rho \qquad \text{(WEAK FIELD)}$$

$$|x| \ll \frac{1}{m_W}, \frac{1}{m_H} \qquad \text{(MASSLES FIELDS, } V(\varphi) \text{ IRRELEVANT)}$$

TO SUMMARIZE, IN MODIFIED SADDLE-POINT APPROX.

$$\boxed{\int [dA\, d\varphi]\, e^{-S[A,\varphi]} = \int d\rho\, e^{-S_{inst}(\rho)}} \left[ \int \text{fluctuations orthogonal to almost flat direc'n} \right]$$

WITH $\boxed{S_{inst}(\rho) = \frac{8\pi^2}{g^2} + \pi^2 v^2 \rho^2}$

FIELDS IN REGION $|x| \ll \frac{1}{m_W}, \frac{1}{m_H}$ ARE

$$\boxed{A^a_{inst,\mu} = \frac{2\rho^2}{g} \frac{\bar{\eta}_{\mu\nu a}(x^\nu - x_0^\nu)}{(x-x_0)^2[(x-x_0)^2 + \rho^2]}}$$

$$\boxed{\varphi_{inst} = v \left[ \frac{(x-x_0)^2}{(x-x_0)^2 + \rho^2} \right]^{1/2} \begin{pmatrix} 0 \\ 1 \end{pmatrix}}$$

THEY MATCH TO FREE FIELDS (EXPONENTIALLY DECAYING) IN THE REGION

$$\rho \ll |x| \ll \frac{1}{m_W}, \frac{1}{m_H}$$

ELECTROWEAK BARYON NUMBER
NON-CONSERVATION IN HIGH ENERGY COLLISIONS

- HIGH RATE OF $\not B$ IN STANDARD MODEL AT HIGH TEMPERATURES (see M$^c$Lerran's lectures)

- TYPICAL SCALE $E_0 \sim \frac{M_W}{\alpha_W} \sim 10$ TeV

$$\Downarrow$$

- SOME (MUTLIPARTICLE) $\not B$ PROCESSES HAVE LARGE PROBABILITY AT HIGH ENERGIES.

MAY ELECTROWEAK $\not B$ CROSS SECTIONS OF two-particle collisions BE LARGE AT HIGH ENERGIES?

QUANTITATIVE STUDIES STARTED BY RINGWALD (89)
STILL NO ANSWER.

OBJECT OF LECTURES:

DEVELOPMENTS IN UNDERSTANDING OF THIS PROBLEM.

OUTLINE

I. INSTANTONS
- ABELIAN HIGGS MODEL IN (1+1) DIM.
- PURE YANG-MILLS IN (3+1) DIM.
- ELECTROWEAK THEORY IN SU(2) SECTOR (YANG-MILLS-HIGGS)

II. LOWEST ORDER (RINGWALD) CALCULATION OF INSTANTON CROSS SECTION
- HIGGS PRODUCTION: $n$-PARTICLE
- HIGGS PRODUCTION: INCLUSIVE
- W-BOSON PRODUCTION

III. CORRECTIONS: GENERALITIES

IV. COHERENT STATE FORMALISM
- S-MATRIX IN COHERENT STATE BASIS
- LEADING ORDER INSTANTON RESULTS
- SOFT-SOFT CORRECTIONS
- CONSTRAINT DEPENDENCE

V. HARD-HARD AND HARD-SOFT CORRECTIONS

IV. TOWARDS THE SEMICLASSICAL CALCULATION OF HIGH ENERGY INSTANTON CROSS SECTIONS

LOWEST ORDER (RINGWALD) CALCULATION OF INSTANTON CROSS SECTION

Ringwald '89
Espinosa '89

xponentiation of HIGGS RODUCTION — McLerran, Vainshtein, Voloshin '90

xponent of W-BOSON RODUCTION — Zakharov '90, Khlebnikov, V.R., Tinyakov '90, Porrati '90

IN SOME APPROXIMATION, THE CROSS SECTIONS OF TWO-PARTICLE COLLISIONS THROUGH INSTANTONS RAPIDLY GROW WITH ENERGY.

APPROXIMATION: (NAIVE) SADDLE POINT.

VALID IN EW THEORY AT $E \ll E_0$

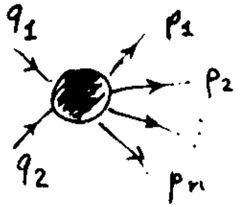
$\left( E_0 \sim \frac{M_W}{\alpha_W} \right)$

THROUGH INSTANTONS: SADDLE POINT = INSTANTON
B IS NOT CONSERVED IF FERMIONS ARE INCLUDED

RAPIDLY: $\sigma_{2\to n} \propto E^{\text{const}\cdot n}$ (POINTLIKE)
for process of $n$-particle production

MAIN IDEA:

CONSIDER PROCESS




$2 \to n$

MAKE USE OF LEHMANN–SYMANZIK–ZIMMERMANN FORMALISM:

$$A(\vec{q}_1, \vec{q}_2; \vec{p}_1, \dots, \vec{p}_n) = \Big[ (q_1^2 - m^2)(q_2^2 - m^2)(p_1^2 - m^2) \cdots \cdot (p_n^2 - m^2)\, G_M(q_1, q_2; p_1, \dots, p_n) \Big]_{\substack{q_i^2 = m^2 \\ p_i^2 = m^2}}$$

SAY, FOR SCALAR BOSONS, generically, $\varphi(x)$)

$$G_M(q_1, q_2; p_1, \dots, p_n) = \langle \varphi(y_1)\varphi(y_2)\varphi(x_1)\dots\varphi(x_n) \rangle_{\text{FOURIER transform}}$$

MINKOWSKI TIME $\Rightarrow$ MINKOWSKIAN MOMENTA

CONSIDER FIRST GREEN'S FUNCTION AT EUCLIDEAN MOMENTA $\Rightarrow$ EUCLIDEAN FUNCTIONAL INTEGRAL

$$G_E(y_1, y_2; x_1 \dots x_n) = \int e^{-S[\varphi]} \varphi(y_1)\varphi(y_2)\varphi(x_1)\dots\varphi(x_n) [d\varphi]$$

(The final answer for $G_E(y; x)$ should be Fourier transformed, analytically continued to

THERE EXIST A MINIMUM OF $S[\varphi] =$
$= \text{INSTANTON}$

Let $\varphi_{inst}(x-x_0)$ BE THE INSTANTON SOLUTION

(CONSIDER SIMPLEST CASE WHEN THE ONLY FREE PARAMETERS ARE COORDINATES OF THE CENTER OF THE INSTANTON – E.G., ABELIAN HIGGS MODEL IN 2D)

TREAT $\varphi(y_1)\dots\varphi(x_n)$ as pre-exponential

$\Uparrow$

"NAIVE" SADDLE POINT (OR "LOWEST ORDER") APPROXIMATION

$\Downarrow$

$$G_E(y,x) = \int e^{-S_{inst}}\, \varphi_{inst}(y_1-x_0)\dots\varphi_{inst}(x_n-x_0)\, d^Dx_0$$

GREEN'S FUNCTION IS NOW EXPLICIT

$$G_E(q,p) = \int \underline{e^{-S_{inst}}}\; e^{iq_1y_1+iq_2y_2-ip_1x_1 - ip_nx_n}\, \varphi_{inst}(y_1-x_0)\dots\varphi_{inst}(x_n-x_0)\, d^Dx_0\, dy_1\dots dx_n$$

NUMBER!

$$= e^{-S_{inst}}\,\delta(\Sigma q_i - \Sigma p_j)\,\varphi_{inst}(q_1)\dots\varphi_{inst}(p_n)$$

EXTRACT A POLE AT $q_i^2 = -m^2,\ p_j^2 = -m^2$

$$\varphi_{inst}(q) = \frac{C}{q^2+m^2} \text{ at } q^2 \to -m^2$$

NUMBER FOR $O(D)$ SYMMETRIC $\varphi_{inst}$

AMPLITUDE

$$A(\vec{q}_i, \vec{p}_j) = e^{-S_{inst}} \cdot C^{n+2}$$

Point-like AMPLITUDE

$\Downarrow$

$$\text{CROSS SECTION} \propto \int \begin{pmatrix}\text{n-particle}\\ \text{phase space}\end{pmatrix} \times e^{-S_{inst}} \cdot C^{n+2}$$

GROWS LIKE $\sim S^{\alpha n}$

($\alpha$ depends on dimensionality of space-time AND ON NATURE OF PRODUCED PARTICLES – SCALARS OR VECTORS)

MORE DETAILS: $\alpha = ?$
ENERGY SCALE $= ?$
ROLE OF $\int d\rho$ IN EW THEORY $= ?$
n-dependence of prefactor in
$\sigma_{2\to n} = F_n S^{\alpha n};\ F_n = ?$
VECTORS VS. SCALARS $= ?$

CALCULATIONS IN LOWEST ORDER INSTANTON APPROXIMATION ARE IN PRINCIPLE STRAIGTFORWARD

MUCH LESS TRIVIAL QUESTION: ROLE OF

At large $n$, integral over $\rho$ is saddle point

Saddle-point value of $\rho$:

$$\rho(n) = \frac{\sqrt{n}}{\pi v}$$

and

$$A_{2\to n} = n! \left(\frac{2}{v}\right)^n e^{-\frac{8\pi^2}{g^2}}$$

(up to a pre-factor that weakly depends on $n$).

Cross section of $2 \to n$:

$$\sigma_{2\to n} = |A_{2\to n}| \cdot (\text{phase space})_n \cdot \frac{1}{n!}$$

prefactor independent of $n$

Bose factor

$$(\text{phase space}) = \int \prod_{i=1}^{n} \frac{d^3p}{2\omega_p} \, \delta^4(\Sigma p - P)$$

If final particles are relativistic ($|\vec{p}_i| \gg m_H$) we have in C.M. frame

$$(\text{phase space}) \propto E^{2n} \quad \text{(dimensional grounds)}$$

$E \equiv \sqrt{s}$ = C.M. energy

In fact, $$(\text{phase space}) = \left(\frac{E}{4\pi}\right)^{2n} \cdot \frac{1}{(n!)^2}$$

$\left(\frac{E}{\cdot}\right)^n$ ...

So,

$$\sigma_{2\to n} = \frac{1}{n!}\left(\frac{E}{2\pi v}\right)^{2n} \cdot e^{-\frac{16\pi^2}{g^2}}$$

$n$-independent prefactor

Calculation is valid provided that:

$$\rho(n) = \frac{\sqrt{n}}{\pi v} \ll \frac{1}{m_H}, \frac{1}{m_W}$$

and $\frac{E}{n} \gg m_H, m_W$ ($m_H \sim m_W$ in what follows)

Main features:

Cross section grows like $E^{2n}$

Instanton size at given $E$ grows like $\sqrt{n}$ at large $n$.

# HIGGS PRODUCTION: INCLUSIVE

$$\sigma_{2\to \text{ANY}} = \sum_n \sigma_{2\to n}$$

$$= \sum_n \frac{1}{n!}\left(\frac{E}{2\pi v}\right)^{2n} e^{-\frac{16\pi^2}{g^2}}$$

$$= e^{-\frac{16\pi^2}{g^2} + \left(\frac{E}{2\pi v}\right)^2}$$

EXPONENTIALLY GROWS WITH ENERGY!

TYPICAL NUMBER OF PARTICLES = SADDLE POINT IN $\sum_n \sigma_{2\to n}$

$$n = \left(\frac{E}{2\pi v}\right)^2 \quad \text{- GROWS WITH ENERGY}$$

RELEVANT INSTANTON SIZE

$$\rho = \frac{\sqrt{n}}{\pi v} = \frac{E}{2\pi^2 v^2}$$

ALSO GROWS WITH ENERGY (UNEXPECTED!)

AVERAGE ENERGY OF FINAL PARTICLES

$$\frac{E}{n} = \frac{4\pi^2 v^2}{E}$$

DECREASES WITH ENERGY (ALSO UNEXPECTED!)

- ENERGY FACTOR BECOMES COMPARABLE WITH ORIGINAL INSTANTON ACTION AT

$$\frac{1}{g^2} \sim \left(\frac{E}{v}\right)^2, \quad \text{i.e.}$$

$$E \sim \frac{v}{g} = \frac{m_W}{g^2}, \quad \text{i.e.}$$

at $E \sim E_0 \propto \frac{m_W}{\alpha_W}$ (sphaleron energy)

THIS IS NOT UNEXPECTED

THESE FEATURES ARE INHERENT IN MANY OTHER LOWEST ORDER INSTANTON CALCULATIONS:

INSTANTON SUPPRESSION IS GOING TO DISAPPEAR DUE TO CREATION OF MANY SOFT PARTICLES THROUGH LARGE SIZE INSTANTONS. THE TOTAL CROSS SECTION GROWS EXPONENTIALLY. TYPICAL ENERGY SCALE IS $\sim$ SPHALERON ENERGY $\frac{m_W}{\alpha_W}$

THIS MUST NOT BE THE WHOLE TRUTH: EXPONENTIAL RISING OF CROSS SECTION MAY NOT PERSIST AT ALL ENERGIES!

# W-BOSON PRODUCTION

[LEADING CONTRIBUTION AT $E \ll E_0$

HERE WE GIVE ONLY ESTMATES. NUMERICAL FACTORS WILL APPEAR LATER.]

RECALL INSTANTON FIELD (AGAIN NEGLECTING $m_W$)

$$A_\mu^a = \frac{2\rho^2}{g} \frac{\bar{\eta}_{\mu\nu a} x^\nu}{x^2(x^2+\rho^2)}$$

CLEARLY, RESIDUE AT $p^2=0$ IS PROPORTIONAL TO

$$\frac{1}{g} \cdot \rho^2 \cdot |\vec{p}\,|$$

AMPLITUDE IS NOW

$$A_{2\to n} \propto \int d\rho \; e^{-\frac{8\pi^2}{g^2} - \pi^2 v^2 \rho^2} \cdot \frac{|\vec{p}_1| \dots |\vec{p}_n|}{g^n} \cdot \rho^{2n}$$

SADDLE-POINT VALUE OF $\rho$ IS AGAIN

$$\rho \sim \frac{\sqrt{n}}{v}$$

AND

$$A_{2\to n} \propto e^{-\frac{8\pi^2}{g^2}} \cdot n! \left(\frac{1}{gv^2}\right)^n |\vec{p}_1| \cdots |\vec{p}_n|$$

PHASE SPACE INTEGRAL IS CHANGED:

$$(\text{"PHASE SPACE"}) \propto \int \prod \frac{d^3p}{2\omega_p} \cdot |\vec{p}\,|^2 \, \delta(\Sigma p - P)$$

from $|A|^2$

$$\propto \frac{E^{4n}}{(n!)^4} \qquad \left(\text{instead of } \frac{E^{2n}}{(n!)^2}\right)$$

[roughly $\left(\frac{E}{n}\right)^4$]

So, $n$-PARTICLE CROSS SECTION

$$\sigma_{2\to n} \propto \frac{1}{n!} \cdot (n!)^2 \left[\frac{1}{gv^2}\right]^{2n} \frac{E^{4n}}{(n!)^4} \, e^{-16\pi^2/g^2}$$

Bose factor

$$\sim \frac{1}{n!} \left(\frac{1}{gv^2}\right)^{2n} \left(\frac{E^2}{n}\right)^{2n} e^{-\frac{16\pi^2}{g^2}}$$

AT FIXED $n$, $\sigma_{2\to n}$ GROWS WITH ENERGY EVEN FASTER THAN HIGGS PRODUCTION.

NEW POINT: EXPLICIT $\left(\frac{1}{g^2}\right)^n$.

## total cross section

$$\sigma_{2\to ANY} \propto \sum_n \frac{1}{n!}\left(\frac{E^2}{g v^2 n}\right)^{2n} \cdot e^{-\frac{16\pi^2}{g^2}}$$

SADDLE POINT VALUE OF $n$:

$$n \sim \left(\frac{E^2}{g v^2}\right)^{2/3} \sim \frac{1}{g^2}\left(\frac{E}{E_0}\right)^{4/3} \quad , \; E_0 \propto \frac{m_W}{g^2} \sim \frac{v}{g}$$

TOTAL CROSS SECTION IS EXPONENTIAL:

$$\sigma_{2\to ANY} \propto e^{-\frac{16\pi^2}{g^2} + \frac{\text{const}}{g^2}\left(\frac{E}{E_0}\right)^{4/3}}$$

AT $E \ll E_0$ THIS IS LARGER THAN HIGGS $e^{\frac{1}{g^2}\left(\frac{E}{E_0}\right)^2}$

NOTICE: $\rho \sim \frac{\sqrt{n}}{v} \sim \frac{1}{M_W}\left(E/E_0\right)^{2/3}$ ( $\ll \frac{1}{M_W}$ AT $E \ll E_0$ AS IT SHOULD )

AT $E \approx E_0$:

$\langle n \rangle$ BECOMES $\sim \frac{1}{g^2}$ ; $\frac{E}{\langle n \rangle}$ BECOMES $\sim M_W$ (NON-RELATIVISTIC BOSONS CREATED)

$\rho$ BECOMES $\sim \frac{\sqrt{n}}{v} \sim \frac{1}{m_W}$

CROSS SECTION BLOWS UP.

THIS IS NOT THE WHOLE TRUTH:

CORRECTIONS BECOME LARGE EXACTLY