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SMR.627-15

**MINIWORKSHOP ON  
STRONGLY CORRELATED ELECTRON SYSTEMS**

**15 JUNE - 10 JULY 1992**

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**"Dynamical Properties of  
Strongly Correlated Electrons"**

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These are preliminary lecture notes, intended only for distribution to participants

# Dynamical Properties of Strongly Correlated Electrons.

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## ● High- $T_c$ Superconductors

- i)  $N(\omega)$ , PES, IPES
  - ii) ARPES, ARI PES
  - iii)  $\chi(\omega)$
- } one band Hubbard  
and  
t-J models.

### iv) Superconductivity?

No indications in Hubbard model.

Results for 2D t-J model at  
 $\langle n \rangle = 1/2$  suggest superconductivity  
near phase separation!

## ● C<sub>60</sub>

QMC study at half-filling using  
Hubbard model.

# How can we study the Hubbard model in strong coupling?

One way: numerical studies

## Exact diagonalization (Lanczos)

Solve exactly a small cluster (4x4)

- $T=0$
- Exact
- No "sign" problems
- Dynamics easy

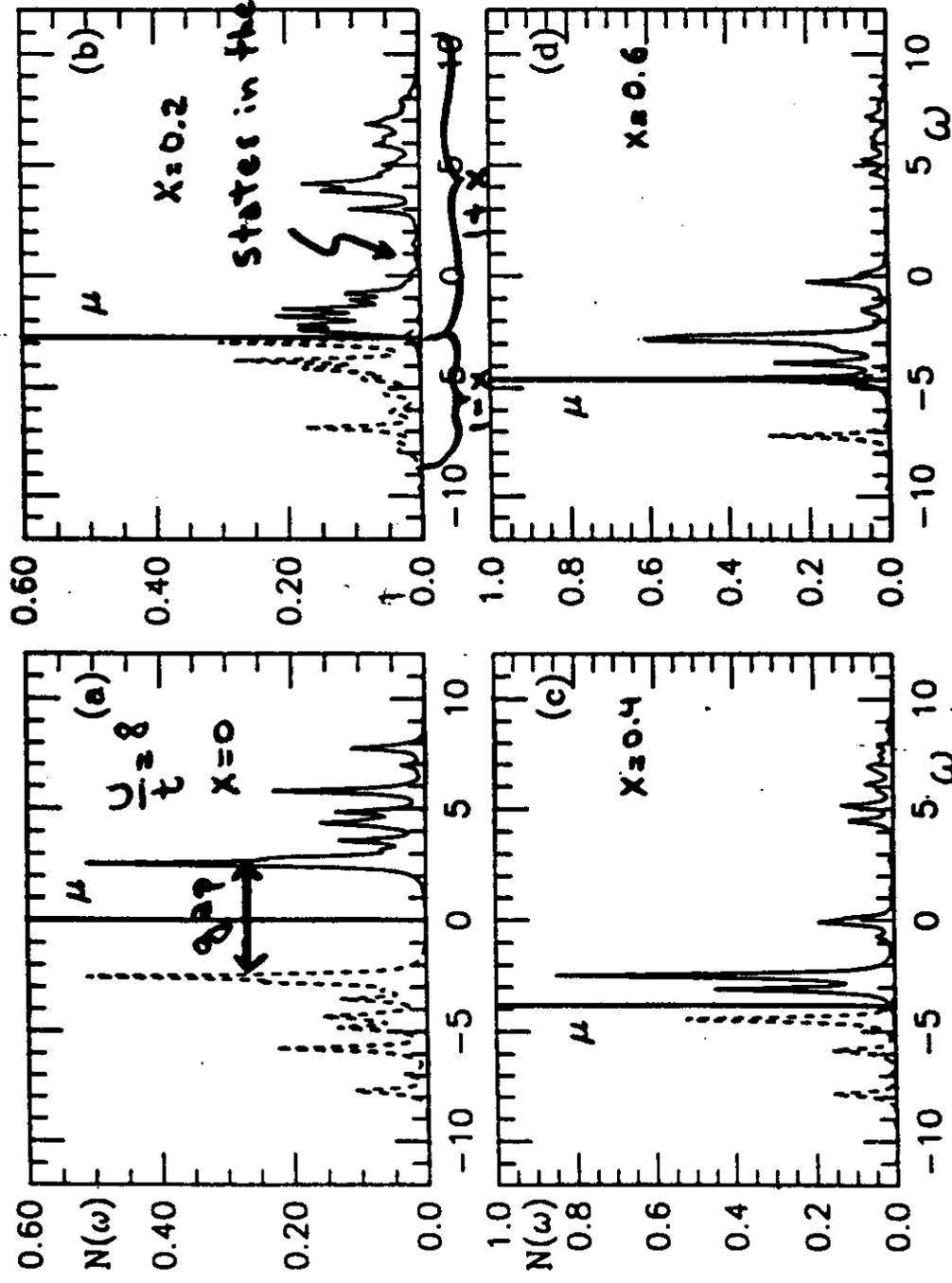
## Q. Monte Carlo

Larger lattices possible (8x8)  
(16x16)

- $T \neq 0$
- Stat. errors
- "sign" problem
- Dynamics difficult

Review:  
(See Int. J. Mod. Phys.)  
B5, 77, 1991.)

Other ways: { mean-field  
variational  
...



E. D., A. Moreo, F. Ortalan, J. Cano, D. Scalapino, PRL 67, 1918 (1991). FIG. 3. Same as Fig. 2 with  $U/t=8$ .

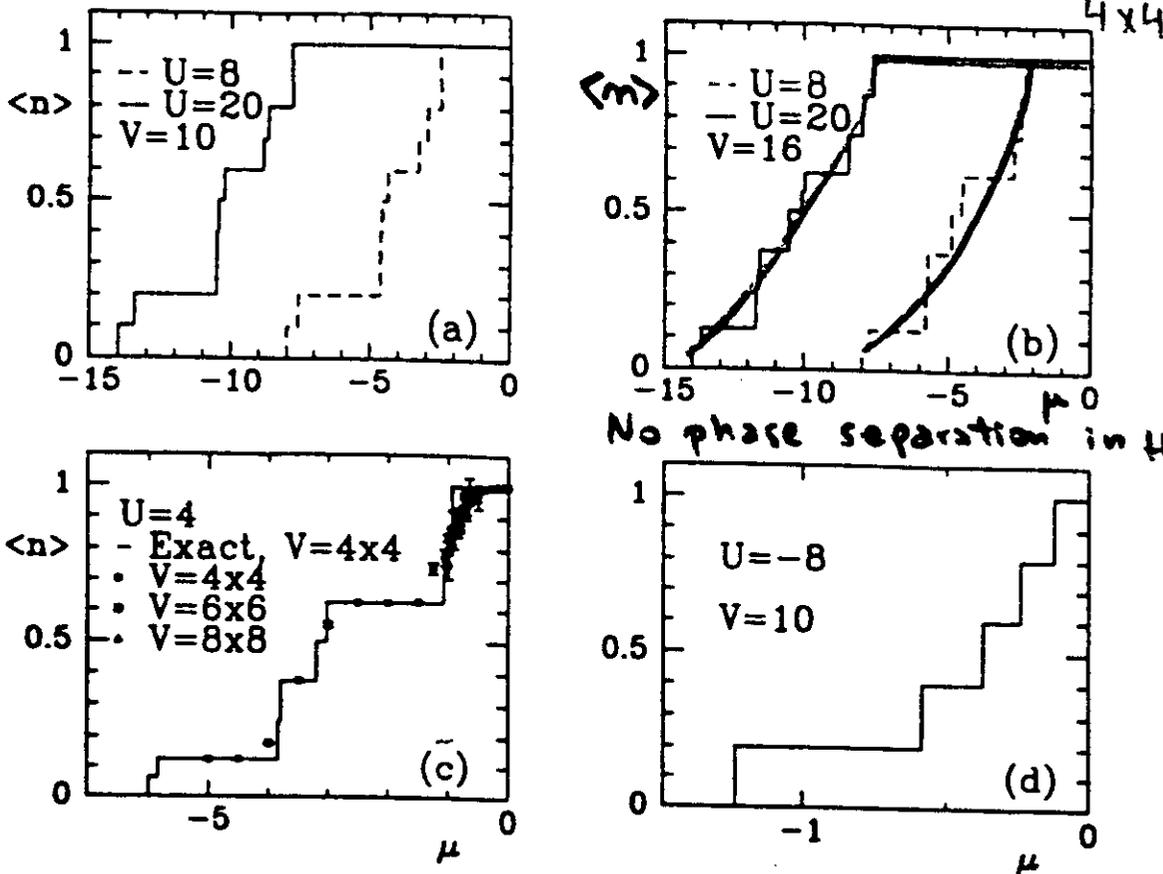
le doping are to remove spectral weight from both the upper and lower Hubbard bands and to create additional density of states in the gap beginning at the edge of the lower Hubbard band. Results for further dopings are shown in Figs. 2(c) and 2(d). With this additional dop-

slope is larger than one (at small  $x$ ) and increases when  $U/t$  is reduced, at least for  $U/t$  large where a genuine gap exists in the IPES spectrum.

Thus we find, as seen in PES experiments [2], that dop-

ing a half-filled Hubbard cluster does not simply produce a rigid shift of the density of states relative to the Fermi

$\mu$  moves across the gap when going from holes to electrons doping



ishes for the the vanishing negative- $U$  chemical potentials slowly move a In order to tion of spectr Hubbard!

$$N_s^{(+)}(\omega) = \frac{1}{\nu}$$

$$N_s^{(-)}(\omega) = -$$

FIG. 1. (a)  $\langle n \rangle$  vs  $\mu$  at zero temperature for the ten-site Hubbard cluster with  $U/t=20$  (continuous line) and  $U/t=8$  (dashed line) using Lanczos techniques; (b) same as (a) for a sixteen-site cluster; (c)  $\langle n \rangle$  vs  $\mu$  at  $U=4$  ( $t=1$ ). The continuous line is the Lanczos result for the sixteen-site cluster. Solid squares denote Monte Carlo results on a  $4 \times 4$  lattice at temperature  $T=t/12$ , while the open squares and the solid triangles are Monte Carlo results for the  $6 \times 6$  and  $8 \times 8$  clusters, respectively, with  $T=t/8$ ; (d)  $\langle n \rangle$  vs  $\mu$  at zero temperature for the ten-site Hubbard cluster with  $U/t=-8$  using Lanczos techniques.

particle-hole transformation, to the statement that the magnetic spin susceptibility of the positive- $U$  Hubbard model at half filling remains finite as the temperature  $T$  goes to zero. Likewise the result that  $\partial \langle n \rangle / \partial \mu |_{\mu=0}$  van-

Here  $N_s^{(+)}(\omega)$  (electron with spin of states for ground state lattice sites, creates a stable ground state its energy.  $\{ N \pm 1$  electrons  $N_i^{(+)}(\omega)$  (so  $\omega$  for  $U/t=$  Fig. 1) is shown half filling (the lower Hubbard bands are removed Comparing I

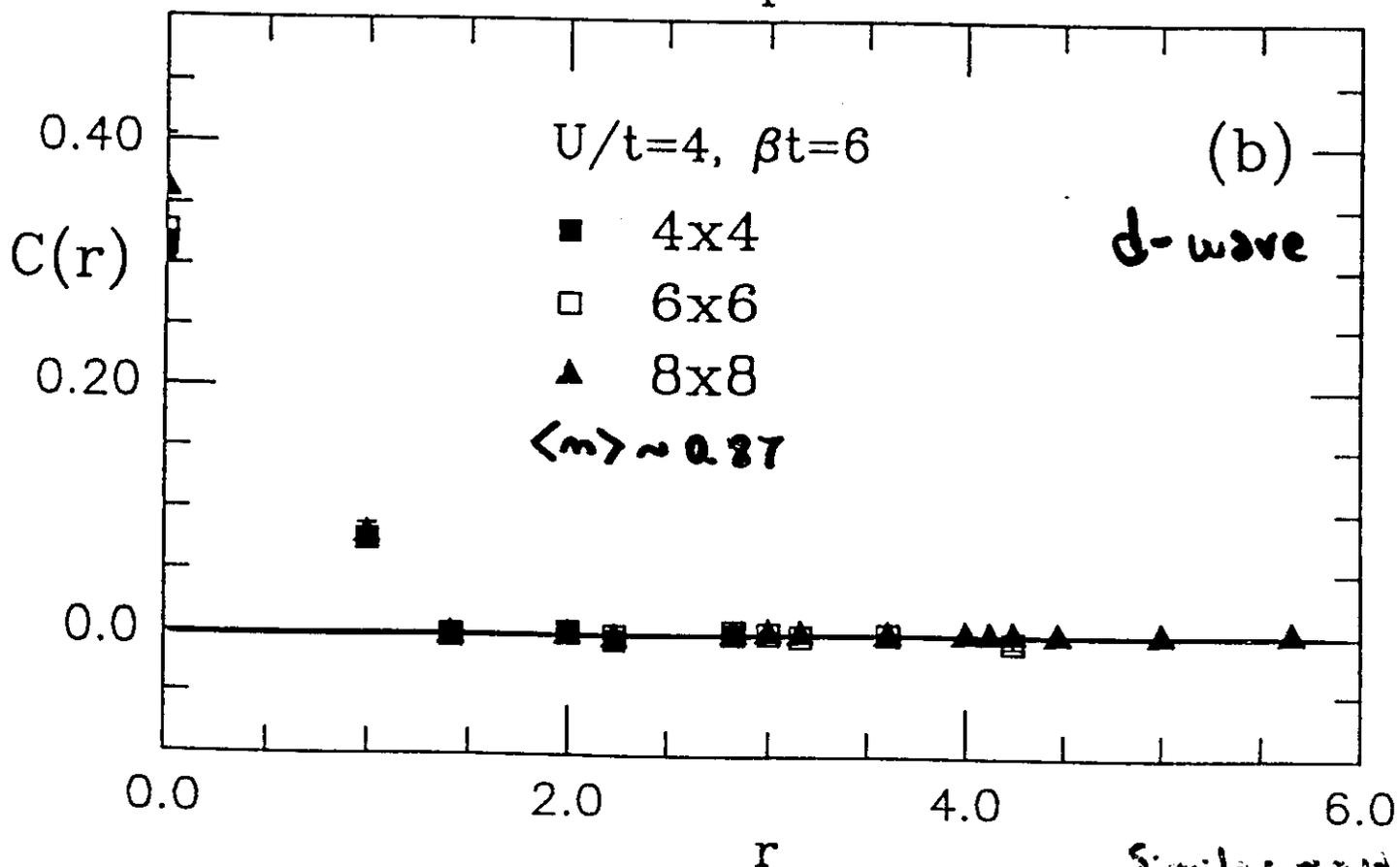
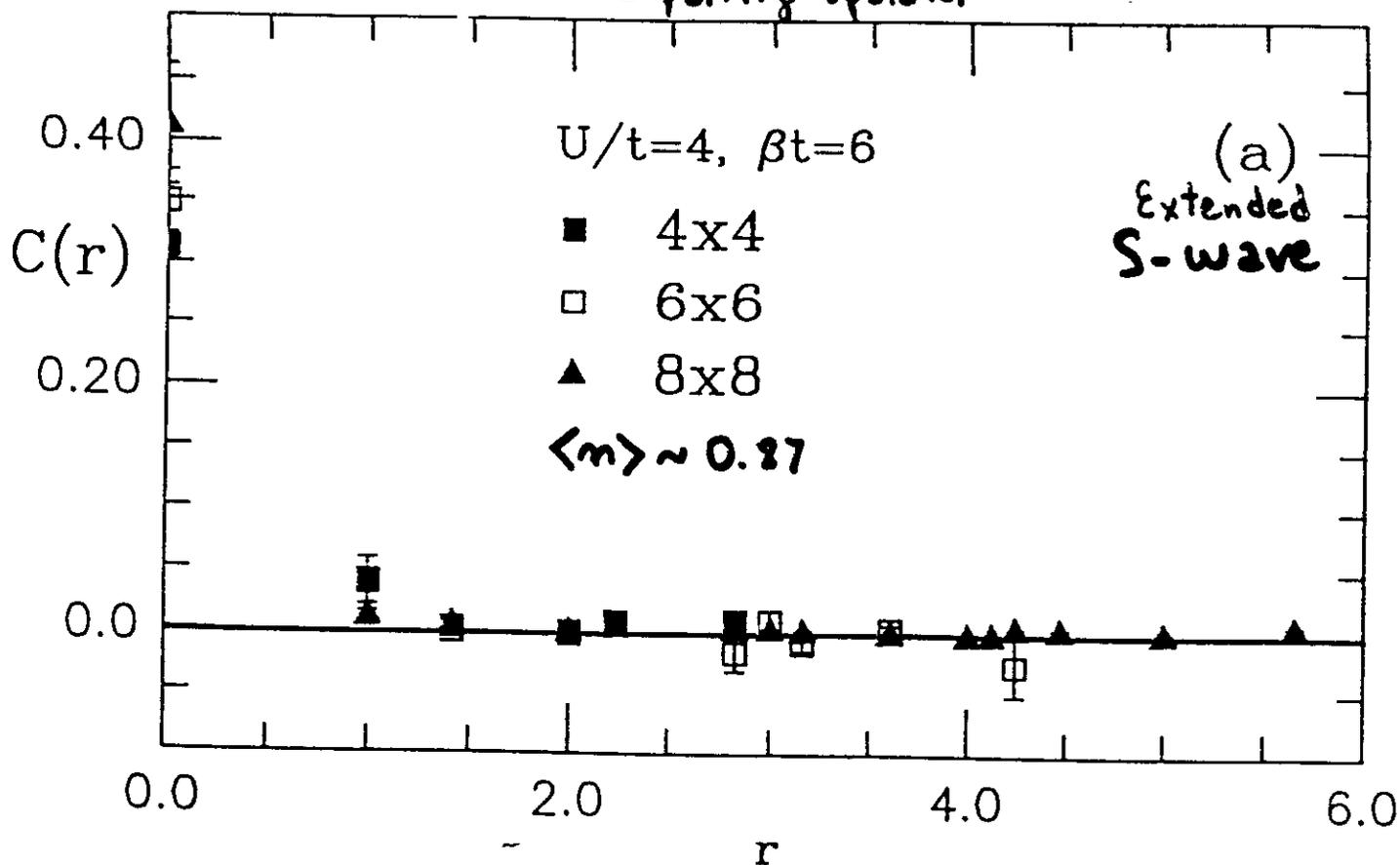
... thus far ...

	Exper.	Theory
$N(\omega)$ { <ul style="list-style-type: none"> <li>New states in gap</li> <li><math>\mu</math> changes when going from hole to <math>e^-</math> doping</li> </ul>	yes	yes
	yes (X-ray) (C. Chen et al. PRL <u>66</u> , 104) no (PES) (J. Allen et al., PRL <u>64</u> , 595)	yes
ARPES { <ul style="list-style-type: none"> <li>quasi particle</li> <li><math>\frac{m^*}{m} \sim 2-3</math></li> </ul>	maybe	yes
	yes (Arko et al.)	yes
Raman { <ul style="list-style-type: none"> <li>half-filling</li> <li>doping</li> </ul>	good agreement	
$G(\omega)$	good agreement mid-IR, Neff, ...	← See A. Tikofsky, Laughlin + Zou Q15 (anyon superc. state)
$S(\vec{q})$ { <ul style="list-style-type: none"> <li>shift of AF peak from <math>(\pi, \pi)</math> to <math>(0, \pi)</math></li> </ul>	yes	yes
Superconductivity	yes	?

$$C(r) = \langle \Delta(0) \Delta^\dagger(r) \rangle$$

↑ pairing operator

(A. Moreo, ~~to appear~~  
in PRR) published



**No superconductivity!**

Similar results  
using other  
methods and  
around Hubbard

Why we don't see superconductivity  
in the Hubbard model?

- $J = \frac{4t^2}{U} \sim 0.1 \text{ eV} \sim 1000 \text{ K}$   
 $t \sim 0.43 \text{ eV} \sim 4300 \text{ K}$   
 $T_c \sim 100 \text{ K}$  More difficult
- temperature in MC too high?
- Hubbard model doesn't superconduct
- we are not using the best operators

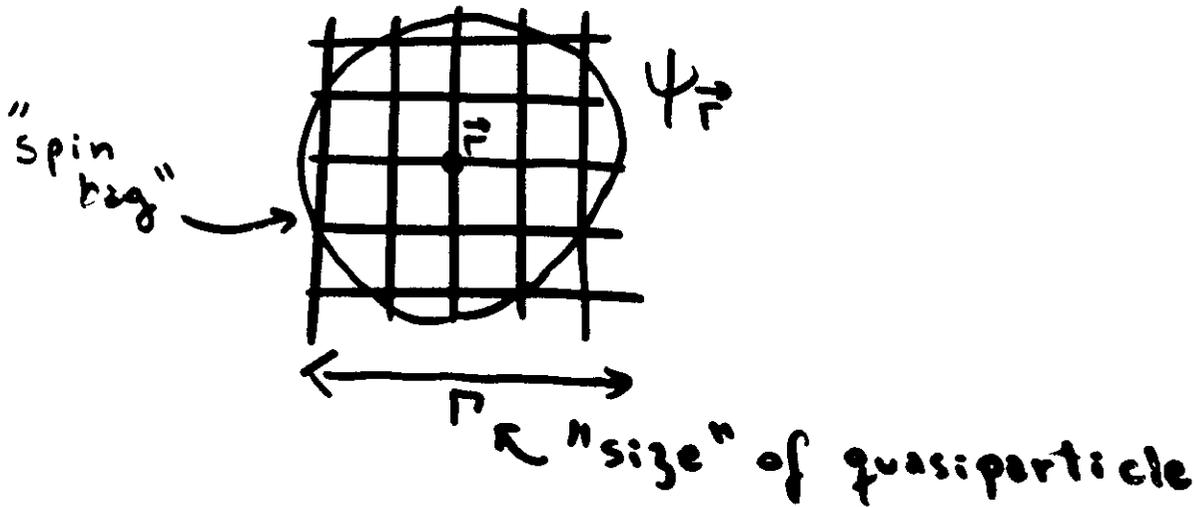
Intuitive idea:

(E.D. + J.A. Schrieffer)  
PRB 43, 2705 (1991);

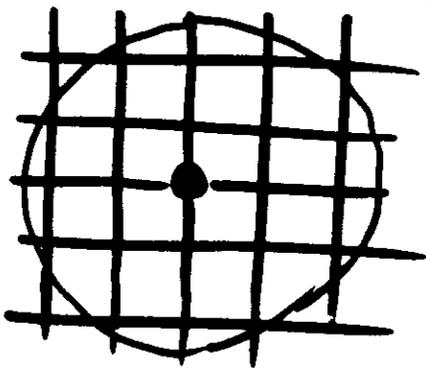
See also: M. Boninsegni  
+ E. Manousakis, PRB 43,

$H \psi_{\vec{q}} = E \psi_{\vec{q}}$  10353

$$\psi_{\vec{q}} = \sum_{\vec{r}} e^{i\vec{q} \cdot \vec{r}} \psi_{\vec{r}},$$



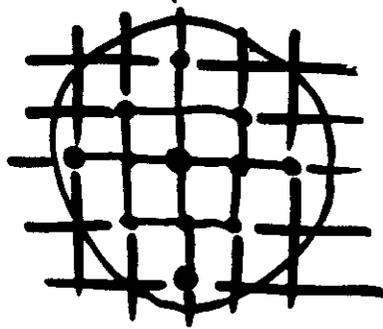
local "antenna"  
 $C_{\vec{r}}^+$



$Z \ll 1$

extended "antenna"

$$C_{\vec{r}}^+ = C_{\vec{r}}^+ + \alpha \sum_{\mu} C_{\vec{r}+\mu}^+ C_{\vec{r}} C_{\vec{r}-\mu}^+ + \dots$$



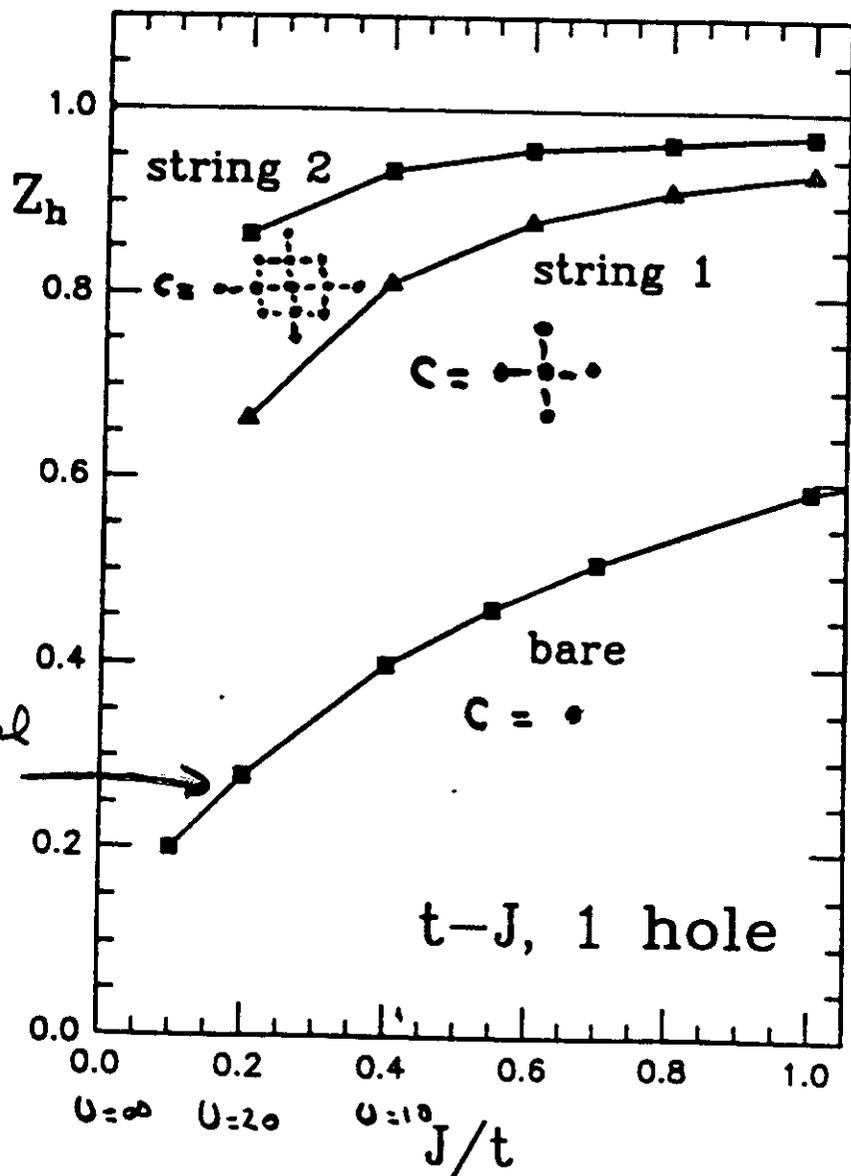
$Z \approx 1$

This concept is valid for any  $H$  and any operator involving quasiparticles. It filters out higher energy states.

Preliminary numerical results show that the idea works!  
( $Z \approx 1$ )

where  $H_t$  ( $H_{\text{Heis}}$ ) is the hopping (exchange) interaction in the  $t$ - $J$  model and  $\beta$  is another constant. The construction guarantees that to this order the spinlike excitations relevant for the dressing of the hole are created in its neighborhood. The hopping term moves the hole

t-J model  
1 hole



Note:  $Z_h \neq 0$  in  $t$ - $J$  model  
 $Z_h \sim J^{1/2}$  at small  $J$ .

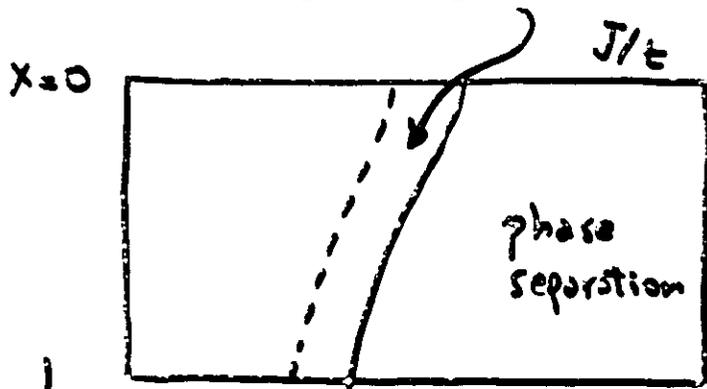
Confirmed by  
D. Poilblanc ( $N=26$ )

(In Hubbard model)  
Sorella claims  $Z_h \rightarrow 0$ )

FIG. 1.  $Z_h$  of one hole as a function of  $J/t$  for the  $t$ - $J$  model on a  $4 \times 4$  lattice. Results are shown using the bare fermionic operator ( $\blacksquare$ ), strings of length 1 ( $\blacktriangle$ ), and strings of length 2 ( $\bullet$ ) in the "dressed" fermionic operator.

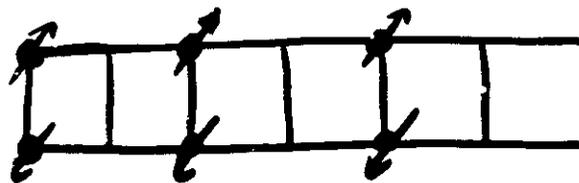
- $t$ - $J$  model is more promising

- In 1D there is a region where supercond. correlations dominate

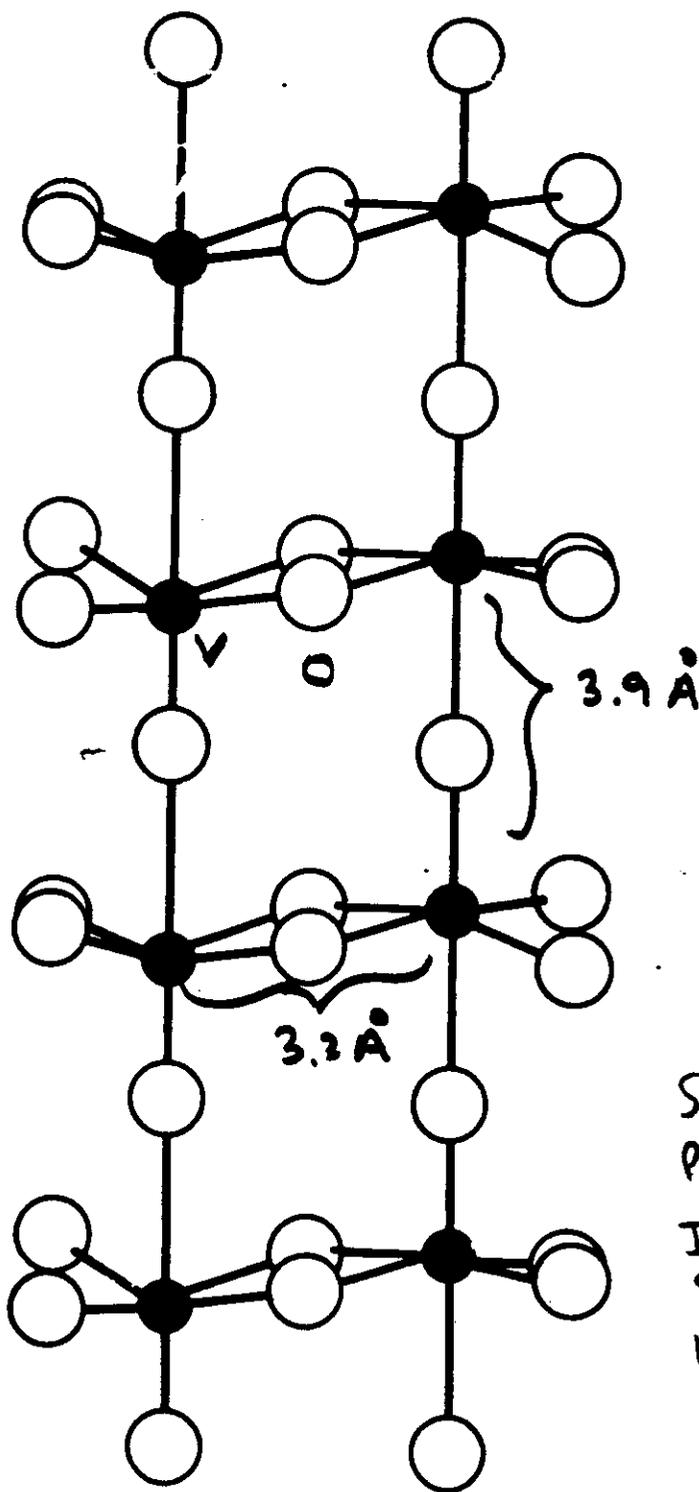
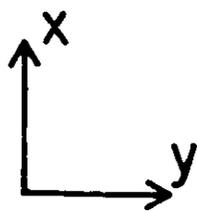
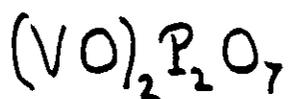


See M. Ogata  
et al., PRL 66  
2382 (91)

- In 2D a study of the spectrum of finite clusters suggest similar results See A. Moreo, PRB
- In "ladders" and  $J/t > 1$ , superconduc. dominates. (dimerization process)

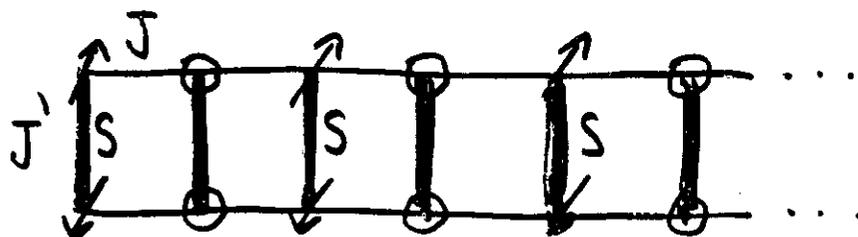


D.C. Johnston  
et al., PRB 35,  
219 (87)



See E.D. et al.,  
PRB, March 92,  
45, 5744  
Imada, J. Phys.  
Soc. Jpn. 60,  
1877 (91)

$$\langle n \rangle = 1/2$$



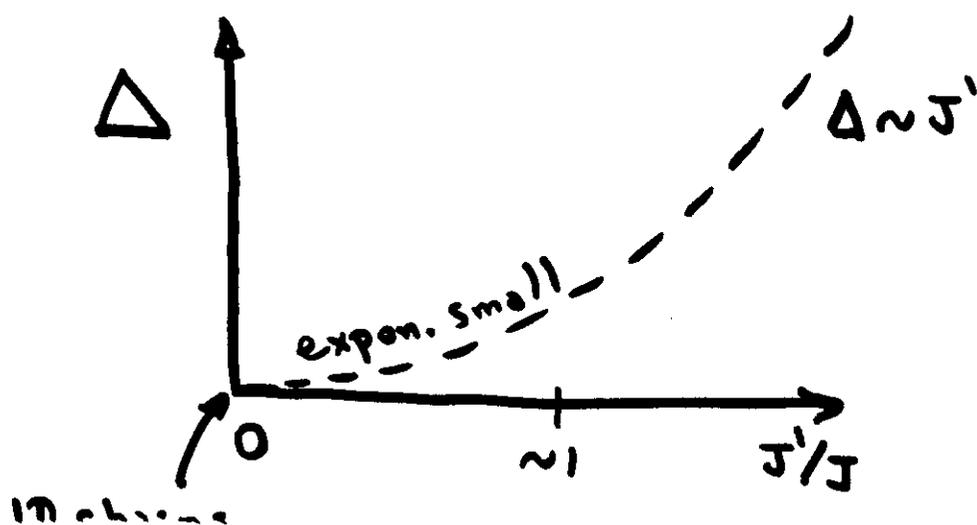
if  $J' \gg J, t \rightarrow$  superconductivity  
(as in  $U < 0$  Hubbard)

Numerical results suggest that  
for  $J' \sim J$  we can still have supercond.

No phonons, purely electronic.

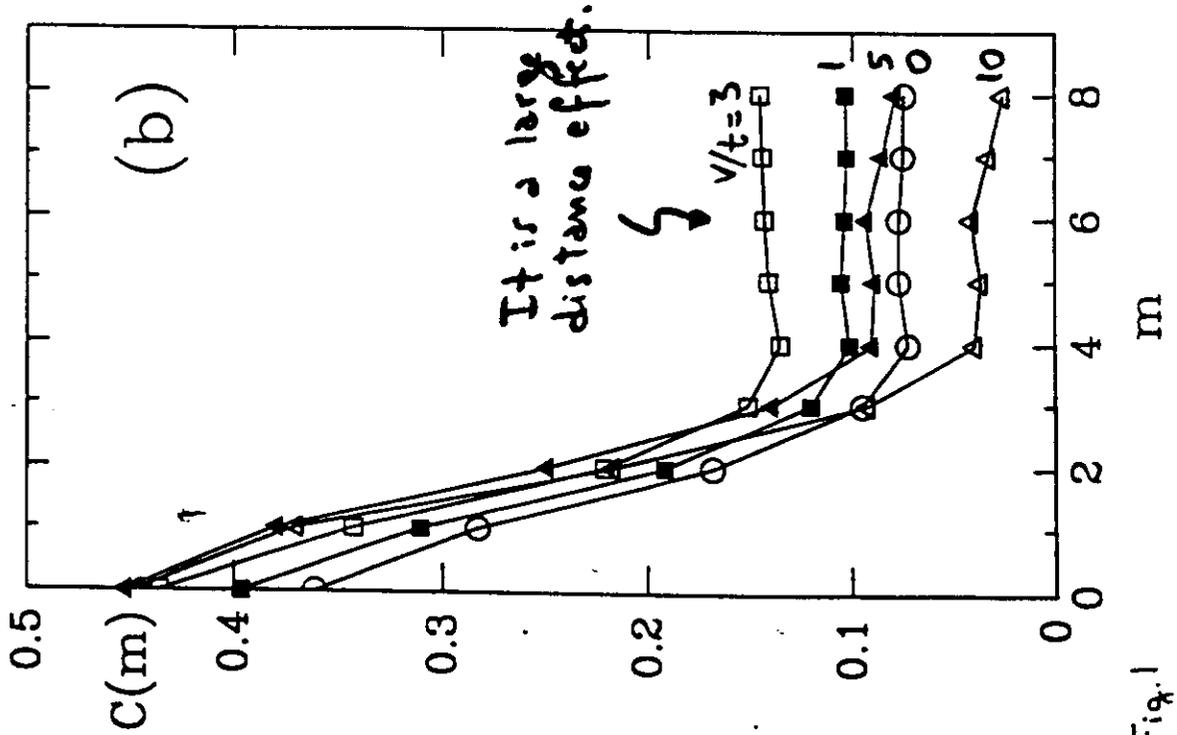
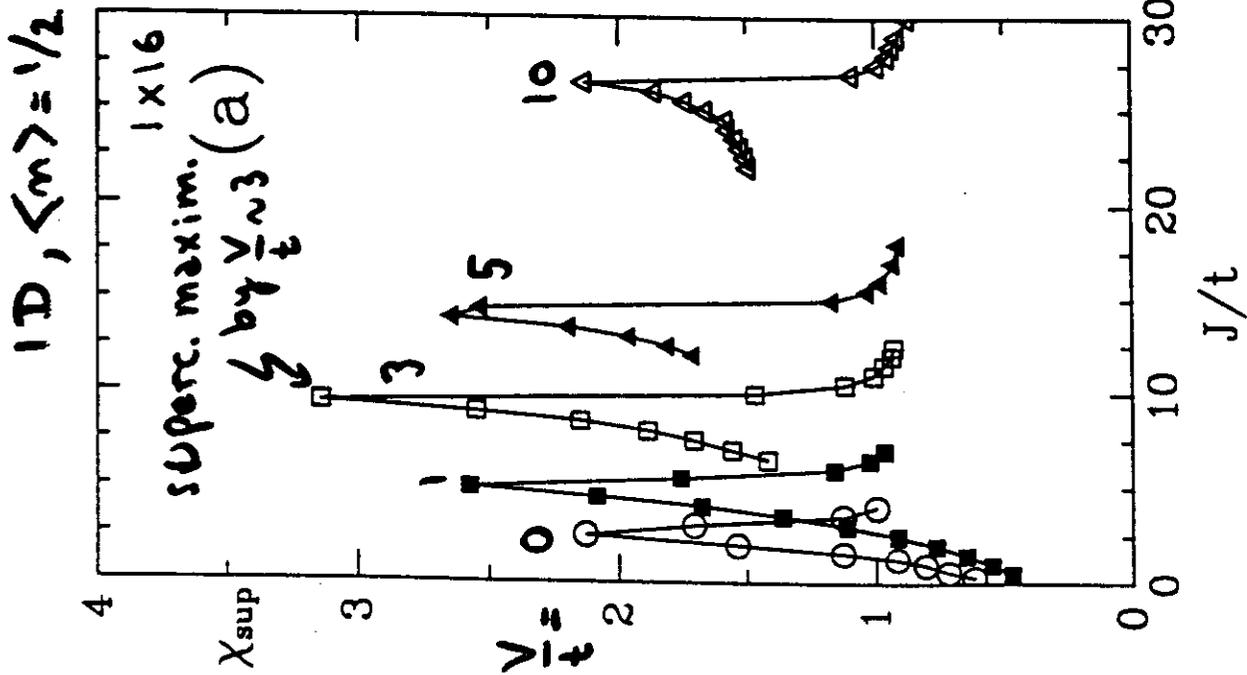
Recent QMC results at  $\langle n \rangle = 1$   
suggest:

(Barnes et al.)



t-J-V model

$\left( \sqrt{\sum_{ij} m_i m_j} \right) \leftarrow$  See also Kovarik + Barnes

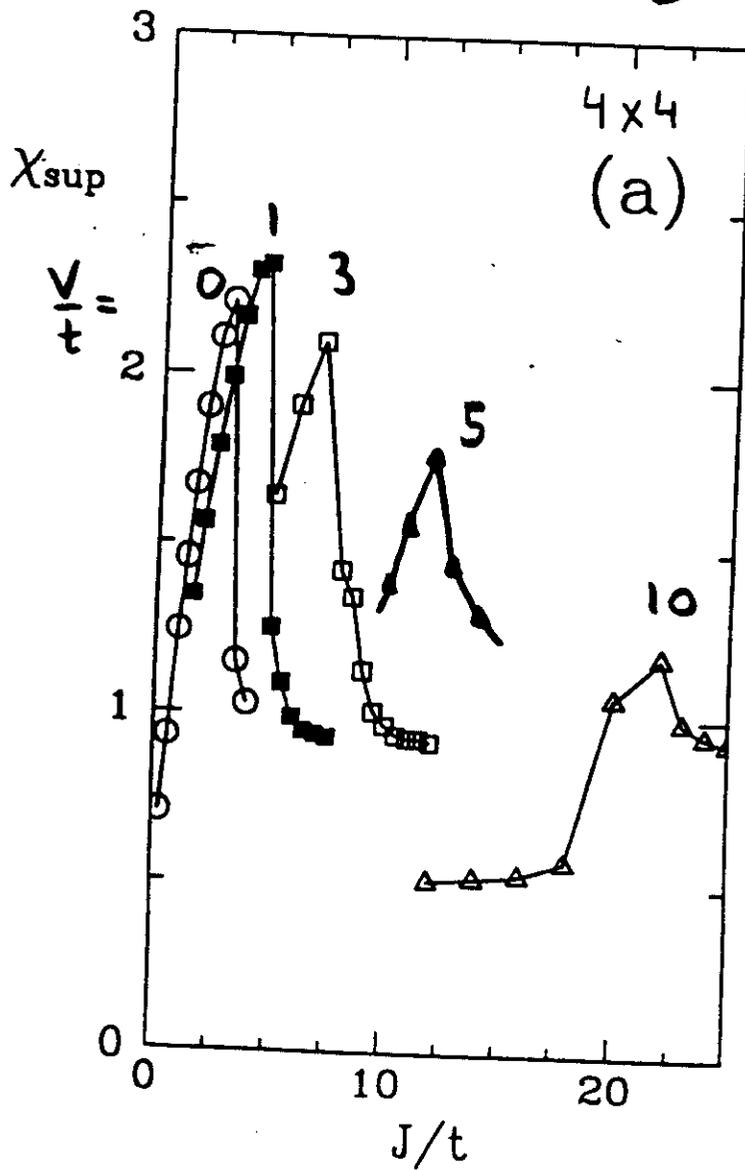


E.D.  
 + Jose Riera,  
 preprint

Fig. 1

2D,  $\langle n \rangle = 1/2$

Very similar to 1D.



There is a spin-gap.

The same quantity near half-filling is flat. So we may have been looking at the wrong doping!

Fig

Why the superconducting correlations are enhanced in 2D near  $\langle n \rangle = 1/2$ ?

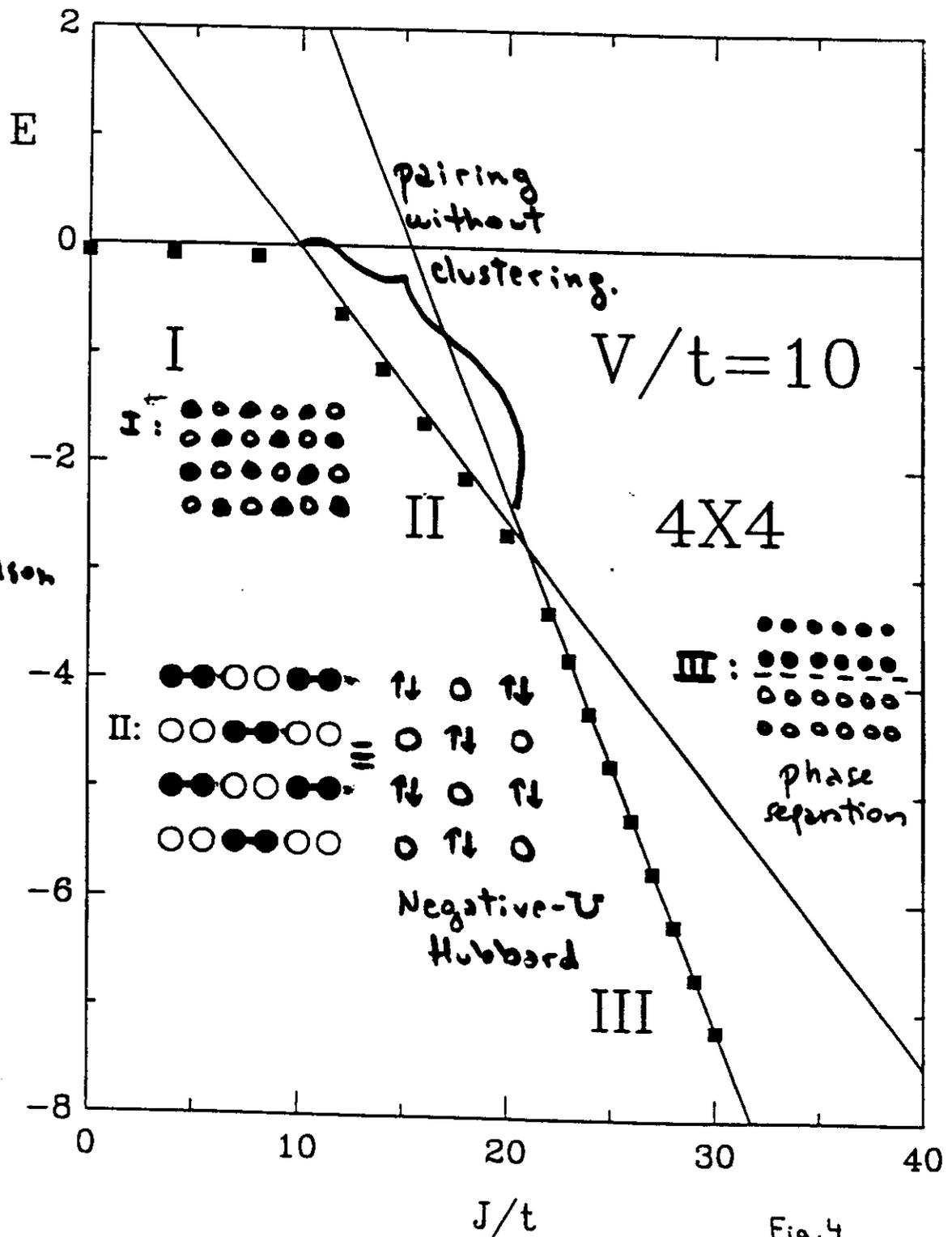
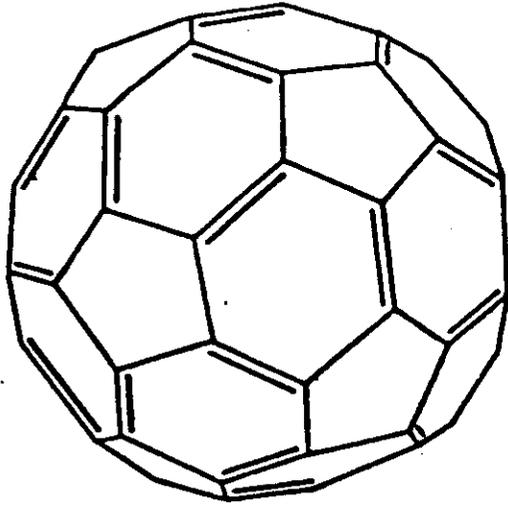
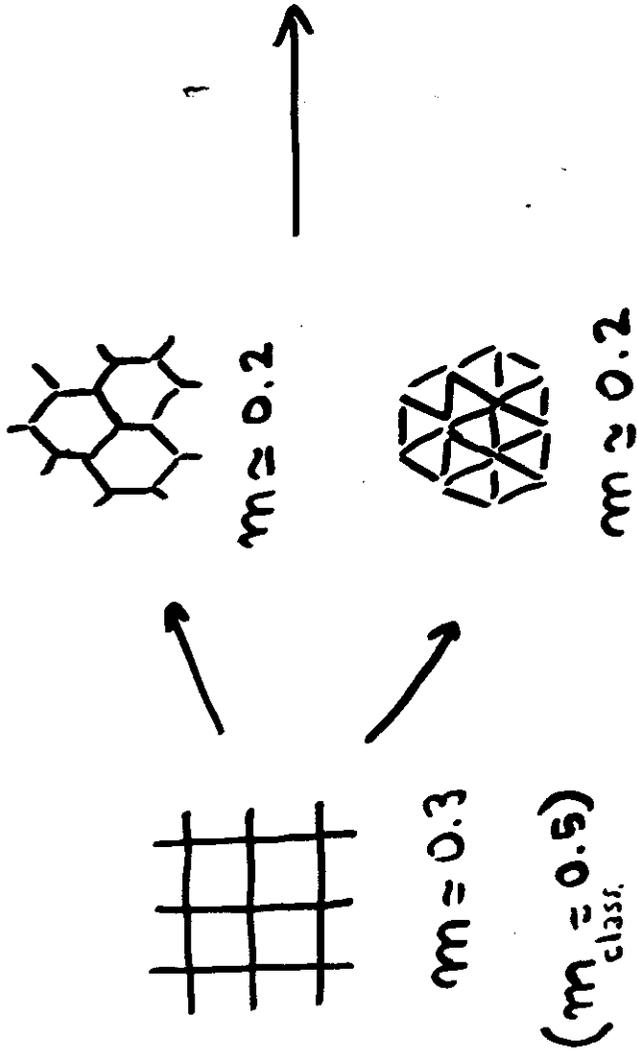


Fig. 4

C60:

Heisenberg model on a ball  
(spin  $1/2$ )



$m = ?$

First results suggest

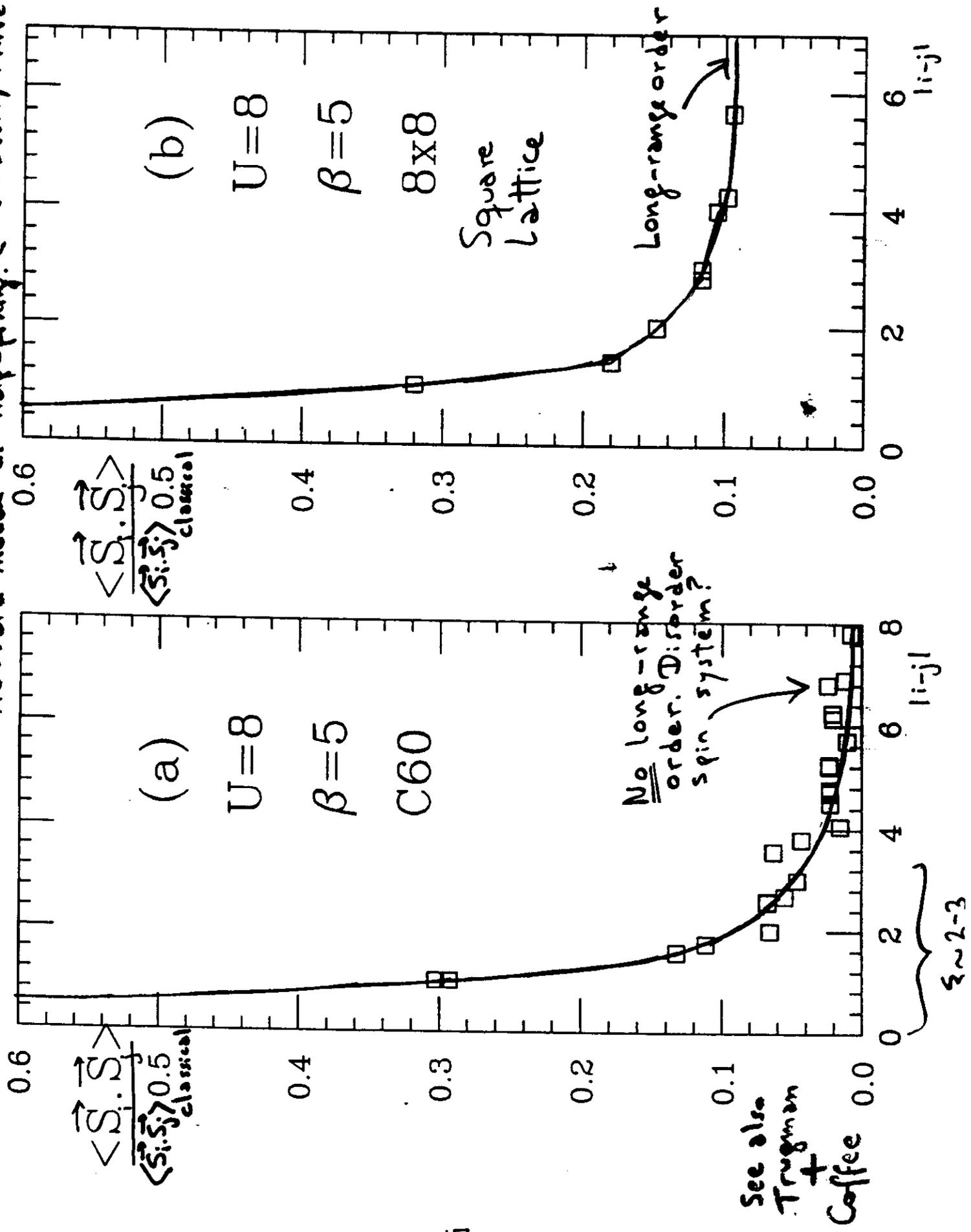
$m \approx 0$

(disordered q. state)

R. Scalettar  
T. Jolicoeur  
H. Monien

E. D., ~~in preparation.~~  
sent to PRL.

Hubbard model at half-filling. (Chakravarty + Kivelson)



# Conclusions

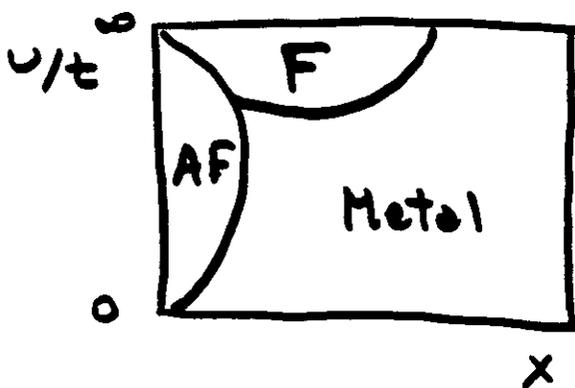
1) Some "unusual" normal state properties may be explained by Hubbard model.

- a) quick loss of AF with  $x$
- b)  $S(q)$  IC
- c)  $N(\omega)$

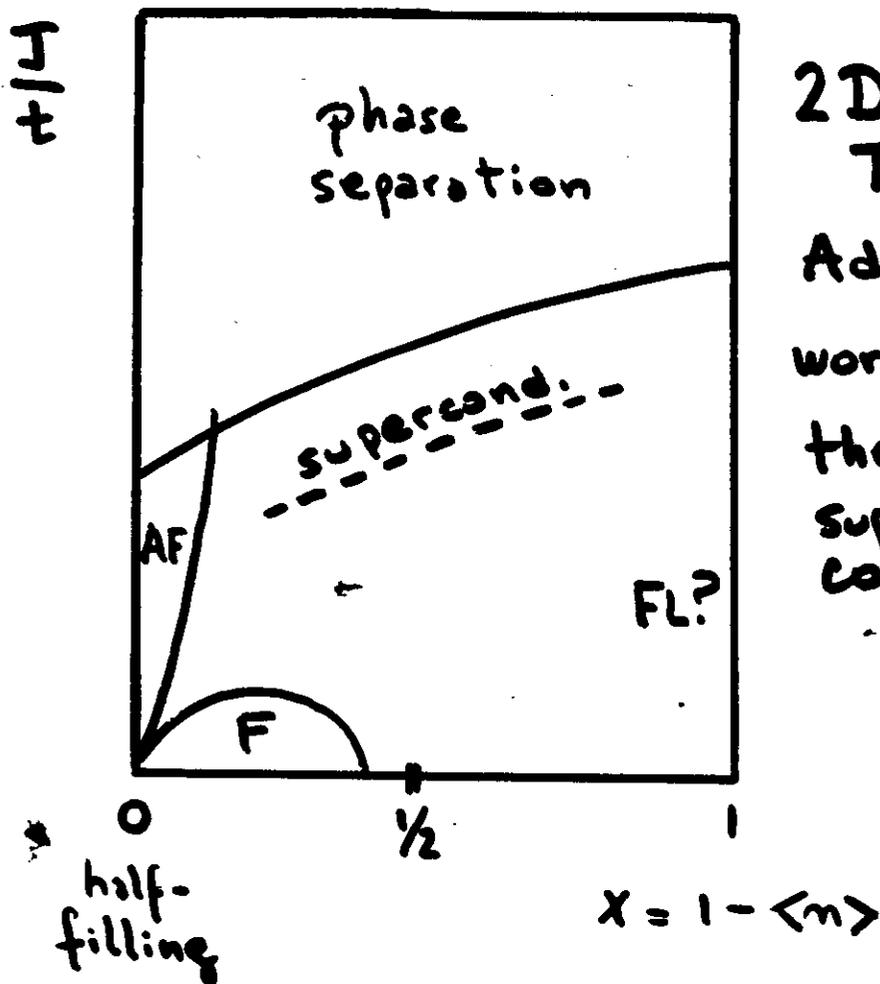


2) Superconductivity in Hubbard model?

Either the model has to be "modified" or new quasiparticle operators will solve the problem.



3) The  $t$ - $J$  model seems to have more structure:



2D  $t$ - $J$   
 $T=0$

Adding  $\frac{V}{t}$  and working at  $\langle n \rangle = \frac{1}{2}$  there are strong superconducting correlations.

