INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
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SMR.627-19

MINIWORKSHOP ON
STRONGLY CORRELATED ELECTRON SYSTEMS

15 JUNE - 10 JULY 1992

"1-d Hubbard model in magnetic field and the equivalent Tomonaga model"

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1-d Hubbard model in magnetic field and the equivalent Tomonaga model

with Karlo Penc

Outline:

1) Motivation.

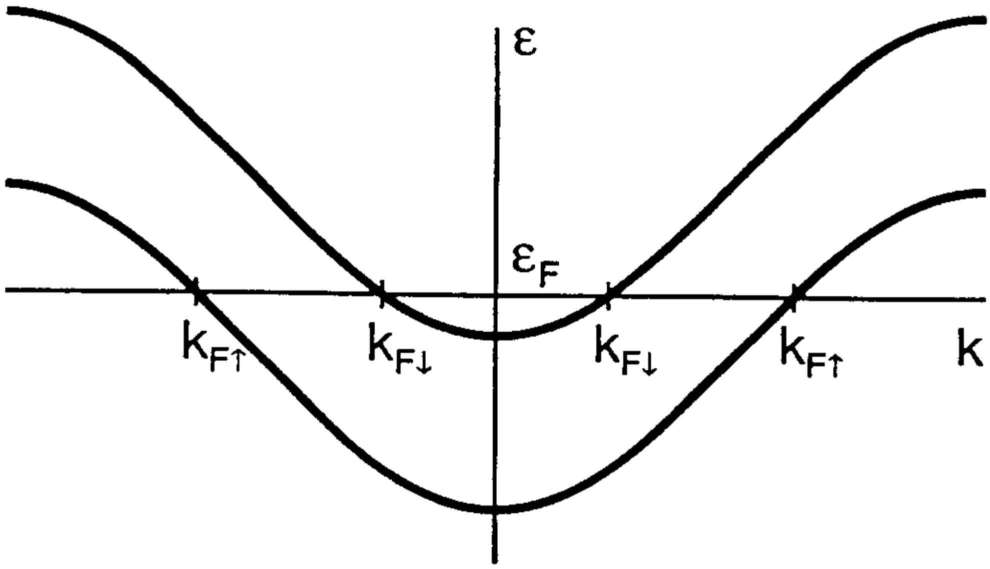
2) Weak coupling limit.

3) Spectrum and correlation functions of the 1-d Hubbard model.

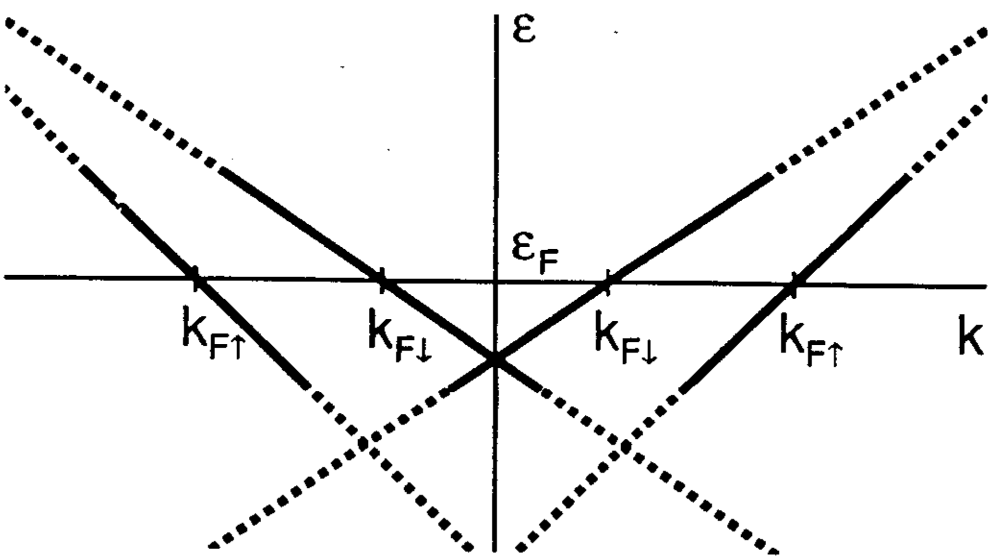
4) Spectrum and correlation functions of generalized Tomonaga models.

5) Comparison of the models, mapping between the couplings.

6) Conclusions

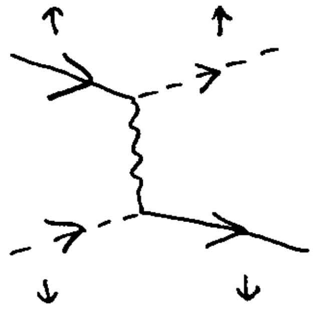

ε
ε$_F$
k$_{F\uparrow}$
k$_{F\downarrow}$
k$_{F\downarrow}$
k$_{F\uparrow}$
k

(a)

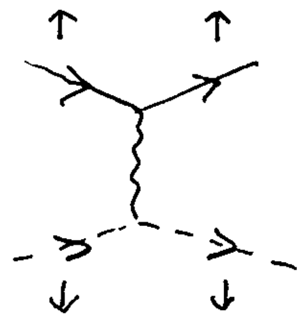

ε
ε$_F$
k$_{F\uparrow}$
k$_{F\downarrow}$
k$_{F\downarrow}$
k$_{F\uparrow}$
k

(b)

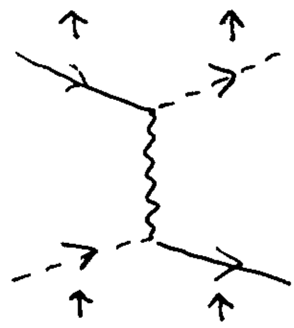
Couplings in the weak coupling limit:



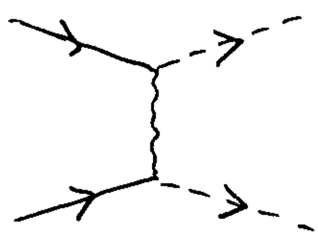
$g_{1\perp}$



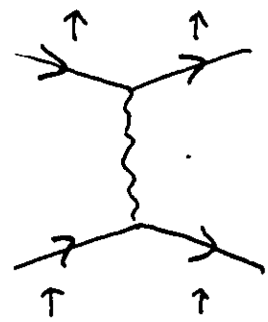
$g_{2\perp}$



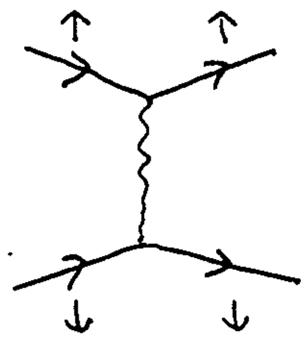
$g_{1\parallel} - g_{2\parallel}$



g_3



$g_{4\parallel}$



$g_{4\perp}$

In magnetic field there is a further splitting

$$g_{\parallel} \begin{array}{l} \nearrow g_{\uparrow} \\ \searrow g_{\downarrow} \end{array}$$

Weak coupling "g-ology" model

$$g_{1\parallel} = g_{2\parallel} = g_{4\parallel} = 0$$

$$g_{1\perp} = g_{2\perp} = g_{4\perp} = U$$

$g_3 = 0$ for non-half-filled band

Scaling theory gives:

$$g_{1\perp} \to 0 \qquad g_{1\parallel} - g_{2\parallel} + g_{2\perp} \to 0$$

$$g_{1\parallel} - g_{2\parallel} - g_{2\perp} = \text{invariant}$$

In the fixed point:

$$g_{1\parallel}^* = g_{1\perp}^* = 0$$

$$g_{2\parallel}^* = g_{2\perp}^* = \frac{U}{2}$$

$$v_c = v_F + \frac{U}{2\pi} \quad , \quad v_S = v_F - \frac{U}{2\pi}$$

Hubbard model in magnetic field

$$H = -\sum_{j,\sigma}\left(c_{j\sigma}^{+} c_{j+1\sigma} + h.c.\right) + U\sum_j n_{j\uparrow}n_{j\downarrow}$$

$$-\mu\sum_j\left(n_{j\uparrow}+n_{j\downarrow}\right) - \frac{h}{2}\sum_j\left(n_{j\uparrow}-n_{j\downarrow}\right)$$

$t = 1$

Lieb - Wu equations:

$N_c = N_\uparrow + N_\downarrow$ pseudomomenta k_j

$N_s = N_\downarrow$ rapidities λ_α

$$N k_j = 2\pi I_j - \sum_{\beta=1}^{N_s} 2\arctan\frac{\sin k_j - \lambda_\beta}{U/4},$$

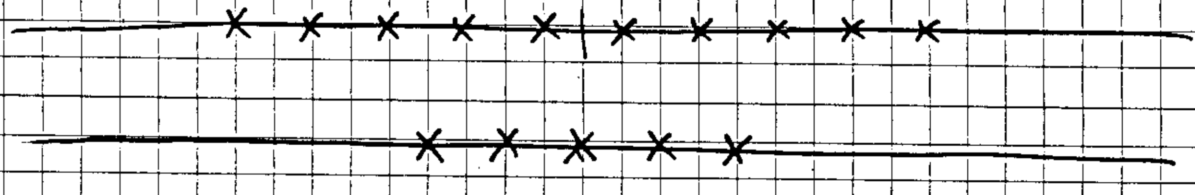
$$\sum_{j=1}^{N_c} 2\arctan\frac{\lambda_\alpha - \sin k_j}{U/4} = 2\pi J_\alpha + \sum_{\beta=1}^{N_s} 2\arctan\frac{\lambda_\alpha-\lambda_\beta}{U/2}.$$

If N_c even and N_s odd,

$$I_j = \pm 1/2,\ \pm 3/2,\ \pm 5/2,\ \dots$$

$$J_\alpha = 0,\ \pm 1,\ \pm 2,\ \dots$$

Ground state:

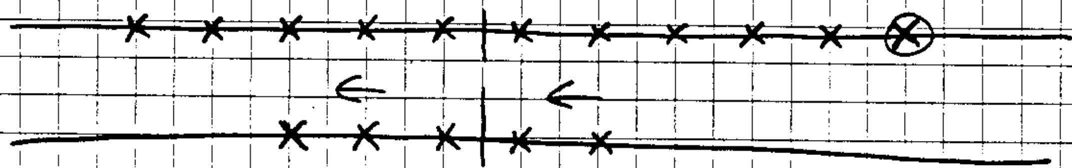


I_j

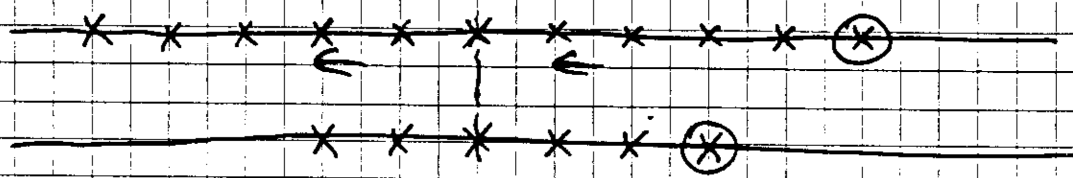
J_α

Excited states:

1) add one spin-up particle

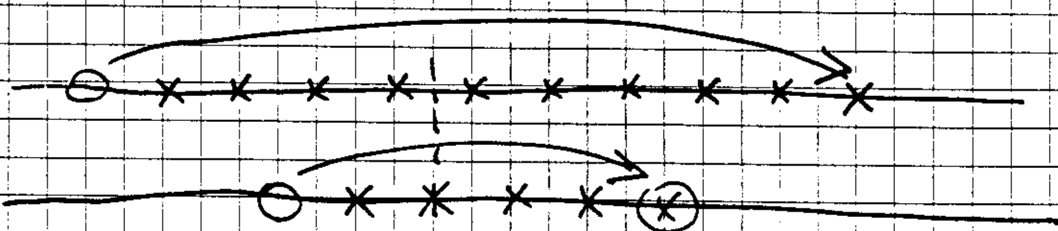


2) add one spin-down particle



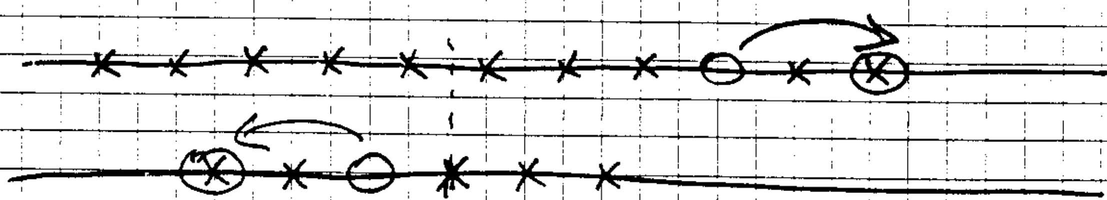
change in number: ΔN_c, ΔN_s

3.) create $2k_F$ excitations



asymmetries D_c, D_s

4) create small momentum excitations



theirs members: n_c^+, n_c^-, n_s^+, n_s^-

To leading order in $1/N$ (Woynarovich)

$$E(\Delta N, D, n) - E_0 =$$

$$= \frac{2\pi}{N} u_c \left(\Delta_c^+ + \Delta_c^- + n_c^+ + n_c^- - \frac{1}{12} \right)$$

$$+ \frac{2\pi}{N} u_s \left(\Delta_s^+ + \Delta_s^- + n_s^+ + n_s^- - \frac{1}{12} \right) + O(1/N)$$

$$P(\Delta N, D, n) = 2D_c (k_{F\uparrow} + k_{F\downarrow}) + 2D_s k_{F\downarrow}$$

$$+ \frac{2\pi}{N} \left(\Delta_c^+ - \Delta_c^- + n_c^+ - n_c^- \right)$$

$$+ \frac{2\pi}{N} \left(\Delta_s^+ - \Delta_s^- + n_s^+ - n_s^- \right) + O(1/N)$$

with

$$2\Delta_c^{\pm} = \left(\pm \frac{Z_{ss}\,\Delta N_c - Z_{cs}\,\Delta N_s}{2 \det Z} + Z_{cc} D_c + Z_{sc} D_s \right)^2$$

$$2\Delta_s^{\pm} = \left(\mp \frac{Z_{sc}\,\Delta N_c - Z_{cc}\,\Delta N_s}{2 \det Z} + Z_{cs} D_c + Z_{ss} D_s \right)^2$$

Z are solution of coupled inhomogeneous integral equations.

With vectors

$$|\Delta N\rangle = \begin{pmatrix} \Delta N_c \\ \Delta N_s \\ 2D_c \\ 2D_s \end{pmatrix}, \quad |\omega^{\pm}_{c,s}\rangle = \begin{pmatrix} Z_\bullet \\ Z_\bullet \\ Z_\bullet \\ Z_\bullet \end{pmatrix} \qquad 2\Delta^{\pm}_{c,s} = \langle \omega^{\pm}_{c,s} | \Delta N \rangle^2$$

Any excited state is characterized by

$$\Delta N_c,\ \Delta N_s,\ D_c,\ D_s,\ n_c^+,\ n_c^-,\ n_s^+,\ n_s^-$$

Alternatively we could use:

$$\Delta N = \Delta N_\uparrow^+ + \Delta N_\uparrow^- + \Delta N_\downarrow^+ + \Delta N_\downarrow^-$$

$$\Delta M = \Delta N_\uparrow^+ + \Delta N_\uparrow^- - \Delta N_\downarrow^+ - \Delta N_\downarrow^-$$

$$J_c = (\Delta N_\uparrow^+ - \Delta N_\uparrow^-) k_{F\uparrow} + (\Delta N_\downarrow^+ - \Delta N_\downarrow^-) k_{F\downarrow}$$

$$J_s = (\Delta N_\uparrow^+ - \Delta N_\uparrow^-) k_{F\uparrow} - (\Delta N_\downarrow^+ - \Delta N_\downarrow^-) k_{F\downarrow}$$

This will indicate spin-charge separation, if $(\Delta N, J_c)$ are decoupled from $(\Delta M, J_s)$.

This will only be valid for

$$h = 0 \ !$$

Correlation functions

$\phi(x,t)$ is a combination of creation and annihilation operators. This determines ΔN_c and ΔN_s and the possible values of D_c and D_s.

$\phi(x,t)$ couples the ground state to several towers.

$$\langle \phi(x,t)\,\phi(0,0)\rangle =$$

$$= \sum_{D_c, D_s} a(D_c D_s) \frac{e^{-2iD_c k_{F\uparrow} x - 2i(D_c+D_s) k_{F\downarrow} x}}{(x-u_c t)^{\alpha_c^+} (x+u_c t)^{\alpha_c^-} (x-u_s t)^{\alpha_s^+} (x+u_s t)^{\alpha_s^-}}$$

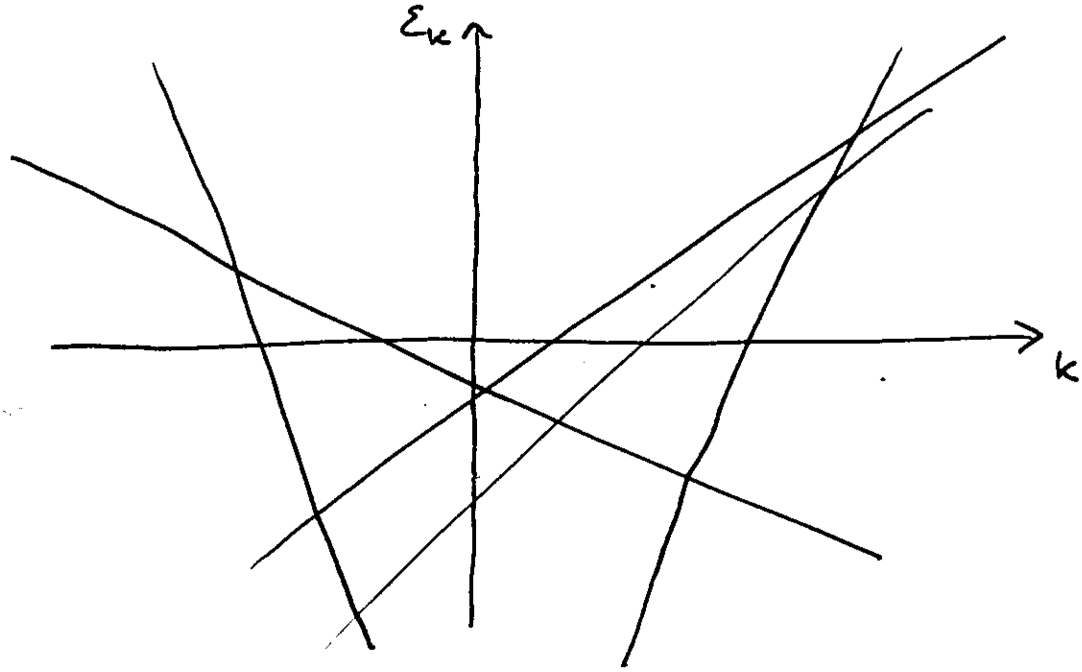
$$\alpha_c^{\pm} = 2\,\Delta_c^{\pm}$$

$$\alpha_s^{\pm} = 2\,\Delta_s^{\pm}$$

What is the equivalent to the Hubbard model in magnetic field?

At $U = 0$ the velocities: $v_\uparrow \neq v_\downarrow$

The Tomonaga model has to be generalized to this case



Branch for particles of "color" λ is characterized by v_λ and $k_{F\lambda}$

The interaction is $g_{\lambda\lambda'}$

Generalized Tomonaga model

$$H = \sum_{\lambda} H_{\lambda} + \sum_{\lambda\lambda'} H_{\lambda\lambda'}$$

$$H_{\lambda} = \sum_{k} v_{\lambda} (k - k_{F\lambda})\, a^{+}_{k,\lambda} a_{k,\lambda}$$

$$H_{\lambda\lambda'} = \frac{1}{2N} \sum_{k,k',q} g_{\lambda\lambda'}\, a^{+}_{k,\lambda} a^{+}_{k',\lambda'} a_{k'+q,\lambda'} a_{k-q,\lambda}$$

$$= \frac{1}{2N} \sum_{q} g_{\lambda\lambda'}\, \rho_{\lambda}(q)\, \rho_{\lambda'}(-q)$$

$$\left[\rho_{\lambda}(-q), \rho_{\lambda'}(q')\right] = q \frac{N}{2\pi} \delta_{\lambda\lambda'} \delta_{qq'} \operatorname{sign} v_{\lambda}$$

Mattis and Lieb have shown that

$$H_{\lambda} = \frac{2\pi}{N} \sum_{q>0} |v_{\lambda}| \begin{cases} \rho_{\lambda}(q)\, \rho_{\lambda}(-q) & v_{\lambda} > 0 \\ \rho_{\lambda}(-q)\, \rho_{\lambda}(q) & v_{\lambda} < 0 \end{cases}$$

Haldane added:

$$\frac{\pi}{N} |v_{\lambda}| \left(\Delta N_{\lambda}\right)^{2}$$

The total Hamiltonian is:

$$H = \frac{2\pi}{N} \sum_{\substack{\lambda\lambda' \\ q>0}} A_{\lambda\lambda'}\, \rho_\lambda(q)\, \rho_{\lambda'}(-q)$$

$$+ \frac{\pi}{N} \sum_{\lambda\lambda'} A_{\lambda\lambda'}\, \Delta N_\lambda\, \Delta N_{\lambda'}$$

with $A_{\lambda\lambda'} = |v_\lambda|\, \delta_{\lambda\lambda'} + \frac{g_{\lambda\lambda'}}{2\pi}$

It can be diagonalized:

$$E(\Delta N_\lambda, n_j) - E_0 = \frac{2\pi}{N} \sum_j |u_j| \left(\Delta^{(j)} + n_j\right) + o(1/N)$$

$$P(\Delta N_\lambda, n_j) = \sum_\lambda \Delta N_\lambda k_{F\lambda} + \frac{2\pi}{N} \sum_j \left(\Delta^{(j)} + n_j\right) \operatorname{sign} u_j$$

where $2\Delta^{(j)} = \langle w^{(j)} | \Delta N \rangle^2$

and u_j and $|w^{(j)}\rangle$ are obtained from an eigenvalue problem

$$(A \cdot B)\, |w^{(j)}\rangle = u_j\, |w^{(j)}\rangle$$

$$B_{\lambda\lambda'} = \delta_{\lambda\lambda'} \cdot \operatorname{sign} v_\lambda$$

The correlation functions could be:

$$\langle \phi(x,t)\,\phi(0,0)\rangle \sim \frac{e^{-i\sum_\lambda k_{F\lambda}\,\Delta N_\lambda \cdot x}}{\prod_j (x-u_j t)^{2\Delta^{(j)}}}$$

ambiguity in $\Delta^{(j)}$. We check this with direct calculation.

$$G_{\lambda_1\lambda_2\ldots\lambda_m}(x_1', x_2', \ldots, x_m'; x_1, x_2, \ldots, x_m) =$$

$$=(-i)^m \langle T\, \psi_{\lambda_1}(x_1')\,\psi_{\lambda_2}(x_2')\ldots\psi_{\lambda_m}(x_m')\cdot$$

$$\cdot\,\psi^+_{\lambda_m}(x_m)\ldots\psi^+_{\lambda_2}(x_2)\,\psi^+_{\lambda_1}(x_1)\rangle$$

The equation of motion gives:

$$\left(\frac{\partial}{\partial t_m} + v_{\lambda_m}\frac{\partial}{\partial x_m}\right) G_{\lambda_1\lambda_2\ldots\lambda_m}(x_1', x_2', \ldots, x_m'; x_1, x_2, \ldots x_m) =$$

$$i\,\delta(x_m'-x_m)\,\delta(t_m'-t_m)\,G_{\lambda_1\lambda_2\ldots\lambda_{m-1}}(x_1', \ldots x_{m-1}'; x_1 x_2 \ldots x_{m-1})$$

$$-\sum_\lambda g_{\lambda_m\lambda}\, F^{\lambda}_{\lambda_1\lambda_2\ldots\lambda_m}(x_1', x_2', \ldots, x_m'; x_1, x_2, \ldots, x_m; x_m^+)$$

where

$$F^{\lambda}_{\lambda_1\lambda_2\ldots\lambda_m}(x_1', x_2', \ldots x_m'; x_1, x_2 \ldots, x_m; x) =$$

$$=(-i)^{m+1}\langle T\,\psi_{\lambda_1}(x_1')\cdots\psi_{\lambda_m}(x_m')\,\rho_\lambda(x)\,\psi^+_{\lambda_m}(x_m)\cdots\psi^+_{\lambda_1}(x_1)\rangle$$

In integrated form:

$$G_{\lambda_1 \ldots \lambda_m}(x_1', \ldots x_m'; x_1, \ldots x_m) =$$

$$G^{(0)}_{\lambda_m}(x_m', x_m)\, G_{\lambda_1 \ldots \lambda_{m-1}}(x_1', \ldots x_{m-1}'; x_1, \ldots x_{m-1})$$

$$+ i \sum_{\lambda} \int d^2x\, g_{\lambda_m \lambda}\, F^{\lambda}_{\lambda_1 \ldots \lambda_m}(x_1', \ldots x_m', x_1, \ldots, x; x^+)\, G^{(0)}_{\lambda_m}(x, x_m)$$

Equation of motion for F gives in Fourier transformed form:

$$(i\omega - i q v_\lambda)\, F^{\lambda}_{\lambda_1 \ldots \lambda_m}(x_1', \ldots x_m', x_1 \ldots x_m, q, \omega) =$$

$$i \sum_{\ell} \left[e^{i(q x_\ell' - \omega t_\ell')} - e^{i(q x_\ell - \omega t_\ell)} \right] \delta_{\lambda \lambda_\ell}\, G_{\lambda_1 \ldots \lambda_m}(x_1', \ldots x_m'; x_1 \ldots x$$

$$+ i q \operatorname{sign} v_\lambda \sum_{\lambda'} \frac{g_{\lambda\lambda'}}{2\pi} F^{\lambda'}_{\lambda_1 \ldots \lambda_m}(x_1' \ldots x_m'; x_1 \ldots x_m, q, t)$$

The same diagonalization problem as before

$$G_{\lambda \lambda_2 \ldots \lambda_m}(x_1', \ldots x_m'; x_1 \ldots x_m) =$$

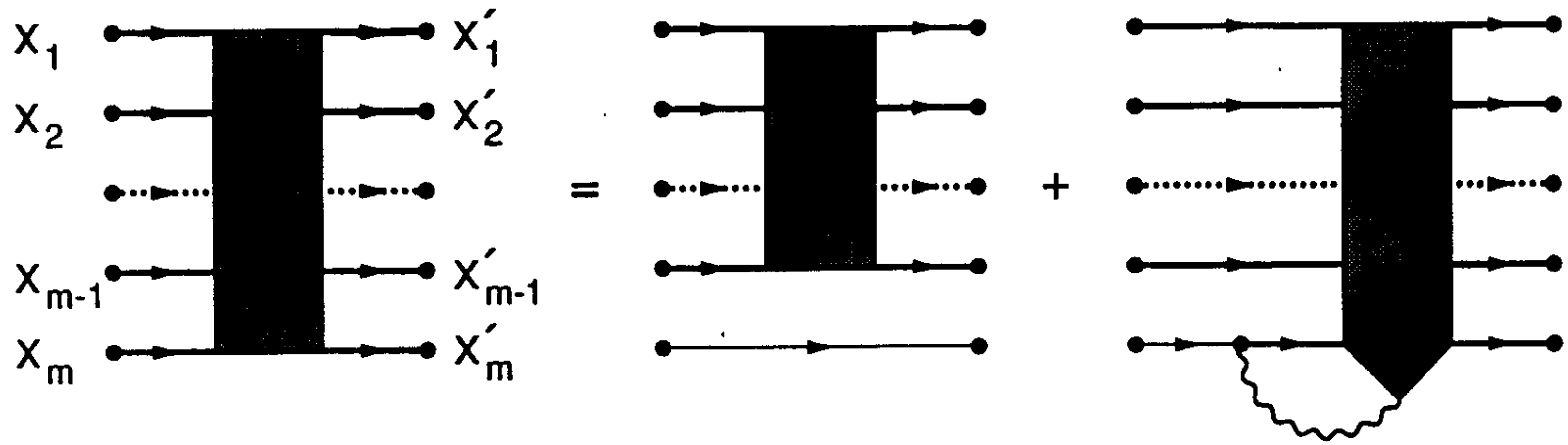
$$= G^{(0)}_{\lambda_m}(x_m', x_m)\, G_{\lambda_1 \ldots \lambda_{m-1}}(x_1', \ldots x_{m-1}'; x_1, \ldots x_{m-1})$$

$$+ i \int d^2x\, G_{\lambda_1 \lambda_2 \ldots \lambda_m}(x_1', \ldots x_m'; x_1, \ldots, x)$$

$$\times \big[K_{\lambda_m \lambda_1}(x_1' - x) - K_{\lambda_m \lambda_1}(x_1 - x) +$$

$$K_{\lambda_m \lambda_2}(x_2' - x) - K_{\lambda_m \lambda_2}(x_2 - x) + \ldots$$

$$\ddot{K}_{\lambda_m \lambda_m}(x_m' - x) - K_{\lambda_m \lambda_m}(x - x) \big]\, G^{(0)}_{\lambda_m}(x, x_m)$$


X_1
X_2
X_{m-1}
X_m
X'_1
X'_2
X'_{m-1}
X'_m
=
+

Effective interactions are introduced that sum the RPA bubbles.

This is exact for the T–L model.

The effective interaction is amputated from the F function:

$$F^{\lambda}_{\lambda_1\lambda_2..\lambda_m}(p_1', .. p_m', p_1, .., p_m, p) =$$

$$= \sum_{l=1}^{m} \left[\delta_{\lambda\lambda_l} + \Pi_\lambda(p) D_{\lambda\lambda_l}(p)\right] \Gamma^{\lambda_l}_{\lambda_1..\lambda_m}(p_1' ... p_m'; p_1 ... p_m; p)$$

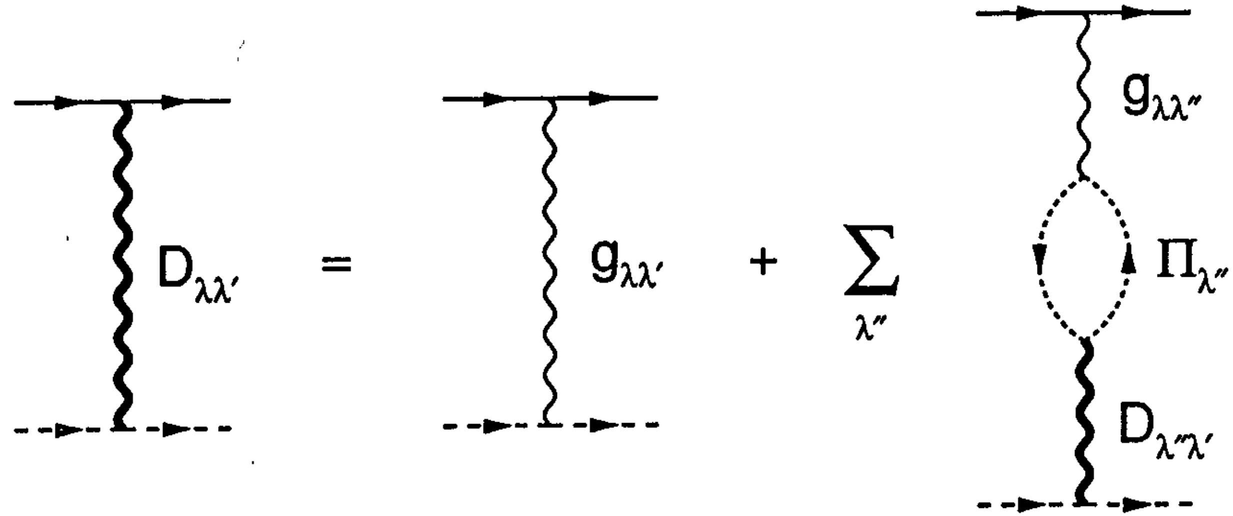
The equation of motion for F gives:

$$\Gamma^{\lambda_l}_{\lambda_1..\lambda_m}(p_1', p_2' .. p_m'; p_1, p_2, .. p_m; p) = \frac{1}{\omega - v_{\lambda_l} q} \times$$

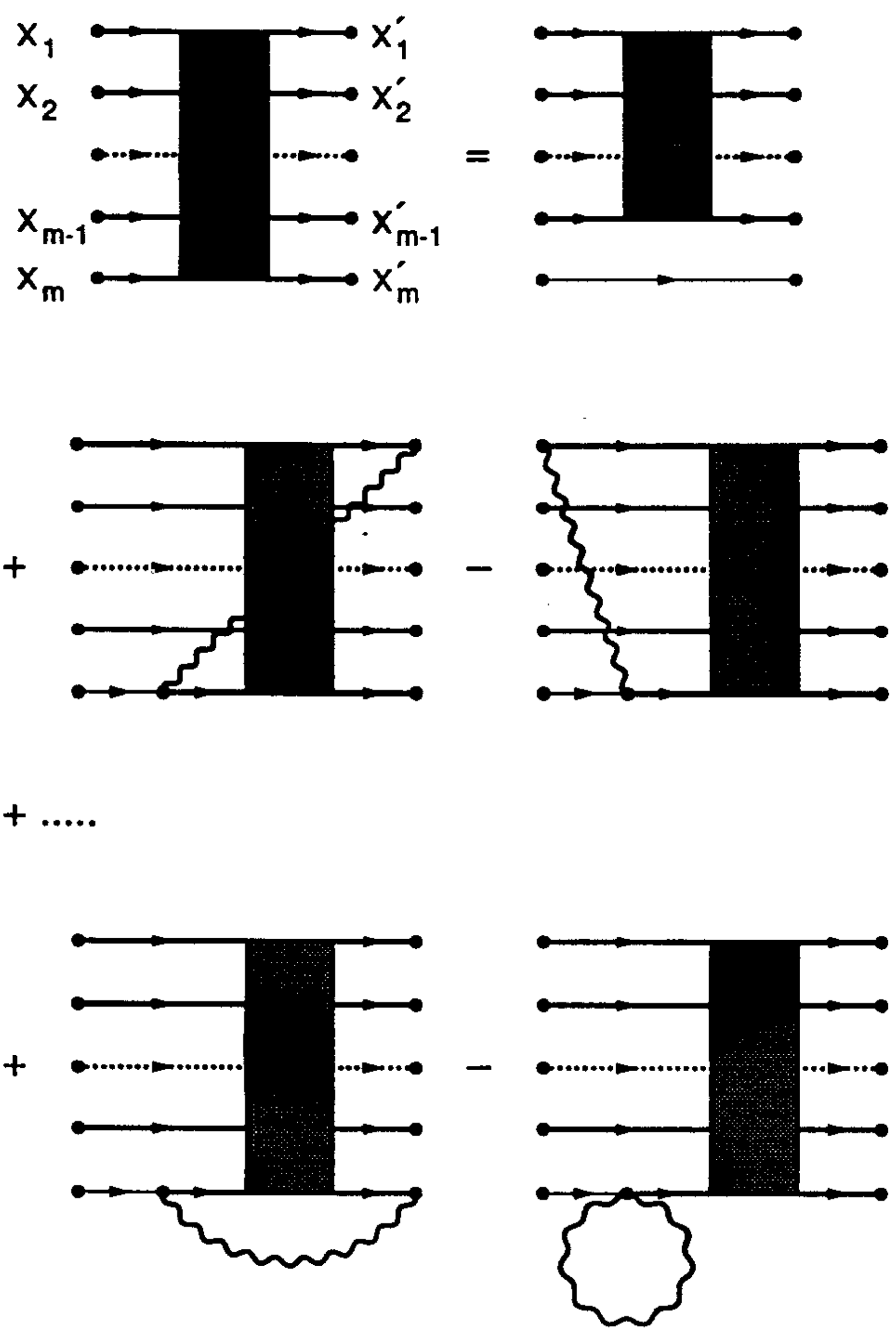
$$\times \Big[G_{\lambda_1..\lambda_m}(p_1', .. p_l' - p, .. p_m'; p_1, .. p_l, .. p_m)$$

$$- G_{\lambda_1..\lambda_m}(p_1', .. p_l', .. p_m'; p_1, .. p_l + p, .. p_m)\Big]$$

Generalized Ward identity!



$D_{\lambda\lambda'}$
=
$g_{\lambda\lambda'}$
+
$\sum_{\lambda''}$
$g_{\lambda\lambda''}$
$\Pi_{\lambda''}$
$D_{\lambda''\lambda'}$


x1
x2
xm-1
xm
x'1
x'2
x'm-1
x'm
=
+
−
+ .....
+
−

In a differential form:

$$\left(\frac{\partial}{\partial t_m} + v_{\lambda_m}\frac{\partial}{\partial x_m}\right) G_{\lambda_1\ldots\lambda_m}(x_1', x_2' \ldots x_m'; x_1 \ldots x_m) =$$

$$i\,\delta(x_m - x_m')\, G_{\lambda_1\ldots\lambda_{m-1}}(x_1', \ldots x_{m-1}'; x_1, \ldots x_{m-1})$$

$$+ \sum_{l=1}^{m}\left[K_{\lambda_m\lambda_l}(x_m - x_l') - K_{\lambda_m\lambda_l}(x_m - x_l)\right]$$

$$\times\, G_{\lambda_1\ldots\lambda_m}(x_1', x_2' \ldots x_m'; x_1 \ldots x_m)$$

For the one-particle Green's function

$$\left(\frac{\partial}{\partial t} + v_\lambda\frac{\partial}{\partial x}\right) G_\lambda(x', x) = i\,\delta(x' - x) + \left[K_{\lambda\lambda}(x - x') - K_{\lambda\lambda}(x - x)\right] G_\lambda(x', x)$$

and for the free Green's function

$$\left(\frac{\partial}{\partial t} + v_\lambda\frac{\partial}{\partial x}\right) G_\lambda^{(0)}(x', x) = i\,\delta(x' - x)$$

$$G_\lambda^{(0)}(x', t'; x, t) = \frac{1}{2\pi}\,\frac{1}{(x' - x) - v_\lambda(t' - t) + i\delta\,\mathrm{sign}\, v_\lambda(t' - t)}$$

Auxiliary problem

$$\left(\frac{\partial}{\partial t} + v_\lambda \frac{\partial}{\partial x}\right) f_{\lambda\lambda'}(x,t) = K_{\lambda\lambda'}(x,t)\, f_{\lambda\lambda'}(x,t)$$

with $K_{\lambda\lambda'}(x,t) = \sum_j (u_j - v_\lambda) \dfrac{\alpha^{(j)}_{\lambda\lambda'}}{x - u_j t + \frac{i}{\Lambda}\,\mathrm{sign}\, u_j t}$

$$\alpha^{(j)} = |\omega^{(j)}\rangle\langle\omega^{(j)}|$$

The solution is:

$$f_{\lambda\lambda'}(x,t) = \left(x - v_\lambda t + \frac{i}{\Lambda}\,\mathrm{sign}\, v_\lambda t\right)^{\delta_{\lambda\lambda'}} \times$$
$$\times \prod_j \left(x - u_j t + \frac{i}{\Lambda}\,\mathrm{sign}\, u_j t\right)^{-\alpha^{(j)}_{\lambda\lambda'}}$$

and

$$G_{\lambda_1\lambda_2\ldots\lambda_m}(x_1', x_2', \ldots x_m'; x_1, x_2, \ldots, x_m) =$$

$$= \prod_\ell G^{(0)}_{\lambda_\ell}(x_\ell', x_\ell)\, f_{\lambda_\ell\lambda_\ell}(x_\ell - x_\ell') \times$$

$$\times \prod_{\ell'<\ell} \frac{f_{\lambda_\ell\lambda_{\ell'}}(x_\ell - x_{\ell'}')\, f_{\lambda_\ell\lambda_{\ell'}}(x_\ell' - x_{\ell'})}{f_{\lambda_\ell\lambda_{\ell'}}(x_\ell - x_{\ell'})\, f_{\lambda_\ell\lambda_{\ell'}}(x_\ell' - x_{\ell'}')}$$

$$\langle \phi(x,t)\, \phi(0,0)\rangle \sim \frac{e^{-ix\sum_\lambda k_{F\lambda}\,\Delta N_\lambda}}{\prod_j (x - u_j t)^{\langle \Delta N | \alpha^{(j)} | \Delta N\rangle}}$$

$$\alpha_j = \langle \Delta N | \alpha^{(j)} | \Delta N\rangle \qquad 2\Delta^{(j)} = \langle \omega^j | \Delta N\rangle^2$$

Specialization to the 2 component case:

$$A_{TL} = \begin{pmatrix} v_\uparrow + \tilde{g}_{4\uparrow} & \tilde{g}_{2\uparrow} & \tilde{g}_{4\perp} & \tilde{g}_{2\perp} \\ \tilde{g}_{2\uparrow} & v_F + \tilde{g}_{4\uparrow} & \tilde{g}_{2\perp} & \tilde{g}_{4\perp} \\ \tilde{g}_{4\perp} & \tilde{g}_{2\perp} & v_\downarrow + \tilde{g}_{4\downarrow} & \tilde{g}_{2\downarrow} \\ \tilde{g}_{2\perp} & \tilde{g}_{4\perp} & \tilde{g}_{2\downarrow} & v_\downarrow + \tilde{g}_{4\downarrow} \end{pmatrix}$$

$$B_{TL} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Eigenvalue problem gives $|\omega_{TL}^{(j)}\rangle$, the exponents are

$$2\Delta^{(j)} = \langle \omega_{TL}^{(j)} | \Delta N_{TL}\rangle^2$$

with $\Delta N_{TL} = \begin{pmatrix} \Delta N_{R\uparrow} \\ \Delta N_{L\uparrow} \\ \Delta N_{R\downarrow} \\ \Delta N_{L\downarrow} \end{pmatrix}$

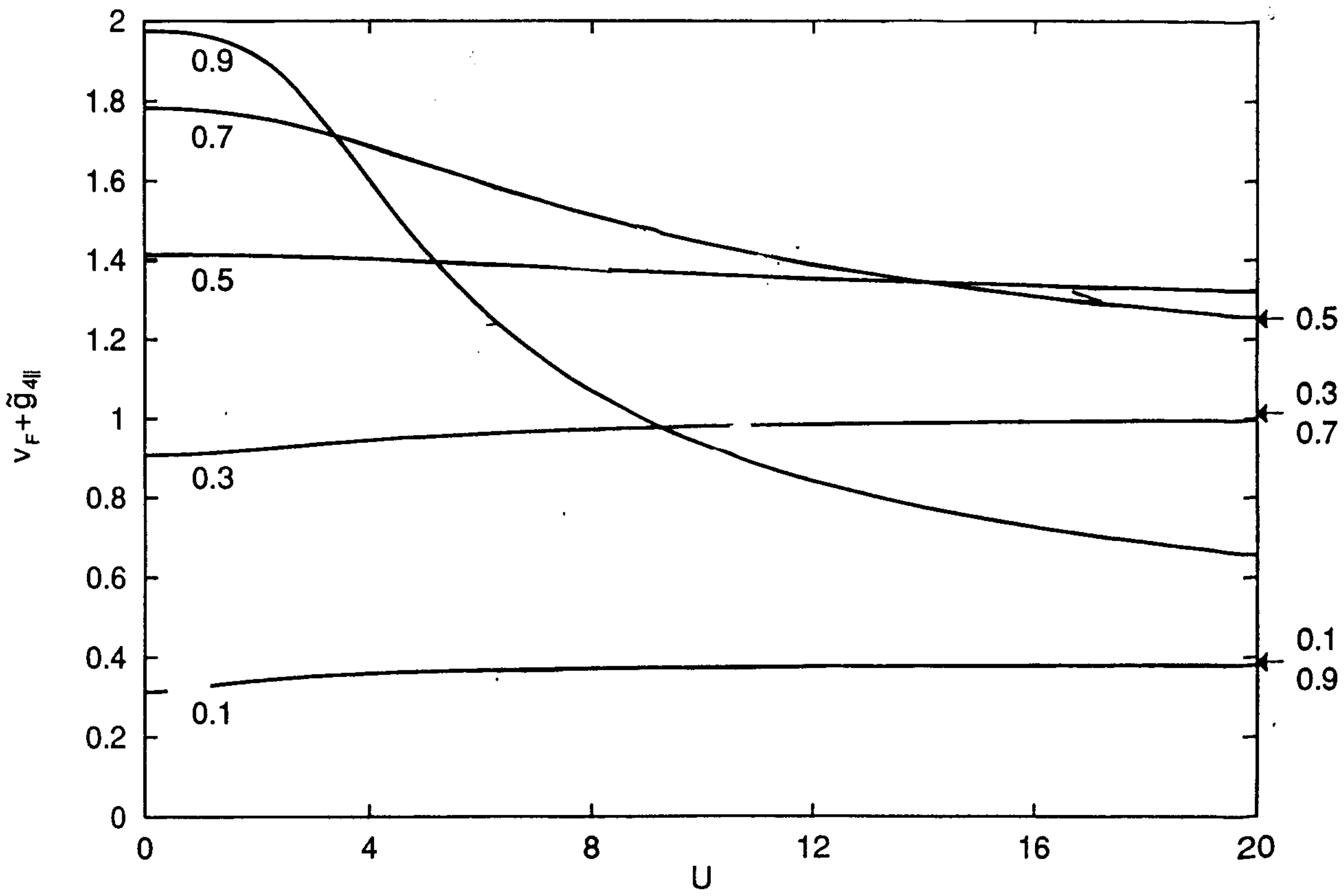
Require that $\langle \omega_{TL}^{(j)} | \Delta N_{TL}\rangle^2 = \langle \omega_H^{(j)} | \Delta N_H\rangle^2$

Since $|\Delta N_{TL}\rangle = \mathcal{U}\,|\Delta N_H\rangle$

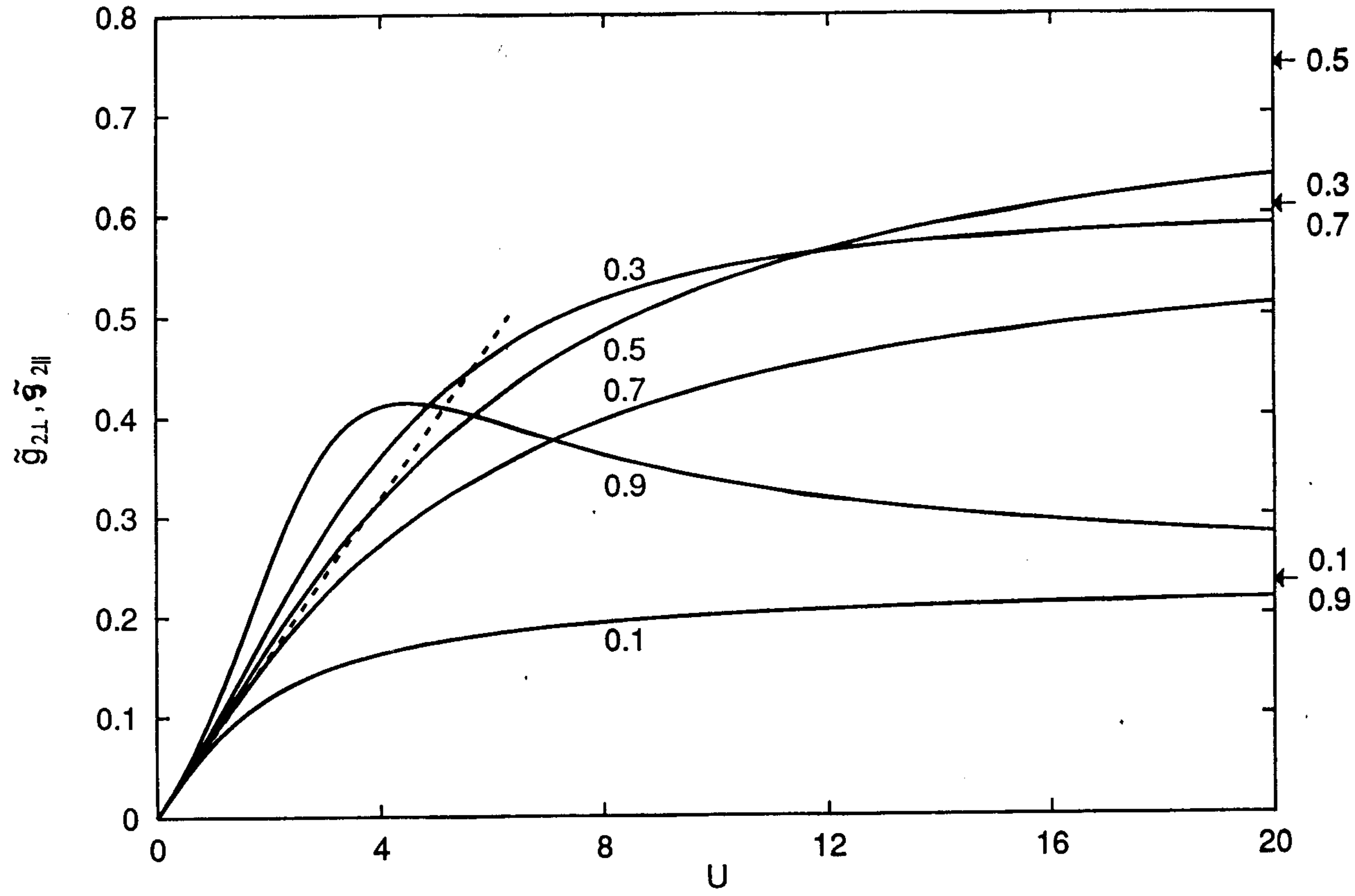
$$|\omega_{TL}^{(j)}\rangle = (\mathcal{U}^{-1})^T\,|\omega_H^j\rangle$$

With the spectral decomposition

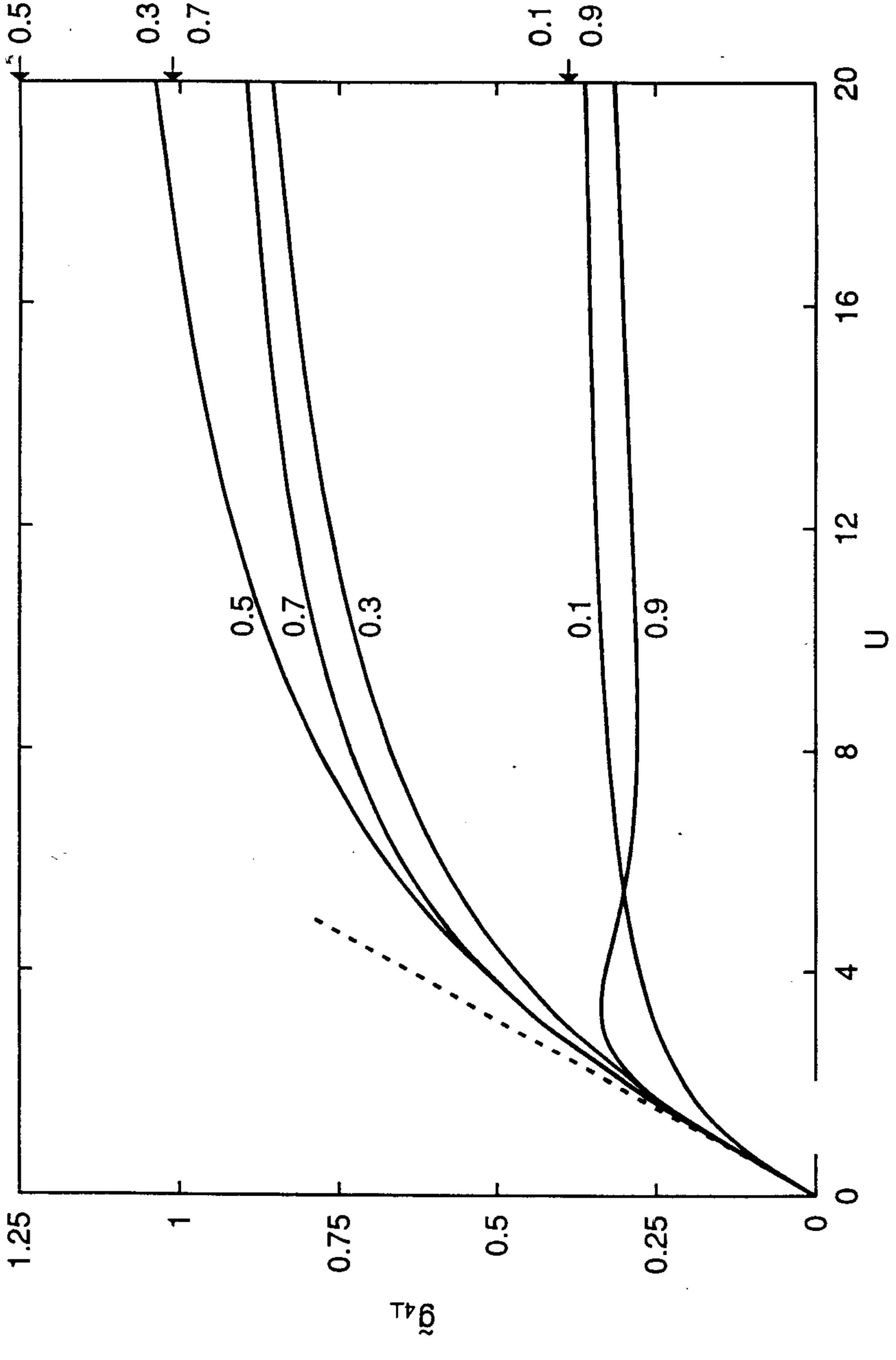
$$\begin{aligned} A_{TL} &= u_c\left(|\omega_{TL}^{(u_c)}\rangle\langle\omega_{TL}^{(u_c)}| + |\omega_{TL}^{(-u_c)}\rangle\langle\omega_{TL}^{(-u_c)}|\right) \\ &\quad + u_s\left(|\omega_{TL}^{(u_s)}\rangle\langle\omega_{TL}^{(u_s)}| + |\omega_{TL}^{(-u_s)}\rangle\langle\omega_{TL}^{(-u_s)}|\right) \\ &= u_c(\mathcal{U}^{-1})^T\left(|\omega_H^{(u_c)}\rangle\langle\omega_H^{(u_c)}| + |\omega_H^{(-u_c)}\rangle\langle\omega_H^{(-u_c)}|\right)\mathcal{U}^{-1} \\ &\quad + u_s(\mathcal{U}^{-1})^T\left(|\omega_H^{(u_s)}\rangle\langle\omega_H^{(u_s)}| + |\omega_H^{(-u_s)}\rangle\langle\omega_H^{(-u_s)}|\right)\mathcal{U}^{-1} \end{aligned}$$

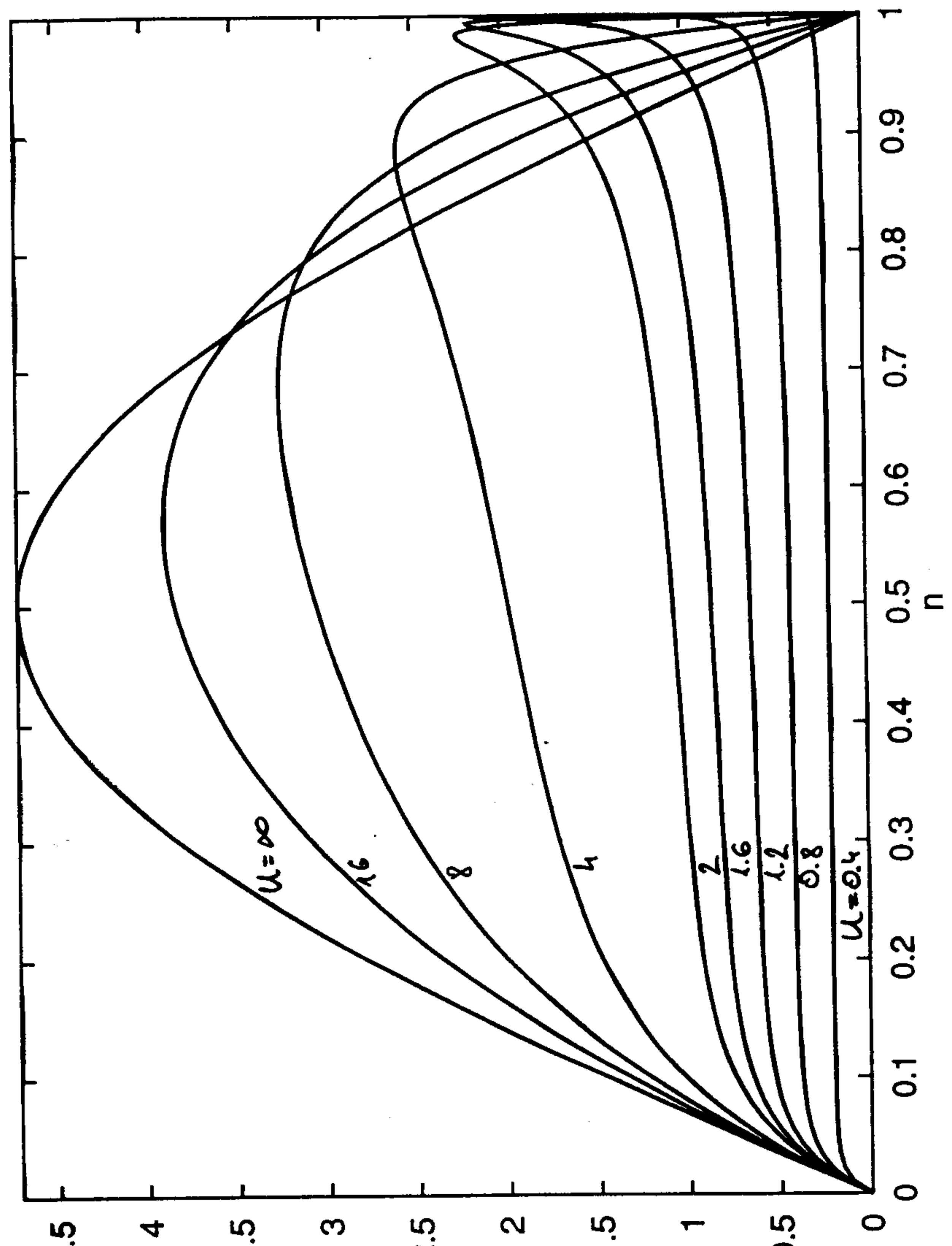


$v_F + \tilde{g}_{4\parallel}$
U
0
4
8
12
16
20
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
0.9
0.7
0.5
0.3
0.1
0.5
0.3
0.7
0.1
0.9

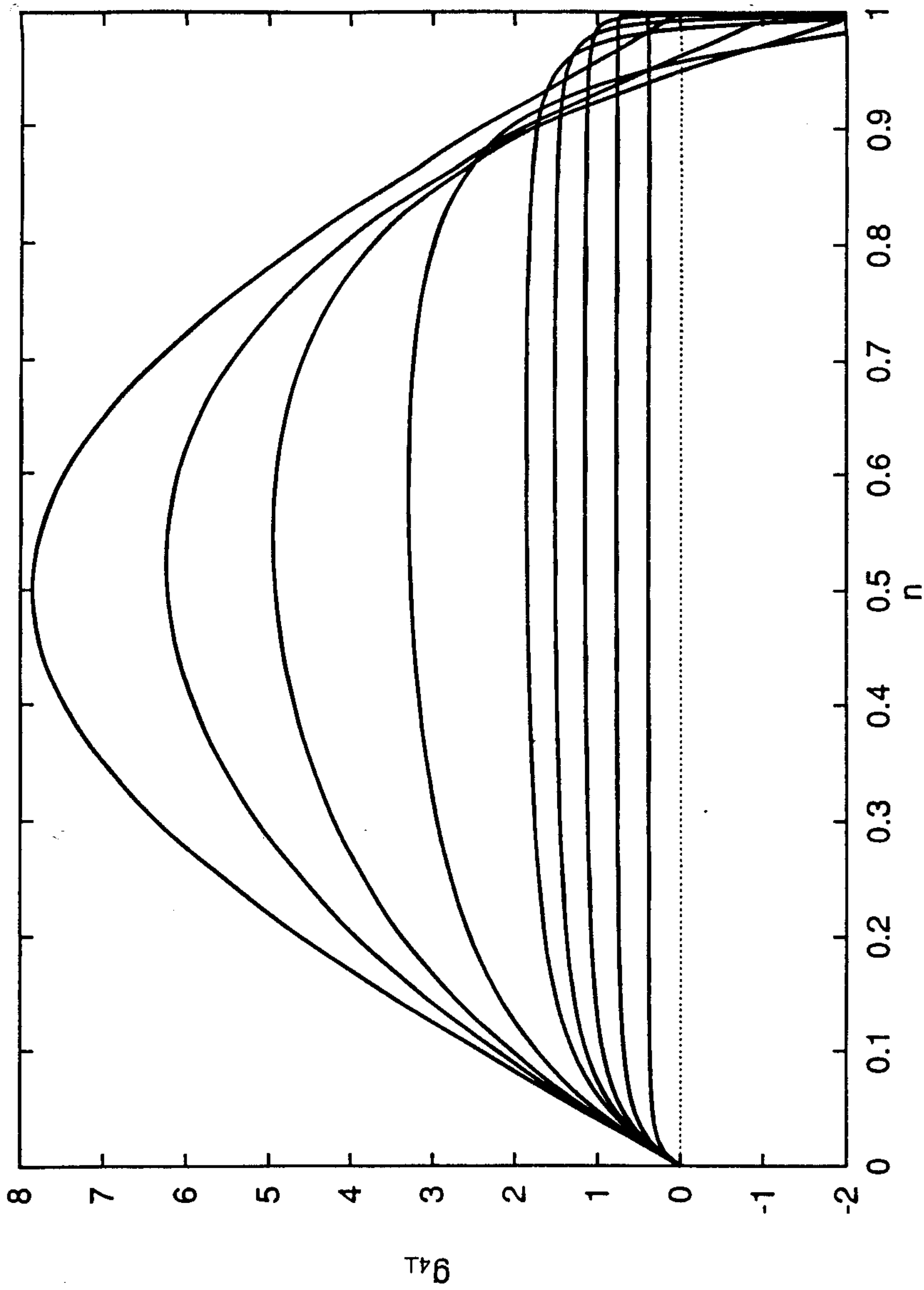


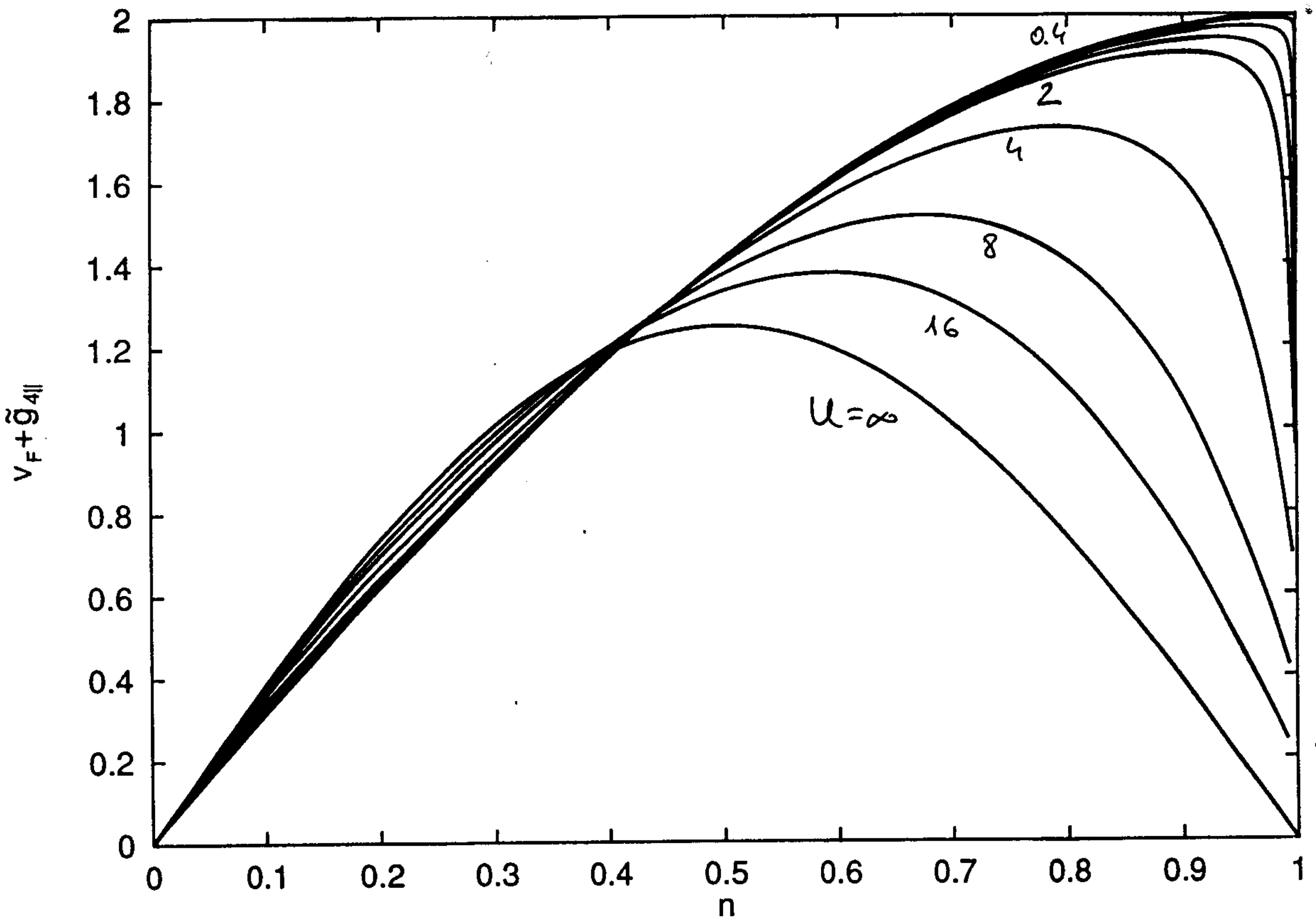
$\tilde{g}_{2\perp}, \tilde{g}_{2\parallel}$
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
4
8
12
16
20
U
0.3
0.5
0.7
0.9
0.1
0.5
0.3
0.7
0.1
0.9


0.5
0.3
0.7
0.1
0.9
0.5
0.7
0.3
0.1
0.9
U
$\tilde{g}_{4L}$
0
4
8
12
16
20
0
0.25
0.5
0.75
1
1.25

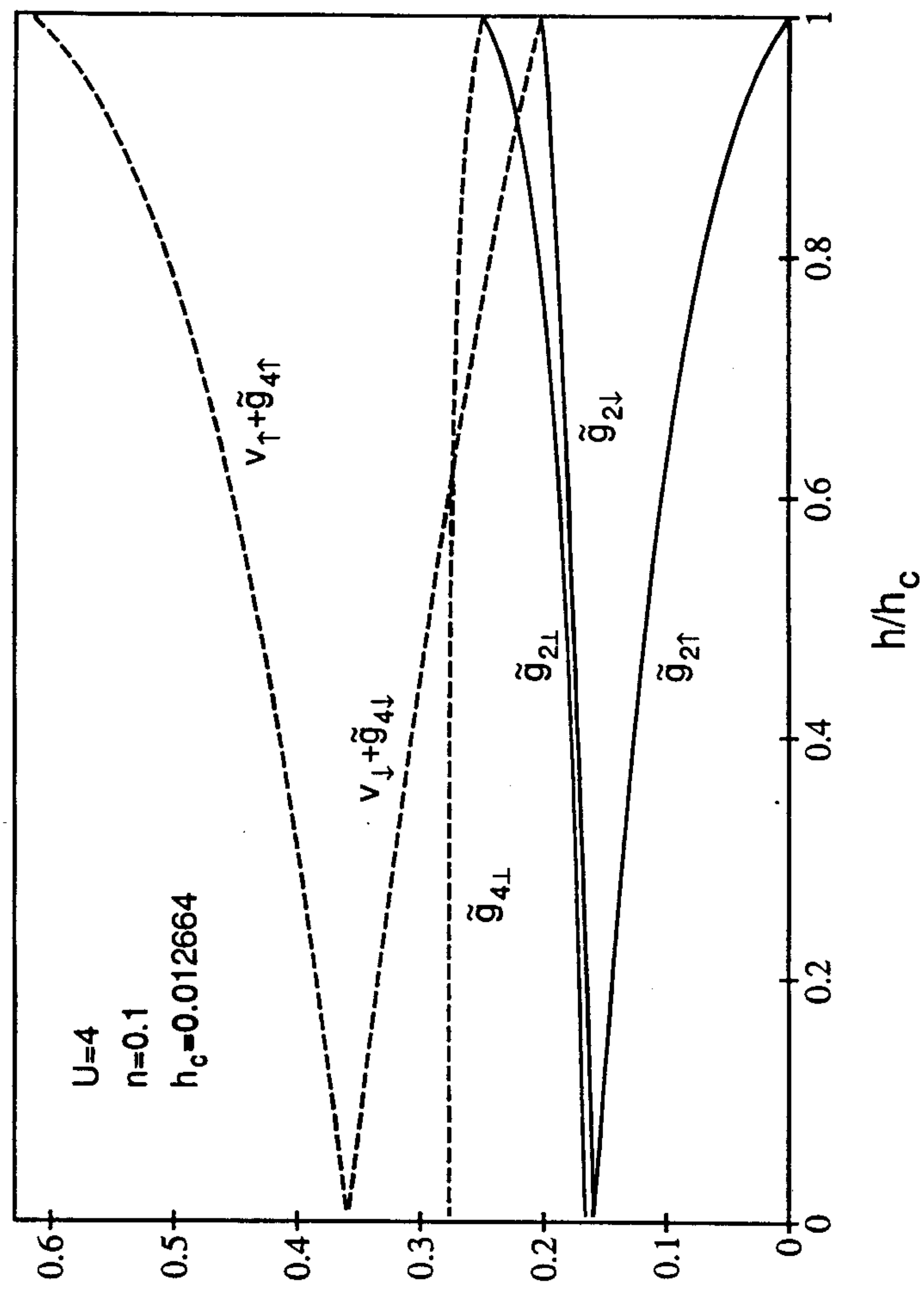


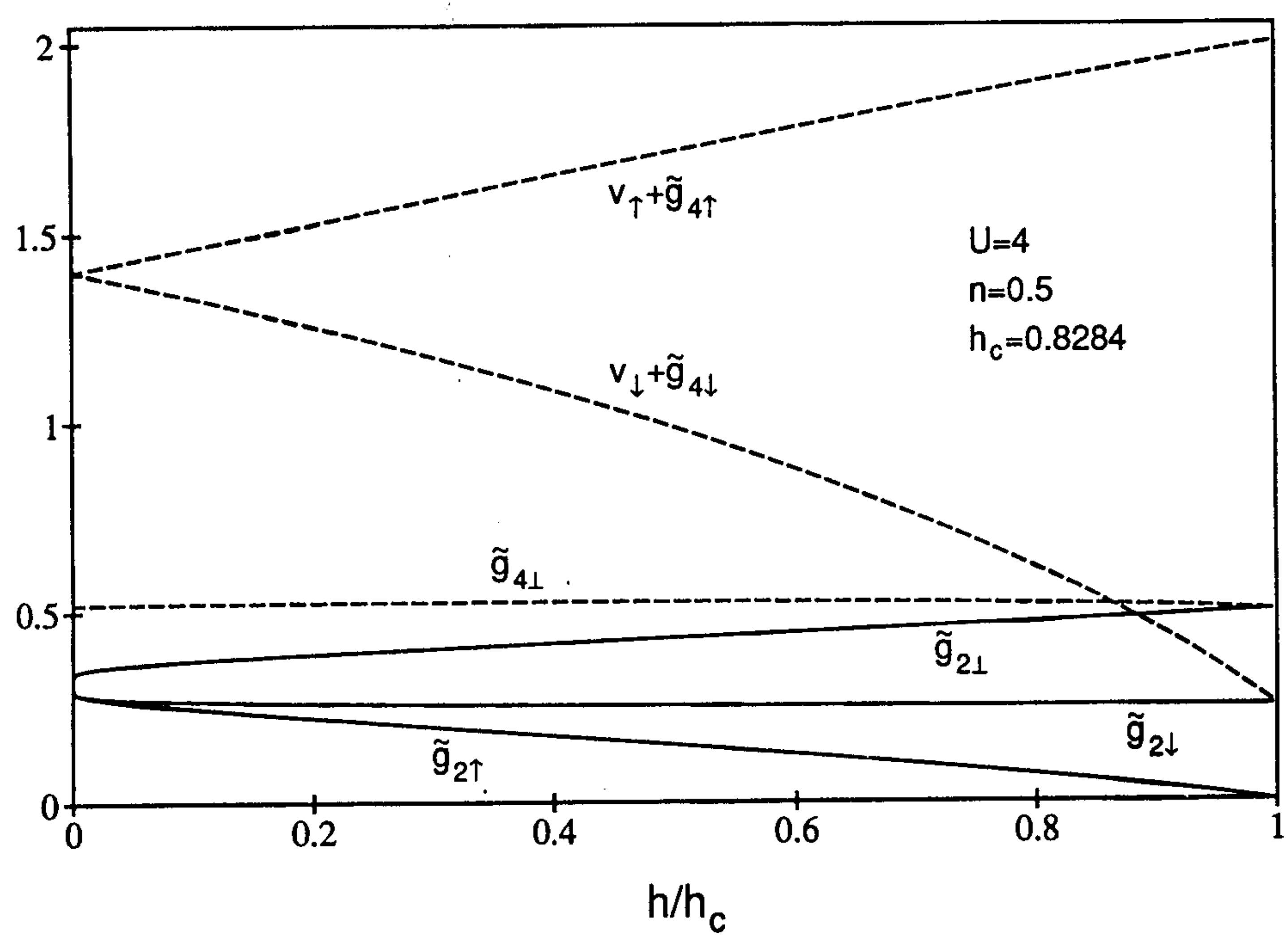
n
$g_{2\perp}, g_{2\parallel}$
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0.5
1.5
2
2.5
3
3.5
4
4.5
U=∞
16
8
4
2
1.6
1.2
0.8
U=0.4


n
g_{4L}
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
-2
-1
0
1
2
3
4
5
6
7
8

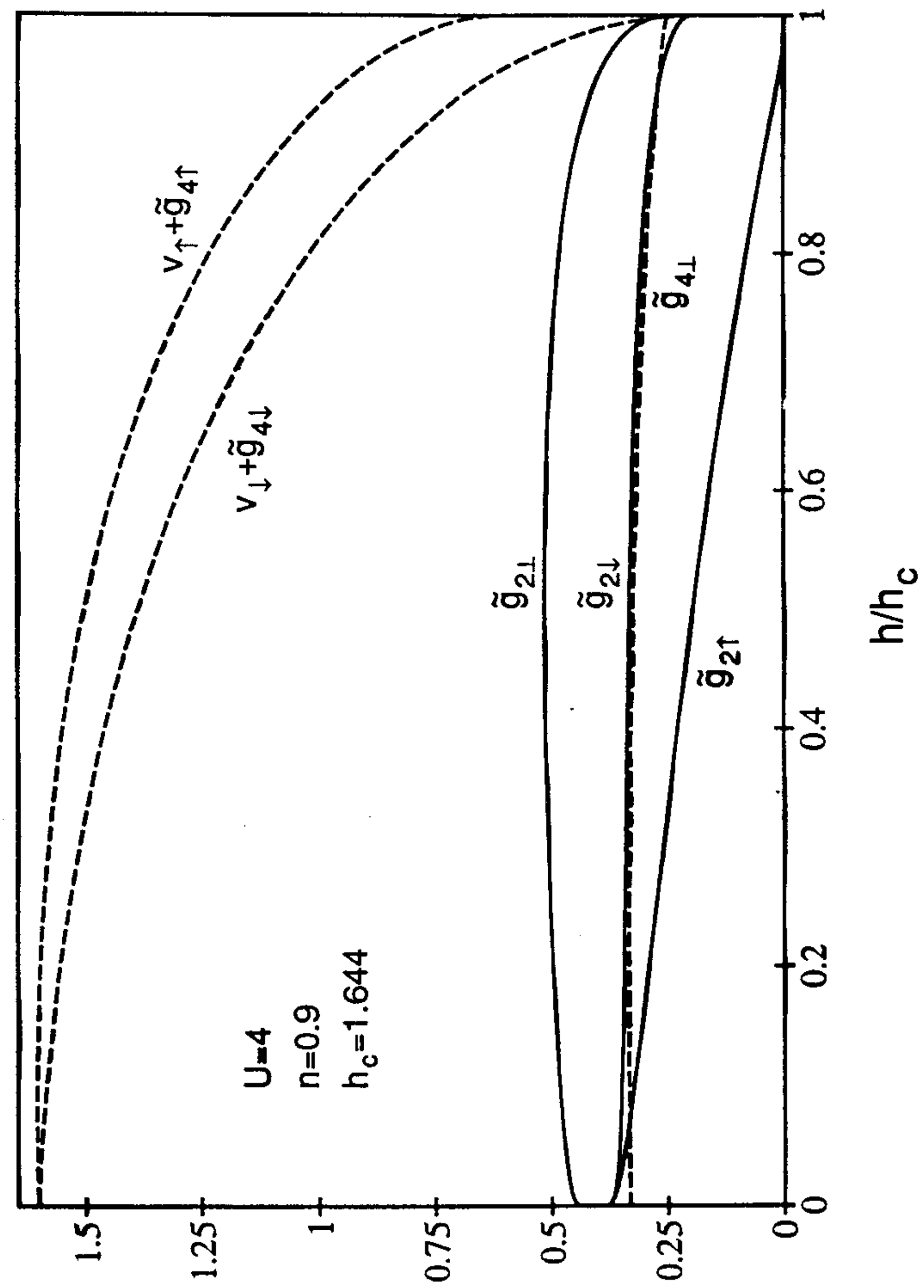


$v_F + \tilde{g}_{4\parallel}$
n
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0.2
0.4
0.6
0.8
1.2
1.4
1.6
1.8
2
0.4
2
4
8
16
U=∞


U=4
n=0.1
h_c=0.012664
$v_\uparrow+\tilde{g}_{4\uparrow}$
$v_\downarrow+\tilde{g}_{4\downarrow}$
$\tilde{g}_{4\perp}$
$\tilde{g}_{2\perp}$
$\tilde{g}_{2\downarrow}$
$\tilde{g}_{2\uparrow}$
h/h_c
0
0.1
0.2
0.3
0.4
0.5
0.6
0.2
0.4
0.6
0.8
1



$v_\uparrow+\tilde{g}_{4\uparrow}$
$v_\downarrow+\tilde{g}_{4\downarrow}$
U=4
n=0.5
h_c=0.8284
$\tilde{g}_{4\perp}$
$\tilde{g}_{2\perp}$
$\tilde{g}_{2\downarrow}$
$\tilde{g}_{2\uparrow}$
0
0.5
1
1.5
2
0
0.2
0.4
0.6
0.8
1
h/h_c



U=4
n=0.9
h_c=1.644
$v_\uparrow+\tilde{g}_{4\uparrow}$
$v_\downarrow+\tilde{g}_{4\downarrow}$
$\tilde{g}_{2\perp}$
$\tilde{g}_{2\downarrow}$
$\tilde{g}_{4\perp}$
$\tilde{g}_{2\uparrow}$
0
0.25
0.5
0.75
1
1.25
1.5
0
0.2
0.4
0.6
0.8
1
h/h_c

Conclusions

For intermediate filling ($0.1 < n < 0.9$) lattice effects start to appear at $U \sim t$.

Weak coupling theory is reasonably good until $U < 4t$.

In this range the couplings are almost independent of filling.

Breakdown for $n < 0.1$ or $n > 0.9$.

Logarithmic singularity in small fields. Affects the spin degrees of freedom.