INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
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SMR.626 - 28
Lect. V

SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

15 June - 31 July 1992

BARYON AND LEPTON NUMBER VIOLATION IN THE ELECTROWEAK MODEL

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Please note: These are preliminary notes intended for internal distribution only.

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SOFT-SOFT CORRECTIONS

SOFT-SOFT CORRECTIONS HAVE EXPONENTIAL FORM AND MAKE SERIES IN $\left(\frac{E}{E_0}\right)^{2/3}$ (EW THEORY)

S-MATRIX WITH SOFT-SOFT CORRECTIONS BUT WITHOUT HARD-HARD AND HARD-SOFT:

$$S(b^*) = \dots \int d^4x_0 \, e^{-S_{inst}} \cdot e^{-Et_0} \times e^{-\int \dots d^3\vec{p}\, b^*(\vec{p}) R(\vec{p})}$$

$$\times \int [d\nu]\, e^{-\nu M \nu - \int d\vec{p} \dots b^*(\vec{p})\, \nu(\vec{p}, T_f) + g \dots \nu^3 + g^2 \dots \nu^4}$$

inverse propagator in instanton background

vertices in instanton background

NOTICE: INTEGRAL OVER FLUCTUATIONS $[d\nu]$ IS USUAL FUNCTIONAL INTEGRAL WITH SOURCE

$$\nu \cdot j[b^*] = \int d\vec{p} \dots b^*(\vec{p})\, \nu(\vec{p}, T_f)$$

$$\Downarrow$$

- IT GENERATES PERTURBATION THEORY (FORMAL) WHICH ELEMENTS ARE:

- PROPAGATOR IN INSTANTON BACKGROUND M^{-1}

- VERTICES IN INSTANTON BACKGROUND.

RESULT OF INTEGRATION ALWAYS HAS THE FORM

$$e^{Q[b^*]}$$

WHERE ONLY CONNECTED GRAPHS CONTRIBUTE TO Q.

- AT $b^* \sim \frac{1}{g}$, INTEGRAL OVER FLUCTUATIONS IS SADDLE POINT:

$$\nu \sim \frac{1}{g} \chi$$

$$\Downarrow$$

$$\int [d\nu]\, e^{-\nu M \nu + \int d\vec{p} \ldots b^* \nu + g \ldots \nu^3 + g^2 \ldots \nu^4}$$

$$= \int d\chi\, e^{\frac{1}{g^2}\left(-\chi M \chi + \int d\vec{p} \ldots \beta^* \chi + \ldots \chi^3 + \ldots \chi^4\right)}$$

WHERE $\beta^* = g b^*$

$\therefore$ RESULT IS $e^{-\frac{1}{g^2} A(\beta^*)}$

ONLY TREE GRAPHS (CONNECTED) CONTRIBUTE TO $A(\beta^*)$.

AT $E \sim \frac{1}{g^2}$ (i.e. $\frac{E}{E_0} = \text{fixed}$)

TOTAL INSTANTON - INDUCED CROSS SECTION

WITH SOFT - SOFT CORRECTIONS INCLUDED

(BUT NOT HARD - HARD AND HARD - SOFT)

HAS EXPONENTIAL FORM

$$\sigma_{2 \to \text{ANY}} \left(\begin{array}{l} \text{NO HARD-HARD} \\ \text{NO HARD-SOFT} \end{array} \right) = e^{-\frac{1}{g^2} F\left(E/E_0\right)}$$

ARGUMENT IS THE SAME AS IN LEADING ORDER:

AT $E \sim \frac{1}{g^2}$ ALL INTEGRALS (FUNCTIONAL

AND ORDINARY) ARE SADDLE POINT

WITH

$$\nu \sim \frac{1}{g} \quad , \quad b^*, b \sim \frac{1}{g}$$

$$x_0^\mu, \rho \sim \frac{1}{M_W}$$

BEHAVIOUR OF $F(E/E_0)$ AT $E/E_0 \ll 1$

IS FOUND BY EXPECTING THE PERTURBATION

SERIES GENERATED BY ν - INTEGRATION.

CONSTRAINT DEPENDENCE: HINT TO HARD-HARD AND HARD-SOFT CORRECTIONS

CONSTRUCTION OF PERTURBATION THEORY AROUND INSTANTON WAS NOT QUITE COMPLETE:

ONE SHOULD CAREFULLY SEPARATE INTEGRATIONS OVER INSTANTON PARAMETERS (SAY, x_c^μ) AND FLUCTUATIONS AROUND INSTANTON

THIS INTRODUCES SOME FREEDOM. FINAL RESULT FOR CROSS SECTION MUST BE INDEPENDENT OF THIS FREEDOM.

BUT, IF SOFT-SOFT CORRECTIONS ARE INCLUDED ONLY, THE FUNCTION $F(E/E_0)$ EXPLICITLY DEPENDS ON THIS FREEDOM

$\Downarrow$

HARD-SOFT AND HARD-HARD CORRECTIONS MUST CONTRIBUTE INTO $F(E/E_0)$.

SEPARATION OF INTEGRATION OVER INSTANTON PARAMETERS:

$$\int[d\varphi]\, e^{-S[\varphi]} \dots = \int dx_0 \int[d\varphi]\, J(\varphi)\, \delta(B(\varphi; x_0))\, e^{-}$$

$B(\varphi, x_0)$ IS 1) TRANSLATION NON-INVARIANT

2) VANISHING AT $\varphi = \varphi_{inst}(x_0)$

IN OTHER RESPECTS ARBITRARY!

$J(\varphi)$ IS Jackobian ensuring $\int dx_0\, J(\varphi)\delta(B) = 1$

FOR EXAMPLE,

$$B(\varphi) = \int \frac{\partial \varphi_{inst}(x-x_0)}{\partial x_0} \cdot \left(\varphi(x) - \varphi_{inst}(x-x_0)\right) W(x)\, dx$$

ARBITRARY MEASURE

PERTURBATION THEORY AROUND INSTANTON:

$$\varphi = \varphi_{inst}(x-x_0) + \nu$$

$$\delta(B(\varphi)) = \int dx\, W(x)\, \frac{\partial \varphi_{inst}(x-x_0)}{\partial x_0}\, \nu(x)$$

$$\Downarrow$$

ν IS ORTHOGONAL TO ZERO MODE $\dfrac{\partial \varphi_{inst}}{\partial x}$ with weight $W(x)$

! PROPAGATOR OF $\nu(x)$ EXPLICITLY DEPENDS ON choice of $W(x)$

FREEDOM IN PROPAGATOR:

NAIVELY $M \cdot \mathcal{D} = \mathbb{1}$ (CONDENSED NOTATI

OR $M \cdot \mathbf{D}(x,y) = \delta(x,y)$.

However, M HAS ZERO MODES $\frac{\partial \varphi_{inst}(x-x_0)}{\partial x_0} \equiv$

(CHANGING THE POSITION OF INSTANTON DOES NOT CHANGE THE ACTION)

$\Downarrow$

$M \cdot D = \mathbb{1}$ IS INCONSISTENT: $(\Psi_0, MD) = 0$

$\Downarrow$

EQN. FOR PROPAGATOR IS MODIFIED:

$$M\mathbf{D}(x,y) = \delta(x,y) + \Psi_0(x)\, U(y)$$

$U(y)$ DEPENDS ON CHOICE OF $W(x)$

(PROPAGATOR MUST BE ORTHOGONAL TO Ψ_0 WITH weight $W(x)$).

$\Downarrow$

VARIATION OF PROPAGATOR UNDER VARIATION OF W MAY BE FOUND EXPLICITLY

$\Downarrow$

CHANGE IN SOFT-SOFT PART OF $F(E/E_0)$ FOUND EXPLICITLY. IT IS NON-ZERO IN THE ORDER $(E/E_0)^{8/3}$ HARD-SOFT CORRECTION MUST COMPENSATE

TOWARDS THE SEMICLASSICAL CALCULATION OF HIGH ENERGY INSTANTON CROSS SECTIONS

SUMMARY OF PERTURBATION THEORY AROUND INSTANTON

- LEADING ORDER + SOFT-SOFT CORRECTIONS:

$$\sigma_{2\to ANY} = e^{-\frac{16\pi^2}{g^2}F(E/E_0)} \qquad \left(g^2\to 0,\ \frac{E}{E_0} = \text{fixed}\right)$$

- SOFT-SOFT CORRECTIONS TO $F\left(\frac{E}{E_0}\right)$ ARE TREE GRAPHS

- HARD-SOFT AND HARD-HARD CORRECTIONS ARE STRONG AT $E \sim E_0$

- DIRECT STUDY OF HARD-SOFT AND HARD-HARD CORRECTIONS (Mueller) +

  CONSTRAINT DEPENDENCE OF SOFT-SOFT CORRECTIONS (KHLEBNIKOV and TINYAKOV)

  $\Downarrow$

  HARD-HARD AND HARD-SOFT CORRECTIONS ARE LIKELY TO SUM UP TO THE SAME EXP. FORM.

$$\sigma_{2\to ANY} \propto e^{-\frac{16\pi^2}{g^2}F\left(\frac{E}{E_0}\right)}\left(1 + g^2 F_1\left(\frac{E}{E_0}\right) + \ldots\right)$$

- HARD-HARD AND HARD-SOFT CONTRIBUTIONS TO $F\left(\frac{E}{E_0}\right)$ INCLUDE LOOPS.

- PERTURBATION THEORY IS USELESS AT $E/E_0 \sim 1$.

$\sigma_{2\to ANY}$ LOOKS SEMICLASSICALLY

$\Downarrow$

HOPE TO CALCULATE $\sigma_{2\to ANY}$ BY SEMICLASSICAL TYPE TECHNIQUE.

MOST DIFFICULT PART OF THE PROBLEM:

HARD INITIAL PARTICLES

FEW SUGGESTIONS UNTIL NOW:

McLerran, Mattis, Yaffe '91

KHLEBNIKOV '91

MUELLER '92

V.R., TINYAKOV '92 (+V.R., Tinyakov, Son '92)

TO BE DISCUSSED IN THIS LECTURE.

SUGGESTION:

INSTEAD OF $2 \to$ ANY, STUDY $n \to$ ANY.

IN THE REGIME $g^2 \to 0$, $\frac{E}{E_0} = \text{fixed}$, $g^2 n \equiv \nu = \text{fixed}$

(i.e. $n \simeq \frac{\nu}{g^2}$)

ARGE CORRECTIONS ARE DUE TO COMBINATORICS ONLY

($\frac{E}{n} = \left(\frac{E}{E_0}\right) \cdot \frac{1}{\nu}$ IS INDEPENDENT OF g^2)

average energy of initial particles

$\Downarrow$

SEMICLASSICAL - TYPE CALCULATION OF PROBABILITY

$$\sigma_{n \to ANY} \propto e^{-\frac{1}{g^2} F\left(\frac{E}{E_0}, g^2 n\right)}$$

IS POSSIBLE.

RELEVANCE TO $2 \to$ ANY :

1) $\sigma_{2 \to any} < \sigma_{n \to any}$

2) Hopefully, exponent for $2 \to$ ANY IS $\lim_{g^2 n \to 0} F\left(\frac{E}{E_0}, g^2 n\right)$

OBJECT TO CONSIDER:

$$\sigma_{n \to ANY}(E) = \sum_{\substack{\text{ALL} \\ \text{FINAL} \\ \text{STATES} \\ |f\rangle}} \; \sum_{\substack{\text{ALL} \\ \text{INITIAL STATES } |i\rangle \\ \text{WITH GIVEN} \\ \text{ENERGY } E \\ \text{AND NUMBER} \\ \text{OF PARTICLES} \\ n}} \left| \langle f | \hat{S} | i \rangle \right|^2$$

S-matrix

INSTANTON-LIKE TRANSITION PROBABILITY FROM MICROCANONICAL ENSEMBLE WITH FIXED ENERGY AND NUMBER OF PARTICLES.

IN OTHER WORDS,

$$\sigma_{n \to ANY}(E) = \sum_{\substack{\text{ALL FINAL} \\ \text{STATES}}} \; \sum_{\substack{\text{ALL} \\ \text{INITIAL} \\ \text{STATES}}} \left| \langle f | \hat{S} \cdot \hat{P}_E \cdot \hat{P}_n | i \rangle \right|^2$$

WHERE P_E = PROJECTOR ON (in-STATES) of given energy E

P_n = PROJECTOR ON (in-STATES) of given number of particles n

KEY OBSERVATION:

IN COHERENT STATE BASIS, PROJECTORS HAVE "EXPONENTIAL FORM":

$$P_E(b^*, b) = \int d\tau \, e^{-iE\tau + \int d^3k \, e^{i\omega_{\vec{k}}\tau} b^*_{\vec{k}} b_{\vec{k}}}$$

$$P_n(b^*, b) = \int d\theta \, e^{-in\theta + \int d^3k \, e^{i\theta} b^*_{\vec{k}} b_{\vec{k}}}$$

THUS,

$(\hat{S} \cdot \hat{P}_E \cdot \hat{P}_n)(b^*, a)$ IS INTEGRAL OVER FIELDS AND EXTRA VARIABLES τ AND θ WITH EXPONENTIAL MEASURE.

$\sigma_{n \to ANY}$ HAS ALSO FORM OF INTEGRAL OF EXPONENT

$$\sigma_{n \to ANY} = \int [d\varphi][da][db^*] d\tau d\theta \ldots\ldots e^{W(\varphi, a, \ldots, \theta, \ldots)}$$

AT $E \sim 1/g^2$ and $n \sim 1/g^2$ THIS INTEGRAL HAS EXPLICITLY "SEMICLASSICAL" (SADDLE-POINT) FORM:

$$W = \frac{1}{g^2} w(\tau, \theta, \alpha, \beta^*, \varphi', \ldots) \quad , \alpha = ga, \beta^* = gb^*$$

$$\varphi' = g\varphi, \ldots$$

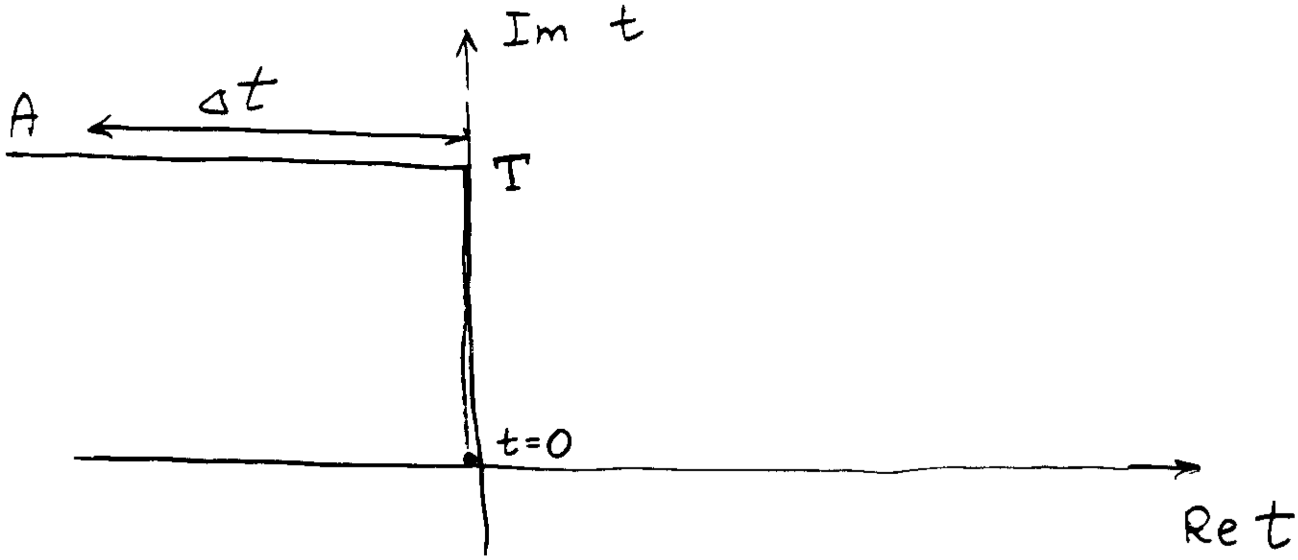
$$\sigma_{n \to ANY}(E) = e^{-\frac{1}{g^2} f\left(\frac{E}{E_0},\, g^2 n\right)}$$

SADDLE-POINT

$\theta \to i\theta$, $\tau \to iT$

(SADDLE-POINT VALUES ARE IMAGINARY)

MOST EASY TO DESCRIBE ON CONTOUR IN COMPLEX TIME PLANE:



SADDLE-POINT FIELD: • SOLUTION TO USUAL FIELD EQS:

$$\frac{\delta S}{\delta \varphi} = 0 \quad \text{ON THE CONTOUR}$$

- TURNING POINT AT $t=0$: $\left.\frac{\partial \varphi}{\partial t}\right|_{t=0} = 0$ for any $\vec{x}$
- Free field at asymptotics A: $(\Delta t = \mathrm{Re}\, t)$

$$\varphi(\vec{k}) = f_{\vec{k}}\, e^{-i\omega_{\vec{k}}(\Delta t)} + g_{\vec{k}}\, e^{i\omega_{\vec{k}}(\Delta t)}$$

- Second boundary condition: $g_{\vec{k}} = e^{\theta} f^*_{-\vec{k}}$

FIELD IS REAL IN EUCLIDEAN PART OF THE CONTOUR, BUT

COMPLEXIFIED IN MINKOWSKIAN PART

(UNLESS $\theta = 0$).

THIS MAY be a good starting point for computer calculations.