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# BARYON AND LEPTON NUMBER VIOLATION IN THE ELECTROWEAK MODEL

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Please note: These are preliminary notes intended for internal distribution only.

## CORRECTIONS: GENERALITIES

CORRECTIONS BECOME LARGE BEFORE LOWEST ORDER CROSS SECTION BLOWS UP

Khlebnikov, V.R., Tinyakov '90; McLerran, Vainshtein, Voloshin '90; Zakharov '90; Khoze, Ringwald '90; ARNOLD, Mattis '90; Yaffe '90; ...

Soft-soft CORRECTIONS EXPONENTIATE
Khlebnikov, V.R., TINYAKOV '90; ARNOLD, Mattis '90; Yaffe '90

HARD-HARD AND HARD-SOFT CORRECTIONS ARE ALSO LARGE AND MAY EXPONENTIATE
Mueller '91

—o—

INTERESTING REGIME:

$$g^2 \to 0, \quad \frac{E}{E_0} = \text{fixed}, \quad n \equiv \text{NUMBER OF FINAL PARTICLES} \sim 1/g^2$$

$E_0 \sim \frac{M_W}{g^2}$

THIS IS INDICATED BY LOWEST ORDER CALCULATIONS

NOTICE: TYPICAL ENERGY OF FINAL PARTICLE

$$\frac{E}{n} = \mathcal{O}(g^0) \cdot m_W \quad \left(\text{CONSTANT AT } g^2 \to 0\right)$$

## POTENTIAL SOURCES OF LARGE CORRECTIONS

i) NUMBER OF FINAL PARTICLES $\sim \frac{1}{g^2}$
(SOFT-SOFT)

ii) ENERGY OF INITIAL PARTICLES $\sim \frac{1}{g^2}$
(HARD-HARD)

FIRST CORRECTIONS:

SOFT-SOFT

$$G_E(y_1, y_2; x_1 \dots x_n) = \int e^{-S[\varphi]} \varphi(y_1)\varphi(y_2)\varphi(x_1)\dots\varphi(x_n)\, d\varphi$$

LOWEST CORRECTION (FINAL PARTICLES)

$$\int e^{-S_{inst}} \varphi_{inst}(y_1)\varphi_{inst}(y_2)\,\varphi_{inst}(x_1)\dots\Delta\varphi(x_i)\dots\Delta\varphi(x_j)\dots\varphi_{inst}(x_n)\; e^{-\Delta\varphi \cdot M \cdot \Delta\varphi}\, d(\Delta\varphi)$$

operator for small fluctuations around instanton

ROUGHLY THIS GIVES

$$e^{-S_{inst}} \varphi_{inst}(y_1)\varphi_{inst}(y_2)\left(\varphi_{inst}(x)\right)^{n-2} \cdot D(x_i - x_j)$$

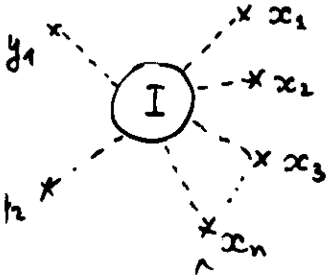
$n-2$ times

AS COMPARED TO LOWEST ORDER

$e^{-S_{inst}} \dots [\dots]^n$

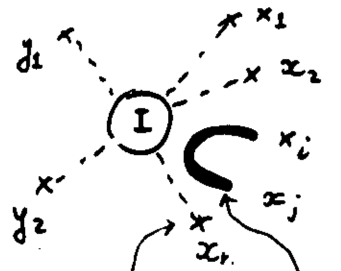
## PICTORIALLY

LOWEST ORDER




classical fields

FIRST CORRECTION




INSTANTON

PROPAGATOR IN INSTANTON BACKGROUND

You loose $\left(\frac{1}{g}\right)^{-2}$ (INSTANTON FIELD IS $\frac{1}{g}$)

You WIN $n^2$ (choose two lines out of $n$)

(FINAL PARTICLES HAVE ENERGIES $\sim \Theta(g^0) \Rightarrow$ NOTHING special in propagator)

$$\boxed{\frac{\text{FIRST CORRECTION}}{\text{LOWEST ORDER}} = g^2 \cdot n^2 \sim \frac{1}{g^2} \quad \text{at } n \sim \frac{1}{g^2}}$$

CORRECTION IS BIG!

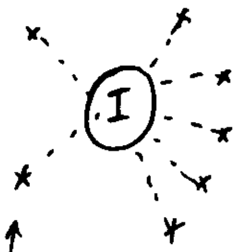
NAIVELY, CORRECTION IS LARGE AT $n \sim \frac{1}{g}$, i.e. at $E \ll E_0$.

BUT THE CORRECTIONS SUM UP INTO EXPONENT AND IN FACT THEY BECOME IMPORTANT AT $E \sim E_0$.

(see sect. 4)

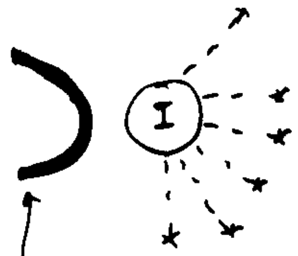
## HARD - HARD

LOWEST ORDER




RESIDUE OF ISTANTON FIELD AT $q^2 = -m_W^2$

FIRST CORRECTION




RESIDUE OF PROPAGATOR IN INSTANTON BACKGROUND

NO COMBINATORICS. BUT PROPAGATOR IN INSTANTON BACKGROUND IS AT LARGE $|\vec{q}| \sim E_0$ (AND AT $q^2 = -m_W^2$). THERE MAY APPEAR – AND THERE DOES APPEAR – ENHANCEMENT OF THE CORRECTION DUE TO HIGH INITIAL ENERGY (see sect. 5)

$$\boxed{\frac{\text{FIRST H.-H. CORRECTION}}{\text{LOWEST ORDER}} \sim g^2 E^2 \ln E^2 \sim \frac{1}{g^2} \quad \text{AT } E \sim \frac{1}{g^2}}$$

These corrections may exponentiate as well. They are strong at $E \sim E_0$ anyway

(NOT PROVED)

C.1

# COHERENT STATE FORMALISM

USEFUL FOR COMBINATORICS OF SOFT-SOFT.

MAY HAVE MORE GENERAL RELEVANCE TO THE NON-PERTURBATIVE CALCULATIONS

COHERENT STATES IN GENERAL TEXTBOOKS

COHERENT STATE FORMALISM FOR INSTANTON TRANSITIONS — KHLEBNIKOV, V.R., TINYAKOV '90

FIRST CORRECTION IN EW THEORY — KHOZE, RINGWALD '91; Mueller '91; DYAKONOV, PETROV '91; ARNOLD, Mattis '91

CONSTRAINT DEPENDENCE — KHLEBNIKOV, TINYAKOV '91

MAIN RESULT:

LEADING ORDER INSTANTON RESULT AND SOFT-SOFT CORRECTIONS SUM UP INTO

$$\sigma_{tot} \propto e^{-\frac{16\pi^2}{g^2} F(E/E_0)}$$

semiclassically looking Result

(REGIME $g^2 \to 0$; $E_0 \sim \frac{M_W}{\alpha_W}$, $\frac{E}{E_0}$ = fixed)

SOFT-SOFT CORRECTIONS (up TO LOGS) MAKE SERIES IN $\left(\frac{E}{E_0}\right)^{2/3}$

A few LOWEST TERMS ARE leading order, first correction

$$F\left(\frac{E}{E_0}\right) = 1 - \frac{9}{8}\left(\frac{E}{E_0}\right)^{4/3} + \frac{9}{16}\left(\frac{E}{E_0}\right)^2 + \dots$$

TOTAL CROSS section does increase with energy at low energies, $E \ll E_0$), $E_0 = \sqrt{6}\,\pi \frac{M_W}{\alpha_W}$

$F(E/E_0)$ is UNKNOWN.

FURTHERMORE, hard-soft and hard-hard corrections are likely to contribute to $F(E/E_0)$. If so, they start at $\left(\frac{E}{E_0}\right)^{8/3}$.

# S-MATRIX IN COHERENT STATE BASIS.

COHERENT STATES:

DEFINED AS EIGENSTATES OF OPERATOR $\hat{a}_{\vec{k}}$ (ANNIHILATING A PARTICLE WITH MOMENTUM $\vec{k}$)

[DROP INDEX $\vec{k}$ IN WHAT FOLLOWS]

$$\hat{a}|a\rangle = a|a\rangle$$ , eigenvalue $a$ is complex

"COORDINATE" REPRESENTATION OF $|a\rangle$:

$$\langle q|a\rangle = e^{-\frac{1}{2}a^2 - \frac{1}{2}\omega q^2 + \sqrt{2\omega}\, a\cdot q}$$

IN FIELD THEORY $q \to \varphi(\vec{k})$, $\omega = \omega_{\vec{k}}$ and integration over $\vec{k}$ is to be made in exponent)

COMPLETENESS RELATION:

$$1 = \int \underbrace{e^{-a^* a}}_{} \, |a\rangle\langle a| \, da\, da^*$$

This measure always appear in coherent state formalism

n-particle state:

WAVE function in coherent state basis

$$\langle a|\vec{k}_1, \dots, \vec{k}_n\rangle = a^*_{k_1} \cdot \dots \cdot a^*_{k_n}$$

Matrix elements of operators

$$\langle b|\hat{A}|a\rangle = A(b^*, a)$$

Relation to usual Fock basis:

$$\langle \vec{k}_1, \dots, \vec{k}_n|\hat{A}|\vec{p}_1, \dots, \vec{p}_m\rangle = \frac{\partial}{\partial b^*_{\vec{k}_1}} \cdots \frac{\partial}{\partial b^*_{\vec{k}_n}} \frac{\partial}{\partial a_{\vec{p}_1}} \cdots \frac{\partial}{\partial a_{\vec{p}_m}} A(b^*, a)$$

Thus, $A(b^*, a)$ is the generating functional for matrix elements of $\hat{A}$ in Fock basis.

EVOLUTION OPERATOR IN COHERENT STATE REP.

$$\langle b|\hat{U}(T_f, T_i)|a\rangle = \int dq_f\, dq_i \, \langle b|q_f\rangle\langle q_f|\hat{U}|q_i\rangle\langle q_i|a\rangle$$

WRITTEN IN PREVIOUS PAGE

$$\int dq\, e^{iS}$$

TO GET S-MATRIX, MULTIPLY $\hat{U}$ BY FREE EVOLUTION OPERATOR

$$S = e^{iH_0T_f}\,\hat{U}\,e^{-iH_0T_i}$$

Now $e^{iH_0T_f}|a\rangle$ IS TRIVIAL $\Rightarrow$

SUBSTITUTE $\quad a \to a\,e^{-i\omega T_i}$

$$b^* \to b^* e^{i\omega T_f}$$

$$\Downarrow$$

$\mathcal{S}$-MATRIX IN COHERENT STATE REPRESENTATION
(FIELD THEORY OF BOSON $\varphi(\vec{x},t)$)

$$\langle b|S|a\rangle = S(b^*,a)$$

field at $T_i < t < T_f$

$$= \int [d\varphi_f\, d\varphi_i\, d\varphi]\, e^{B_i(\varphi_i,a) + B_f(\varphi_f,b^*) + i\mathcal{S}(\varphi)}$$

at $t = T_f$ — field at $t = T_i$ — BOUNDARY TERMS — ORDINARY ACTION

$$B_i(\varphi_i,a) = -\frac{1}{2}\int d\vec{k}\; a_{\vec{k}}\,a_{-\vec{k}}\,e^{-2i\omega_k T_i} - \frac{1}{2}\int d\vec{k}\,\omega_{\vec{k}}\,\varphi_i(\vec{k})\varphi_i(-\vec{k})$$

$$+ \int \sqrt{2\omega_{\vec{k}}}\; e^{-i\omega_{\vec{k}}T_i}\, a_{\vec{k}}\,\varphi_i(\vec{k})$$

ANALOGOUSLY FOR $B_f(\varphi_f, b^*)$



$S$-MATRIX FOR 2-PARTICLE COLLISION

$$S(b^*) = \frac{\partial}{\partial a_{\vec{q}}}\frac{\partial}{\partial a_{-\vec{q}}}\mathcal{S}(b^*,a)\Big|_{a=0}$$

TOTAL CROSS SECTION $\quad 2 \to$ ANY

$$\sigma_2(E) = \frac{1}{j}\int db^*_{\vec{k}}\,db_{\vec{k}}\; e^{-\int d\vec{k}\, b^*_{\vec{k}} b_{\vec{k}}}\,|S(b^*)|^2$$

ADVANTAGE: $\sigma_2(E)$ HAS INTEGRAL REPRESENTATION like $\int e^{F(b,b^*,\varphi\dots)}\,db\,db^*\,d\varphi\dots$ up to initial particles.

FORMAL TRICK TO GO TO EUCLIDEAN TIME

AT LARGE $T_i, T_f$, $S(b^*,a)$ is independent of $T_i, T_f$
$(T_i \to -\infty,\ T_f \to +\infty)$

ANALYTICALLY CONTINUE $T_i \to -iT_i$, $T_f \to -iT_f$

AND $t \to -it$ in functional integral

$$\Downarrow$$

$$S(b^*,a) = \int d\varphi_f\, d\varphi_i\, d\varphi\; e^{B_i + B_f - \mathcal{S}}$$

# LOWEST ORDER INSTANTON RESULTS.

REOBTAIN EXPONENTIAL BEHAVIOUR IN COHERENT STATE FORMALISM.

$$S(b^*) = \int d\varphi\, d\varphi_f\, \varphi_i(\vec{q})\varphi_i(-\vec{q})\, e^{B_f(\varphi_f, b^*) - S(\varphi)}$$

STRATEGY: EXPAND $\varphi$ AROUND INSTANTON

$$\varphi(x) = \varphi_{inst}(x - x_0) + \nu(x)$$

$$S(\varphi) = S_{inst} + \nu M \nu + \ldots \nu^3 + \ldots \nu^4 + \ldots$$

$$\varphi_i(\vec{q}) = e^{-\omega_{\vec{q}} t_0 + i\vec{q}\vec{x}_0}\, \varphi_{inst}(\vec{q}, t \to -\infty)\ (t_0 \equiv x_0^0)$$

position of instanton in time

$$+ \underbrace{\nu(\vec{q}, t \to -\infty)}$$

DISREGARD THIS TERM UNLESS HARD-HARD AND HARD-SOFT CORRECTIONS ARE INCLUDED

$$\Downarrow$$

$$\varphi_i(\vec{q})\varphi_i(-\vec{q}) = e^{-Et_0} \times (\text{KNOWN FUNCTION OF } E)$$

THIS is PRE-EXPONENTIAL

$$\varphi_f(\vec{p}) = e^{\omega_p t_0 - i\vec{p}\vec{x}_0}\, \varphi_{inst}(\vec{p}, T_f) + \nu(\vec{p}, T_f)$$

Recall

$$B_f = -\frac{1}{2}\int d\vec{p}\, \sqrt{2\omega_{\vec{p}}}\, e^{\omega_{\vec{p}} T_f}\, \varphi_f(\vec{p})\, b^*(\vec{p})$$

+ quadratic terms

$$\Downarrow$$

$$B_f = -\frac{1}{2}\int d\vec{p}\, \sqrt{2\omega_{\vec{p}}}\, e^{\omega_p t_0 - i\vec{p}\vec{x}_0}\, b^*(\vec{p})\left(e^{\omega_{\vec{p}} T_f}\varphi_{inst}(\vec{p}, T_f) + \nu(\vec{p}, T_f)\right)$$

NOW, AT LARGE $T_f$

$$\sqrt{2\omega_{\vec{p}}}\, e^{\omega_{\vec{p}} T_f}\, \varphi_{inst}(\vec{p}, T_f) = R(\vec{p})$$

Residue of instanton field at $p^2 = -m^2$

(NO CONTRIBUTION FROM $(-\infty)$ BECAUSE $\varphi_{inst}$ dies there)

PROOF:

$$\varphi_{inst}(\vec{p}, T_f) = \frac{e^{-\omega_{\vec{p}} T_f}}{2\omega_p}\int_{-\infty}^{T_f} d\tau\, \partial_\tau\left[e^{\omega_{\vec{p}}\tau} \overleftrightarrow{\partial_\tau}\, \varphi_{inst}(\vec{p}, \tau)\right]$$

$$= \frac{-e^{-\omega_{\vec{p}} T_f}}{(2\pi)^{3/2}\, 2\omega_{\vec{p}}}\int_{-\infty}^{T_f} dx\, e^{+\omega_p \tau - i\vec{p}\vec{x}}\left(\omega_{\vec{p}}^2 - \partial_\tau^2\right)\varphi_{inst}(x)$$

[PROOF CONT'D]

$$= \frac{e^{-\omega_p T_f}}{(2\pi)^{3/2}\, 2\omega_{\vec p}} \Big[ \big( p^2+m^2 \big)\, \varphi_{inst}(p^\mu) \Big]_{p^2=-m^2}$$

AS DESIRED.

THUS,

$$B_f = -\frac{1}{2} \int d\vec p\, e^{\omega_{\vec p} t_0 - i\vec p \vec x}\, b^*(\vec p) R(\vec p)$$

$$+ \int \cdots\; b^*(\vec p)\, \nu(\vec p, T_f)$$

THUS, WE HAVE THE INSTANTON-INDUCED S-MATRIX WITH SOFT-SOFT CORRECTIONS BUT WITHOUT HARD-HARD AND HARD-SOFT:

$$S(b^*) = \ldots \int d^4x_0\, e^{-S_{inst}} \cdot e^{-Et_0} \cdot$$

$$(C.9) \qquad \cdot\, e^{-\frac{1}{2} \int d\vec p\, e^{\omega_{\vec p} t_0 - i\vec p \vec x_0}\, b^*(\vec p) R(\vec p)}$$

$$\cdot \int [d\nu]\, e^{-\nu M \nu - \int d\vec p \ldots\; b^*(\vec p) \cdot \nu(\vec p, T_f) + \mathcal{O}(\nu^3)}$$

LOWEST ORDER RESULT IS RECOVERED IF WE DISREGARD THE LAST FACTOR (INTEGRAL OVER FLUCTUATIONS) ALTOGETHER:

$$\boxed{S_{\text{LOWEST ORDER}}(b^*) = e^{-S_{inst}} \times \int d^4x_0\, e^{-Et_0 - \frac{1}{2} \int d\vec p\, e^{\omega_{\vec p} t_0 - i\vec p \vec x} \cdot b^*(\vec p) R(\vec p)}}$$

EVERYTHING IN THIS FORMULA IS EXPLICIT.

CROSS SECTION TO LEADING ORDER

$$\sigma_{2\to ANY} = \ldots \int d^4x_0\, d^4x_0' \quad e^{-2S_{INST}} \times$$
$$\times\, e^{-E(t_0-t_0') - \frac{1}{2}\int d\vec{p}\, e^{\omega_{\vec{p}} t_0 - i\vec{p}\vec{x}}\, b^*(\vec{p}) R(\rho)}$$
$$\times\, e^{-\frac{1}{2}\int d\vec{p}'\, e^{-\omega_{\vec{p}'} t_0' + i\vec{p}'\vec{x}'}\, b(\vec{p}') R^*(\vec{p}')}$$
$$\times\, e^{-\int d\vec{p}\, b(\vec{p}) b^*(\vec{p})} \quad [db][db^*]$$

- TRANSLATIONAL INVARIANCE $\Leftrightarrow$ INTEGRAND DEPENDS ONLY ON $(x_0 - x_0')$.

- ALL OTHER INTEGRATIONS ARE SADDLE POINT

$\Downarrow$

RESULT IS EXPONENTIAL

- SCALING ARGUMENT: IF $E \sim \frac{1}{g^2}$

AND $R \sim \frac{1}{g}$ ( gauge SECTOR OF E.W. THEORY)

THEN CHANGING VARIABLES $b \to \frac{1}{g}\beta$

ONE FINDS THE INTEGRAND

$$e^{-1/g^2}\, W\left(\frac{E}{E_0}, (x_0 - x_0'), \beta\right)$$

ELECTROWEAK THEORY:

THERE ARE ALSO INTEGRATIONS OVER $\rho, \rho'$.

AND INTEGRATIONS OVER ORIENTATIONS.

SADDLE POINT:

$$(t_0 - t_0') = \left(\frac{24E}{\pi^2 g^2 v^4}\right)^{1/3} \sim \frac{1}{M_W}\left(\frac{E}{E_0}\right)^{1/3}$$

$$\rho = \rho' = \left(\frac{3E^4}{8\pi^8 g^2 v^{10}}\right)^{1/6} \sim \frac{1}{M_W}\left(\frac{E}{E_0}\right)^{2/3}$$

IN THE SADDLE POIN APPROXIMATION

$$\sigma_{2\to ANY}^{\text{LOWEST ORDER}} = e^{-\frac{16\pi^2}{g^2}\left[1 - \frac{9}{8}\left(\frac{E}{E_0}\right)^{4/3}\right]}$$

$$E_0 = \sqrt{6}\,\pi\,\frac{M_W}{\alpha_W}$$

NOTICE THAT $\tau \equiv (t_0 - t_0')$ HAS MEANING OF SEPARATION BETWEEN INSTANTON (RELEVANT for $S(b^*)$) AND ANTI-INSTANTON (RELEVANT for $S^*(b)$).

CORRECTIONS EXPECTED TO BE OF ORDER $\tau^2/\rho^2 \sim (E/E_0)^{2/3}$