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SMR.626 - 27

SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

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NON-PERTURBATIVE ELECTROWEAK THEORY

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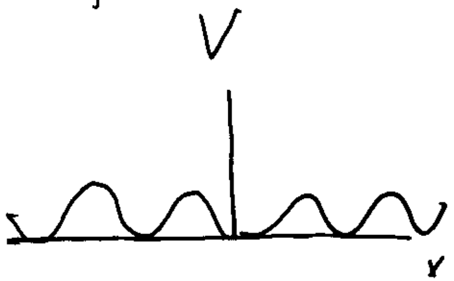
Please note: These are preliminary notes intended for internal distribution only.

(1)

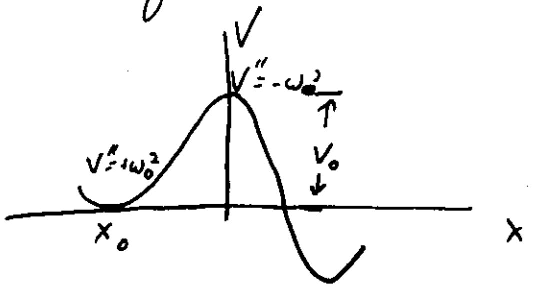
Lecture

We argued



Want to compute tunneling rate - transition rate

Consider quantum mechanics



Some thermal ensemble of states located around false vacuum

(2)

Case I

$T \ll \omega_0$

Transitions only by tunneling

Rate for each state is $\Gamma(E) = 2\,\mathrm{Im}$

(Rate $\sim |A|^2 \to e^{-2P(E)}$)

$\therefore$ Total rate is given by thermal average

$\Gamma = 2\,\mathrm{Im}\,F$

$T \gg V_0 \Rightarrow$ unsuppressed

$T \lesssim V_0$ classical thermal fluctuations (actually true for $T \gg \omega_0$)

$$\Gamma = \langle \delta(x - x_{\text{BARRIER}})\, \Theta(v)\, v \rangle$$

$$= \frac{\int dp\,dx\; e^{-\beta(P^2/2 + V(x))}\; \delta(x - x_{\text{BARRIER}})\, v\, \Theta(v)}{\int dp\,dx\; e^{-\beta(P^2/2 + V(x))}}$$

$$= \frac{\int_0^\infty dp\; e^{-\beta P^2/2}\, p\; e^{-\beta V_0}}{\int_{-\infty}^{\infty} dp\; e^{-\beta P^2/2} \int dx\; e^{-\beta/2\, \omega_0^2 x^2}}$$

(3)

$$= \frac{\omega_0}{2\pi} e^{-\beta V_0}$$

∴ On the other hand

$$\mathrm{Im}\, F = \frac{1}{\beta} \mathrm{Im} \ln Z$$

$$= \frac{1}{\beta} \frac{\mathrm{Im}\, Z}{Z} = \frac{1}{\beta} \frac{\mathrm{Im} \int dp\, dx\, e^{-\beta(p^2/2 + V_0 - \frac{1}{2}\omega_-^2 x^2)}}{\int dp\, dx\, e^{-\beta(p^2/2 + \frac{1}{2}\omega_0^2 x^2)}}$$

$$= \frac{\omega_0}{2\omega_- \beta} e^{-\beta V_0}$$

∴

$$\Gamma = \frac{\omega_- \beta}{\pi} \mathrm{Im}\, F$$

True in Gaussian approximation $\left(\frac{g^2 T}{M_W} \ll 1\right)$

⇒ $T \ll M_W/\alpha_W$

(4) What about more degrees of freedom

$$\Gamma \sim \frac{\int dp_1\, dx_1\, e^{-\beta(\overset{\downarrow \text{ AROUND MAXIMUM}}{p_1^2/2 + \frac{1}{2}\omega_1^2 x_1^2})}}{\int dp_1\, dx_1\, e^{-\beta(\underset{\uparrow \text{ AROUND MINIMUM}}{p_1^2/2 + \frac{1}{2}\bar{\omega}_1^2 x_1^2})}} = \frac{\bar{\omega}_1}{\omega_1}$$

Many degrees of freedom

$$\Gamma = e^{-\beta V_0} \frac{\omega_-}{2\pi} \mathrm{Im} \left(\prod_i \frac{\bar{\omega}_i^2}{\omega_i^2} \right)^{1/2}$$

On the other hand, this is also

$$\frac{\omega_-}{\pi} \frac{\mathrm{Im}\, Z^{\text{BARRIER}}}{Z_0} = \frac{\omega_- \beta}{\pi} \mathrm{Im}\, F$$

∴

$$\Gamma = \frac{\omega_- \beta}{\pi} \mathrm{Im}\, F \text{ is true in field theory}$$

(5) Now, what do we do:

Find sphaleron.
Compute modes of small fluctuations
Compute Im F
Technical problem is from zero modes (see Valerie's lectures) which must be treated carefully.

Answer is roughly

$$\frac{\Gamma}{V} = \frac{1}{R_{sp}^4} e^{-E_{sp}/T} F\left(\frac{g^2 T}{M_W}\right)$$

↑ computable

$\frac{\Gamma}{V}$ has been computed for $T \ll \frac{M_W(T)}{\alpha}$,

result is rate is big

(6) What happens for $T \gtrsim M_W/\alpha$

Strong coupling region

$$e^{-S} \to e^{-\frac{1}{g^2}\int_0^\beta dt \int dx\, S}$$

$$\to e^{-\frac{M_W}{g^2 T}\int dx\, S_0}$$

Fluctuations are big.

Another argument: Power counting

Finite $T \Rightarrow \phi(t) = \phi(t+\beta) \Rightarrow p_0 \to T\sum_n$

Each term in loop expansion

$$\alpha_W^N \left(T \textstyle\sum_n\right)^{N_L} \sim \frac{\alpha_W^N T^N}{M^N}$$ to get dimension right

(Note that for fermions, there is

7

$\Rightarrow$ coupling constant expansion is OK.

Note that when $M \simeq \alpha_W T$, perturbation theory begins to break down

$R \sim 1/\alpha_W T$ length scale for n.p. physics

Expect

$$\Gamma/V \sim \alpha_W^4 T^4$$

— — — — —

Another way to compute rate

Suppose we assume transitions classical at high T $\Rightarrow$ $[\Pi, \phi] = 0$. Therefore evolution determined by classical equations given initial conditions $\Rightarrow$ Start with an ensemble of initial conditions $\rightarrow$ evolve

Can explicitly see sphaleron transitions
Can compute rate in high T region in principle.

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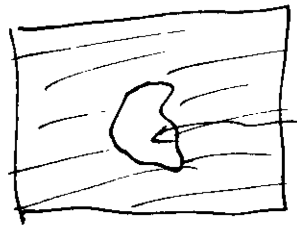
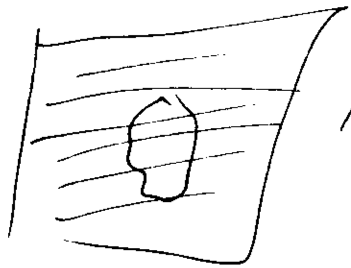
Problems: Why is this stuff hard to buy?



Large classical object

Prob reduced by $e^{-S/V} = e^{-T^3 V_{sp}}$
true if sphaleron excludes high energy radiation

If radiation penetrates sphaleron & doesn't interact



Rate $\sim e^{-E_{sp}/T}$

When we do the computations include distortion of radiation field. Is small at high ω $\Rightarrow$ Rate close to Boltzmann value

(9)

An instanton theorem

$\langle qqq\,l\rangle = \int [d\bar{q}\,dq\,d\bar{l}\,dl\,dA]\; qqq\,l\;e^{-S}$

Make a B+L global rotation

$[d\bar{q}\,dq\,d\bar{l}\,dl] \to e^{-i\alpha N_{B+L} Q_{TOP}}$

$qqq\,l \to e^{i\alpha N_{B+L}}$

$\int d\alpha \Rightarrow N_{B+L} = Q_{TOP} = 1$

$\Rightarrow$ Must have ~~instanton~~ $Q_{TOP} \neq 0$, but if instanton, then

$(F - F^d)^2 > 0$

$F^2 > FF^d$ ($= FF^d$ for instanton)

$(D\phi)^\dagger (D\phi) > 0$

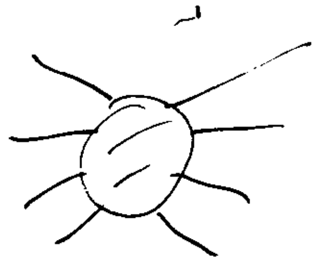
$V(\phi) > 0$

(10)

$\therefore R \lesssim e^{-S_{INST}} = e^{-2\pi/\alpha_W}$

True also if we add boson fields

Boson fields can however be coherent



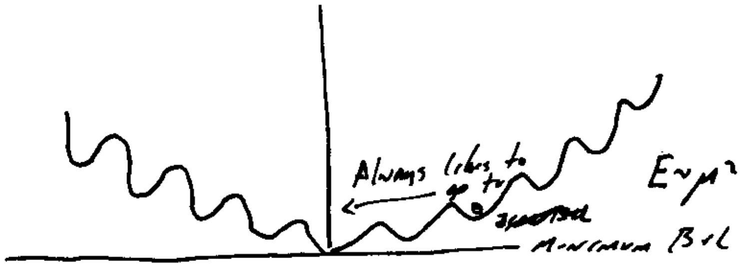
Strength of field $\sim N$

$\langle \phi_1 \cdots \phi_N \rangle \sim N^N$

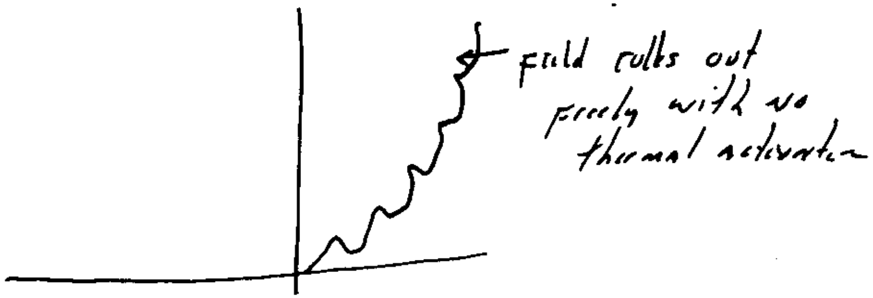
$A \sim e^{-2\pi/\alpha_W + N \ln N}$ big when $N \sim 1/\alpha$

Lots of bosons $\to$ Lots of bosons $+ \Delta(B+L)$

(11) Considerations including effects of fermions

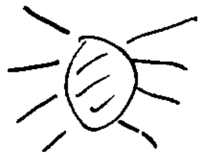


What about high density and zero T?
(Rubakov argument)



$\Rightarrow$ $\exists$ a critical density at zero T where $\Delta B{+}L$ takes place and unsuppressed rate

(12) Problem: Fermi gas has no bosons



Cannot get N^N because fermions are in different states

~~Should be dominated by~~
2 particle processes as good as multi?

Diakonov & Petrov

Probably <u>Not</u>, but needs work & understanding

①

Generating the Baryon Asymmetry at EW Phase Transition

Shaposhnikov; McLerran; Dine, Susskind et al, Caplan; Cohen & Nelson

Sakharov Conditions

① $\Delta B \neq 0$

② $\Delta CP \neq 0$

③ Out of equilibrium

① is OK

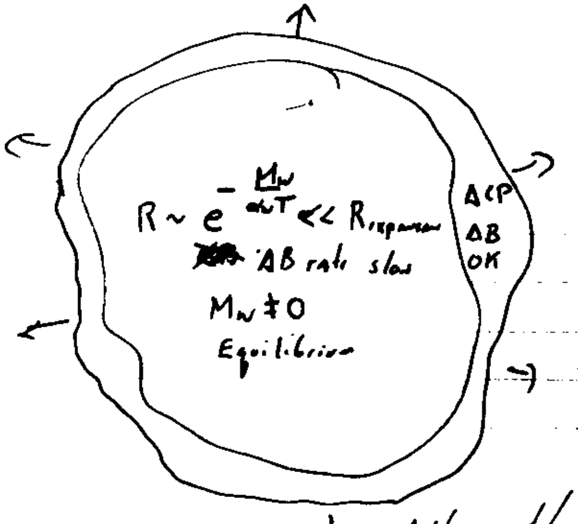
② Might have to extend EW theory

~~(Standard condition)~~

③ Will see is true in EW theory

First: A picture to understand what happens

Nucleation Bubble



$\langle \phi \rangle = 0$

$M_W = 0$

$R \sim \alpha_W^4 T^4$

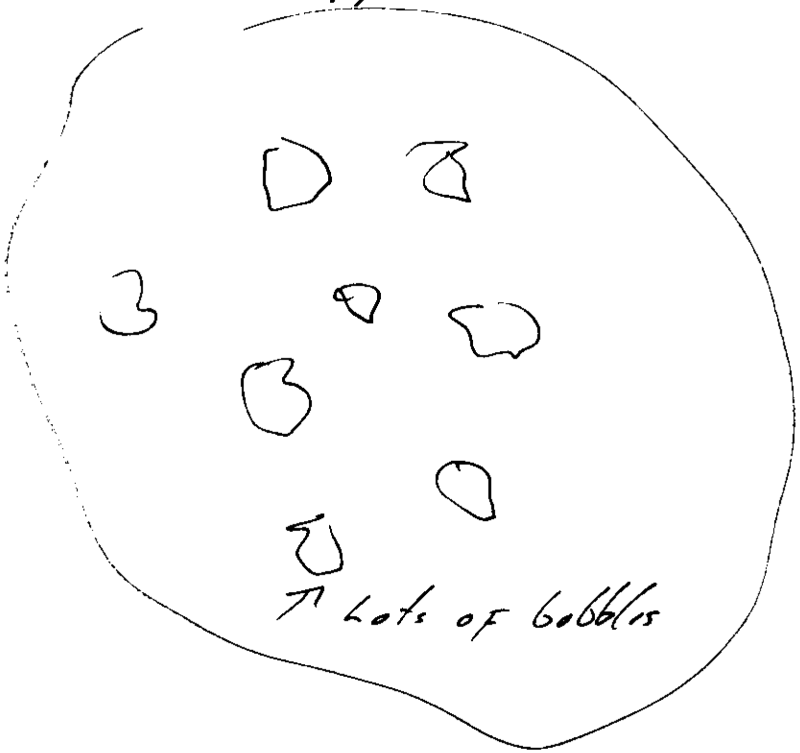
Equilibrium

ΔB big

All matter must be " "

②

In cosmology



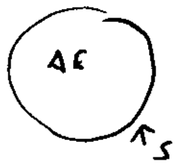
Assuming $v_{wall} \sim c$, and using classical nucleation theory I can estimate size of bubbles when they collide

Find $R \sim 10^{-4} R_{event} \sim 10^{10} M_W^{-1}$

(3) How does nucleation theory work

$R \sim e^{-S}$

S = action of critical bubble



Gain ΔE, loose S.

$R > R_0$ grows, $R < R_0$ shrinks

=> unstable static soln

$P \sim e^{-E/T}$,

Compare rate for this compared to expansion rate

$T \to T_c$ degenerate

$\Delta E \to 0$, Bubble has infinite size

$P \to 0$

=> At some time, when T is small enough ,,

(3') Note that

$E = -\Delta E V + \mathcal{E}_s S$

$\Rightarrow \frac{dE}{dL} = 0 \qquad 3\Delta E = 2\mathcal{E}_s / L$

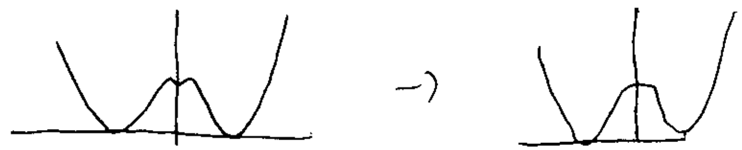
$L = \frac{2}{3} \mathcal{E}_s / \Delta E$

$E \sim \mathcal{E}_s^3 / \Delta E^2$

This comes from solving classical equations in Electroweak theory => $E \sim 1/\alpha_w$

will never happen

unless at some T



This also always happens in EW theory!

(4)

In order that inside the wall, Rate is less than expansion rate of universe, must have

$\frac{M_W}{\alpha_W T} >> 1 \Rightarrow$ transition takes place at low $T \Rightarrow$ will see gives constraint on scalar particle masses.

Crude estimation of ~~...~~ N_B/S

Measurements

$N_B/_S \sim 10^{-10}$

(Estimate $N_p \sim 1/m^3$
$N_\gamma \Rightarrow \sim T^3$
$T \sim 3°K \sim 10^{-2}\, 300°K \sim 10^{-2}\times10^{-2}\,eV \sim 10^{-4}\,eV$
$\approx 10^{-10}\,MeV$
$(1\,MeV)^{-1} = 200\,Fm$ $\quad T \sim \frac{10^{-10}}{2\times10^{4}\,cm} \sim 1/mm$
$T^3 \sim 1/mm^3 \sim 10^{-9}/m^3$
$N_p/N_\gamma \sim 10^{-9}$)

We have

$$\frac{N_B}{S} \sim \frac{\alpha_W^4}{N_{DOF}} \sin\Theta_{cp}\,\Lambda$$

Λ a dynamical factor which depends on details in surface of bubble

$$\sim 10^{-8} \sin\Theta_{cp}\,\Lambda$$

$\Rightarrow$ Need high efficiency $\Lambda \sim 1$

Θ_{cp} big $\gtrsim 10^{-1}$-10^{-2} or so

(5)

To get big effect must go beyond standard model, since here all CP violating effects are masked by large powers of Yukawa couplings (recall no effect of CP violation if one of quark masses vanishes $\Rightarrow$ can be absorbed into redef of mass)

Typically need multiple Higgs doublets, CP violation in Higgs sector

Phenomenological Consequences:
Rare decays ?
e, n electric dipole moments

— — — — — — —

We will find that typically, the thickness of the wall

$$\Delta X \sim 1/\alpha_W T$$

is $\sim R_{sph}$

so it is not hard to imagine high efficiencies.

(5)'
How does symmetry restoration work at high T.

Consider

$V = \lambda(\phi^2 - v^2)^2 =$

$\phi = \phi_0 + \eta$

$V = \lambda(\phi_0^2 - v^2 + 2\phi_0\eta + \eta^2)^2$

$\frac{dV}{d\phi_0} = 0 \Rightarrow \phi_0 = v$ at zero T

$V = \lambda(2v\eta + \eta^2)^2 = 4v^2\eta^2\lambda + \dots$

$M = \sqrt{8v\lambda}$

What happens at finite T?

$F = V + T\int\frac{d^3p}{(2\pi)^3}\ln(1 - e^{-\beta E})$

$E = \sqrt{p^2 + M^2(\phi_0)}$

$M(\phi_0) = \sqrt{8\lambda}\,\phi_0$

(6) Find

$F \sim V - \pi A T^4 + B\lambda\phi_0^2 T^2 - C\lambda^{3/2}|\phi_0|^3 T + \dots$

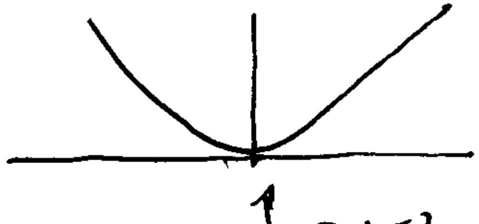
Two effects: first

~~$F = \lambda 2\phi_0^2$~~

$\eta = 0$

$V = \lambda(\phi_0^2 - v^2)^2 + B\lambda\phi_0^2 T^2 - C\lambda^{3/2}|\phi_0|^3 T$

At some high T



$B\lambda T^2 > 2\lambda v^2$

$T^2 > \frac{2v^2}{}$

(6')

Shape of potential

$$V \sim \tfrac{1}{2} M^2(T)\phi^2 - \delta T \phi^3 + \lambda/_4 \phi^4$$

$$\delta \sim g^3, \quad M^2 \sim g^2(T^2 - T_c^2)$$

$T = T_c$ curvature at origin changes sign

$\smile \;\to\; \frown$ $\Rightarrow$ must be a transition

Is it first order

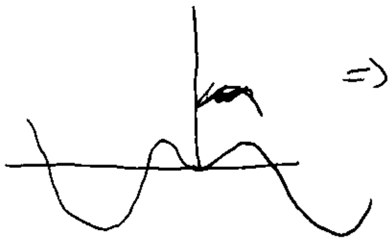
Near $M^2 = 0$

$$V \sim -\delta T \phi^3 + \lambda/_4 \phi^4$$

$$\frac{dV}{d\phi} = 0 \Rightarrow \lambda \phi^4 = 3\delta T \phi^2$$

$$\phi = \frac{3\delta T}{\lambda}$$

$$V \sim -\frac{27\delta^4}{\lambda^3} T^4 + 27 \cdot \frac{3}{4} \frac{\delta^4 T^4}{\lambda^3} < 0$$



$\Rightarrow$ 1st order transition of exactly type needed

(7) Find:

$\langle \phi \rangle = 0$ restored

First order phase transition?

Must be careful $|\phi_0|^3 T$ term is of order $\lambda^{3/2}$. There are higher order diagrams which contribute here.

are important. At high T, generate

$$E = \sqrt{p^2 + M^2(T)} = \sqrt{p^2 + 8\lambda\phi_0^2 + \kappa\lambda T^2}$$

$T \gg \phi_0 \Rightarrow$

$$\lambda^{3/2} |\phi_0|^3 T \to \lambda^{3/2} T^4 \; !$$

Effect is washed out in $\lambda\phi^4$ theory since region of metastability is same as T where this occurs

In EW theory: more complicated. Here only longitudinal propagator is modified $\Rightarrow$ contribution from 2 massive, without finite T correction modes $\Rightarrow$ $g^{3/2} |\phi_0|^3 T$ term survives

(8)

See recent papers by M. Carrington, Dine Lhode Huet, Linde Linde & Leigh

Bottom line: in EW theory ∃ a first order symmetry restoration transition

Constraints on Higgs Lagrangian: Shaposhnikov

E_{sph}/T large

$M_W(T) \gtrsim 15\, \alpha_W T$

(final rate is big when $M_W \gtrsim 7 \alpha_W T$!! Very sharp function

$$\Gamma/V = \kappa\,(1/g^2)\, T^4 \left(\frac{\alpha_W}{4\pi}\right)^4 N_{tr} N_{rot} \left(\frac{2 M_W(T)}{\alpha_W T}\right)^7 e^{-E_s(T)/T}$$

$$V(\phi) = \tfrac{1}{2} M^2(T)\,\phi^2 - \delta T\,\phi^3 + \tfrac{1}{4}\phi^4$$

$$M^2(T) = 5/16\, g^2 (T^2 - T_0^2)$$

$$\delta \sim 3/66\pi\, g^3 \cdot ?$$

$$\phi(T) = \frac{\delta}{3\lambda} T = \frac{g^3}{12\pi\lambda} T \qquad (M^2(T) \simeq 0 \text{ at minimum})$$

(9)

$\therefore\ g^2/\lambda \gtrsim 60 \Rightarrow M_H^2/M_W^2 \le .15$!!

If standard model $\Rightarrow M_H \lesssim \cancel{20}\ 30$ GeV

$\Rightarrow$ Ruled out

Must have more complicated set of doublets.

Is the wall thick or thin

If thick, baryogenesis in wall

If thin, then can generate an asymmetry in reflection of t vs $\bar{t}$

(If thick, hard to do since WKB limit

$\Rightarrow$ depending only on mass and energy particle reflected or transmitted)

(10) What does wall look like?

$V' \ \frac{1}{2}M^2(T) - \delta T\phi^3 + \lambda/4\, \phi^4$

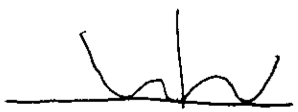
If we take

$x = \delta/\sqrt{\lambda}\, T r$

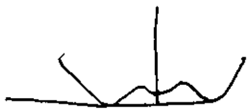
$g = \lambda/\delta T \phi$

$$S \to 4\pi \left(\frac{\delta^2}{\lambda^3}\right)^{1/2} \int x^2 dx \left[\frac{1}{2}\frac{dg}{dx} + \left(\frac{1}{4}g^4 - g^3 + \zeta/2\, g^2\right)\right]$$

$\zeta = 2 \Rightarrow$ Minima degenerate



At ~~ep~~ phase transition T



Kink should still be ~ fair approximation

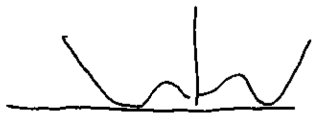
Note that $\Rightarrow r \sim \frac{\sqrt{\lambda}}{\delta T} \sim \sqrt{\lambda}\frac{4}{3\alpha_w}\frac{M_H}{M_W}\frac{1}{T}$

$\sim 20\text{-}40\ 1/T \sim 1/\alpha_w T$

(11)

$\therefore$ Wall is thick and

$L \sim R_{split}$

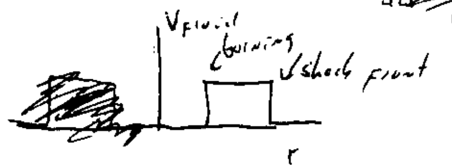


To compute moving wall, must solve

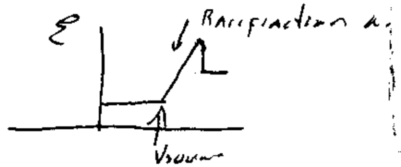
$$-\Box\phi + \partial V/\partial\phi + Q_x = 0$$

If we ignore loop term, wall accelerates to speed of light. Expect ~~stop~~ finds some finite velocity. Will check later. Suppose it does? Shape of bubble

if $V < V_{sound}$ ~~propagate~~



$V > V_{sound}$



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(12) How to compute velocity? Hard!

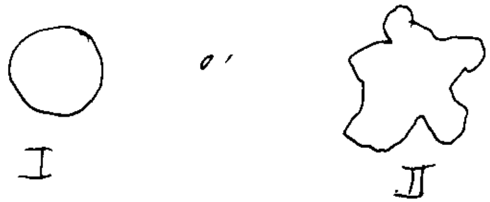
Can prove damping is due to viscous effects in fluid ⇒ out of equilibrium

$\gamma_v \sim 1$

Must compute

for out of equilibrium thermal distributions

How does bubble grow?



II corresponds to a convective instability

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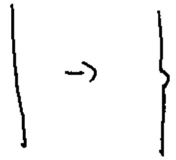

small perturbation in $\delta T, \delta v, \delta S_{vacuum}$

If $\delta v > 0$ ⇒ blip grows

Such instabilities typical of slow burning
example flame front

Find (Huet, Liu, Leigh, Kajantie, McLerran)
That stable for $v_{burning} \gtrsim .04$
(v we already know $v \sim 1$)