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A MODEL FOR A UNIFICATION OF SCALES. FROM M_{Planck} TO m_ν

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It is proposed that the hierarchical scales – from M_{Planck} to m_ν – have a common origin. Using M_{Planck} and the coupling constant associated with a preonic metacolor gauge force as the only input parameters, it is shown how large ratios such as (M_{Pl}/M_I), $(M_{Pl}/\delta m_s)$, (M_{Pl}/m_W), (M_{Pl}/m_t) and even $(M_{Pl}/m_\nu) \gtrsim 10^{27}$ can arise naturally. Here M_I denotes an intermediate scale $\sim 10^{11}$ GeV, which is identified with the scale parameter of the metacolor force, while δm_s denotes SUSY-breaking mass splittings ~ 1 TeV. Local supersymmetry together with an inhibition in the breaking of global SUSY (index theorem) as well as compositeness of quarks, leptons and Higgs play crucial roles in this approach. Two key features of the model are the natural origins of composite vector-like families with masses of order of a few hundred GeV to 1 TeV and the consequent see-saw mechanism for the generations of quark–lepton masses and CP violation.

1. Introduction

The diversity of scales encountered in particle physics which include (i) M_{Pl}, (ii) a possible intermediate scale $M_I \sim 10^{12\pm2}$ GeV associated perhaps with supersymmetry breaking, Peccei–Quinn symmetry breaking and/or inflation and baryogeneis, (iii) SUSY breaking mass splittings $\delta m_s \sim 1$ TeV, (iv) the electroweak scale $m_W \sim 100$ GeV and (v) the hierarchical quark–lepton masses spanning from m_t to m_ν is among the deep mysteries in particle physics. Can all or almost all of these scales have a *common origin* despite their diversity? To be specific, can they all be related to perhaps just one input scale – e.g. M_{Pl} – and one dimensionless parameter – e.g. a gauge coupling? This idea, if successful, would constitute a unification of scales, which is fundamentally as important as the unifications of diverse particles and of their forces [1].

If quarks, leptons and Higgs are elementary, with or without (the attractive) idea of a radiative origin of inter-family hierarchies [2], one inevitably ends up introducing – in the context of the standard model and its extensions – more than a dozen of parameters associated with the Higgs sector just to accommodate the fermion masses and mixings. The intermediate scale (in the context of SUSY breaking) and/or the large ratio (M_{Pl}/m_W) are addition inputs. One may hope that an underlying economical theory – for example, a superstring theory [3] – would lead to just the desired choice of parameters at an effective level. While such a hope may be entertained, it seems to be a heavy burden nevertheless since no convincing evidence has yet emerged to support it. To add to this, the problems of consistent supersymmetry breaking and the origin of $(M_{Pl}/m_W) \sim 10^{+17}$ are still unresolved within superstring theories – with elementary quarks and leptons.

The purpose of this paper is to present an alternative picture which is manifestly economical in its fundamental parameters and building blocks and seems promising to address the issues raised above. It arises as a *variant* within a class of locally supersymmetric composite models which are viable and predictive [4,5] [#1].

Consistent with our goal of economy, we attempt

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For footnote see next page.

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here to provide a broad scenario for deriving all scales listed above by introducing only *two fundamental parameters*, i.e. $M_{Pl}=10^{19}$ GeV and a gauge coupling $\bar{\alpha}_M \equiv \bar{g}_M^2/4\pi$, associated with an asymptotically free locally supersymmetric preonic "metacolor" force. We choose $\bar{\alpha}_M$ to have a "natural" value ~ 0.1 (say) at $M_{Pl}/10$, such that it grows to become of order unity at a scale $\Lambda_M \sim 10^{11}$ GeV. This in turn is identified with the intermediate scale M_I. The first and one of the biggest steps in the hierarchical ladder – i.e. $(M_{Pl}/M_I) \sim 10^8$ – thus arises naturally due to the slow logarithmic variation of the running coupling $\bar{\alpha}_M$.

In attempting to realize a unification of scales, we propose (as a variant to refs. [4,5] [#1]) that *not only SUSY breaking but also the binding of all families* (e, μ *and* τ), *electroweak symmetry breaking and even quark–lepton masses and CP violation arise dynamically and entirely at this heavy scale* Λ_M *and that there is no other preonic force other than metacolor.* The large hierarchies between $\Lambda_M \sim 10^{11}$ GeV versus $\delta m_s \sim 1$ TeV and that between Λ_M versus m_W are attributed [6] to an inhibition in SUSY breaking which must vanish, owing to the index theorem [7], in the limit of global SUSY (i.e. $M_{Pl} \to \infty$). Following the arguments of ref. [6], we obtain $\delta m_S \sim \Lambda_M(\Lambda_M/M_{Pl}) = M_{Pl}(\Lambda_M/M_{Pl})^2 \sim 1$ TeV and $m_W \sim m_Z \sim \frac{1}{10}\Lambda_M(\Lambda_M/M_{Pl}) \sim 100$ GeV, which constitute the second and third steps in the hierarchical ladder.

Fermion masses need new considerations beyond those of ref. [6]. It was noted in ref. [6] that, due to constraints on SUSY breaking, the mass parameters connecting composite chiral quarks and leptons – i.e. $m^{(0)}(q_L \leftrightarrow q_R)$ – are not just small compared to Λ_M, but they are too small: $m_q^{(0)} \lesssim m_W(\Lambda_M/M_{Pl}) \lesssim 1$ MeV. (This is one reason why a second hypercolor force with a scale $\Lambda_H \sim 1$ TeV was introduced in refs. [4,5] to break the electroweak symmetry and generate quark–lepton masses as large as of order m_W). One main point of this letter, which provides the basis for the variant model, is the observation that there exists within these SUSY composite models an attractive – but hitherto unutilized – *see-saw mechanism* for the generation of quark–lepton masses of the desired magnitude. This comes about as follows. The SUSY composite models produce not only composite chiral families $q_{L,R}$ but also *vector-like families* $Q_{L,R}$ and $Q'_{L,R}$, which couple vectorially to W_L and W_R respectively. We show that these vector-families acquire $SU(2)_L$-preserving flavor–color independent masses $m_Q = m_{Q'} \sim \Lambda_M(\Lambda_M/M_{Pl}) \sim 1$ TeV and that the chiral fermions $q_{L,R}$ can mix appreciably with these vector families and, thereby, acquire masses of the desired magnitude. Through the see-saw mechanism, the top quark acquires a mass $m_t \sim 2\,(\frac{1}{5}-\frac{1}{6})^2 m_Q \simeq 80$–$60$ GeV, where the factor $(\frac{1}{5}-\frac{1}{6})$, we argue, arise simply because the metacolor force is expected to be more attractive between particles in the adjoint than those in the fundamental representation.

Family replication is attributed to varying internal composition of the composites. The mixing of a chiral family with a given vector family depends upon the compositions of these two families, and, therefore, so does its mass. In this note, I will indicate only certain possibilities in this regard and remark how inter-family splittings $(m_\tau > m_\mu > m_e)$ could well arise due to such varying internal structures.

The composite right-handed neutrinos acquire heavy Majorana masses of order Λ_M, while the left-handed neutrinos becomes extra light due to a double see-saw mechanism which yields $m(\nu_L^i) \sim \eta_i^2 \Lambda_M \times (\Lambda_M/M_{Pl})^2 \sim \eta_i^2$ (10^4 eV). The factor η_i is explained later.

In this way, the variant model ends up explaining how a cascading of scales leading to large hierarchies – in particular the large ratios (M_{Pl}/M_I), $(M_{Pl}/\delta m_S)$, (M_{Pl}/m_W), (M_{Pl}/m_t) and even (M_{Pl}/m_ν) – can arise naturally, in the sense of Dirac and 't Hooft, without introducing any new parameter – large or small – beyond the two already mentioned – i.e. M_{Pl} and $\bar{\alpha}_M$.

The mixing of chiral and vector-like families provides an attractive new source of spontaneous *CP* violation, through the coupling of W_L's to right-handed currents of known quarks, which is relevant even when W_R's are superheavy [#2].

[#1] Although fairly economical, the models presented in these papers (refs. [4,5]) introduce two preonic gauge forces – metacolor and hypercolor – one to break SUSY and bind the e and the μ families at a high scale $\Lambda_M \sim 10^{11}$–10^{13} GeV and the other to break the electroweak symmetry and bind the replicated τ and τ′ families at a low scale $\Lambda_H \sim 1$ TeV. See, however, the appendix of ref. [5] for a brief mention of possible variants.

[#2] For preliminary remarks along these lines, see ref. [8].

2. The one-scale model

To develop the variant based on the one-scale idea, it is useful to recall a few salient features of the dynamics [6] of preon models of the type proposed in refs. [4,5]. These models introduce an asymptotically free "metacolor" gauge force based on a vectorial gauge symmetry $G_M = SU(N)$ (for example) possessing $N=1$ local supersymmetry. The metacolor gauge multiplet $V=(v_\mu, \lambda_L, D)$ couples to a set of n_p massless ($m_o=0$) chiral superfields $\Phi_+^{a,\beta} = (\varphi_L, \psi_L, F_L)^{a,\beta}$ in representat[illegible] N and an equal number $[\Phi_-^{a,\beta}]^\dagger = [(\varphi_R, \psi_R, F_R)^{a,\beta}]^\dagger$ in representation N^* of $SU(N)$; β runs over metacolor and a over preon "flavor" indices including ordinary flavors and colors. Thus $\beta = 1, 2, ..., N$, while $a = 1, 2, ..., n_p$, with $n_p = n_f + n_c$, where n_f and n_c denote the numbers of basic flavors and colors respectively. Minimally, $n_f = 2$ for (u, d) flavors and $n_c = 4$ for the four colors (r, y, b, ℓ) including leptonic color [1], and thus, $n_p = 6$ [9].

The metacolor gauge force is thus invariant under $SU(n_p)_L \times SU(n_p)_R \times U(1)_V \times U(1)_X \times G_M$, where $U(1)_V$ denotes preon number and $U(1)_X$ is the non-anomalous R symmetry. An anomaly-free part $\mathcal{G}_0$ of $SU(n_p)_L \times SU(n_p)_R$, e.g. $SU(2)_L \times SU(2)_R \times SU(4)^C_{L+R}$, or just $SU(2)_L \times U(1)_Y \times SU(3)_C$ is gauged [#3] [10].

It was argued in ref. [6] that SUSY breaking – at least for a class of models – is damped by powers of (Λ_M/M_{Pl}) since it must vanish, owing to the index theorem, in the limit of global SUSY (i.e. $M_{Pl} \to \infty$). Since the condensates $\langle \bar\psi_R^a \psi_L^b \rangle$, $\langle \lambda \cdot \lambda \rangle$ and even $\langle \varphi_R^{a*} \varphi_L^b \rangle$ (for $n_p \geq N$) break global SUSY for $m_0 = 0$, these condensates, if they form, must be damped by M_{Pl}:

$$\langle \lambda \cdot \lambda \rangle = a_\lambda \Lambda_M^3 (\Lambda_M/M_{Pl})^{n_\lambda},$$

$$\langle \bar\psi_R^a \psi_L^b \rangle = a_\psi A_{ab} \Lambda_M^3 (\Lambda_M/M_{Pl})^{n_\psi}, \qquad (1)$$

$$\langle \varphi_R^{a*} \varphi_L^b \rangle = a_\varphi M_{ab} \Lambda_M^2 (\Lambda_M/M_{Pl})^{n_\varphi}. \qquad (1 \text{ cont'd})$$

Here $|a_\lambda|$, $|a_\psi|$ and $|a_\varphi|$ are each, independently, zero or order unity. The exponents n_λ, n_ψ and n_φ are expected to be ≥ 1 [#4]. The matrices A_{ab} and M_{ab} operate on flavor–color indices. For SUSY QCD, they need not be unit matrices. For consistency, we assume that $\langle \lambda, \lambda \rangle$ and $\langle \bar\psi\psi \rangle$ form with a_ψ and $a_\lambda = O(1)$ and $n_\lambda = n_\psi = 1$, while $\langle \varphi_R^* \varphi_L \rangle$ is induced "perturbatively" through $\langle \lambda \cdot \lambda \rangle$ and $\langle \bar\psi\psi \rangle$ so that $a_\varphi \propto a_\lambda a_\psi$ and $n_\varphi = n_\lambda + n_\psi$. These induce soft SUSY-breaking mass splittings, characterized, for example, by scalar preon masses:

$$\delta m_s \sim m_\varphi \sim a_\lambda \Lambda_M (\Lambda_M/M_{Pl}) = a_\lambda M_{Pl} (\Lambda_M/M_{Pl})^2 \sim 1\ \text{TeV}. \qquad (2)$$

Now $\langle \bar\psi_R^a \psi_L^b \rangle$ break $SU(2)_L \times U(1)_Y$ when (a, b) span over flavor indices (u, d). These give masses to $W_L^\pm$ and Z^0 [6].

$$(m_{W_L^\pm}, m_{Z^0}) \sim g_2 a_\psi \Lambda_M (\Lambda_M/M_{Pl}) \sim 100\ \text{GeV}. \qquad (3)$$

Here, g_2 denotes the $SU(2)_L$ gauge coupling constant. Typically, we expect $a_\psi \sim (\frac{1}{3}-\frac{1}{6})\ a_\lambda$ (see below): Thus the second and third steps of the hierarchy – i.e. $m_W < \delta m_s \ll \Lambda_M \ll M_{Pl}$ – have emerged.

3. See-saw mechanism for quark–lepton masses

The model produces [4,5] composite quarks and leptons $q_{L,R}$ as spin-$\frac{1}{2}$ components of *chiral* composite superfields: $q_L^{fc*} =$ "$\psi_L^f \varphi_R^{c*} + \varphi_L^f \psi_R^{c*}$" $\subset \Phi_+^{fc*} =$ "$\Phi_+^f \Phi_-^{c*}$"; $q_R^{fc*} =$ "$\psi_R^f \varphi_L^{c*} + \varphi_R^f \psi_L^{c*}$"; $\subset \Phi_-^{fc*} =$ "$\Phi_-^f \Phi_+^{c*}$". The superscripts f and c denote flavor (u, d) and color (r, y, b, ℓ) indices respectively. The quotation marks signify that the composites may, in general, have additional constituents which are neutral under flavor and color leading to new families (see later). The symbol q here denotes quarks as well as leptons. As noted in ref. [6], a quark-mass term like $(\bar q_R q_L + \bar q_L q_R)$ can form provided *both* $\langle \bar\psi_R \psi_L \rangle$ and $\langle \varphi_R^* \varphi_L \rangle$ with appropriate flavor–color indices

[#3] While such a gauging introduces two (or three) gauge coupling parameters, which enter into the standard model, these do not play significant roles in the non-perturbative dynamics that is relevant to this paper. Furthermore, these couplings may well be related to the metacolor coupling through an underlying theory such as that of superstrings, which may lead to preons rather than elementary quarks and leptons (see e.g. ref. [10]).

[#4] Naively, we expect that a single graviton exchange superimposed on metacolor dynamics would suffice to form some of the condensates, which would then be proportional to $\sqrt{\kappa^2} \simeq 1/M_{Pl}$ [6].

are non-vanishing. Thus, $m^{(0)}(q_L \leftrightarrow q_R) \sim a_\psi a_\varphi (\Lambda_M/M_{Pl})^{n_\psi + n_\varphi} < 1$ MeV.

The metacolor force produces [4–6] two other types of "two-body" metacolor-singlet composites given by "$\Phi^f_+ \Phi^{c*}_+$" and "$\Phi^f_- \Phi^{c*}_-$" which define general superfields. Each of these is reducible under SUSY to a sum of positive, negative and vector superfields: $\hat{\Phi}^{fc*} \equiv \Phi^f_+ \Phi^{c*}_+ = \hat{\Phi}^{fc*}_+ + \hat{\Phi}^{fc*}_- + \hat{\Phi}^{fc*}_1$; $\hat{\chi}^{fc*} \equiv \Phi^f_- \Phi^{c*}_- = \hat{\chi}^{fc*}_+ + \hat{\chi}^{fc*}_- + \hat{\chi}^{fc*}_1$. By left–right symmetry and SUSY of the metacolor force, $\hat{\Phi}_+$ and $\hat{\Phi}_-$ must form together, so also $\hat{\chi}_+$ and $\hat{\chi}_-$. The vector superfields $\hat{\Phi}_1$ and $\hat{\chi}_1$ may or may not form. We will ignore these for the moment. Let us denote the fermionic components of $\hat{\Phi}^{fc*}_+$ by Q_L and Q_R and those of $\hat{\chi}^{fc*}_+$ and $\hat{\chi}^{fc*}_-$ by Q'_L and Q'_R respectively. these have the flavor–color quantum numbers defined by their constituents:

$$Q_L \sim \text{“}\psi^f_L \varphi^{c*}_L\text{”}, \quad Q_R \sim \text{“}\varphi^f_L \psi^{c*}_L\text{”},$$

$$Q'_L \sim \text{“}\varphi^f_R \psi^{c*}_R\text{”}, \quad Q'_R \sim \text{“}\psi^f_R \varphi^{c*}_R\text{”}. \tag{4}$$

Thus Q_L and Q_R have identical flavor–color quantum numbers but opposite helicities. They couple vectorially to W_L's. Likewise, Q'_L and Q'_R couple vectorially to W_R's. Hence the name "vector" families.

It is easy to verify that the condensate $\langle\lambda\lambda\rangle$ induces the transitions $Q_L \leftrightarrow Q_R$ and $Q'_L \leftrightarrow Q'_R$, though not $Q_L \leftrightarrow Q'_R$, $Q'_L \leftrightarrow Q_R$ and $Q_L \leftrightarrow q_R$, etc. Since $\langle\lambda\lambda\rangle$ is blind to flavor and SU(4)-color, the vector families Q_L and Q_R, (and likewise Q'_L and Q'_R) combine to obtain *flavor–color independent* Dirac masses: $m_Q = m_{Q'} \sim a_\lambda \Lambda_M (\Lambda_M/M_{Pl}) \sim 1$ TeV. *Within each vector family all the fermions are, thus, degenerate barring electroweak radiative corrections* ($\sim(\alpha/\pi)$1 TeV–10 GeV).

The new observation is that the chiral quarks and leptons, $q_{L,R}$ can mix with the vector families utilizing matter fermion condensates $\langle\bar{\psi}^a_R \psi^a_L\rangle$. Define $\Lambda_M^{-2}\langle\bar{\psi}^u_R \psi^u_L\rangle \equiv S^u_\psi$, $\Lambda_M^{-2}\langle\bar{\psi}^d_R \psi^d_L\rangle \equiv S^d_\psi$, $\Lambda_M^{-2}\langle\bar{\psi}^i_R \psi^i_L\rangle_{i=r,y,b} \equiv S^r_\psi$ and $\Lambda_M^{-2}\langle\bar{\psi}^\ell_R \psi^\ell_L\rangle \equiv S^\ell_\psi$. In SUSY QCD, the four condensates $S^{u,d,r,\ell}_\psi$ may be comparable but not necessarily equal to each other. Barring electroweak and QCD radiative corrections, and allowing for generation structure for the chiral and vector-like families whose possible origin will be mentioned, the mass-matrices for the (u, d, ℓ)-sectors have the following block form:

$$M^a = \begin{array}{c} \\ \bar{q}^{a,m}_R \\ \bar{Q}^{a,p}_R \\ \bar{Q}'^{a,p}_R \end{array} \begin{array}{c} \begin{array}{ccc} q^{a,j}_L & Q^{a,k}_L & Q'^{a,k}_L \end{array} \\ \begin{pmatrix} 0 & X\cdot S^a_\psi & X\cdot S^r_\psi \\ X^\dagger\cdot(S^r_\psi) & A\cdot S_\lambda & 0 \\ X^\dagger(S^a_\psi) & 0 & A\cdot S_\lambda \end{pmatrix}_{a=u,d} \end{array} . \tag{5}$$

In arriving at this block form, left–right symmetry of the binding force, which interchanges $q^i_L \leftrightarrow q^i_R$, $Q^k_L \leftrightarrow Q'^k_R$ and $Q^k_R \leftrightarrow Q'^k_L$, is assumed. The mass-matrix M^ℓ for charged leptons is obtained by replacing S^r_ψ by S^ℓ_ψ in M^d and that for the Dirac masses of the neutrinos (M^ν_D) by replacing S^d_ψ by S^u_ψ in M^ℓ. The superscripts (j, m) and (k, p) denote generation-labels for the chiral and the vector families respectively; X and A are matrices in these generation-spaces with entries which vary between zero and order one.

Note that the block form exhibited above is already rather non-trivial. It asserts that (a) the same block form and the same matrices AS_λ and X hold for all four sectors M^u, M^d, m^ℓ and M^ν_D, and that (b) the entries in the blocks which connect $q_L \to q_R$, $Q_L \to Q'_R$ and $Q'_L \to Q_R$ are *naturally* very small (< 1 MeV), which are denoted by zeroes. *These features following from the constraints of supersymmetry and compositeness simplify the mass-matrix and reduce even the effective parameters considerably.*

Observe that $\langle\lambda\cdot\lambda\rangle$ is expected to be effectively larger than $\langle\bar{\psi}\psi\rangle$ because the gauginos are in the adjoint while the matter fermions are in the fundamental representation of the metacolor group. On this basis, one can argue that for a metacolor symmetry like SU(4), $[S^m_\psi] \simeq (\frac{1}{3}-\frac{1}{6})S_\lambda$. Here, the superscript m signifies maximum of $[S^i_\psi]_{i=u,d,r,\ell}$. *This together with a zero in the upper left block in (5) imply that the model provides a natural see-saw mechanism for the origin of quark–lepton masses.* Denoting the Dirac mass matrices for the known quarks, leptons and neutrinos by $\mathcal{M}^a_q$, $\mathcal{M}^\ell$ and $\mathcal{M}^\nu_D$ respectively, we have

$$\mathcal{M}^a_q \simeq -(X^\dagger A^{-1} X)[2(S^r_\psi \cdot S^a_\psi)/S_\lambda]_{a=u,d}, \tag{6}$$

to obtain $\mathcal{M}^\ell$, replace S^r_ψ by S^ℓ_ψ in $\mathcal{M}^d_q$ and to obtain $\mathcal{M}^\nu_D$, replace S^d_ψ by S^u_ψ in $\mathcal{M}^\ell$.

Very briefly, family replication would arise in the model by keeping the flavor–color constituents (ψ^f, φ^{c*}) and (φ^f, ψ^{c*}) the same but varying additional constituents which are neutral under flavour and color. Such constituents are needed since massless spin-$\frac{1}{2}$ composites can not, strictly speaking, be formed as *two-body* composites of massless ψ and φ^* (see refs. [4,5]). The additional constituents are provided most naturally by members of the metacolor-gauge multiplet (i.e. v_μ and λ). It will be discussed elsewhere [#5] that one possible scenario in the presence of SUSY-breaking is this: $\psi^c_L \varphi^{c*}_R v$, $\psi^c_L \varphi^{cc2}_R vv$ and $\varphi^c_L \varphi^{c*}_R \bar{\lambda}$ define three left-handed chiral families q^τ_L, q^μ_L and q^e_L respectively. Likewise, there is a replication in the vector-families, whose number need not equal that of the chiral families. For consistency, with renormalization group analysis [#5], we assume that the spectrum of "massless" composites saturate with three chiral, two (or one) vectorial Q-like plus two (or one) vectorial Q′-like families. Given that $\langle\bar{\psi}\psi\rangle \gg \langle\varphi^*\varphi\rangle$, one can argue plausibly that q^τ mixes more efficiently than q^μ and q^μ more efficiently than q^e with the vector families. With rather modes and plausible differences – by factor of $\frac{1}{6}-\frac{1}{30}$ (say) – between these family-dependent mixing elements (i.e. the elements in X) – one can obtain, owing to see-saw, rather large inter-family mass-hierarchies: $\hat{m}_\tau : \hat{m}_\mu : \hat{m}_e \sim 1 : \frac{1}{36} : \frac{1}{1000}$ (say). We are thus led to interpret that a structure in X leading to inter-family hierarchy ($m_e < m_\mu < m_\tau$) is a consequence of *SUSY breaking and differing internal compositions* of the families [#6]. Details of this discussion are beyond the scope of this paper and will be covered in a longer paper [#5]. Regardless of the mechanism for replication, however, a few important features may be deduced from the general form of the mass-matrices derived above, which are among the main points of this letter:

(i) Given that the compositions of the would-be heaviest chiral and vector families are similar (e.g. $\psi\varphi^* v$, see above), one can argue [#5] that the leading eigenvalue of $X^\dagger A^{-1} X$ is comparable to that of A. It then follows that the heaviest chiral fermion (e.g. top) would have a mass $m_t \simeq 2\ (S^m_\psi/S_\lambda)^2 M_Q \sim 2(\frac{1}{3}-\frac{1}{6})^2\ (1\ \text{TeV}) \simeq (80\text{–}60)$ GeV [#7]. Thus arises naturally a fourth step in the hierarchy: $M_{Pl} \gg M_1 = \Lambda_M \gg \delta m_s > m_{W,Z} \gtrsim m_t$.

(ii) Since $X^\dagger A^{-1} X$ is common to all three sectors (u, d, ℓ), the relative masses for up:down:lepton must be the same for all three families, barring radiative corrections: $m_t : m_b : m_\tau = m_c : "m_s" : "m_\mu" = "m_u" : "m_d" : m_e$". Quotation marks signify that radiative effects, which may be as much as tens of MeV, are important for the light fermions. Without a reliable evaluation of these corrections it is difficult to judge the validity of these relations. The relation $(m_t/m_b) \simeq (m_c/m_s)$ should still hold reasonably well (say to 20%) since corrections to m_t, m_b and m_c should be negligible, and that to m_s should be moderate ($\lesssim 20\%$).

(iii) Intra-family up-down splittings (i.e. $m_t > m_b$, $m_c > m_s$, etc.) may be attributed to $S^u_\psi > S^d_\psi$, or equivalently to larger effective coupling of $\langle\bar{\psi}^u\psi^u\rangle$ to $\bar{q}^U Q^u$ than that of $\langle\bar{\psi}^d\psi^d\rangle$ to $\bar{q}^d Q^d$ (see ref. [11] for an analogous result), which may have its origin in the isospin breaking condensates of order Λ_M such as Δ_R which transforms as (1, 3, 10) under $SU(2)_L \times SU(2)_R \times SU(4)^c$. Quark–lepton mass-splittings within a family (i.e. $m_b > m_\tau$, $m_s > m_\mu$, etc.) may arise mostly due to QCD radiative effects and in part due to $S^d_\psi \neq S^\ell_\psi$.

(iv) At the tree level, $\mathcal{M}^u_q$, $\mathcal{M}^d_q$ and $\mathcal{M}^\ell_q$ are nearly

[#5] Since our understanding of the dynamics of SUSY gauge theories is still poor, we must content ourselves at present with possible alternative scenarios for saturation and compositions of light or massless spin-$\frac{1}{2}$ composites. For instance, with the metacolor symmetry being SU(4), it is perfectly possible that in addition to mesonic type composites "$\psi\varphi^*$", there are "baryonic" type composites of the type $\psi^c_L \varphi^c_L \varphi^c_L \varphi^c_L$, etc., which transform as $(2_L, 4^c_c)$ under $SU(2)_L \times SU(4)^c_{L+R}$. These may provide a family instead of or in addition to $\psi\varphi^* vv$. A longer paper addressing to alternative scenarios for family compositions and their phenomenological consequences and consistency with renormalization group analysis will be presented in collaboration with B. Balakrishna and H. Stremnitzer.

[#6] With three chiral and two vector-like families, one would still end up having (in the context of our approximation) $\hat{m}_\tau > \hat{m}_\mu > \hat{m}_e = 0$. If e and τ families have the same strong interaction quantum numbers, the e family can still get a mass from the τ family via a two-metagluon loop, which (in a specific model) gives $\hat{m}_e \sim O(\alpha^4_M)\hat{m}_\tau$. For a relatively *short-range* two-gluon process, one may reasonably have $\alpha_M \sim 0.2$ and thus $\hat{m}_e \sim 10^{-3}\ \hat{m}_\tau$. I thank B. Balakrishna for discussions on this point. These remarks will be elaborated in a forthcoming paper (see footnote 5).

[#7] Admittedly, an understanding of the gross pattern involving the hierarchical ordering of masses from M_{Pl} to m_ν, rather than that of the precise value of any one mass, is the main objective of this paper. Notwithstanding the facts that we expect $(S^m_\psi/S_\lambda) < 1$ and we can argue plausibly that $\frac{1}{10} < (S^m_\psi/S_\lambda) < \frac{1}{3}-\frac{1}{6}$, we an in fact determine (S^m_ψ/S_λ) by using the ratio $(m_W/\delta m_s)$ or (m_W/m_Q) once SUSY partners and/or vector families are discovered. This will enable us to predict m_t more reliably. Alternatively, given m_t, we can predict δm_s and m_Q more reliably.

proportional to each other. This feature is altered, however, by electroweak radiative effects on q–Q mixings, which, in turn, lead to non-vanishing Cabibbo angles and may lead to up–down reversal in the light e family. This needs further study.

(v) Denoting the neutrino-like members of the vector families $Q_{L,R}$ and $Q'_{L,R}$ by $\mathcal{N}_{L,R}$ and $\mathcal{N}'_{L,R}$ respectively, we note that a condensate Δ_R mentioned above gives a heavy majorana mass $\sim \Lambda_M$ to ν_R and to *both* $\mathcal{N}'_L$ and $\mathcal{N}'_R$, which couple to W_R. In the presence of a small Dirac mass due to $\langle\lambda\cdot\lambda\rangle \neq 0$ (i.e. $\mathcal{M}_D^{N'} \sim (\frac{1}{10}-1)\Lambda_M(\Lambda_M/M_{Pl}) \ll \Lambda_M$), both $\mathcal{N}'_L$ and $\mathcal{N}'_R$ remain superheavy with masses of order Λ_M. On the other hand, $\mathcal{N}_{L,R}$, which couple to W_L, acquire only Dirac mass $\mathcal{M}_D^{1} \sim (\frac{1}{10}-1)\Lambda_M(\Lambda_M/M_{Pl}) \sim (1$ to few$)\times 100$ GeV. As a result, Z^0 and $W^{\pm}$ cannot decay to $\mathcal{N}'_{L,R}$ in any case and they may not be able to decay to $\mathcal{N}_{L,R}$ either. The off-diagonal mixing of $\mathcal{N}_{L,R}$ and $\mathcal{N}'_{L,R}$ with the chiral neutrinos $\nu_{L,R}$ (see eq. (5)) through $\langle\bar{\psi}\psi\rangle \neq 0$ induces, via the see-saw mechanism, Dirac masses $(m_D^{\nu}) \sim \eta_i\Lambda_M(\Lambda_M/M_{Pl})$ for the chiral neutrinos, as mentioned before. These, together with a heavy Majorana mass $\sim\Lambda_M$ for the ν_R's, yield (via a second see-saw) masses for the left-handed neutrinos $\mathcal{M}(\nu_L^i) \sim \eta_i^2\Lambda_M(\Lambda_M/M_{Pl})^2 \sim \eta_i^2 M_{Pl}(\Lambda_M/M_{Pl})^3 \sim \eta_i^2(10^4$ eV), where $\eta_i = K_i^2[(2)(\frac{1}{3}-\frac{1}{6})^2]$. Note that the second factor within the square bracket for η_i enters also into the mass of the top quark. The factor K_i depends on the family-dependent q^i–Q^i mixing and could plausibly have a value of nearly $1-\frac{1}{2}$ for ν_τ, $\frac{1}{3}-\frac{1}{10}$ for ν_μ and $\frac{1}{30}-\frac{1}{60}$ for ν_e. As a result, we may expect $m(\nu_\tau) \approx (40–2.5)$ eV, $m(\nu_\mu) \approx 4(10^{-2}-10^{-3})$ eV and $m(\nu_e) \approx (4-\frac{1}{4})\times 10^{-5}$ eV. Note the natural emergence of ultralight neutrinos owing to the double see-saw mechanism. Bulk of the damping in neutrino masses is due to the factor $(\Lambda_M/M_{Pl})^3 \sim 10^{-24}$, while a significant suppression is due to the family-dependent factor $K_i^4 \sim 1-10^{-6}$.

(vi) *CP*. Since physical quarks $q_{L,R}$ are now mixtures of chiral ($q^0_{L,R}$) and vectorlike quarks ($Q^0_{L,R}$, $Q'^0_{L,R}$) – i.e. symbolically, $q_R = q_R^{(0)}\cos\alpha + Q_R^{(0)}\sin\alpha$ (ignoring Q', for simplicity) – even q_R's couple to W_L's through their $Q_R^{(0)}$-component with a strength $\propto g\sin^2\alpha$. From constraints of mass-matrix, which includes *family-dependent* q – Q mixing through the matrix X, it is possible to argue plausibly [12] that $\sin\alpha_i \approx (S_\psi/S_\lambda)\xi_i \sim (\frac{1}{3}-\frac{1}{6})\xi_i$ where $\xi_i \sim \frac{1}{2}$ for the μ and $\frac{1}{10}$ for the e family. First of all, such admixture of V + A coupling is fully compatible with known data. Second, such an admixture permits spontaneous *CP* violation through phases in condensates such as $\langle\bar{\psi}'\psi'\rangle$. *This is analogous to CP violation in L–R symmetric models* [12], *except that the present mechanism survives even when* W_R's *are superheavy* (≫ 100 *TeV, say*). This was noted in ref. [8]. The value of $|\eta_{+-}|$ and $|\eta_{00}|$ arising through the standard box graph for dd → ss including V + A coupling of $W_L^{\pm}$ at two vertices is of order $(\sin^2\alpha_e) \times (\sin^2\alpha_\mu)(\sin\delta)\cdot(Y \approx 10^3) \sim (\frac{1}{60})^2\cdot(\frac{1}{15})^2(\sin\delta)\cdot(10^3) \sim \frac{1}{600}$, as desired, where the phase parameter $\sin\delta \sim O(1)$ and Y includes the Beall–Bander–Soni (430) and QCD (≈ 2.5) enhancement factors.

This mechanism of *CP* violation would give a sizeable electric dipole moment of the neutron $d_n \sim (\sin^2\alpha_e)\sin\delta(10^{-22}\, e\,\text{cm}) \sim (10^{-25}-10^{-26})\, e\,\text{cm}$ in contrast to the superweak and the KM models.

(vii) q–Q mixing necessarily induces flavor mixing processes such as K^0–$\bar{K}^0$, B^0–$\bar{B}^0$ and $K_L \to \bar{\mu}e$. These are under study and will be reported in a longer paper [#5].

The full consistency of the present model with (a) renormalization group analysis for the runing coupling constants below the scale Λ_M, which is tied to the spectrum of low-mass composites; (b) flavor-changing neutral current processes and (c) detailed structure of quark–lepton masses and KM mixings remain to be shown [#5]. An unambiguous prescription for saturation and composition of light-composites is still missing due to our poor understanding of the dynamics of SUSY QCD. The non-perturbative aspects of the model and its possible origin within superstring theories need to be explored. A locally supersymmetric preonic force in the observable sector may well be necessary for consistent breaking of supersymmetry within superstring theories.

Meanwhile, the model has produced an attractive broad picture for a common origin of the hierarchical scales from M_{Pl} to m_ν in terms of just two input parameters: M_{Pl} and $\bar{\alpha}_M$. Nowhere a large or a small dimensionless parameter (even effective ones [#8]) was fed in, in accord with Dirac and 't Hooft's philosophy of naturalness.

For footnote see next page.

The model introduces a novel concept that electroweak symmetry breaking including quark–lepton masses may be dynamical like technicolor; yet, unlike technicolor, they may have their origins in a gauge force with a superheavy scale Λ_M far above 1 TeV. Local supersymmetry and compositeness of quarks and leptons play *crucial roles* in this new approach. Two key features of the model are the *natural origins* of the vector-like families and the consequent see-saw mechanism for the generations of quark–lepton masses and *CP* violation. Presence of vector-like families with almost degenerate (within ≈ 10 GeV) up-down and quark–lepton members within such a family having a mass in the range of 100–1000 GeV, is a compelling prediction of the model. This should be of interest to LEP II, LHC, SSC and especially future e^-e^+ machines with $E_{CM} \approx 1$ TeV. The model, of course, requires the presence of SUSY partners of quarks, leptons and gluons with masses of the order of a few hundred GeV to 1 TeV, as in some alternative approaches. Finally, although the Higgs mechanism is dynamical, we expect light "left-over" Higgs-like composite scalars in the mass range 100 GeV–1 TeV, whose masses are protected by SUSY and which play the roles of providing good high energy behavior, consistent with unitarity for processes such as $WW \to WW$, $q\bar{q} \to WW$, etc.

[#8] The number of effective parameters, all of which are in principle calculable are also (relatively speaking) few. These include S_λ and $S_\psi^{u,d,e,l}$ of which $[S'_\psi]_{max}/S_\lambda$ is determined, on plausible theoretical grounds, to be $\approx \frac{1}{3}-\frac{1}{6}$ and conservatively to lie between $\frac{1}{2}$ and $\frac{1}{10}$ (see footnote 7). In addition, it turns out that the six elements in X and four elements in A (with three chiral plus two vector families) may be described with some assumption about the compositions of the families, in terms of only two or three effective parameters. This amounts to some seven or eight in-principle calculable parameters, all of which have "natural" values between $\frac{1}{10}$ and 1. This is to be contrasted with some 19 or 20 arbitrary parameters for the standard model and its extensions, which are not calculable.

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A simple reason based on supersymmetry for replication of chiral families

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In the context of the miminal flavon–chromon preon model, we show that supersymmetry, because of fermion–boson pairing in its field content, provides a rather simple reason for replication of composite quark–lepton families. At the level of minimum number of core constituents, which turns out to be three, it also provides a good reason why one may expect to have just three light chiral families. One crucial prediction is that there must exist complete vector-like families with mass of order 1 TeV for quark-like and few hundred GeV for lepton-like members. This can be tested at SSC, LHC and future high energy e^-e^+ machines.

Unravelling the reason for family replication and an understanding of the origin of the hierarchial mass scales (spanning from M_{Planck} to m_ν) are among the major challenges confronting particle physics at present. The superstring theories together with the standard presumption that they yield elementary quarks and leptons have not shed any clear light on these issues yet [#1]. In a recent paper [1] it was shown that the combination of the idea of local supersymmetry with the idea of compositeness of quarks and leptons seems particularly promising to address these issues – especially the one of hierarchical masses. An economical preon model combining these two ideas was presented which seems capable of generating all the diverse scales in terms of just the Planck mass serving as the unit of scale and one fundamental input parameter: the coupling constant associated with the preon-binding force [#2]. A longer paper elaborating on the issue of hierarchical mass scales (in the context of such a model) is in preparation [3].

The purpose of this note is to focus primarily on the issue of family replication in such a model. We show that supersymmetry, because of fermion ↔ boson pairing in its field content, provides a rather simple reason for replication of quark–lepton families. As in ref. [1], we start with a preonic theory possessing $N=1$ local supersymmetry and assume that the preonic superfields possess a metacolor degree of freedom which is gauged. The metacolor force is assumed to be asymptotically free. It becomes strong at a scale $\Lambda_M \gg 1$ TeV and binds preons to make composites at that scale which are singlest under metacolor. We use the very old idea (see footnote 7 in

[#1] Although they provide the intriguing possibility that the number of chiral families is associated with the topology of the compact manifold, their ability to make contact with the low-energy world and thereby to account unambiguously for the origin of hierarchial mass scales as well as SUSY breaking is far from clear.

[#2] Because of these promising features, it seems prudent to us to keep an open mind at the represent stage as to whether the "right" superstring theory yields massless preons rather than elementary quarks and leptons near the Planck scale. This point of view has been expressed elsewhere [2].

ref. [4] [#3] that the preons carry, in addition to metacolor, either the flavor *or* the color attribute (including lepton color) but not both. Since quarks and leptons carry both flavor and color they must be composites of at least two preons – one carrying flavor and the other carrying color.

Following our desire that we must not put in family replication by hand (e.g. by proliferating the flavor attributes) we start with the minimum number of preonic attributes which suffice to provide the ingredients (i.e. the quantum numbers) of just one chiral quark-lepton family $q_{L,R}$ and ask whether the system can provide a compelling reason for replication. We, therefore, presume (as in ref. [1]) that there are just two massless primordial flavor-carrying preons (x, y) called "flavons" and four massless color carrying preons (r, y, b, l) called "chromons" each of which comes with both left and right chirality. The x and y flavons correspond to up- and down-type flavors in each family. For this minimal model there are then six positive and six negative chiral superfields each of which transforms as a fundamental representation of the metacolor group which for concreteness is taken to be SU(N). Their representation content under the familiar flavor–color gauge symmetry $SU(2)_L \times SU(2)_R \times SU(4)^c_{L+R}$ group and the metacolor symmetry $SU(N)_{L+R}$ is given below:

	$SU(2)_L$	$\times SU(2)_R$	$\times SU(4)^c_{L+R}$	$\times SU(N)_{L+R}$
$\Phi^{f,\alpha}_{+} = (\varphi^f_L, \psi^f_L, F^f_L)^\sigma \sim$	2_L,	1,	1,	N
$\Phi^{f,\alpha}_{-} = (\varphi^f_R, \psi^f_R, F^f_R)^\sigma \sim$	1,	2_R,	1,	N
$\Phi^{c,\alpha}_{+} = (\varphi^c_L, \psi^c_L, F^c_L)^\sigma \sim$	1,	1,	4_c,	N
$\Phi^{c,\alpha}_{-} = (\varphi^c_R, \psi^c_R, F^c_R)^\sigma \sim$	1,	1,	4_c,	N .

(1)

Here f stands for two flavors (x and y) and c for four colors (r, y, b, and l), while σ stands for the metacolor index which runs from 1 to N. The fields $\varphi_{L,R}$ and $\psi_{L,R}$ are the spin-0 and the spin-$\frac{1}{2}$ partners of a given chiral superfield $\Phi_\pm$ and $F_{L,R}$ are the auxiliary components.

We see that the preonic content presented above is indeed the minimum that is needed just to define the anomaly-free chiral $SU(2)_L \times SU(2)_R$ and vectorial $SU(4)^c_{L+R} \times SU(N)_{L+R}$-gauge interactions with the requirement that all the preonic field must be non-trivial under SU(N). In a relative manner, this system is clearly far more economically than an elementary quark-lepton theory with elementary Higgs. In particular, note that there is no repetition of any entity at the preonic level, unlike the case of quark–lepton families.

Is there a natural reason within this minimal system for replication at the composite level? Naively, one might have imagined that such a replication could arise simply through something like radial or orbital excitations as in the case of the atomic and the nuclear composites. But we believe that the analogy with the atomic and the nuclear systems does not apply to the quarks and the leptons because, unlike the case of the former, the masses of quarks and leptons and in particular their mass difference are much too small compared to their scale of compositeness – i.e. their inverse size – which by far exceeds 1 TeV ($m_q \ll 1/r_0$). If the μ and the τ were merely radial and/or orbital excitations of the electron in any sense at all, one would have expected their mass difference to be of the order of the compositeness scale >1 TeV which is not the case. For this reason, it seems far more plausible to us that all three families (e, μ and τ) are essentially on par as regards the dynamics of their binding and one is not be regarded as some form of radial or orbital excitation of the other.

There must then be a reason, based primarily on symmetry, which protects their masses compared to their scale of compositeness and, furthermore, it must be essentially symmetry (together perhaps with some dynamics) that provides the reason for at least the major step in the inter-family mass hierarchy (i.e. $\bar{m}_e \ll \bar{m}_\mu \ll \bar{m}_\tau$).

The reason for the protection of composite quark–lepton masses (i.e. $m_q \ll 1/r_0$) has been shown to exist [6] because of supersymmetry and the index theorem [#4] whereas that for the inter-family hierarchy has been related in part to SUSY and in part to the chiral quantum numbers of the quark–lepton families which leads to a sec-

[#3] For a supersymmetric extension of this idea with global SUSY, see ref. [5].

[#4] The argument here is based on the index theorem which inhibits dynamical breaking of SUSY and in turn the formation of the chiral symmetry breaking condensate $\langle\bar{\psi}\psi\rangle$ which happens to break SUSY as well.

saw type mass matrix [1,3]. Details on the question of fermion masses may be found in ref. [3].

With these preliminary remarks, we return to the task of the present paper and seek for a simple reason for family replication other than radial, orbital or even quantum-pair excitations. With this in view, we construct the minimum dimensional composite operators consisting of constituent preons which can a priori serve as candidates for "massless" composite chiral quarks and leptons. We show that the demands of supersymmetry and metacolor gauge invariance automically provides a compelling reason for replication of such families.

Construction of composite quark–lepton families. Since the chiral quark–lepton families q_L and q_R should transform as singlets of metacolor and as $(2_L, 1, 4_c^2)$ and $(1, 2_R, 4_c^*)$ respectively under the symmetry $SU(2)_L \times SU(2)_R \times SU(4)^c_{L+R}$, it would seem that at least one such family of quarks and leptons and their superpartners can be made by taking minimally the bilinear superfield combinations $\Phi_-^{c*}\Phi_+^f$ and $\Phi_+^*\Phi_-^f$ which transform irreducibly under SUSY as positive and negative chiral superfields respectively in the limit of zero-splitting between the superspace coordinates of the two constituents. Their spin-$\frac{1}{2}$ components, denoted temporarily by "q_L" and "q_R, are given by

$$``q_L" = (\Phi_-^{c*}\Phi_+^f)|_\theta = \psi_L^f \varphi_R^{c*} + \varphi_L^f \psi_R^{c*}, \quad ``q_R" = (\Phi_+^{c*}\Phi_-^f)|_{\bar\theta} = \psi_R^f \varphi_L^{c*} + \varphi_R^f \psi_L^{c*}. \qquad (2)$$

Note that we are using the symbols "$q_{L,R}$" to denote collectively quarks *and* leptons. The reason for quotation marks will be seen shortly. We noted earlier that quarks and leptons remain essentially massless in the scale of Λ_M [6,1,3] in the class of SUSY gauge theories under consideration (see footnote 4). Now, it has been noted elsewhere [7], however, that massless spin-$\frac{1}{2}$ objects cannot, strictly speaking, be formed as composites of two-body systems consisting of massless spin-$\frac{1}{2}$ (i.e. ψ or ψ^*) and spin-0 (i.e. φ or φ^*) constituents. This is because the residue for the massless quark-pole appearing in the scattering of $\psi_L + \varphi^* \to \psi_L + \varphi^*$ which involves the on-shell vertex $\psi_L + \varphi^* \to q_L$ vanishes [#5] as $(m_q/\Lambda_M) \to 0$. If, on the other hand, a quark is regarded as an n-body composite with $n \geq 3$ – e.g. of the type $\psi\varphi^*v$ (say), where v is a metagluon – no such difficulty as regards the vanishing of the vertex for the transition $q_L \to \psi_L + \varphi^* + v$ arises. This is the reason why quotation marks are put around q_L and q_R in eq. (2).

Adding a metagluon to the "valence" preons as an essential constituent (not just as part of a cloud) does not, of course, alter the flavor–color quantum numbers of the composites. Furthermore, metacolor singlet (gauge invariant) combinations of $\psi_L^f\varphi_R^{c*}v$ and $\varphi_L^f\psi_R^{c*}v$ can, of course, be formed since v is in the adjoint and ψ and φ^* are in the $\underset{\sim}{N}$ and $\underset{\sim}{N}^*$ representations of SU(N). In a SUSY theory, once we permit the metagluon as a constituent we must, of course, permit the metagaugino (λ or $\bar\lambda$) in appropriate combinations like $\varphi_L^f\varphi_R^{c*}\bar\lambda$ and $\psi_L^f\psi_R^{c*}\lambda$ etc. as a consistent as well. While these a priori define different combinations, it remains to be seen whether supersymmetry and the demand of gauge invariance groups them into just one or more than one combination. It is the latter possibility which would imply replication.

We, therefore, proceed to construct all possible lowest dimensional composite superfields (which will turn out to have dimensions four) consisting of a Φ_+^f, a Φ_-^{c*} (or Φ_+^{c*}) and a member of the metagluon vector multiplet $V = (v_\mu, \lambda$ or $\bar\lambda$ and $D)$ in metacolor gauge-invariant combinations, subject to the constraints of SUSY [#6]. These will serve as candidates for quark–lepton type composites transforming as $(2_L, 1, 4_c^*)$ under $SU(2)_L \times SU(2)_R \times SU(4)^c_{L+R}$. Because they are $SU(2)_L$-doublets, we will call these, regardless of their chirality, $SU(2)_L$-families. The $SU(2)_R$-families transforming as $(1, 2_R, 4_c^*)$ can be formed in an analogous manner be replacing $\Phi_\mp^{f,c}$ by $\Phi_\pm^{f,c}$.

Now, keeping the requirements of gauge invariance in mind, the gauge multiplet for $SU(2)_L$- families can be

[#5] We are aware that this argument may appear to be naive because of confinement of preons. But it seems to us that evasions of the forbiddeness of the $\psi_L + \varphi^* \to q_L$-vertex due to confinement amounts to the necessity of additional consistent(s) like the metagluons.

[#6] Sometimes, gauge invariance would, of course, force two-gauge-field combinations accompanying one gauge field as in $v_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu + ig[v_\mu, v_\nu]$.

introduced, in general, only in the following ways: (i) through a required factor of e^V; (ii) through the gauge-covariant spinorial chiral superfield W_α; (iii) through the gauge-covariant derivatives (∇_α, $\bar\nabla_{\dot\alpha}$, ∇_μ) which are defined by [#7] (see ref. [8] for notations and definitions)

$$e^V = 1 + V + \tfrac{1}{2}V^2, \quad [V]_{WZ} = -(\theta\sigma^\mu\bar\theta)v_\mu + i\theta\theta\bar\theta\bar\lambda - i\bar\theta\bar\theta\theta\lambda + \tfrac{1}{2}\theta\theta\bar\theta\bar\theta\,D\,, \tag{3}$$

$$\nabla_\alpha \equiv e^{-V}D_\alpha e^V = D_\alpha + (e^{-V}D_\alpha e^V) \equiv D_\alpha + \Gamma_\alpha, \quad \bar\nabla_{\dot\alpha} \equiv \bar D_{\dot\alpha}\,, \tag{4}$$

$$\nabla_\mu \equiv \frac{1}{4i}\{\bar D, \bar\sigma^\mu \nabla\} = \partial_\mu + \frac{1}{4i}(\bar D\bar\sigma^\mu\Gamma)\,, \tag{5}$$

$$W_\alpha = -\tfrac{1}{4}\bar D\bar D(e^{-V}D_\alpha e^V) \equiv -\tfrac{1}{2}i\sigma^\mu_{\alpha\dot\alpha}[\bar\nabla^{\dot\alpha}, \nabla_\mu]\,. \tag{6}$$

Since the preonic superfields $\Phi_\pm$ are in the fundamental representation N of SU(N) and the gauge fields are in the adjoint, the metacolor gauge transformation properties of the fields and of the covariant derivatives are given by

$$\begin{aligned} &\Phi_+ \to e^{-i\Lambda}\Phi_+, \quad \Phi_- \to e^{-i\Lambda^\dagger}\Phi_-, \quad e^V \to e^{-i\Lambda^\dagger}(e^V)e^{i\Lambda}, \quad W_\alpha \to e^{-i\Lambda}W_\alpha e^{i\Lambda}\,, \\ &\nabla_\alpha \to e^{-i\Lambda}\nabla_\alpha e^{i\Lambda}, \quad \bar\nabla_{\dot\alpha} \to \bar\nabla_{\dot\alpha}, \quad \nabla_\mu \to e^{-i\Lambda}\nabla_\mu e^{i\Lambda}\,. \end{aligned} \tag{7}$$

Here Λ and $\Lambda^\dagger$ are chiral and antichiral superfields respectively with $\bar D\Lambda = 0$, $D\Lambda^\dagger = 0$ and $\Lambda \equiv T^a\Lambda_a$, T^a being the generators of the metacolor group SU(N). All the derivatives in superspace are covariant with respect to Λ-transformations only. This so-called gauge chiral representation necessarily implies an asymmetry between chiral and antichiral object (see ref. [8] for details). However, since most of our left-handed quarks are made up of chiral constituents superfields, this representation turns out to be very convenient.

Although in reality the constituents within a composite would be separated from each other by distances of order Λ_M^{-1}, to see the multiplicity of the composites, in particular the origin of replication, in the simplest manner we will take the superspace coordinates of all the constituents to be the same (i.e. zero-splitting between these coordinates) [#8].

One may now verify that, in general, there are altogether nine gauge invariant composite operators, each of dimension four, consisting of a Φ^f_+, a Φ^{c*}_- and a gauge field (in appropriate form) [#9], dimension four being the lowest permissible. In addition, there is just one gauge-invariant composite operator of the lowest dimension, which is two, consisting of a Φ^f_+, a Φ^{c*}_- and a gauge field. Each of these ten composite operators transforms as $(2_L, 1, 4^*_c)$ and thus provides a candidate for an SU(2)$_L$-family. The ten combinations are

$$X_1 \equiv \Phi^{c*}_- W^\alpha(\nabla_\alpha\Phi^f_+), \quad X_2 \equiv (\nabla^\alpha\Phi^{c*}_-)W_\alpha\Phi^f_+, \quad X_3 \equiv (\Phi^{c*}_-)(\nabla^\alpha W_\alpha)\Phi^f_+\,, \tag{8}$$

$$A_1 \equiv \tfrac{1}{16}\bar D\bar D[\Phi^{c*}_-(\nabla^\alpha\nabla_\alpha\Phi^f_+)], \quad A_2 \equiv \tfrac{1}{16}\bar D\bar D[(\nabla^\alpha\nabla_\alpha\Phi^{c*}_-)\Phi^f_+], \quad A_3 \equiv \tfrac{1}{16}\bar D\bar D[(\nabla^\alpha\Phi^{c*}_-)(\nabla_\alpha\Phi^f_+)]\,, \tag{9}$$

$$B_1 \equiv \Phi^{c*}_-(\nabla^\mu\nabla_\mu\Phi^f_+), \quad B_2 \equiv (\nabla^\mu\nabla_\mu\Phi^{c*}_-)\Phi^f_+, \quad B_3 \equiv (\nabla^\mu\Phi^{c*}_-)(\nabla_\mu\Phi^f_+)\,, \tag{10}$$

$$C \equiv \Phi^{c*}_+ e^V \Phi^f_+\,. \tag{11}$$

[#7] Here, $D_\alpha = \partial/\partial\theta^\alpha - i(\sigma^\mu\bar\theta)_\alpha\partial_\mu$; $\bar D_{\dot\alpha} = -\partial/\partial\bar\theta^{\dot\alpha} + i(\theta\sigma^\mu)_{\dot\alpha}\partial_\mu$.

[#8] If we allow for separations between these coordinates we could still make Talor expansions in these separations. This would in fact amount to taking higher dimensional operators involving extra derivatives. We wish to explore, however, replication at the level of the lowest dimensional operators. The analog in QCD is the construction of baryons as qqq-composites.

[#9] The proof for this can be found by simply writing down all possible nonzero products of two covariant derivatives ∇'s and two $\bar D$'s, operating in any arbitrary order on the chiral fields Φ^{c*}_- and Φ^f_+. This yields nine nonzero products as given in eqs. (8)–(11). Whereas the nine composite operators A_i, B_i, X_i are the only ones possible by operating with two ∇'s and two $\bar D$'s on a flavon and a chromon superfield, one may wonder if there are other operators with dimension four possible by application of either four ∇'s or four $\bar D$'s. While the latter is trivially zero, the former yields a composite operator $E = \nabla\nabla\Phi^{c*}_- \cdot \nabla\nabla\Phi^f_+$, whose components are made up entirely of auxiliary fields $F_{L,R}$ which do not contribute to the on-shell vertex $q_L \to$ preons. We therefore do not consider it further.

Note that each of the first nine combinations has dimension four while the tenth one has dimension two because $[\Phi]=1$, $[\nabla]=[D]=[\bar{D}]=\frac{1}{2}$, $[\nabla_\mu]=1$, $[W_\alpha]=\frac{3}{2}$ and $[V]=0$. Of these ten combinations, A_1, A_2 and A_3 are purely positive chiral while the remaining seven – i.e. $X_{1,2,3}$, $B_{1,2,3}$ and C – are general superfields which are reducible under SUSY to a sum of a positive chiral, negative chiral and vector superfields, i.e.

$$A_i=A_i^{(+)},\quad X_i=X_i^{(+)}+X_i^{(-)}+X_i^{(1)},\quad B_i=B_i^{(+)}+B_i^{(-)}+B_i^{(1)},\quad C=C^{(+)}+C^{(-)}+C^{(1)}. \tag{12}$$

These three components can be projected out by using respectively the projectors $\pi^{(+)}$, $\pi^{(-)}$ and $\pi^{(1)}$ given by [8]

$$\pi^{(+)}=\frac{\bar{D}\bar{D}DD}{16\Box},\quad \pi^{(-)}=\frac{DD\bar{D}\bar{D}}{16\Box},\quad \pi^{(1)}=-\frac{D^\alpha\bar{D}\bar{D}D_\alpha}{8\Box}. \tag{13}$$

Thus $X_i^{(\pm)}\equiv\pi^{(\pm)}X_i$ etc. Of course, it should be stressed that not all of these components are on a par with each other on dynamical grounds and, therefore, not all of them need to form as composites. For instance, it turns out that $\pi^{(-)}(B_1+B_2)$ give only four and five-particle combinations [#10] while some of the others given four as well as three-particle combinations. On the other hand, some of these composites, e.g. A_1 and A_2 and likewise $C^{(+)}$ and $C^{(-)}$, are on par because of the symmetry of the metacolor force under the interchange of flavor and color. So, if A_1 forms, A_2 would be expected to form also and similarly if $C^{(+)}$ forms, so should $C^{(-)}$. We will assume that the vector composite superfields $X_i^{(1)}$ and $B_i^{(1)}$ do not form in any case because they would contain spin-$\frac{1}{2}$ and spin-1 components as superpartners and the formation of non-gauge massless spin-1 composites does not seem to be a consistent possibility on dynamical grounds [9].

If the chiral components of all ten combinations do form, using the reduction under SUSY given by (12), we see that there can be at most ten positive chiral (i.e. $A_i^{(+)}$, $X_i^{(+)}$, $B_i^{(+)}$ and $C^{(+)}$) and seven negative chiral (i.e. $X_i^{(-)}$, $B_i^{(-)}$ and $C^{(-)}$) composite superfields. Recall that each of these is an $SU(2)_L$-doublet. Not all of these composite superfields are, however, linearly independent. Using some algebra (details of these will be given in a longer paper [3]) we obtain the following relations between them:

$$A_1=-\tfrac{1}{2}X_1-\tfrac{1}{4}X_3+B_1,\quad A_2=\tfrac{1}{2}X_2+\tfrac{1}{4}X_3+B_2,\quad A_3=\tfrac{1}{4}(X_1-X_2)+B_3, \tag{14}$$

$$A_1+A_2+2A_3=B_1+B_2+2B_3=\Box(\Phi_-^{c}\Phi_+^{f}), \tag{15}$$

$$\pi^{(\pm)}(X_1+X_2+X_3)=0,\quad \pi^{(-)}X_3=0, \tag{16}$$

$$\pi^{(-)}X_1=-\pi^{(-)}X_2=2\pi^{(-)}B_1=2\pi^{(-)}B_2, \tag{17}$$

$$\pi^{(+)}(B_1-B_2)=A_1-A_2. \tag{18}$$

Using (14), we see that X_1-X_2 and X_3 can be eliminated in terms of A_i's and B_i's. This still leaves X_1+X_2 as an independent variable. Based on our argument that two-body massless spin-$\frac{1}{2}$ composites do not form [7], we see from (15) that the two specific combinations $A_1+A_2+2A_3$ and $B_1+B_2+2B_3$ do not form. taking $A_1\pm A_2$ and $A_1+A_2+2A_3$ as independent superfields, we thus see that only $A_1\pm A_2$ may be retained and $A_1+A_2+2A_3$ and, therefore, A_3 may be dropped. Similarly, $B_1\pm B_2$ may be retained and B_3 dropped. We are thus left with $A_1\pm A_2$, $B_1\pm B_2$, X_1+X_2 and C as independent variables which are six in number instead of ten with which we started.

$A_1\pm A_2$ are purely chiral and give two positive chiral superfields only. X_1+X_2 also gives a positive chiral but no negative chiral superfield because of (16). So does B_1-B_2 because of (17) which, however, is just A_1-A_2 (see eq. eq. (18)). B_1+B_2 gives a positive as well as a negative chiral superfield (see (17)) and so does C.

Thus, out of the ten gauge invariant composite operators (A_i, X_i, B_i and C) which could give a maximum of ten positive and seven negative chiral superfields, we have altogether five linearly independent positive chiral and two negative chiral superfields, each of which transforms as an $SU(2)_L$-doublet. Specifically, they are

[#10] After some algebra, one finds that $\pi^{(-)}(B_1+B_2)=-(DD/16\Box)(\Phi_-^{c}W^\alpha W_\alpha\Phi_+^{f})$ which is purely a four- and a five-body composite unlike the other superfields.

$$A_1+A_2,\quad A_1-A_2,\quad \pi^{(+)}(B_1+B_2),\quad \pi^{(+)}(X_1+X_2),\quad (\Box\pi^{(+)})C,\quad \pi^{(-)}(B_1+B_2),\quad (\Box\pi^{(-)})C. \tag{19}$$

We have defined $\Box\pi^{(\pm)}C$ by inserting an extra overall factor $\Box$ so that they would have the same dimension as the other superfields (see later the explicit expressions for the spin-$\frac{1}{2}$ components of these fields). Note that two of these, $\pi^{(-)}(B_1+B_2)$ and $\Box\pi^{(-)}C$, serve as $SU(2)_L$-mirror families because they give right chiral spin-$\frac{1}{2}$ components which are $SU(2)_L$-doublets, while the other five give standard $SU(2)_L$-families.

The compositions of the spin-$\frac{1}{2}$ members of the composite superfields listed in (19) are given as follows [#1]

$$\begin{aligned} q_L^1=(A_1+A_2)|_\theta &= \mathrm{i}(\psi_R^{c*}\cdot\lambda\psi_L^f-\psi_R^{c*}\lambda\cdot\psi_L^f) \\ &\quad+\sqrt{2}\,\{\psi_R^{c*}\mathrm{D}^\mu\mathrm{D}_\mu\varphi_L^f+(\mathrm{D}^\mu\mathrm{D}_\mu\psi_R^{c*})\cdot\varphi_L^f+\varphi_R^{c*}\mathrm{D}^\mu\mathrm{D}_\mu\psi_L^f+(\mathrm{D}^\mu\mathrm{D}_{\mu n}\varphi_R^{c*})\psi_L^f\} \\ &\quad-\frac{\mathrm{i}}{\sqrt{2}}(\varphi_R^{c*}\sigma^{\mu\nu}v_{\mu\nu}\psi_L^f-\psi_R^{c*}\sigma^{\mu\nu}v_{\mu\nu}\varphi_L^f)-\sigma^\mu[\varphi_R^{c*}\mathrm{D}_\mu(\bar\lambda\varphi_L^f)-\mathrm{D}_\mu(\varphi_R^{c*}\bar\lambda)\varphi_L^f]\,, \end{aligned} \tag{20a}$$

$$\begin{aligned} q_L^2=(A_1-A_2)|_\theta &= \mathrm{i}(\psi_R^{c*}\cdot\lambda\psi_L^f+\psi_R^{c*}\lambda\cdot\psi_L^f)+\sqrt{2}\,\{\psi_R^{c*}\mathrm{D}^\mu\mathrm{D}_\mu\varphi_L^f-(\mathrm{D}^\mu\mathrm{D}_\mu\psi_R^{c*})\cdot\varphi_L^f+\varphi_R^{c*}\mathrm{D}^\mu\mathrm{D}_\mu\psi_L^f-(\mathrm{D}^\mu\mathrm{D}_\mu\varphi_R^{c*})\psi_L^f\} \\ &\quad-\frac{\mathrm{i}}{\sqrt{2}}(\varphi_R^{c*}\sigma^{\mu\nu}v_{\mu\nu}\psi_L^f+\psi_R^{c*}\sigma^{\mu\nu}v_{\mu\nu}\varphi_L^f)-\sigma^\mu[\varphi_R^{c*}\mathrm{D}_\mu(\bar\lambda\varphi_L^f)+\mathrm{D}_\mu(\varphi_R^{c*}\bar\lambda)\varphi_L^f]\,, \end{aligned} \tag{20b}$$

$$q_L^3=\pi^{(+)}(B_1+B_2)|_\theta=\sqrt{2}\,\Box(\psi_R^{c*}\varphi_L^f+\varphi_R^{c*}\psi_L^f)-2\sqrt{2}\,(\mathrm{D}^\mu\psi_R^{c*}\cdot\mathrm{D}_\mu\varphi_L^f+\mathrm{D}^\mu\varphi_R^{c*}\cdot\mathrm{D}_\mu\psi_L^f)+\frac{\mathrm{i}\sigma^\mu\partial_\mu}{\Box}\cdot\{...\}\,, \tag{20c}$$

$$q_L^4=\pi^{(+)}(X_1+X_2)|_\theta=-2\varphi_R^{c*}\sigma^\mu(\mathrm{D}_\mu\bar\lambda)\varphi_L^f+\frac{\mathrm{i}\sigma^\mu\partial_\mu}{\Box}\cdot\{...\}\,, \tag{20d}$$

$$\tilde q_R=\pi^{(-)}(B_1+B_2)|_\theta=\frac{\mathrm{i}\bar\sigma^\mu\partial_\mu}{\Box}\,[\varphi_R^{c*}\sigma^{\rho\nu}\{\lambda, v_{\rho\nu}\}\varphi_L^f-(\psi_R^{c*}\cdot\lambda\lambda\varphi_L^f+\varphi_R^{c*}\lambda\lambda\cdot\psi_L^f)]\,, \tag{20e}$$

$$\begin{aligned} Q_L &= (\Box\pi^{(+)})C|_\theta \\ &= \sqrt{2}\,(\mathrm{D}^\mu\varphi_L^c\cdot\mathrm{D}_\mu\psi_L^f+\varphi_L^{c*}\cdot\mathrm{D}^\mu\mathrm{D}_\mu\psi_L^f-2\mathrm{D}^\mu\varphi_L^{c*}\cdot\sigma_{\mu\nu}\mathrm{D}^\nu\psi_L^f-\tfrac{1}{2}\mathrm{i}\varphi_L^{c*}\sigma^{\mu\nu}v_{\mu\nu}\psi_L^f)-\partial_\mu(\varphi_L^{c*}\sigma^\mu\bar\lambda\varphi_1^f)\,, \end{aligned} \tag{20f}$$

$$\begin{aligned} Q_R &= (\Box\pi^{(-)})C|_\theta \\ &= \sqrt{2}\,\{(\mathrm{D}^\mu\mathrm{D}_\mu\bar\psi_L^c)\cdot\varphi_L^f+\mathrm{D}^\mu\bar\psi_L^c\cdot\mathrm{D}_\mu\varphi_L^f+2\sigma^{\mu\nu}\mathrm{D}_\mu\bar\psi_L^c\mathrm{D}_\nu\varphi_L^f+\tfrac{1}{2}\mathrm{i}\sigma^{\mu\nu}\bar\psi_L^c v_{\mu\nu}\varphi_L^f\}+\partial_\mu(\varphi_L^{c*}\bar\sigma^\mu\lambda\varphi_L^f)\,. \end{aligned} \tag{20g}$$

Here D_μ stands for the covariant derivative [#12], $\mathrm{D}_\mu=\partial_\mu+\frac{1}{2}\mathrm{i}v_\mu$, and $v_{\mu\nu}=\partial_\mu v_\nu-\partial_\nu v_\mu+\frac{1}{2}\mathrm{i}[v_\mu, v_\nu]$, where $v_\mu\equiv v_\mu^a[T^a]$. The symbol $\{...\}$ in q_L^3, q_L^4 stands for additional terms having one dimension more than the others, which arise from the chiral projection operator $\sigma\partial/\Box$ on a general superfield. They contain higher derivatives and/or four-body states and are not given here. Thus, we have all together five linearly independent left-chiral families given by $q_L^{1,2,3,4}$ and Q_L as well as two right-chiral (mirror) families given by Q_R and $\tilde q_R$, all of which transform as doublets of $SU(2)_L$. Furthermore, we can obtain an analogous set of $5+2$ $SU(2)_R$-families by making the following replacements: (i) interchange the subscripts + and − in the constituent superfields carrying flavor or color; (ii) choose the gauge antichiral representation for the derivative operations in superspace (see eqs. (3)–(6)) by making the following replacement: $e^V\to e^{-V}$, $\bar{\mathrm{D}}_{\dot\alpha}\to\mathrm{D}_\alpha$, $\nabla_\alpha\to\bar\nabla_{\dot\alpha}$, $\nabla_\mu\to\nabla_\mu$ and $W_\alpha\to\bar W_{\dot\alpha}$ in the definitions of A_i, X_i and B_i.

This will yield five right-chiral and two left-chiral families – i.e. $q_R^{1,2,3,4}$, Q'_R, Q'_L and $\tilde q'_L$ – each of which transforms as a doublet of $SU(2)_R$. The possible spectrum of composite quarks and leptons with minimum dimensional composite operators is then given by

[#11] These expressions for the quark fields are not normalized. Furthermore, terms containing auxiliary components $F^a_{L,R}$ and D are not exhibited. They do not, of course, change the character of the composites.

[#12] The usual form of the covariant derivative and the non-abelian field strength is obtained by rescaling the gauge superfield $V\to 2gV$.

$$(q_L^{1,2,3,4}, Q_L, Q_R, \tilde{q}_R) \sim (2_L, 1_R, 4_c^*), \quad (q_R^{1,2,3,4}, Q'_R, Q'_L, \tilde{q}'_L) \sim (1_L, 2_R, 4_c^*) . \qquad (21)$$

The following remarks are in order:

(1) By inspection we see that the two families q_L^1 and q_L^2 are essentially on par as regards the dynamics of their binding. Each one of these contains a three-fermion term of the type $\psi_R^{c*}\lambda\psi_L^f$ without a space–time derivative which corresponds to pure S-wave binding. If one of these two families forms, we would expect that the other one should form as well and thus would given rise to replication.

In retrospect, we might have anticipated replication in a SUSY preon model as follows. Given that a massless spin-$\frac{1}{2}$ composite made of massless preons must consista of a minimum of three constituents and that the fermionic constituents may be replaced by bosonic ones in a SUSY theory without altering the internal quantum (i.e. $\psi \leftrightarrow \varphi$, $v_\mu \leftrightarrow \lambda$, etc.), there exist several alternative three-particle combinations which could make a left-chiral spin-$\frac{1}{2}$ quark–lepton family. For example, they are: (i) $\sigma^{\mu\nu}\psi_L^f\varphi_R^{c*}v_{\mu\nu}$; (ii) $\sigma^{\mu\nu}\varphi_L^f\psi_R^{c*}v_{\mu\nu}$; (iii) $\psi_L^f\psi_R^{c*}\lambda$ (with different spinor-contractions) and (iv) $\varphi_L^f\varphi_R^{c*}\sigma^\mu\partial_\mu\bar{\lambda}$, etc. All ot these possess the same flavor–color metacolor and $U(1)_x$ quantum numbers. The plurality of these combinations is in essence the origin of replication. Now, SUSY and gauge invariance might have grouped these terms into just one grand combination through one irreducible SUSY multiplet thereby giving only one family. But as we saw that is in fact not the case. Thus, even at the level of minimum number of constituents which is three, supersymmetry seems to provide a compelling reason for replication of quark–lepton families other than radial, orbital and quantum-pair excitations [#13].

(2) While we believe that we have a good reason why quark–lepton families should replicate in a SUSY preon model, we do not yet have a mechanism to determine precisely the number of families which actually form. It is, however, clear by inspection that not all seven combinations listed above are on par with each other as regards the dynamics of their binding and thus not all seven need form. In this sense, the number of families which do form may depend on dynamics. But it may also depend on additional factors such as anomaly-matching [10]. Now, chiral anomalies at the levels of constituents and composites match trivially and therefore do not pose any constraint on the number of massless families, if the residual symmetry after dynamical symmetry breaking at the metacolor scale is no more than a subgroup of $\mathcal{G}_0 = SU(2)_L \times SU(2)_R \times SU(4)_{L+R}^c$. This is because $\mathcal{G}_0$ is anomaly-free. So anomalies match trivially because they vanish at both the preon and the constituent levels. This is what is in fact assumed for simplicity in refs. [1,3]. But in reality this may not be the case. This issue needs to be pursued further.

(3) While we are unable to determine precisely the number of families which form, it is nevertheless remarkable that the spectrum (listed in eqs. (19), (21)) contains an excess of left- over right-chiral $SU(2)_L$-families and likewise an excess of right- overleft-chiral $SU(2)_R$-families. It is furthermore remarkable that the excess in each case happens to be precisely $5-2=3$. The excess is important in the following sense.

Even if all seven $SU(2)_L$-families listed in eq. (21) and the corresponding seven $SU(2)_R$-families did form, one can conceive a plausible mass-generation mechanism through the formation of the metacolor gaugino condensate $\langle\lambda\cdot\lambda\rangle$ [1,3] which would combine the two mirror $SU(2)_L$-families Q_R and $\tilde{q}_R$ with two non-mirror ones Q_L and $\tilde{q}_L$ respectively (where $\tilde{q}_L$ is a certain linear combination of the four q_L^i's) to make two relatively heavy [#14] four-component $SU(2)_L$-families with masses of order 1 TeV. These two families (Q and $\tilde{q}$) would

[#13] By contrast, note that for a non-supersymmetry QCD, with only spin-$\frac{1}{2}$ quarks, which also needs a minimum of three-particle combination to yield a spin-$\frac{1}{2}$ composite baryon, there is just one possible SU(3)-color singlet (qqq)-combination which yields a given SU(3)-flavor representation with a certain permutation symmetry – i.e. a singlet, a decuplet or an octet.

[#14] It is discussed in refs. [6,1] that a metacolor gaugino condensate $\langle\lambda\cdot\lambda\rangle$ and also the fermionic condensate $\langle\bar{\psi}\psi\rangle$ would be damped by M_{Pl}, because both condensates break SUSY and, therefore, must vanish, due to the index theorem, in the absence of gravity (i.e. $M_{Pl}\to\infty$). Thus we expect $\langle\lambda\cdot\lambda\rangle = a_\lambda \Lambda_M^3(\Lambda_M/M_{Pl})$ and $\langle\bar{\psi}\psi\rangle = a_\psi \Lambda_M^3(\Lambda_M/M_{Pl})$, where a_λ and a_ψ are of order one, but one can argue that $a_\psi < a_\lambda$ (see ref. [1]). With $\Lambda_M \sim 10^{11}$ GeV, the gaugino condensate gives masses to Q, $\tilde{q}$ of order $a_\lambda\Lambda_M(\Lambda_M/M_{Pl}) \sim 1$ TeV but leaves the $q_{L,R}$ massless.

have vectorial coupling to W'_L's. Hence they are referred to as vector-like families. Likewise the two $SU(2)_R$-mirror families Q'_L and $\tilde{q}'_L$ would combine, utilizing the same metagaugino condensate $\langle \lambda \cdot \lambda \rangle$, with two non-mirror one Q'_R and $\tilde{q}'_R$ respectively (where $\tilde{q}'_R$ is a certain linear combination of the four q'_R's) to make two relatively heavy vector-like $SU(2)_R$-families (Q' and $\tilde{q}'$). These would have vectorial coupling to W'_R which are superheavy [1].

The metagaugino condensate cannot, however, give masses to the remaining families – i.e. the three q^i_L's and the three q^i_R's. These acquire Dirac-type masses that are much lighter than 1 TeV via a see-saw mechanism [1,3] by utilizing the matter-fermion condensates $\langle \bar{\psi}^a \bar{\psi}^a \rangle$ which mix the three $q^i_{L,R}$'s with the heavier vector-like families $Q_{R,L}$ and $Q'_{R,L}$. Thus, we are left with three light four-component families made essentially of $q^i_{L,R}$ which have chiral couplings to $W_{L,R}$ and may be identified with the e, μ and τ families. Note that the number three corresponds precisely to the excess of non-mirrors over mirrors. Details of the fermion-mass-generation mechanism and the origin of interfamily mass hierarchy are beyond the scope of this paper. They are presented in ref. [3].

In summary, we have shown the supersymmetry, because of fermion↔boson pairing in its field-content, provides a rather simple reason for replication of composite quark–lepton families. At the level of a minimum number of constituents [#15] within each composite which is three, a SUSY preon model also provides a good reason why one may expect to have just three-light chiral families $q^i_{L,R}$. This is because we found that at this level the excess of non-mirror over mirror families is just three.

These considerations also suggest that there must exist some relatively heavy – as many as two but at least one [#16] – vector-like families which are doublets of $SU(2)_L$ and the same number which are doublets of $SU(2)_R$. Various considerations [1,3] pin down the masses of quark-like members of these families to be nearly 1 TeV within a factor of two and those of lepton-like members to be nearly 150–300 GeV. They would thus provide a very rich source of new discoveries in the TeV region. Their existence is a crucial prediction of the model because, without at least one vector-like $SU(2)_L$ and one vector-like $SU(2)_R$- family, the familiar quarks and leptons would remain massless. In this sense, SSC, LHC and a possible high-energy e^-e^+ machine could either confirm the prediction or exclude what seems to us a very attractive idea. The discovery of complete vector-like families in the TeV region may also shed some light whether some underlying superstring theory gives rise to elementary quarks or preons.

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[#15] While we have no dynamical argument to ignore quantum pair-excitations in these considerations, it is conceivable that such excitations merely provide cloud-effects without generating new families.

[#16] We discuss elsewhere [11,3] that the renormalization group analysis of the standard model gauge coupling constants is consistent with observation there are no more than one $SU(2)_L$ and one $SU(2)_R$-vector-like families. With two $SU(2)_L$ and two $SU(2)_R$-vector-like families, together with three chiral families, and their SUSY partners, the QCD coupling constant grows much too rapidly with increasing momentum above 1 TeV and becomes confining at about 10^7 GeV. Thus, consistency with renormalization group analysis and fermion masses demands that just one vector-like $SU(2)_L$ and one vector-like $SU(2)_R$-family should form, should form, but not two. Such a spectrum is at least not implausible once we observe that there is a clear distinction between the dynamics of the composites $\tilde{q}_R$ and q^4_L and those of the other five: $q^{1,2,3}_L$, Q_L and Q_R. As mentioned before, $\tilde{q}_R$ is made entirely of four- and five-body systems (see footnote 10) in contrast to ($q^{1,2,3}_L$, Q_L and Q_R). Now q^4_L is essentially on the same footing as $\tilde{q}_R$ in the sense that the first term in q^4_L contributes only as a four-body composite – i.e. as $(\varphi^{c*}_R \sigma^\mu v_\mu \bar{\lambda})\varphi^c_L$ – to the on-shell vertex for the transitions $q^4_L \to$ preons, for which $\sigma^\mu \partial_\mu \bar{\lambda} = 0$. The second term in q^4_L, which is proportional to $\sigma^\mu \partial_\mu / \Box$, contains higher derivatives and/or four-body states inside the curly bracket. It is conceivable, although we have no convincing argument in this respect, that because of these differences, the two combinations $\tilde{q}_R$ and q^4_L do not bind but the other five $SU(2)_L$-families – i.e. $q^{1,2,3}_L$, Q_L and Q_R – do. In this case, $\tilde{q}'_L$ and q^4_R would not bind either while $q^{1,2,3}_R$, Q'_L and Q'_R will. Thus, we will have altogether five four-component families made of the three chiral families $q^{1,2,3}_{L,R}$, and the two vector families $Q_{L,R}$ and $Q'_{L,R}$. Considerations of fermion masses based on this specific spectrum may be found in ref. [3].

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Fermion Masses and *CP* Violation in a Model with Scale Unification

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It is observed that the scale-unifying model based on supersymmetry and compositeness provides a natural reason for the family mass hierarchy $\bar{m}_e \ll \bar{m}_\mu \ll \bar{m}_\tau$ and links spontaneous *CP* violation to the nonvanishing mass of the electron family. Some of its predictions include (i) K^0-$\bar{K}^0$ and $K_L \to \bar{\mu}e$ are normal, but (ii) $Z \to t\bar{c}, c\bar{u}$, and $\mu\bar{e}$ have observably large strengths *allowing for single top production at the CERN e^+e^- collider LEP II*, and (iii) $d_n \approx (0.5-10)\times 10^{-26}$ *e* cm.

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It has recently been shown [1,2] that the idea that quarks, leptons, and Higgs bosons are composites and that their constituents possess local supersymmetry can be realized in the context of a *viable* and most economical preon model which has many attractive features. These include (i) a common origin of all the diverse scales from M_{Pl} to m_ν [1]; (ii) a simple and compelling reason, based on supersymmetry, for replication of chiral families [2]; and (iii) an explanation based on the index theorem for the protection of quark-lepton masses [3].

The purpose of this Letter is to probe into the origins of interfamily mass hierarchy, family mixing, and *CP* violation, within this scale-unifying model. In the process, we observe that the model not only provides a natural reason for the progressive hierarchy $\bar{m}_e \ll \bar{m}_\mu \ll \bar{m}_\tau$, but links *CP* violation that arises spontaneously within the model to the small but nonvanishing mass of the electron family, and leads to a host of testable predictions.

To discuss the fermion masses, we first need to recall a few salient features of the model [1,2]. The model assumes $N=1$ local supersymmetry (SUSY) at the Planck scale. It introduces six positive and six negative massless chiral preonic superfields $\Phi^{a,\sigma}_{\pm} = (\varphi,\psi,F)^{a,\sigma}_{L,R}$, each belonging to the representation **N** of a metacolor gauge symmetry SU(N). Here σ denotes the metacolor index running from 1 to N; a denotes flavor-color quantum numbers having six values (x,y,r,y,b,l), where (x,y) provide up and down flavors and (r,y,b,l) the four colors including lepton color [4]. The symmetry $SU(N)\times SU(2)_L \times SU(2)_R \times SU(4)^C_{L+R}$ is gauged. Corresponding to an input value for the metacolor coupling $\bar{\alpha}_M \approx 0.07$ to 0.05 at $M_{Pl}/10$, the asymptotically free metacolor force becomes strong and confining at a scale $\Lambda_M \simeq 10^{11}$ GeV, for $N=5$–6. At that point, it serves many purposes.

(i) It makes three light chiral families of composite quarks and leptons $(q^i_{L,R})_{i=1,2,3}$ *and two relatively heavy (mass ~200 GeV–2 TeV) vectorlike families $Q_{L,R}$ and $Q'_{L,R}$ that couple vectorially to W_L's and W_R's, respectively* [2,5]. There are thus altogether five $SU(2)_L$-doublet and five $SU(2)_R$-doublet families, each having the transformation properties under $SU(2)_L\times SU(2)_R \times SU(4)^C_{L+R}$ as noted below:

$$(q_L^{1,2,3},Q_L,Q_R) \sim (2_L,1,4^{*C}),\quad (q_R^{1,2,3},Q'_R,Q'_L) \sim (1,2_R,4^{*C})\,. \quad (1)$$

The members of these families are denoted by $q^e_{L,R} = (u,d,\nu_e,e)_{L,R}$, $q^\mu_{L,R} = (c,s,\nu_\mu,\mu)_{L,R}$, $q^\tau_{L,R} = (t,b,\nu_\tau,\tau)_{L,R}$, $Q_{L,R} = (U,D,N,E)_{L,R}$, and $Q'_{L,R} = (U',D',N',E')_{L,R}$.

(ii) It is assumed that the metacolor force makes a *SUSY-preserving* condensate Δ_R of the scale of Λ_M which transforms as $(1,3_R,10^{*C})$ under $SU(2)_L \times SU(2)_R \times SU(4)^C_{L+R}$. This gives superheavy Majorana masses of order $\Lambda_M \sim 10^{11}$ GeV to the three right-handed neutrinos ν^i_R's belonging to the chiral families q^i_R's and breaks $SU(2)_L\times SU(2)_R\times SU(4)^C_{L+R}$ to $SU(2)_L \times U(1)_Y \times SU(3)^C_{L+R}$ [6].

(iii) It is furthermore assumed that the metacolor force makes a few SUSY-breaking condensates as well. These include the metagaugino condensate $\langle\lambda\cdot\lambda\rangle$ and the matter fermion condensates $\langle\bar{\psi}^a\psi^a\rangle$, each of which breaks SUSY [3]. Noting that, within the class of models under consideration, the index theorem prohibits a dynamical breaking of supersymmetry in the absence of gravity [3,7], however, the formation of these condensates must need the collaboration between the metacolor force and gravity. As a result, each of these condensates is expected to be damped by one power of Λ_M/M_{Pl} relative to Λ_M [3,8]:

$$\langle\lambda\cdot\lambda\rangle = a_\lambda \Lambda_M^3(\Lambda_M/M_{Pl});\quad \langle\bar{\psi}^a\psi^a\rangle = a_{\psi_a}\Lambda_M^3(\Lambda_M/M_{Pl})\,. \quad (2)$$

There are four $\langle\bar{\psi}\psi\rangle$ condensates corresponding to a having the values x, y, (r,y,b), or l [8]. The coefficients a_λ and a_{ψ_a}, *a priori*, are expected to be of order unity within a factor of 10 (say), although a_λ is expected to be larger than the a_ψ's, typically by a factor of 3–10, because the ψ's are in the fundamental and the λ's are in the adjoint representation of the metacolor group [1].

The condensates $\langle\lambda\cdot\lambda\rangle$ and $\langle\bar{\psi}^a\psi^a\rangle$ induce SUSY-breaking mass splittings $\delta m_S \sim a_\lambda\Lambda_M(\Lambda_M/M_{Pl}) \sim 1$ TeV. The condensates $\langle\bar{\psi}^a\psi^a\rangle$, for $a=x$ and y, break not only SUSY but also the electroweak symmetry $SU(2)_L \times U(1)_Y$. The resulting masses of W and Z bosons are $m_W, m_Z \sim g_2 a_\psi \Lambda_M(\Lambda_M/M_{Pl}) \sim 100$ GeV, where g_2 is the $SU(2)_L$ gauge coupling constant.

Masses of the vectorlike families $Q_{L,R}$ and $Q'_{L,R}$ are protected by $U(1)_X$. They acquire flavor-color-independent masses of order $a_\lambda\Lambda_M(\Lambda_M/M_{Pl}) \sim 1$ TeV only through the condensate $\langle\lambda\cdot\lambda\rangle$, which breaks $U(1)_X$ just as needed [1,3]. But the chiral families $q^i_{L,R}$ acquire masses primarily through their mixings with the vectorlike families $Q_{L,R}$ and $Q'_{L,R}$ which are induced by $\langle\bar\psi^a\psi^a\rangle$. This is because the direct mass terms $m^{(0)}_{\rm dir}(q^i_L\to q^i_R)$ cannot be induced through either $\langle\lambda\cdot\lambda\rangle$ or $\langle\bar\psi\psi\rangle$. These receive small contributions $\lesssim 1$ MeV at Λ_M from effective four-body condensates like $\langle\bar\psi\psi\varphi^*\varphi\rangle$, which are, however, damped by $(\Lambda_M/M_{\rm Pl})^2$. Thus, ignoring QCD corrections and $m^{(0)}_{\rm dir}$ for now, the Dirac mass matrices of all four sectors—i.e., q_u, q_d, l, and ν—have the form

$$M^{(0)}_{f,c}=\begin{array}{c} \\ \bar q^i_R \\ \bar Q_R \\ \bar Q'_R\end{array}\begin{array}{c}\begin{array}{ccc} q^i_L & Q_L & Q'_L\end{array}\\ \left[\begin{array}{ccc} 0 & X\kappa_f & Y\kappa_c \\ Y'^{\dagger}\kappa_c & \kappa_\lambda & 0 \\ X'^{\dagger}\kappa_f & 0 & \kappa_\lambda\end{array}\right]\end{array}. \quad (3)$$

Here, $f=x$ or y and $c=(r,y,b)$ or l. The index i runs over three families. The entities X, Y, X', and Y' are column matrices in the family space having entries of ~ 1 to $\frac{1}{10}$. In the above, $\kappa_f\equiv O(a_{\psi_f})\Lambda_M(\Lambda_M/M_{\rm Pl})$, $\kappa_c\equiv O(a_{\psi_c})\Lambda_M(\Lambda_M/M_{\rm Pl})$, and $\kappa_\lambda\equiv O(a_\lambda)\Lambda_M(\Lambda_M/M_{\rm Pl})$. Following the remarks made above, we expect $\kappa_\lambda\simeq(3\text{-}10)\kappa_{f,c}$. *Thus the Dirac mass matrices of all four sectors have at least an approximate seesaw structure.*

In the absence of electroweak corrections [$\sim$(5-10)%], left-right symmetry and flavor-color independence of the metacolor force guarantee (a) $X=X'$ and $Y=Y'$, and (b) the same X, Y, and κ_λ apply to q_u, q_d, l, and ν [see Eq. (3)]. This results in an enormous reduction of parameters.

We first observe that by ignoring electroweak corrections one can always rotate the chiral fermions q^i_R and q^i_L to bring the row matrices $Y^T=Y'^T$ to the simple form (0,0,1) and simultaneously $X^T=X'^T$ to the form $(0,p,1)$, with redefined κ_f and κ_c. As a result, *the* 5×5 *mass matrices of the four sectors—i.e.,* q_u, q_d, l, *and* ν*—which in general could involve a hundred parameters, are essentially determined (barring electroweak corrections and contributions from* $m^{(0)}_{\rm dir}$*) by just six effective parameters*—i.e., p, κ_u, κ_d, κ_r, κ_l, *and* κ_λ. Furthermore, we know their approximate values (within a factor of 10, say). Examining the relevant preon diagrams, one can argue that p is less than but not very much smaller than unity; $p\approx\frac{1}{2}$ to $\frac{1}{4}$ is quite natural [9].

Since we expect $\kappa_f,\kappa_c\lesssim\kappa_\lambda/3$ (see above), we obtain the following eigenvalues in the leading seesaw limit (neglecting electroweak corrections and $m^{(0)}_{\rm dir}$):

$$\begin{aligned}
&m_u^{(0)}=m_d^{(0)}=m_e^{(0)}=(\tilde m_{\nu_e}^{(0)})=0\,,\\
&(m_c^{(0)},m_s^{(0)})\approx(\kappa_u,\kappa_d)(\kappa_r/\kappa_\lambda)(p^2/2)\eta_{\rm QCD}\,,\\
&(\tilde m_{\nu_\mu}^{(0)},m_\mu^{(0)})\simeq(\kappa_u,\kappa_d)(\kappa_l/\kappa_\lambda)(p^2/2)\,,\\
&(m_t^{(0)},m_b^{(0)})\simeq(\kappa_u,\kappa_d)(\kappa_r/\kappa_\lambda)(2)\eta_{\rm QCD}\,,\\
&(\tilde m_{\nu_\tau}^{(0)},m_\tau^{(0)})\simeq(\kappa_u,\kappa_d)(\kappa_l/\kappa_\lambda)(2)\,,\\
&m(U,D,U',D')\simeq\kappa_\lambda\eta_{\rm QCD}\,,\\
&m(E,E')\approx\tilde m(N,N')\approx\kappa_\lambda\,.
\end{aligned}\quad (4)$$

The tildes on neutrino masses denote that they are Dirac masses. Combined with the superheavy Majorana masses of ν_R's, they yield light ν_L's [10]. The QCD renormalization factors for quarks are momentum dependent. With five families and their superpartners (masses ~ 1 TeV), we obtain $\eta_{\rm QCD}(\mu)\approx 2.9$, 3.3, 4.1, and 5.2 for $\mu=1$ TeV, 100 GeV, 5 GeV, and 1 GeV, respectively.

We see that despite the fact that the electron family is made of the same stuff as the μ *and the* τ *families, it is guaranteed to remain massless [barring contributions from* $m^{(0)}_{\rm dir}\sim(1\text{ MeV})\eta_{\rm QCD}$*]—a fact which is not far from the truth.* The reason is simply the rank of the matrix $M^{(0)}$. We also see that the μ-τ mass ratios (evaluating $\eta_{\rm QCD}$ at a fixed momentum for all quarks) are

$$\frac{m_c^{(0)}}{m_t^{(0)}}\approx\frac{m_s^{(0)}}{m_b^{(0)}}\approx\frac{m_\mu^{(0)}}{m_\tau^{(0)}}\approx\frac{p^2}{4}\,. \quad (5)$$

Thus, for $p\approx\frac{1}{3}$ to $\frac{1}{4}$, which is natural (see remarks above), *we obtain a rather large* μ-τ *hierarchy of about* $\frac{1}{40}$ to $\frac{1}{64}$. In this way, the model provides a natural reason for the *interfamily hierarchy* $\bar m_e\ll\bar m_\mu\ll\bar m_\tau$.

To accommodate the observed features of quark-lepton mass splittings within a family (e.g., m_b/m_τ) including the $\eta_{\rm QCD}$ factor and the up-down ratios [11] (e.g., m_t/m_b), one needs to assume $\kappa_r/\kappa_l\approx 0.6\pm0.2$ and $\kappa_d/\kappa_u\approx 1/(30\pm5)$. The first ratio is in a natural range but the second is outside. It is conceivable that κ_d is so small because it is generated only radiatively through κ_u [12]. To see the kind of masses which could be obtained at the tree level, consider the following choice of parameters which turns out to be near optimum: $p\approx0.31$, $\kappa_u\approx 80$ GeV, $\kappa_l/\kappa_\lambda\approx\frac{1}{3}$, $\kappa_r/\kappa_l\approx0.6$, $\kappa_d/\kappa_u\approx1/30$, and $\kappa_\lambda\approx(3\text{-}5)\kappa_u\approx 200\text{-}400$ GeV. These yield (including QCD corrections) $m_u^{(0)}=m_d^{(0)}=m_e^{(0)}=0$, $m_t^{(0)}\approx110$ GeV, $m_b^{(0)}\approx4.7$ GeV, $m_c^{(0)}\approx3.9$ GeV, $m_s^{(0)}\approx130$ MeV, $m_\tau^{(0)}\approx1.7$ GeV, and $m_\mu^{(0)}\approx40$ MeV, while $m(U,D,U',D')\approx1.5\text{-}3$ TeV and $m(E,E')\approx\tilde m(N,N')\approx200\text{-}400$ GeV.

While these results possess at least the desired gross pattern—i.e., $\bar m_e\ll\bar m_\mu\ll\bar m_\tau$, with $\bar m_e\approx0$—they are off in details by a factor of 2-3. In particular, m_c is too high and m_μ too low; all the other masses are reasonable. The tree-level mass matrix $M^{(0)}$ has an additional shortcoming: The Cabibbo-Kobayashi-Maskawa (CKM) matrix in the 3×3 light-family sector is found to be essentially unity, rendering $\theta_{e\mu}\approx\theta_{\mu\tau}\approx\theta_{e\tau}\approx0$. The results improve dramatically, however, with regard to both the masses and the CKM matrix by including the electroweak corrections at Λ_M and $|m^{(0)}_{\rm dir}|\sim1$ MeV.

The $SU(2)_L\times U(1)_Y$ interactions distinguish between left and right, up and down, and quarks and leptons. These corrections, evaluated at Λ_M, through preon diagrams [9], alter X^T, Y^T, $(X')^T$, and $(Y')^T$ to the general forms

$$X^T=(0,p+\delta_2,1),\quad Y^T=(0,0,1+\delta_3),\quad (X')^T=(\bar\delta_1,p+\bar\delta_2,1+\bar\delta_3),\quad (Y')^T=(0,0,1)\,. \tag{6}$$

The parameters δ_i and $\bar\delta_i$ are in principle calculable. There are eight δ's—i.e., $\delta_{2,3}^{u,d,l,\nu}$—and twelve $\bar\delta$'s—i.e., $\bar\delta_{1,2,3}^{u,d,l,\nu}$. Each of these δ's is a sum of several δ's (evaluated in the preon basis), and is expected to be nearly a few to 10%. (Note that $(\alpha_2/2\pi)\ln[\Lambda_M/(100\text{ GeV})]\sim\frac{1}{200}20\sim10\%$.) Including the δ's, the mass eigenvalues are altered as follows ($\eta_{\rm QCD}$ is suppressed):

$$\hat m_e^j=0,\quad \hat m_\mu^j\approx\kappa_f\left(\frac{\kappa_c}{\kappa_\lambda}\right)\left(\frac{p^2}{2}\right)\left(1+\frac{\delta_2^j+\bar\delta_2^j}{p}\right), \tag{7}$$
$$\hat m_\tau^j\approx\kappa_f\left(\frac{\kappa_c}{\kappa_\lambda}\right)2\left(1+\frac{p^2}{4}+\frac{\delta_3^j+\bar\delta_3^j}{2}+\frac{p(\delta_2^j+\bar\delta_2^j)}{4}-\frac{\kappa_f^2+\kappa_c^2}{\kappa_\lambda^2}\right).$$

Here, $j=u,d,\nu,l$. The carets indicate that $m_{\rm dir}^{(0)}\lesssim1$ MeV is not included. We see that while the corrections to the τ-family masses are only of order (5–10)%, those to the muon family can be substantial because they are proportional to $1+(\delta_2^j+\bar\delta_2^j)/p\approx1\pm(5\text{–}20)\%/0.3\approx1\pm(16\text{–}66)\%$. For example, if $\delta_2^u+\bar\delta_2^u\approx-20\%$ and $\delta_2^l+\bar\delta_2^l\approx+22\%$ (say), which are within a reasonable range, m_c could be reduced by a factor of 3 and m_μ enhanced by about 1.7, compared to the tree-level solutions, just as desired. Assuming, conservatively, that each individual $|\delta_i^j|\lesssim(5\text{–}12)\%$, and using observed values of $m_\tau/m_\mu\approx17$, $m_c(1\text{ GeV})\approx1.4\pm0.1$ GeV, and $m_t(\text{phys})\gtrsim89$ GeV, we find from Eq. (7) that $0.29\lesssim p\lesssim0.33$ and $m_t(\text{phys})\lesssim180$ GeV. The model, however, typically prefers much lower values of $m_t\lesssim130$ GeV.

The CKM elements, ignoring $m_{\rm dir}^{(0)}$ still, are given by

$$\hat V_{us}\approx[(\bar\delta_1^d-\bar\delta_1^u)/p][1+O(\delta/p)]\,,$$
$$\hat V_{ub}\approx[(\bar\delta_1^d-\bar\delta_1^u)/2][1+O(\delta/p)]\,, \tag{8}$$
$$\hat V_{cb}\approx[(\bar\delta_2^d-\bar\delta_2^u)/2][1+O(p)]+(p/2)(\kappa_u^2-\kappa_d^2)/\kappa_\lambda^2\,.$$

These can yield a reasonable set of mixing angles for the δ's (a few to 10%). Note especially that $\hat V_{us}$, enhanced by $1/p$, is expected to be larger than both $\hat V_{ub}$ and $\hat V_{cb}$.

Let us now include the contributions from $m^{(0)}(q_L^i\to q_R^j)$, which are induced only by effective four-body condensates $\langle\bar\psi_L\psi_R\varphi_L^*\varphi_R\rangle$ and are thus of order (1 MeV) $\times\eta_{\rm QCD}$. These lead to $m_e,m_u,m_d\neq0$. Most importantly, they also permit spontaneous CP violation through the fermion mass matrix which would vanish as $m_{\rm dir}^{(0)}\to0$.

To see this, first set $m_{\rm dir}^{(0)}=0$ and introduce phases into the condensates $\langle\lambda\cdot\lambda\rangle$ and $\langle\bar\psi\psi\rangle$ or equivalently into the κ's—i.e., $\kappa_f=|\kappa_f|e^{i\xi_f}$, $\kappa_c=|\kappa_c|e^{i\xi_c}$, and $\kappa_\lambda=|\kappa_\lambda|e^{i\xi_\lambda}$. Simultaneously, impose the following transformations: $q_L^{f,c}\to q_L^{f,c}$, $q_R^{f,c}\to e^{i(\xi_f+\xi_c-\xi_\lambda)}q_R^{f,c}$, $Q_L^{f,c}\to e^{i(\xi_c-\xi_\lambda)}Q_L^{f,c}$, $Q_R^{f,c}\to e^{i\xi_c}Q_R^{f,c}$, $Q_L'^{f,c}\to e^{i(\xi_f-\xi_\lambda)}Q_L'^{f,c}$, and $Q_R'^{f,c}\to e^{i\xi_f}Q_R'^{f,c}$. These do not introduce any phase into V_{KM}^L because the left chiral fields are unchanged while Q_L and Q_R transform the same way for up and down. It is easy to verify that the mass matrix $M_{f,c}$ including electroweak corrections, subject to the transformations mentioned above, is rendered real if $m_{\rm dir}^{(0)}=0$. This says that neither the mass matrix nor the gauge interactions (ignoring $W_R^\pm$, which are superheavy) can generate observable CP violation if $m_{\rm dir}^{(0)}=0$. However, with $m_{\rm dir}^{(0)}$ being nonvanishing and complex, the reality of the mass matrix is in general lost and, thereby, CP conservation as well. *We thus see an interesting connection between the nonvanishing masses of the electron family and the spontaneously generated CP violation in the model.*

To explore the consequences of $m_{\rm dir}^{(0)}$, we write the mass matrix for the 3×3 light d-quark sector in the form $M^{(d)}\equiv\hat M^{(d)}+m_{\rm dir}^{(0)d}$ and choose the basis such that $\hat M^{(d)}$ (which includes electroweak corrections) is diagonal: $\hat M^{(d)}=(0,\hat m_s,\hat m_b)$. In the same basis, we denote $(m_{\rm dir}^{(0)d})_{ij}\equiv\Delta_{ij}^{(d)}$, where the Δ_{ij}'s are complex. For quarks, we expect $|\Delta_{ij}^{(q)}|\sim(1\text{ MeV})\eta_{\rm QCD}(1\text{ GeV})\approx$ a few to 15 MeV. The CKM elements for $W_L^\pm$ are now altered to

$$V_{ud}=V_{cs}^*\approx1-\frac{\bar\delta_1^{u\,2}+\bar\delta_1^{d\,2}}{2p^2}-\frac{\bar\delta_1^d-\bar\delta_1^u}{p}\left(\frac{\Delta_{12}^{*d}}{m_s}-\frac{\Delta_{12}^u}{m_c}\right),$$
$$V_{us}\approx\hat V_{us}+\left(\frac{\Delta_{12}^d}{m_s}-\frac{\Delta_{12}^u}{m_c}\right)-\frac{\Delta_{23}^{d*}}{m_b}\frac{\bar\delta_1^d-\bar\delta_1^u}{2},$$
$$V_{cd}\approx-\hat V_{us}+\left(\frac{\Delta_{12}^{*u}}{m_c}-\frac{\Delta_{12}^{*d}}{m_s}\right)-\frac{\Delta_{13}^{*d}}{m_b}\frac{\bar\delta_2^d-\bar\delta_2^u}{2},$$
$$V_{cb}\approx\hat V_{cb}+\Delta_{23}^d/m_b,\quad V_{ub}\approx\hat V_{ub}+\Delta_{13}^d/m_b\,.$$

The phase-invariant parameter $J\equiv\text{Im}(V_{ud}V_{cs}V_{us}^*V_{cd}^*)$, relevant for CP violation in $K\to2\pi$ decay, is given by

$$J\approx\tfrac12(\bar\delta_2^d-\bar\delta_2^u)\text{Im}\{[(\bar\delta_1^d-\bar\delta_1^u)/p+\Delta_{12}^{d*}/m_s]\Delta_{13}^d/m_b\}\,. \tag{9}$$

This leads to $J\approx[0.05\text{–}0.07](1\text{–}2)(0.1\text{–}0.15)(2\times10^{-3})\xi\approx(1\text{–}4)\times10^{-5}\xi$, where ξ is the phase of Δ_{13}/m_b. This gives $|\epsilon|\sim\frac{1}{600}$ with $\xi\sim1$ [13]. Thus the suppression of ϵ is naturally explained because, essentially, $|\epsilon|\sim|\Delta_{13}^d/m_b|\sim m_d/m_b\approx2\times10^{-3}$, with a maximal ξ. As regards ϵ', it is found to receive contributions primarily from the penguin graph as in the CKM model.

Turning attention to the electric dipole moment of the neutron, d_n, it is a special property of this model that although $W_R^{\pm}$ are superheavy, right chiral currents couple to W_L's because q_R's mix with Q_R's [see Eq. (3)] belonging to the vectorlike family Q which couple to W_L's. The dominant contribution comes from $d_L \to d_R + \gamma$ with charm quark and W_L^- in the loop. This involves the vertex $d_R \to c_R + W_L^-$, for which the CKM element is given by $(\kappa_u \kappa_d/\kappa_\lambda^2) p^2 \Delta_{12}^{*d}/m_s$. Thus, we obtain

$$d_n \simeq \left[\left(\frac{e a_2}{4\pi} \right) \left(\frac{m_c}{m_W^2} \ln \frac{m_c^2}{m_W^2} \right) \sin\theta_C \right] \left(\frac{\kappa_u \kappa_d}{\kappa_\lambda^2} \right) p^2 \left| \frac{\Delta_{12}^{d*}}{m_s} \right| \sin\eta \,, \qquad (10)$$

where η is the phase of Δ_{12}^d. Allowing for $\kappa_d/\kappa_u \approx 1/30$, $\kappa_u/\kappa_\lambda \approx \frac{1}{3} - \frac{1}{5}$, $|\Delta_{12}^{d*}/m_s| \approx (\frac{1}{2} - 1.5) \times 10^{-1}$ and $\eta \approx 1 - \frac{1}{10}$, we expect $d_n \approx 10^{-25}$ to $\frac{1}{2} \times 10^{-26}$ e cm [14]. This is a relatively large d_n which should be observable.

Finally, as regards flavor-changing processes, arising from the mixing of q^i's with Q and Q', we find [9] that the new contributions to processes such as K^0-$\bar{K}^0$, $K_L \to \mu^+\mu^-$, and $K_L \to \bar{\mu} e$ (through box and tree graphs) are smaller typically by 1 to 2 orders of magnitude than that of the standard model, while those for B^0-$\bar{B}^0$ are comparable to that of the standard model [15]. However, the model predicts intriguing new processes and effects such as the following: (i) $Z \to t\bar{c}$ with a coupling $\approx (g_2/\cos\theta_W)(\kappa_u/\kappa_\lambda)^2 p/2 \approx (g_2/\cos\theta_W)(2-\frac{1}{2})\%$, which provides the genuine scope for observing a $t\bar{c}$ "resonance" in e^+e^- annihilation. This is, of course, the only way the top can be observed at the CERN e^+e^- collider LEP II if $m_t \gtrsim 100$ GeV. (ii) $Z \to c\bar{u}$ with a coupling $\approx (g_2/\cos\theta_W)(\kappa_u/\kappa_\lambda)^2 (p/2)\delta_1^u$ which gives $\Delta m(D\text{-}\bar{D}) \simeq (10-3) \times 10^{-14}$ GeV. This is at least 10 times larger than the standard model prediction and is in range for experimental detection [16]. (iii) $Z \to \bar{\mu} e$ with a coupling $\approx (g_2/\cos\theta_W)(\kappa_d/\kappa_\lambda)^2 (p/2)\delta_1^l$ leading to $B(\mu \to 3e) \approx (1-5) \times 10^{-13}$. (iv) Significant departures from unitarity in certain combinations occurring within the 3×3 part of the full CKM matrix which would imply a (4–10)% increase in top and τ lifetimes compared to standard model predictions.

Our dramatic prediction and hallmark of the model is, of course, the existence of vectorlike families Q and Q' [1,2] whose charged lepton and quark members have masses $\simeq$200–500 GeV and 0.6–1.5 TeV, respectively. This should provide rich new physics to be probed at the Superconducting Super Collider, the CERN Large Hadron Collider, and TeV-range e^+e^- colliders. All these show that the model not only provides a natural reason for the interfamily mass hierarchy and an attractive framework for CP violation [17], but (a) it is safe at present (unlike standard technicolor) and (b) it can be falsified in many ways, even at low energies.

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[4] J. C. Pati and A. Salam, Phys. Rev. D **10**, 275 (1974).
[5] The q_L^i's are made of preonic combinations such as $\psi_L^f \varphi_R^{c*} v$, $\varphi_L^f \psi_R^{c*} v$, $\psi_L^f \psi_R^{c*} \lambda$, and $\varphi_L^f \varphi_R^{c*} (\sigma_\mu \bar{\lambda})$; q_R^i's are obtained by switching $L \leftrightarrow R$ and $\lambda \leftrightarrow \bar{\lambda}$; while $Q_L \sim \psi_L^f \varphi_L^{c*} v$, $Q_R \sim \varphi_L^f \psi_L^{c*} v$, $Q_R' \sim \psi_R^f \varphi_R^{c*} v$, and $Q_L' \sim \varphi_R^f \psi_R^{c*} v$ [2]. Here f stands for flavor indices (x,y) and c for color indices (r,y,b,l) and $(v_\mu, \lambda, \bar{\lambda})$ denote metacolor gauge fields.
[6] Hereby, we are assuming a breakdown of global vectorial symmetries such as $SU(2)_{L+R}$ in SUSY QCD, which would be forbidden in ordinary QCD by the Vafa-Witten theorem. Whether such a breaking is permitted in SUSY QCD for which the proof of Vafa-Witten theorem does not apply is still an open problem.
[7] E. Witten, Nucl. Phys. **B185**, 513 (1981); **B202**, 253 (1983); E. Cohen and L. Gomez, Phys. Rev. Lett. **52**, 237 (1984).
[8] This is based on contributions from a single graviton exchange to the $(\text{mass})^2$ of the corresponding composite Higgs boson [1,3]. It is worth noting that if the leading contributions to $\langle \lambda \cdot \lambda \rangle$ and $\langle \bar{\psi} \psi \rangle$ were damped by $(\Lambda_M/M_{Pl})^2$, involving two-graviton exchange, the values of δm_S, m_W, m_Z, and the masses of quarks and charged leptons would still be unaltered if one chooses $\bar{\alpha}_M(M_{Pl}/10)$ such that $\Lambda_M^3/M_{Pl}^2 \sim 1$ TeV, i.e., $\Lambda_M \sim 10^{13.7}$ GeV.
[9] Details of these will be given in a forthcoming paper by K.S. Babu, J. C. Pati, and H. Stremnitzer.
[10] K. S. Babu, J. C. Pati, and H. Stremnitzer, Phys. Lett. B (to be published).
[11] We allow $89 \lesssim m_t \lesssim 150$ GeV, where the lower limit is experimental and the upper is theoretical, in the model.
[12] A possible mechanism leading to $\kappa_d \approx \kappa_u (\alpha/2\pi) \ln[\Lambda_M/(100 \text{ GeV})]$ will be discussed elsewhere.
[13] See, e.g., C. Albright, C. Jarlskog, and B. A. Lindholm, Phys. Rev. D **38**, 872 (1988).
[14] We have estimated that additional contributions to d_n through either an induced three-gluon or an induced θ term do not exceed the estimate of Eq. (10).
[15] As in all SUSY models, box graphs involving squarks could introduce additional contributions to K^0-$\bar{K}^0$. These would be suppressed, however, either if the squarks of the first two families with masses of order few TeV are highly degenerate (to within 10%) or if they are superheavy ($\gg$TeV)—a possibility that arises for a new allowed scenario for SUSY breaking [9].
[16] For a phenomenological discussion see P. Langacker and D. London, Phys. Rev. D **38**, 886 (1988).
[17] In addition, as noted in Refs. [1] and [2], it also provides a good reason for the origin of diverse mass scales from M_{Pl} to m_ν and of families.