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SMR.626 - 30

SUMMER SCHOOL IN HIGH ENERGY PHYSICS AND COSMOLOGY

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COBE DMR ANISOTROPY DETECTION

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Please note: These are preliminary notes intended for internal distribution only.

SMR.626 - 27

COBE DMR ANISOTROPY DETECTION

- The DMR Instrument
- Maps
- Correlation function analysis
- Cosmic variance and best fit n and Qrms_ps

COBE DMR
Cosmic Background Explorer Differential Microwave Radiometers

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Papers Submitted for these results:

Smoot et al., "Structure in the COBE-DMR First Year Maps" *(Astrophysical Journal Letters)*

Kogut et al., "COBE-DMR: Preliminary Systematic Error Analysis" *(Astrophysical Journal)*

Bennett et al., "Preliminary Separation of Galactic and Cosmic Emission for the COBE-DMR" *(Astrophysical Journal Letters)*

Wright et al., "Interpretation of the CMB Anisotropy Detected by the COBE-DMR" *(Astrophysical Journal Letters)*

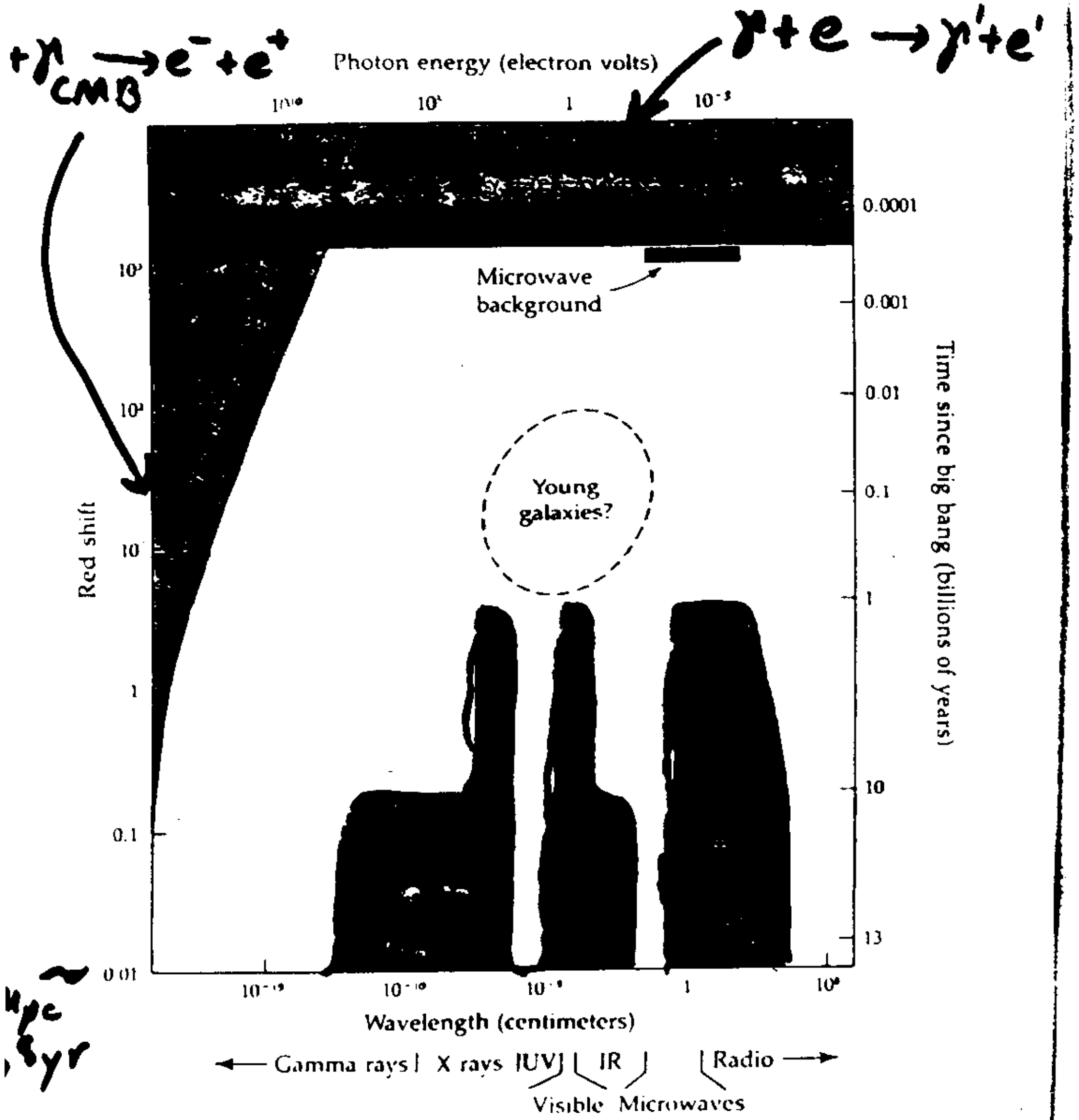
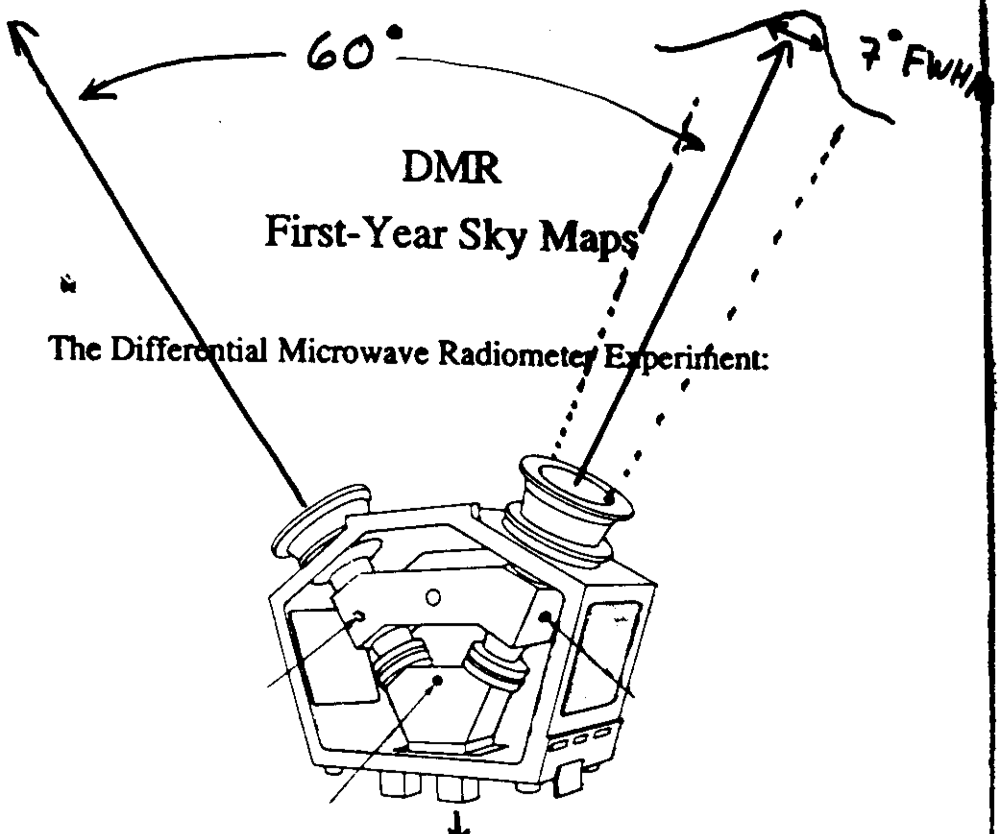


Figure 9-1. This figure shows the extent to which we can explore the universe throughout the spectrum of electromagnetic radiation (in terms of either the red shift of sources or, equivalently, how far back in time we see them). The darkly shaded areas show the extent of our present knowledge. The lightly shaded area shows the region that we can never view directly because the photons are either scattered by electrons ($\gamma + e \rightarrow \gamma' + e'$) or collide with other photons, producing electron-positron pairs ($\gamma + \gamma \rightarrow e^- + e^+$). The dashed boundary surrounds the region where we may see galaxies in their early phase of development.

DMR
First-Year Sky Maps

The Differential Microwave Radiometer Experiment:



$V_{out} \sim T_1 - T_2$

- Radiometers observe temperature differences 60° apart.
- Full sky coverage every six months.
- Maps obtained by least-squares fits to the observed temperature differences.
- Observe at 3 frequencies: 31.5, 53, and 90 GHz. (9, 6, 3 mm)
- Two channels per frequency: 31A, 31B,..., 90B.
- **Spin period 75 s**
- **Orbit period 103 min**

Differential measurements $\longrightarrow$ Map

single measurement $T_i - T_j = D_1$

$$\begin{array}{c} \xleftarrow{\quad 6144 \quad} \\ \#\text{ of obs} \downarrow \begin{pmatrix} 0,0,1,0,\dots 0,-1,0\dots \\ 0,1,0\dots,-1,0,0,\dots \\ \vdots \\ \\ \end{pmatrix} \end{array} \begin{pmatrix} T_1 \\ T_2 \\ \vdots \\ T_{6144} \end{pmatrix} = \begin{pmatrix} D_1 \\ D_2 \\ \vdots \\ \vdots \end{pmatrix} \begin{array}{c} \#\text{ of} \\ \text{obs} \\ \downarrow \end{array}$$

$$M \vec{T} = \vec{D}$$

$$A \equiv \boxed{M^T M} \vec{T} = M^T \vec{D} \qquad \text{normal equations}$$

$$\chi^2 = (M^T M \vec{T} - M^T \vec{D})^2 \qquad \text{cf Smoot et al references to Torres et al '9?, Jansen + Gulkis '92}$$

$$\vec{T} = A^{-1} M^T \vec{D}$$

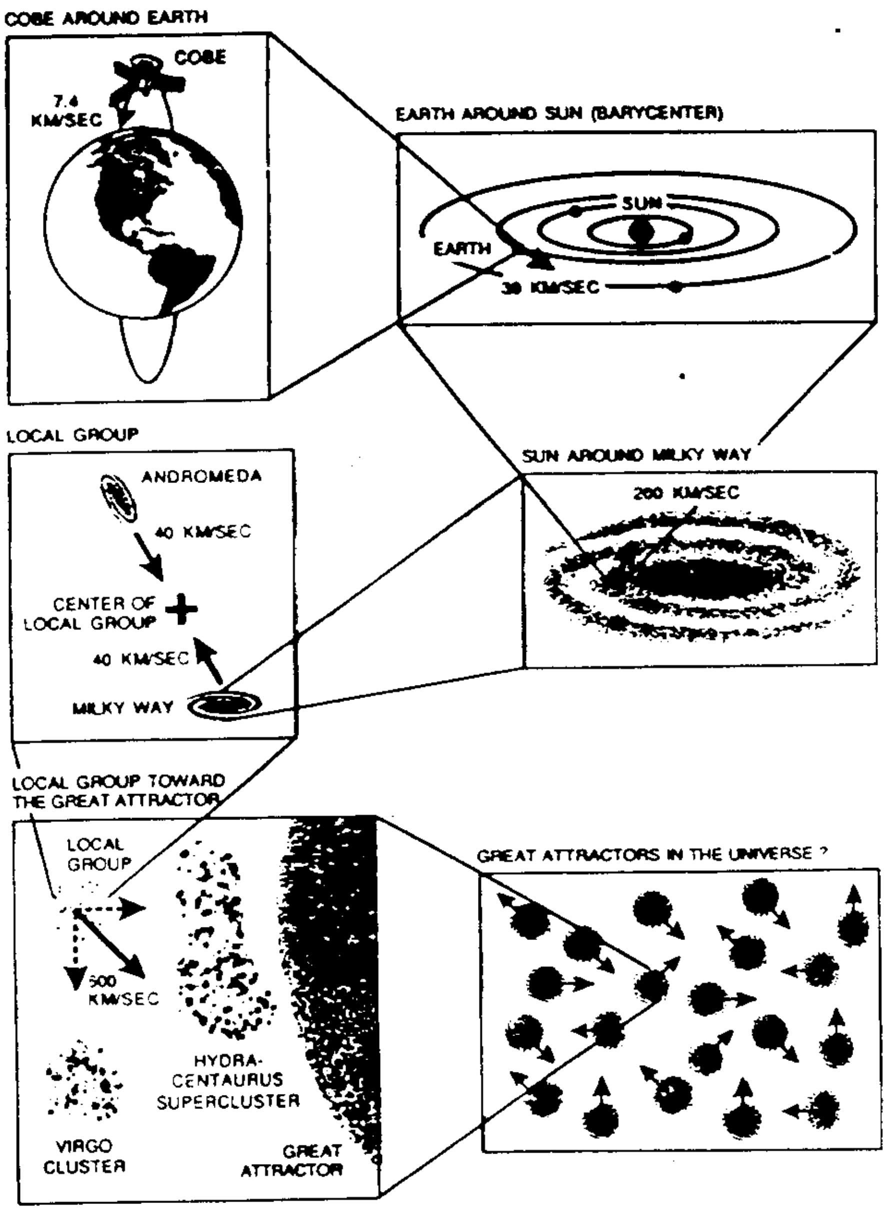
Differential measurements $\longrightarrow$ power spectrum (ie $a_{\ell m}$'s)

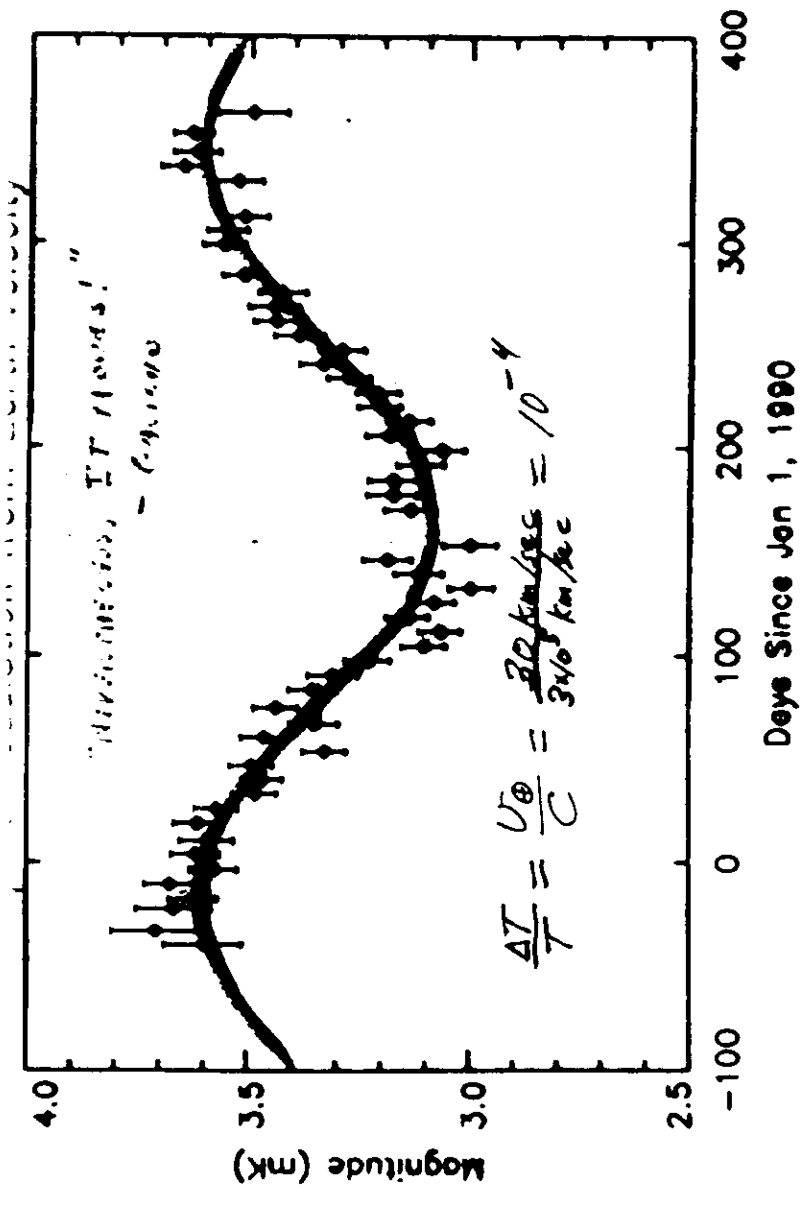
$$T(\hat{n}_1) = \sum a_{\ell m} Y_{\ell m}(\hat{n}_1)$$

$$\sum a_{\ell m} \left(Y_{\ell m}(\hat{n}_1) - Y_{\ell m}(\hat{n}_2) \right) = T_{\hat{n}_1} - T_{\hat{n}_2} = D_1$$

$$\begin{array}{c} \xleftarrow{\qquad 2\ell+1 \qquad} \\ \begin{pmatrix} [Y_{11}(\hat{n}_1) - Y_{11}(\hat{n}_2)] & [Y_{10}(\hat{n}_1) - Y_{10}(\hat{n}_2)] & [Y_{1-1}(\hat{n}_1) - Y_{1-1}(\hat{n}_2)] \\ [Y_{11}(\hat{n}_3) - Y_{11}(\hat{n}_4)] & [Y_{10}(\hat{n}_3) - Y_{10}(\hat{n}_4)] & [Y_{1-1}(\hat{n}_3) - Y_{1-1}(\hat{n}_4)] \\ & \vdots & \end{pmatrix} \end{array} \begin{pmatrix} a_{11} \\ a_{10} \\ a_{1-1} \end{pmatrix} = \begin{pmatrix} D_1 \\ D_2 \\ \vdots \\ \vdots \end{pmatrix} \begin{array}{c} \#\text{ of} \\ \text{obs} \\ \downarrow \end{array}$$

VELOCITY COMPONENTS OF THE OBSERVED CMB DIPOLE





COSMIC MICROWAVE BACKGROUND

DIPOLE

AMPLITUDE	$T_D = 3.36 \pm 0.1$ mK
SPEED	$v = 369 \pm 11$ km/sec
	$\frac{v}{c} = 0.00123 \pm 0.00004$
DIRECTION	$l^{II} = 264.7^\circ \pm 0.8^\circ$ $b^{II} = 48.2^\circ \pm 0.5^\circ$
SPECTRUM	consistent with <u>thermal</u> spectrum

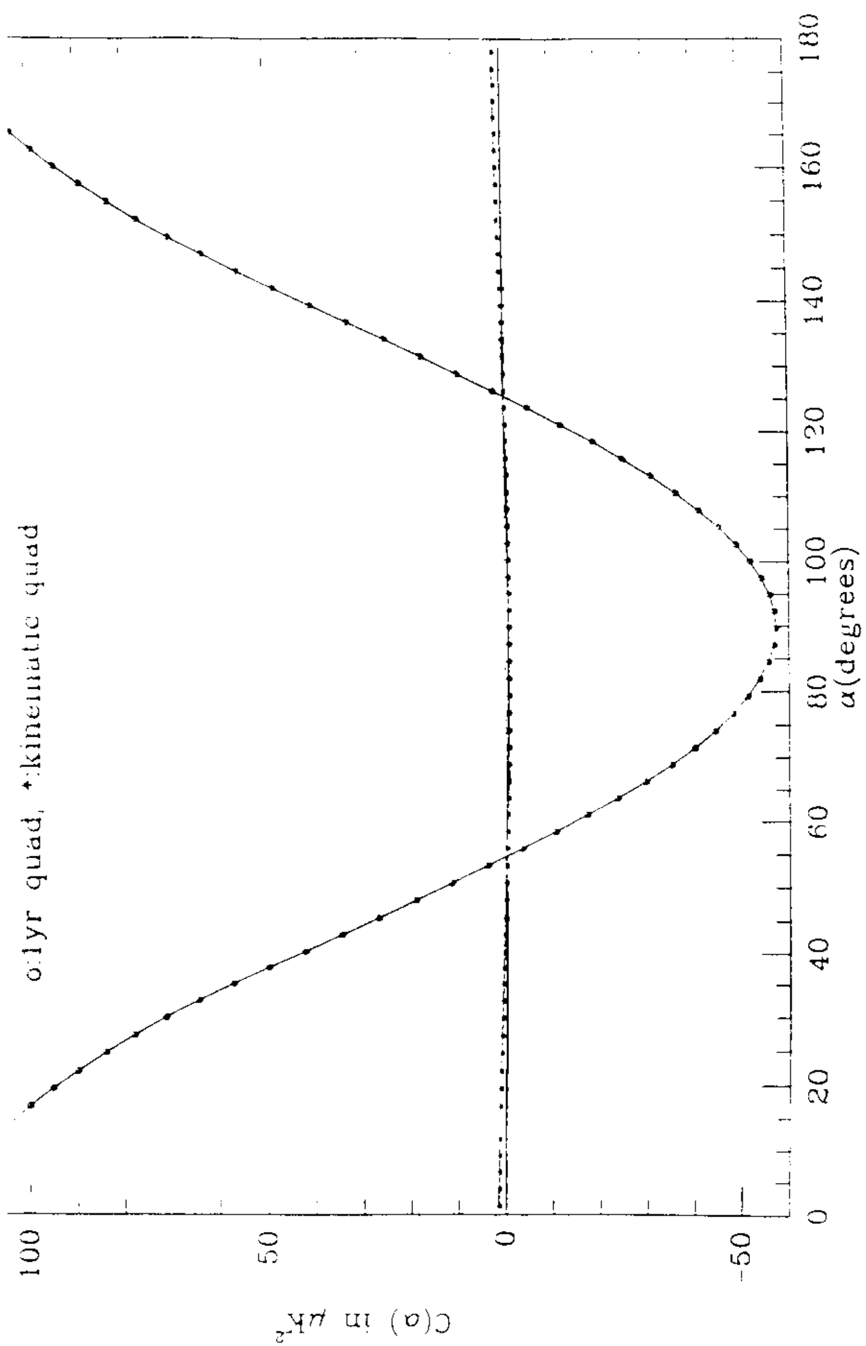
KINEMATIC QUADRUPOLE

$$T(\theta) = T_o(1-\beta^2)^{1/2}/(1-\beta\cos\theta)$$

$$= T_o\left[1+\beta\cos\theta+(\beta^2/2)\cos 2\theta+\cdots\right]$$

$$Q_{rms} = 1.2\ \mu K$$

$$(Q_1,Q_2,Q_3,Q_4,Q_5) = (0.9, -0.2, -2.0, -0.9, 0.2)\ \mu K$$



What is signal, what is noise?

$$\sigma_{i_N} = \frac{\sigma_i}{\sqrt{N_i}}$$

channel noise per observation

$$\underset{\text{map}}{\Delta T_i} = \frac{20\,mK}{\sqrt{N_i}}$$

# of times pixel i was observed

sec in yr, # of yrs, 2 obs per sec, 2 pixels per observation

$$N_i = \frac{(3.14 \times 10^7)(n)(2)(2)}{6144}$$

# of pixels

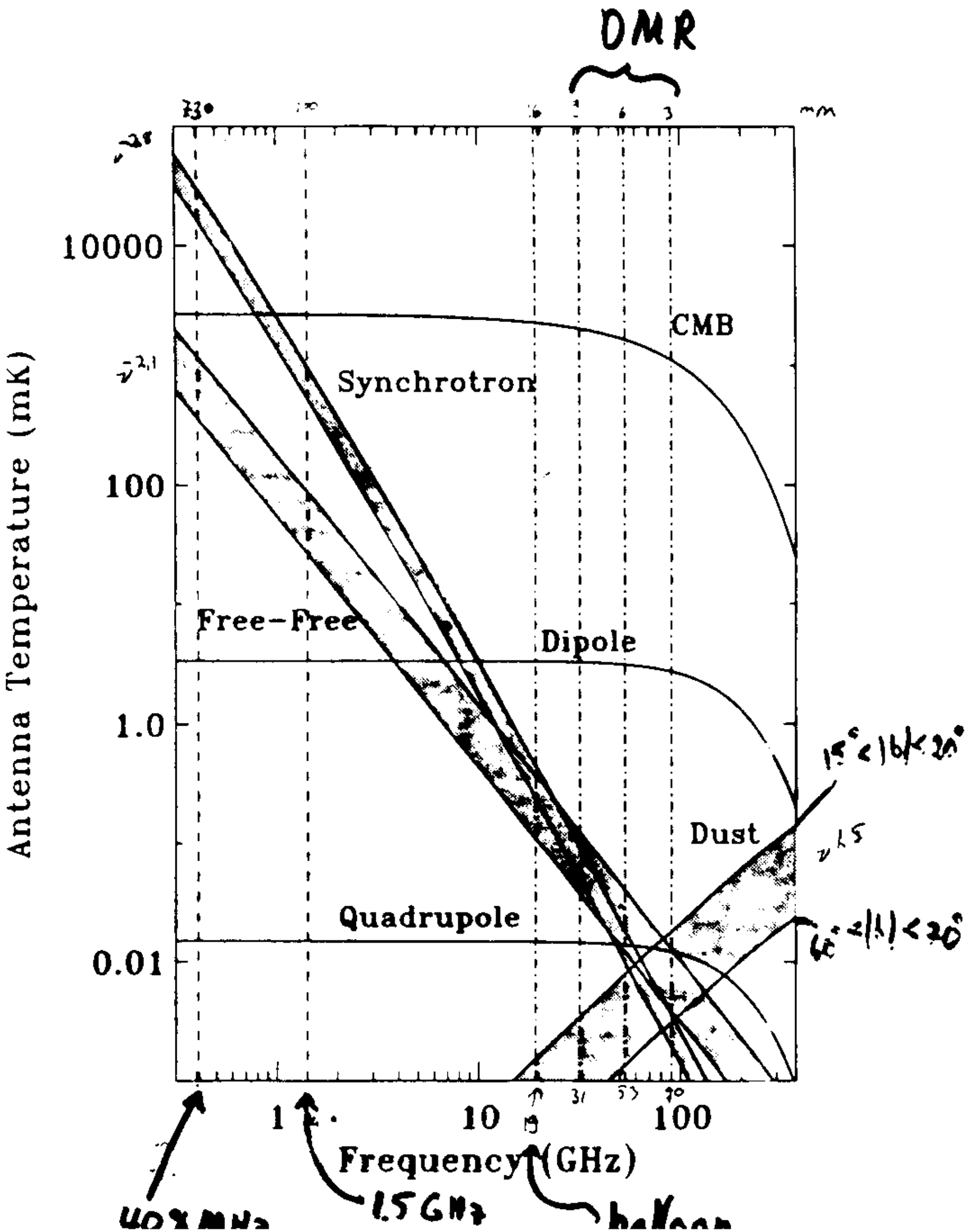
~2.6°

pixel

$$N_i \simeq (20{,}000)\, n$$

$\therefore$ $\boxed{\Delta T_i \sim 100\mu K \text{ in 1yr maps}}$

Size	Noise μK		Signal	Systematics
	1yr	4yr		
1 pixel	~100	~50	(>30)	< 6
10° ~ 16 pixels	~25	~13	30±5	
90° ~ quad ~ 1000 pixels	~3	~1.5	13 ± 4	< 3

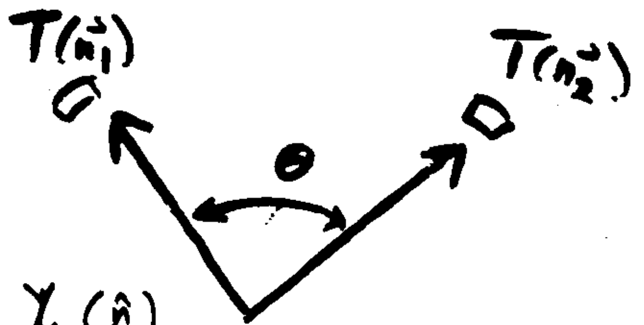


Quantifying Clumpiness

Correlation Function

$$C(\theta) = < T(\vec{n_1})\, T(\vec{n_2}) > = \sum_{i,j} n_i T_i n_j T_j \Big/ \sum_{i,j} n_i n_j$$

where $\vec{n_1} \cdot \vec{n_2} = \cos(\theta)$



$\hat{n}_1) = \sum a_{\ell m} Y_{\ell m}(\hat{n}_1)$

$T(\hat{n}_1)\ T(\hat{n}_2) >$

Average product of temperatures separated by angle θ

DMR data show unambiguous detection of anisotropy

$$C(\theta) = \sum_{\ell=2}^{\infty} \Delta T_\ell^2\, P_\ell(\cos\theta)\, e^{-(\ell+\frac{1}{2})^2 \sigma_{smooth}^2}$$

$$\Delta T_\ell^2 = \frac{1}{4\pi} \sum_m |a_{\ell m}|^2 \qquad Q_{rms} = \Delta T_2$$

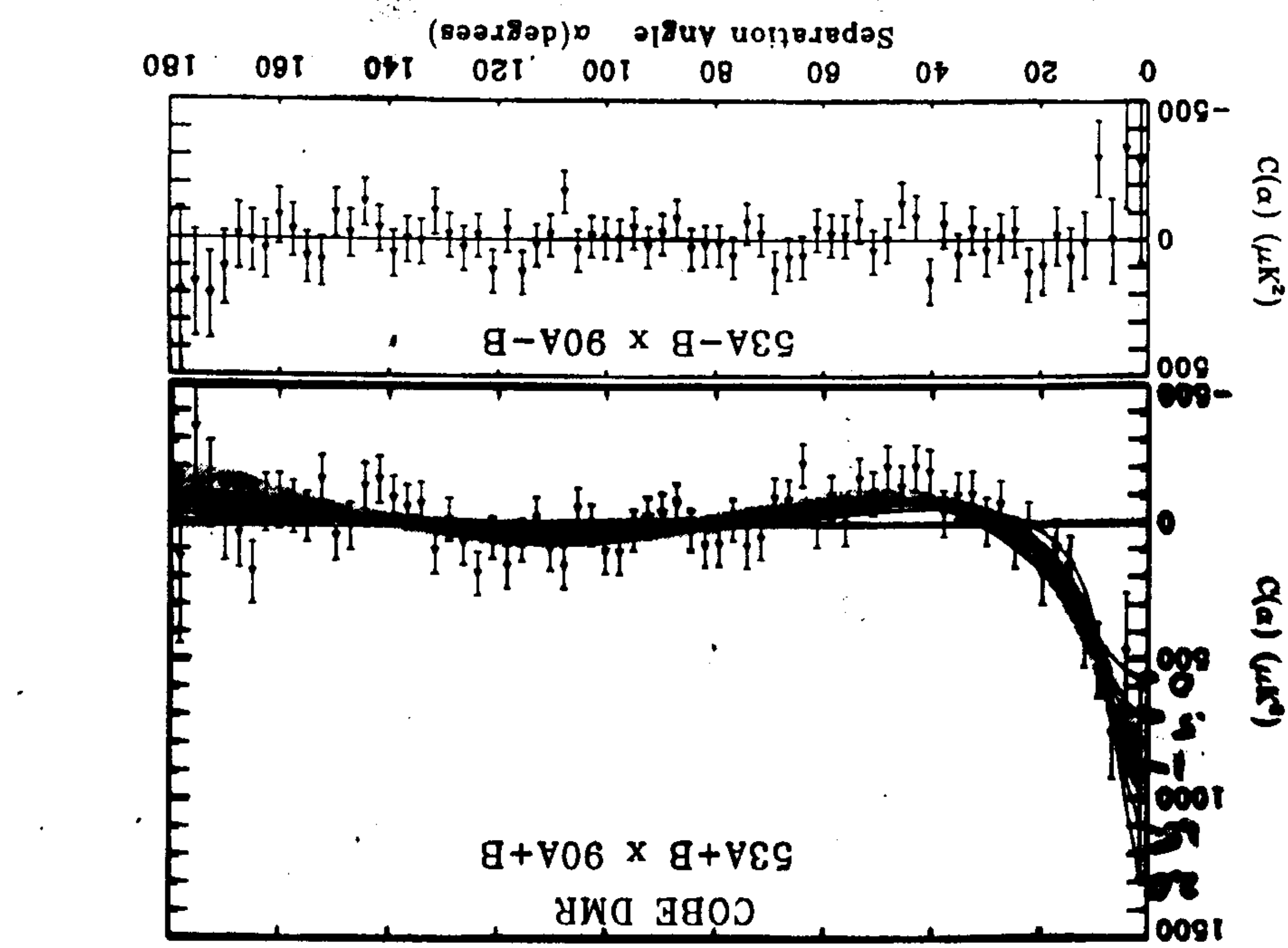
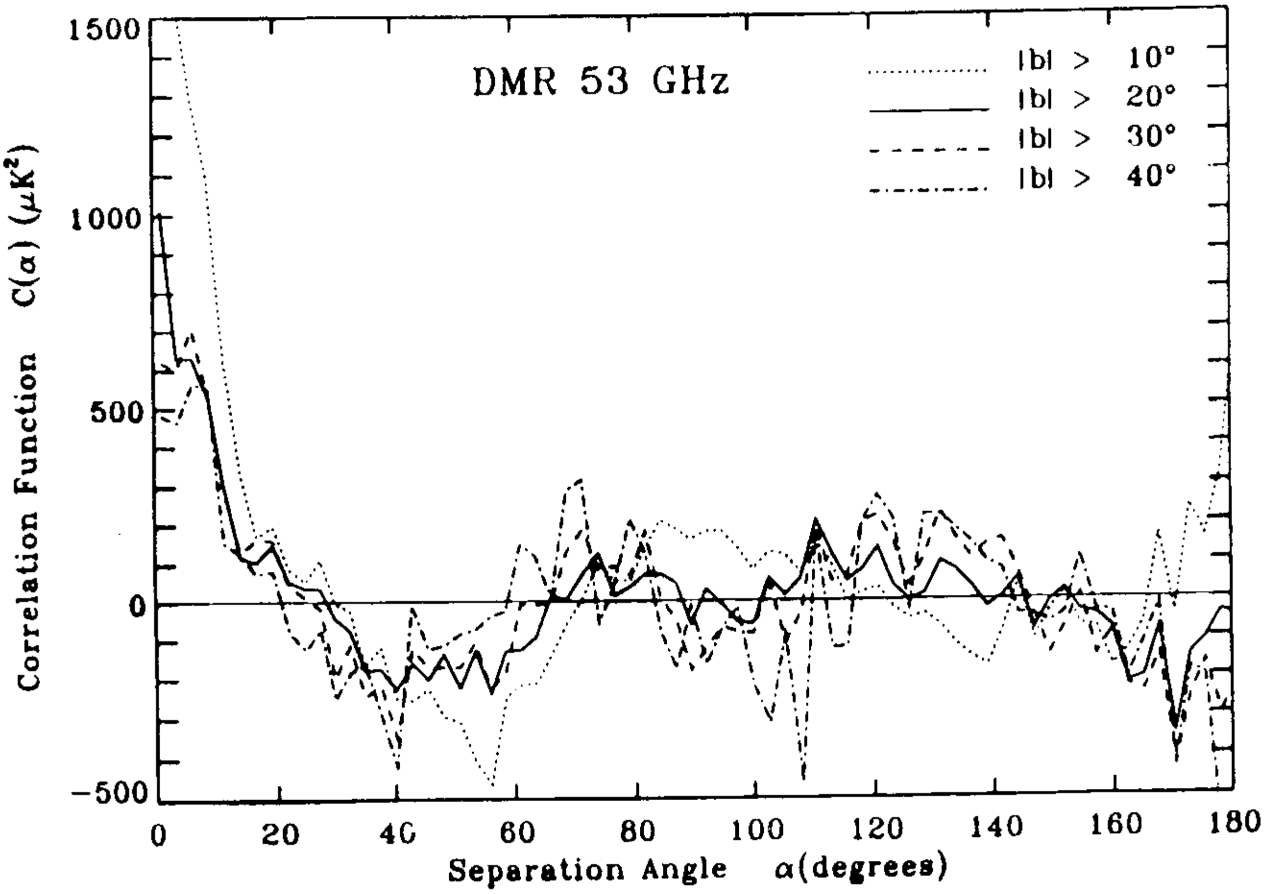
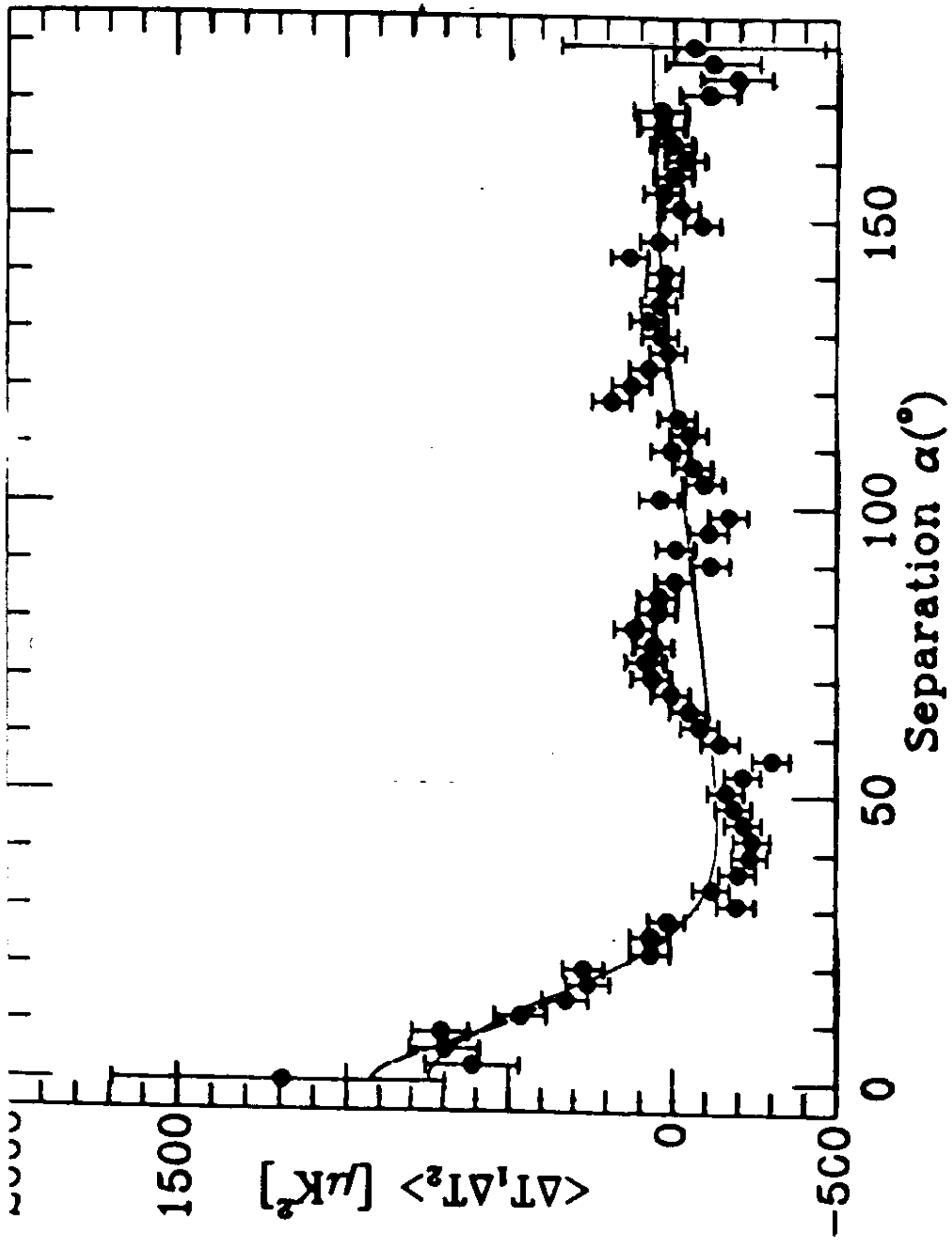


Fig. 2 of Smoot et al

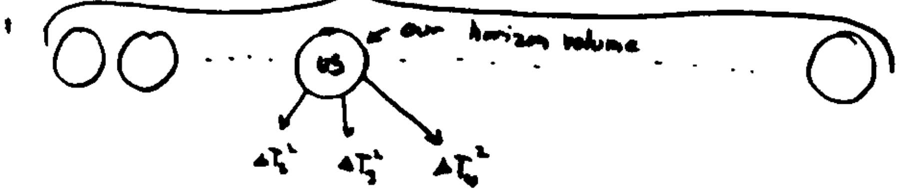




Cosmic Variance

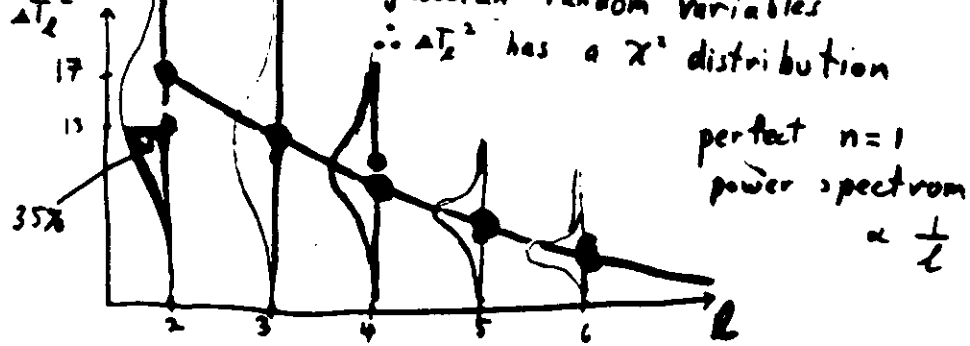
Cosmic variance is the statistical error on the power spectrum of our horizon volume. It is due to the fact that on the largest scales ($L = 2, 3, ..$) our horizon volume represents a small sample of the universe.

Theory makes predictions about the entire universe – not just our horizon volume.



$\Delta T_\ell^2 = \frac{1}{4\pi} \sum_m |a_{\ell m}|^2$ ← sum of the squares of $2\ell+1$ gaussian random variables

$\therefore$ ΔT_ℓ^2 has a χ^2 distribution



$\boxed{\text{Getting } n \text{ from } C_d(\alpha)}$ (one way)

NWB

) assume the universe is described by a power law $P(k) \propto k^n$

) $C_n(\alpha) = \sum_\ell \langle \Delta T_\ell^2 \rangle_n \, P_\ell(\cos\alpha) \, W_\ell^2$

beam cut off

power spectrum

Legendre polynomials

here

$$\langle \Delta T_\ell^2 \rangle_n = \underbrace{\langle \Delta T_2^2 \rangle}_{= Q^2_{rms-ps}} \frac{2\ell+1}{5} \frac{\Gamma\left(\ell + \frac{n-1}{2}\right) \Gamma\left(\frac{9-n}{2}\right)}{\Gamma\left(\ell + \frac{5-n}{2}\right) \Gamma\left(\frac{3+n}{2}\right)}$$

cf. Bond + Efstathiou ApJ '87

Q^2_{rms-ps} ← sets the amplitude of the entire power spectrum

) $$\chi_n^2 = \sum_i \frac{\left[C_d(\alpha_i) - C_n(\alpha_i)\right]^2}{\sigma_d(\alpha_i)^2 \left(+ \sigma_n(\alpha_i)^2\right)} \rightarrow \frac{\partial \chi_n^2}{\partial Q^2_{rms-ps}} = 0 \rightarrow \text{solve for } Q_{rms-ps} \rightarrow \chi_n^2$$

cosmic variance

$$\left.\begin{matrix} n_1 \rightarrow \chi^2_{n_1} \\ n_2 \rightarrow \chi^2_{n_2} \\ n_3 \rightarrow \chi^2_{n_3} \\ \vdots \end{matrix}\right\} \text{choose lowest} \rightarrow \begin{matrix}\text{best fit } n \\ \text{best fit } Q_{rms-ps}\end{matrix}$$

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re sophisticated technique using monte carloing covariance matrices handles the correlated errors rectly but yields the same answers

approach*	n	Q_{rms-PS} (μK)
best fit (no quad, no cosmic variance)	1.1±0.5	16±4
best fit (no quad, with cosmic variance)	$1.15^{+0.45}_{-0.65}$	16.3±4.4
best fit (with quad)	1.5	14
forced n=1 (no quad, with cosmic var)	1.0	17±5

*|b|>20° for all

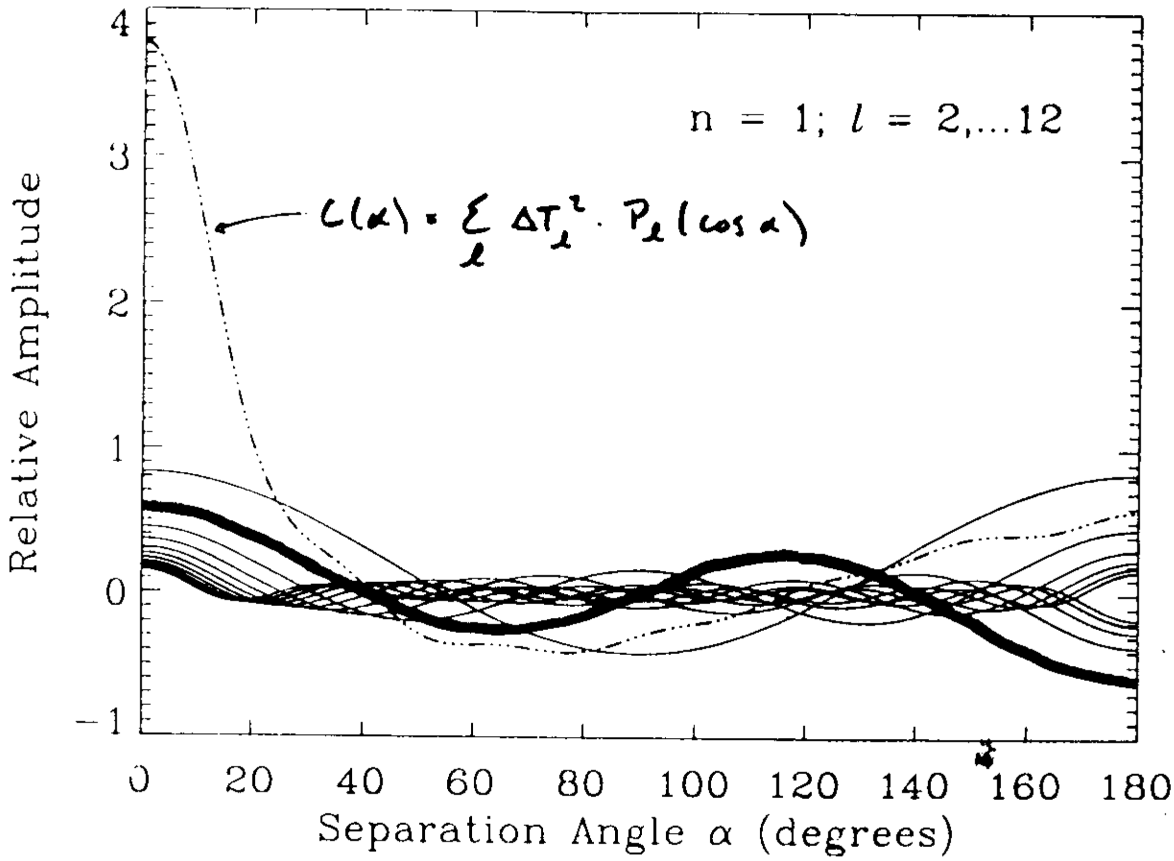
measured Q_{rms} = 13±4 μK

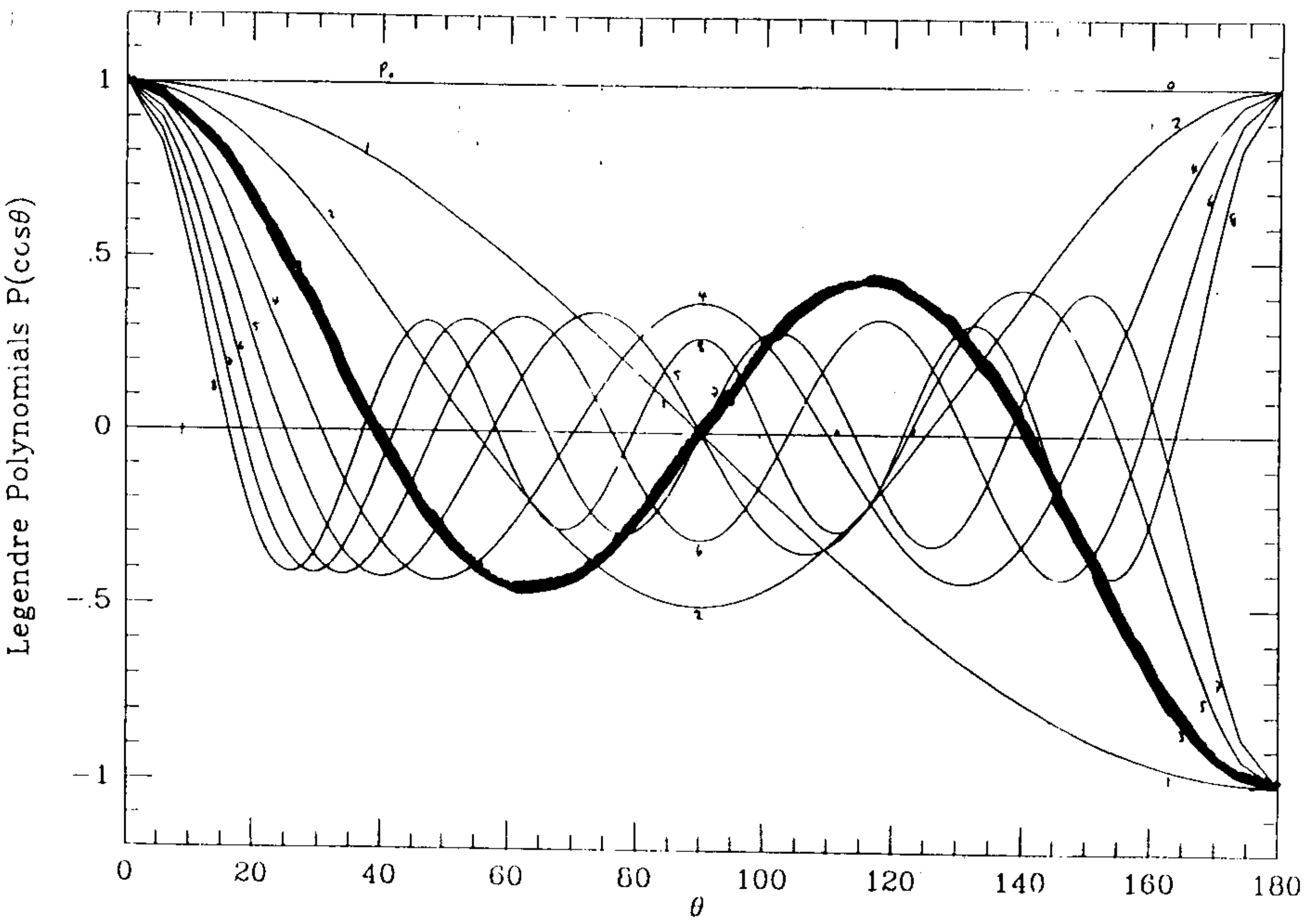
σ(@10° smoothing) = 30 ± 5 μK

σ(@10° smoothing) = 2 Q_{rms-PS} for n=1

$\frac{30 \pm 5 \ \mu K}{2} = 15 \ \mu K$

~1% of dipole is cosmic


n = 1; l = 2,...12
C(α) = Σ_l ΔT_l² · P_l(cos α)
Relative Amplitude
Separation Angle α (degrees)


Legendre Polynomials P(cosθ)
θ