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MINIWORKSHOP ON
STRONGLY CORRELATED ELECTRON SYSTEMS

15 JUNE - 10 JULY 1992

"COMPOSITE FERMION THEORY OF THE FRACTIONAL QUANTUM HALL EFFECT"

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These are preliminary lecture notes, intended only for distribution to participants

COMPOSITE FERMION THEORY OF THE FRACTIONAL QUANTUM HALL EFFECT

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USEFUL DISCUSSIONS WITH: N. READ, V. EMERY

SUNY STONY BROOK

STATEMENT OF PROBLEM

SOLVE THE SCHRÖDINGER EQUATION

$$H\Psi = E\Psi$$

$$H = \frac{1}{2m}\sum_i \left(\vec{p}_i + \frac{e}{c}\vec{A}_i\right)^2 + \frac{1}{2}\sum_{i\neq j}\frac{e^2}{r_{ij}}$$

$$\vec{\nabla}\times\vec{A} = B\hat{z}$$

in the limit $B\to\infty$

but $\nu \equiv \frac{nhc}{eB}$ fixed.

Solve $\quad H\Psi = E\Psi \quad$ in 2D

$$H = \underbrace{\frac{1}{2m}\left(\vec{p}+\frac{e\vec{A}}{c}\right)^2}_{H_0} + \underbrace{\frac{1}{2}\sum_{i\neq j}\frac{e^2}{r_{ij}}}_{V}$$

$$\vec{\nabla}\times\vec{A} = B\hat{z}$$

in the limit $B\to\infty$ (but ν fixed)

Consider $\quad H_0 = \frac{1}{2m}\left(\vec{p}+\frac{e}{c}\vec{A}\right)^2$

— $E = (n+\frac{1}{2})\hbar\omega_c \qquad$ Landau levels (LL)

— $\frac{\#\text{ states in a LL}}{\text{area}} = \frac{eB}{hc} = \frac{\Phi}{\Phi_0} \qquad \phi_0 = \frac{hc}{e}$

— Filling factor $= \nu \equiv \frac{n\,hc}{eB}$

— When $\hbar\omega_c >> V$, $n=0$

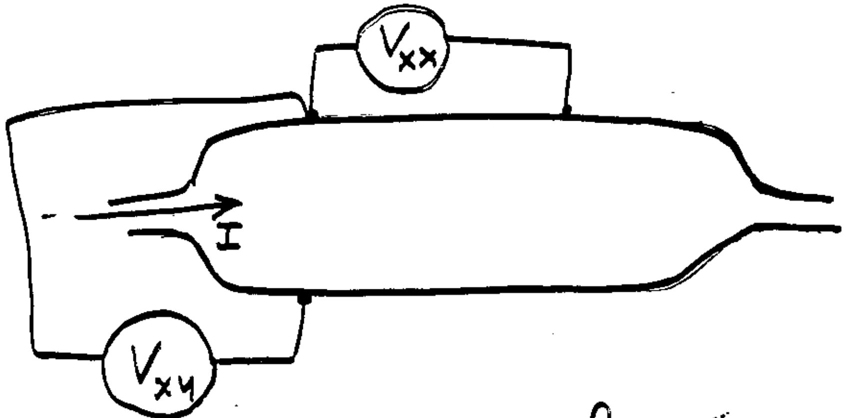
$$H = \frac{1}{2}\sum_{i\neq j}\frac{e^2}{r_{ij}} \quad (+\text{constant})$$

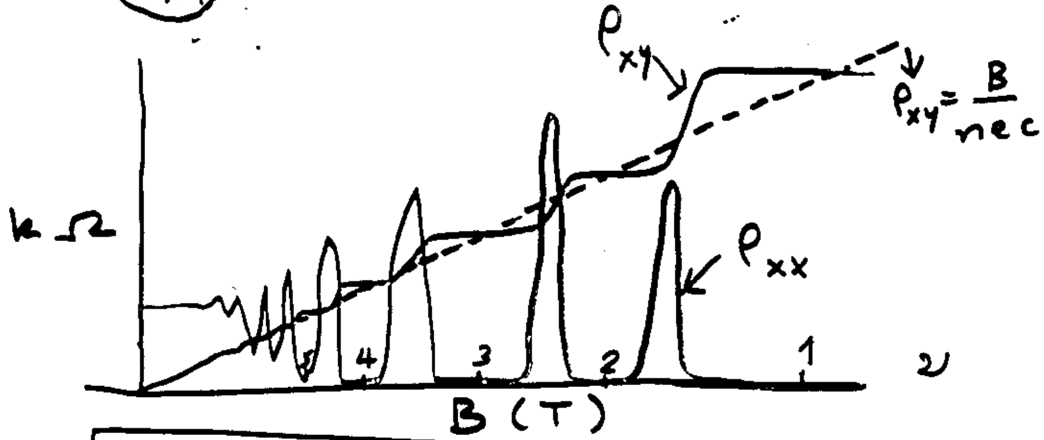
— No small parameter. Perturbation theory not possible. Without interactions large degeneracy of the G.S.

No standard techniques available. Difficult problem.

— Will obtain approximate solutions for the low energy eigenstates. These contain the essential features of the exact solutions and explain experiments

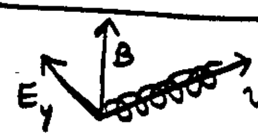
Quantum Hall effect (2D)





$$\boxed{\rho_{xx}\to 0 \;,\; \rho_{xy} = \frac{h}{ie^2}}$$

classically



$v_x = \frac{cE_y}{B}$

$$I_x = e\,[nW]\,\frac{cE_y}{B} = \frac{nec}{B}V_H \qquad (V_H = E_y W)$$

Discoverer: K. von Klitzing $R_H = h/3e^2$

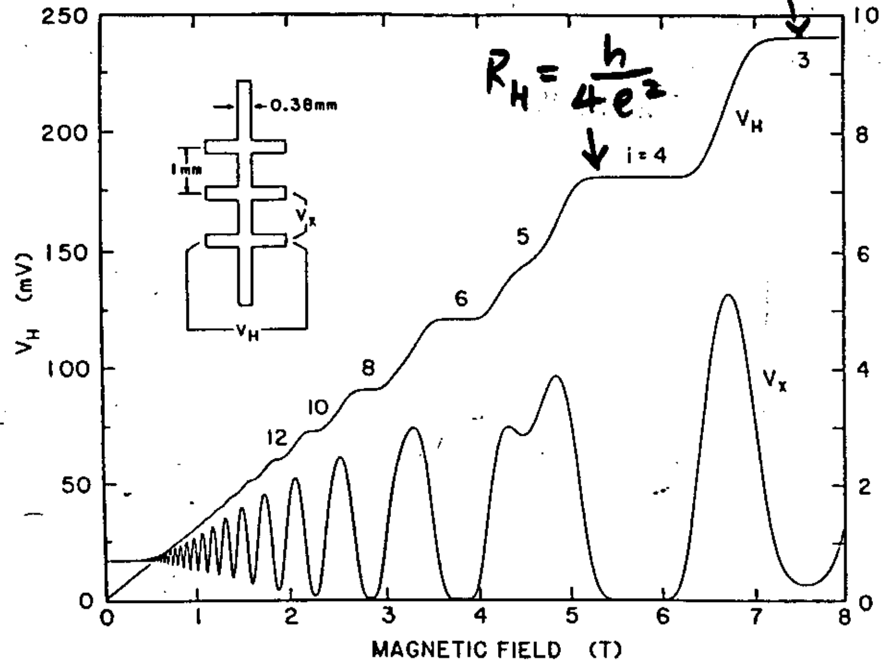


Figure 2.2. Chart recordings of V_H and V_x vs. B for a GaAs-AlGaAs heterostructure cooled to 1.2 K. The source-drain current is 25.5 μA and $n = 5.6 \times 10^{11}$ electrons/cm^2. Cage et al. (1985).

Since c is known very accurately, this is a very precise (error < 1 part in 10^8) measurement of the fine structure constant

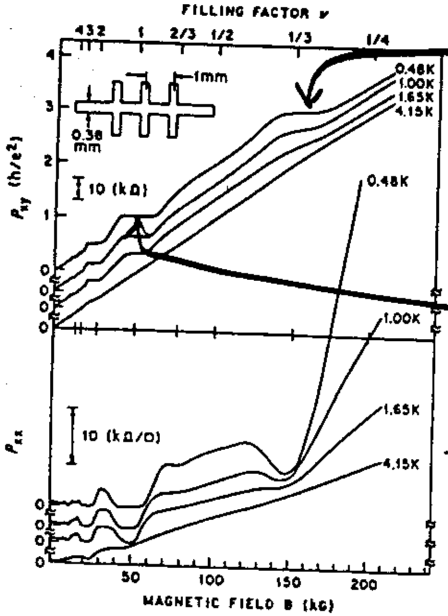
$$\alpha = \frac{e^2}{\hbar c} \approx \frac{1}{137}.$$

This is already used as the standard of resistance.

Tsui, Störmer, Gossard (1983)

$\rho_{xy} = R_H$

$\boxed{\frac{h}{\frac{1}{3}e^2}}$

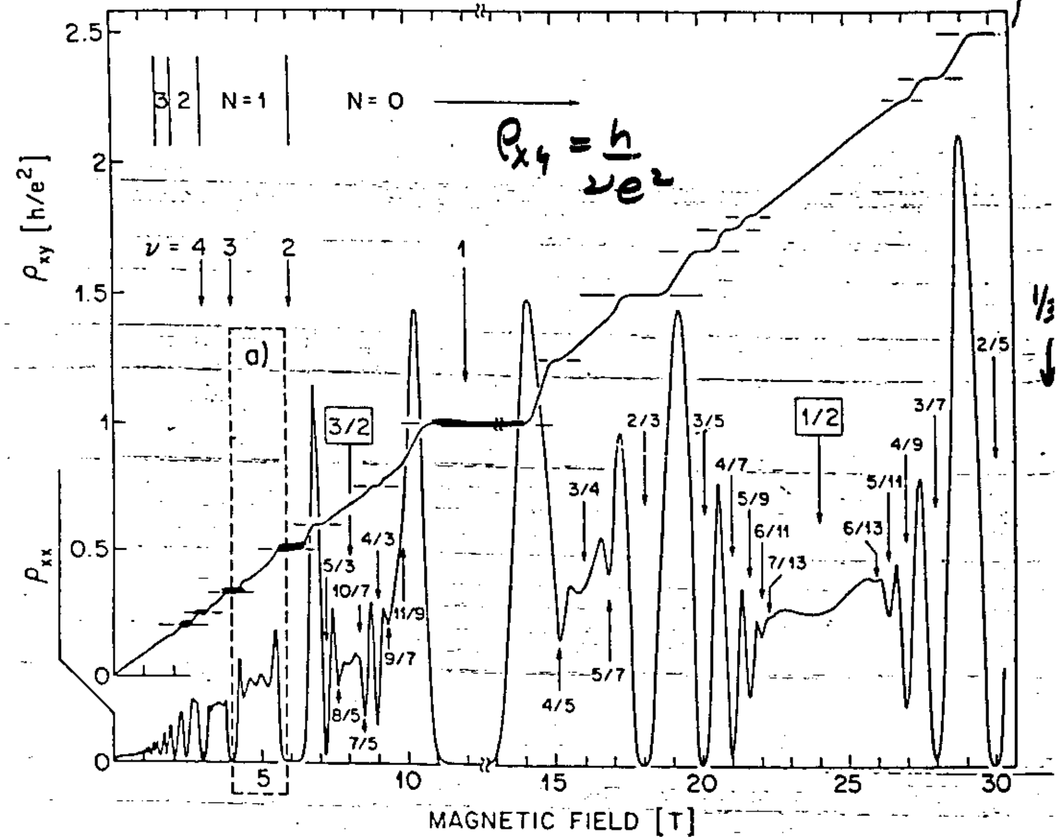


$\frac{h}{1e^2}$

ρ_{xx}

Figure 1. ρ_{xy} and ρ_{xx} of sample 1 vs. B for different temperatures between 0.48K and 4.15K. Notice zero-offsets in both graphs. Insert shows sample geometry. The Landau level filling factor is defined by $\nu = nh/eB$.

WILLETT ET AL



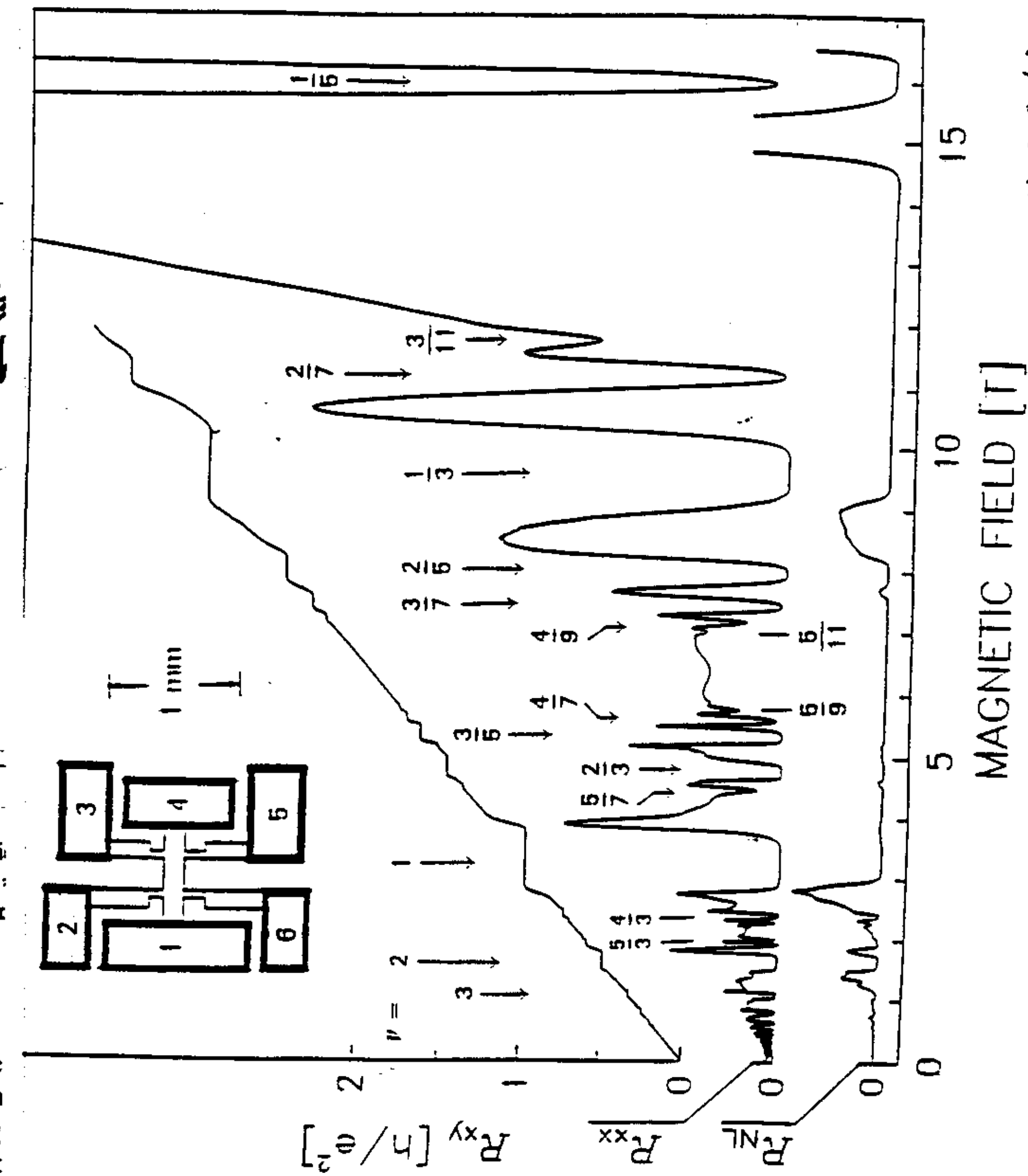
rview of diagonal resistivity ρ_{xx} and Hall resistance ρ_{xy} of sample described in text. The use of a hybri required composition of this figure from four different traces (breaks at $\simeq 12$ T). Temperatures were ;h-field Hall trace at $T=85$ mK. The high-field ρ_{xx} trace is reduced in amplitude by a factor 2.5 for (ındau levels N are indicated.

$R_H = \frac{h}{\nu e^2}$; $\nu = \frac{n}{m}$; m ODD

measurements were performed at magnetic i0 T and at temperatures down to 20 mK :rent dilution-refrigerator–magnet systems. ıs been exercised in order to assure thermal etween the 2D electrons and the crystal lat- arge changes in resistivity were observed of the crystal lattice from 40 to 25 mK (as h a nearby carbon resistance thermometer)

tions $\frac{32}{13}$ and $\frac{33}{13}$ are indicated ($\frac{5}{2} \pm 1.5\%$). are found in ρ_{xy} at these fractions which lie the observed $\frac{5}{2}$ plateau. From this it can that the $\frac{5}{2}$ plateau is not likely the conseq high-order odd-denominator plateaus blenc to form an apparent, but spurious, plateau a

Figure 2 also shows that the strong dependence of the $\frac{5}{2}$ minimum in ρ_{xx} comp



V. J. Goldman et al

FACTS

** Quantization of $R_H = \frac{h}{p e^2}$

p occurs in certain prominent sequences

$p = n \quad : 1, 2, 3, \ldots$

$p = \frac{n}{2n+1} : \frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \ldots$

$p = 1 - \frac{n}{2n+1} : \frac{2}{3}, \frac{3}{5}, \frac{4}{7}, \ldots$

$p = \frac{n}{4n+1} : \frac{1}{5}, \frac{2}{9}, \ldots$

Denominator odd.

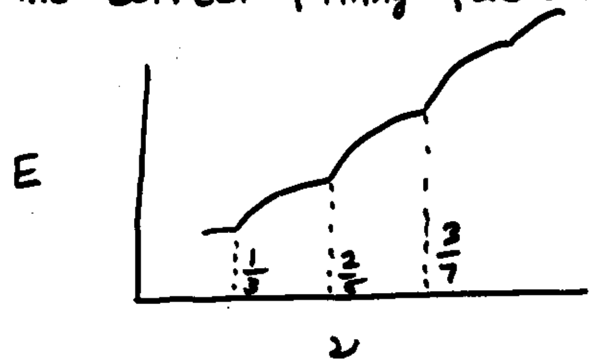
** R_{xx} activated at p

$R_{xx} \sim \exp\left[-\frac{\Delta}{kT}\right]$

Gap in the excitation spectrum

** Gap at $\nu = n$ is $\hbar\omega_c$.

** Numerical studies show gaps at the correct filling factors

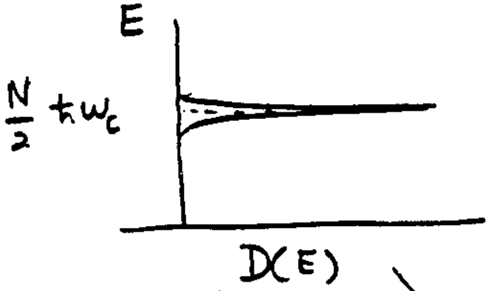


- numerical experiments provide a good representation of the real experiments; allow the possibility of an extremely rigorous test of any theory.

- gap occurs due to repulsive interaction

Consider filling factor ν.

N electrons $\quad$ N/ν states

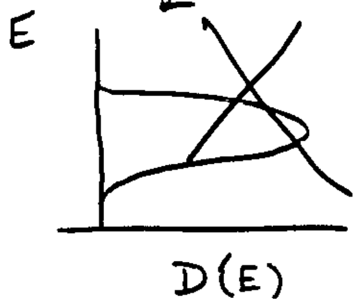


No interactions

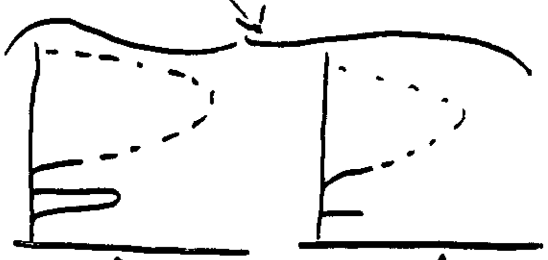
enormous degeneracy

$$\frac{(N/\nu)!}{N!\,(\frac{N}{\nu}-N)!}$$

turn on interactions



Naive expectation



generically $\qquad$ special cases

actual situation

The Hilbert space of low energy states is severely restricted.

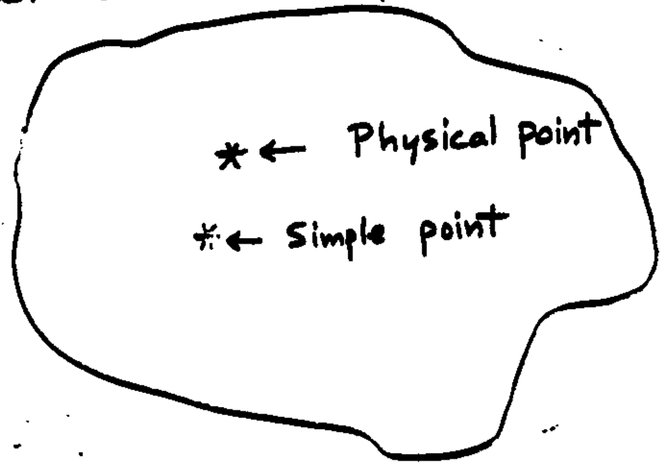
<u>Other</u>

<u>Constraints on Theory</u>

1. Simplicity
2. Must describe all QHE in a unified fashion

MICROSCOPIC THEORY

— Exact solution not possible.



— What do we want from a microscopic theory? Take the example of BCS.

(i) Idea. — Formation of pairs + Bose condensation

- tells us how to think about the phenomenon
- explains some qualitative features — $\phi_0 = \frac{h}{2e}$

(ii) Microscopic implementation of the idea

$$|\phi_{GS}\rangle = \prod_k (u_k + v_k c_k^+ c_{-k}^+)|0\rangle$$

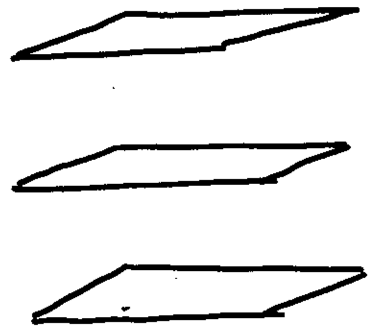
$$|Exc\rangle = \gamma_{k_1}^+ \dots \gamma_{k_N}^+ |\phi_{GS}\rangle$$

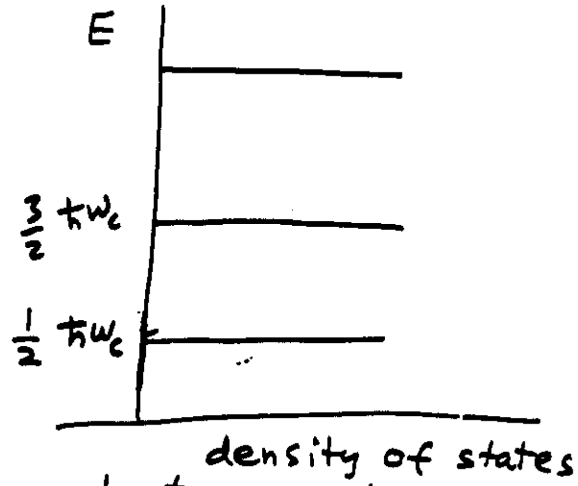
- detailed predictions

(iii) Agreement with Experiment !!

IQHE Well understood

Landau levels





A non-interacting electron system exhibits IQHE.
It results from the fact that there is a gap in the excitation spectrum (i.e., the system is incompressible) when $\nu = i$, since i LLs are completely filled.

Disorder required for plateaus at $R_H = \frac{h}{ie^2}$. Role of disorder fairly well understood.

$\boxed{\nu = \frac{1}{m}}$

Laughlin wave functions

$$- \Psi_{\frac{1}{m}} = \prod_{j<k} (z_j - z_k)^m \, e^{-\frac{1}{4}\sum_i |z_i|^2}$$

$$z_j = x_j + i y_j$$

$$- \Psi_{1/m}^{\text{quasihole}} = \prod_i (z_i - z_0)\, \Psi_{1/m}$$

$$- \Psi_{1/m}^{\text{quasielectron}} = \prod_j \left(\frac{\partial}{\partial z_j} - z_0^* \right) \Psi_{1/m}$$

filling factor	N	(overlap)^2	$\frac{E_{tr}-E_0}{E_0}$
1/3	4	0.99608	0.044%
	5	0.99812	0.024%
	6	0.99289	0.049%
	7	0.99273	0.048%
	8	0.99082	0.053%
	9	0.98816	0.058%

Fano, Ortolani, and Colombo

$\boxed{\nu = \frac{n}{m}}$??

ANYON CONDENSATION APPROACH (HALDANE-HALPERI[N])

— VIEW THE SYSTEM NEAR $1/m$ AS $\chi_{1/m}$ + QUASIPARTICLES (ANYONS)

— INCOMPRESSIBILITY IS OBTAINED WHEN THE ANYONS THEMSELVES CONDENSE INTO A LAUGHLIN TYPE STATE

— LEADS TO A HIERARCHY

— $$\nu = \cfrac{1}{m + \cfrac{\alpha_1}{p_1 - \cfrac{\alpha_2}{p_2 - \cfrac{\alpha_3}{\ddots}}}}$$

$$\alpha_i = \begin{cases} +1 & \text{(QUASIHOLES)} \\ -1 & \text{(QUASIELECTRONS)} \end{cases}$$

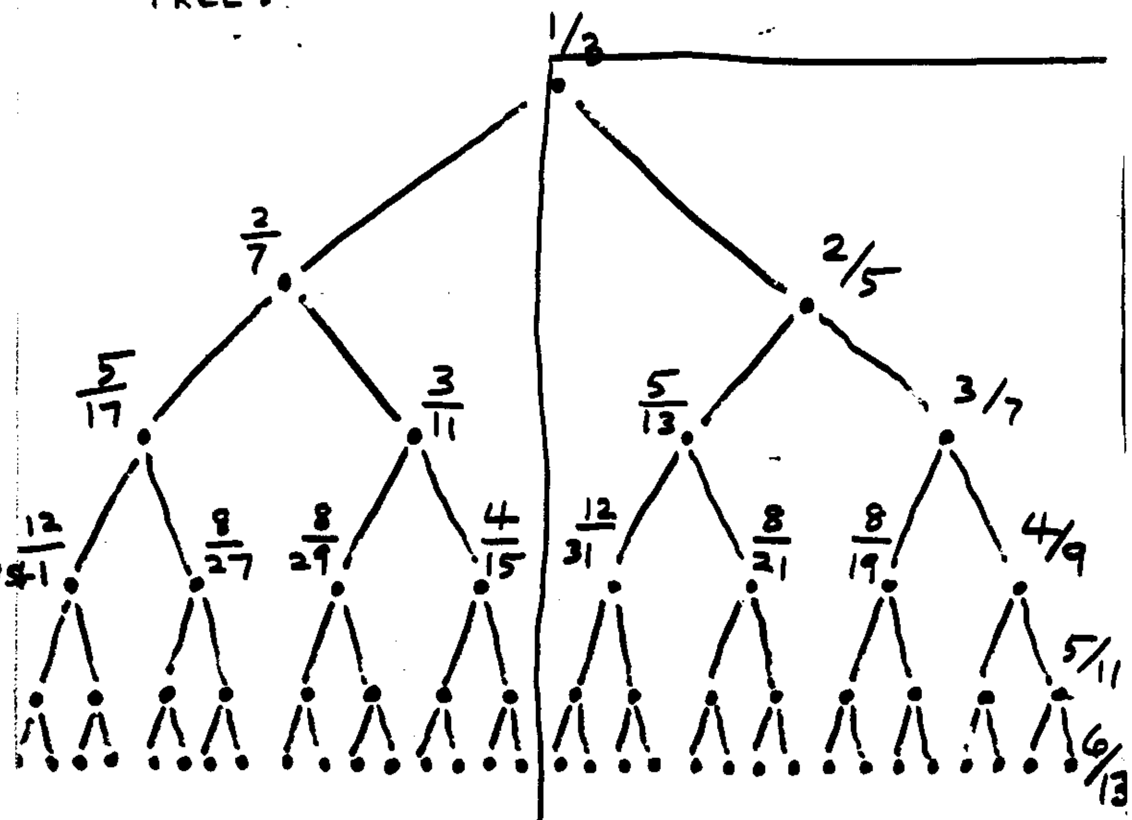
p_i = EVEN INTEGER

— WITH ODD m, ALL RATIONAL FRACTIONS WITH ODD DENOMINATORS ARE OBTAINED

Continued fractions (Haldane)

$$\nu = \cfrac{1}{m + \cfrac{\alpha_1}{p_1 - \cfrac{\alpha_2}{p_2 - \cfrac{\alpha_3}{\ddots}}}} \qquad \alpha_i = \begin{cases} -1 & \text{q. electrons} \\ +1 & \text{q. holes} \end{cases} \qquad p_i = \text{even}$$

For $m=3$, $p_i = 2$, ANYON CONDENSATION APPROACH PRODUCES THE FOLLOWING HIERARCHY TREE:

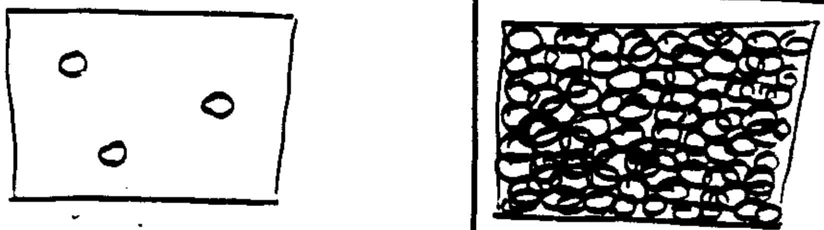


Only a few of the fractions are observed

PROBLEMS WITH THIS SCENARIO

1. CANNOT SAY WHICH FRACTIONS ARE PROMINENT

2. QUASIPARTICLES HAVE A FINITE SIZE



DILUTE LIMIT

QUASIPARTICLES WELL DEFINED

DENSE LIMIT

??

QUASIPARTICLE CONDENSATION OCCURS IN THE DENSE LIMIT

$$\frac{2}{5} = \frac{1}{3} + \frac{N}{2} \text{ QUASIELECTRONS}$$

$$\frac{1}{3} \rightarrow \frac{2}{5} \rightarrow \frac{3}{7} \rightarrow \frac{4}{9} \rightarrow \frac{5}{11} \rightarrow \frac{6}{13} \quad ?!!$$

ARGE & STATISTICS NOT WELL DEFINED IN THE DENSE LIMIT

3. NO MICROSCOPIC JUSTIFICATION

Starting point

Physics of FQHE & IQHE must be related.

There must exist a single theoretical framework that provides a unified description of IQHE & FQHE.

(Must abandon the lowest Landau level restriction — at least at the intermediate steps.)

UNIFICATION OF IQHE AND FQHE

FQHE FROM IQHE (INTUITIVE PICTURE)

NEED TO KNOW

- ν^{-1} = # OF FLUX QUANTA PER ELECTRON
  ($\phi_0 = hc/e$)
- A FLUX QUANTUM IS UNOBSERVABLE
- ELECTRON + $(2p)\phi_0$ = "COMPOSITE" FERMION
- ELECTRON + $(2p+1)\phi_0$ = "COMPOSITE" BOSON

- START WITH Φ_n

  $\nu_i = n$; $\frac{\text{\# OF FLUX QUANTA}}{\text{ELECTRON}} = \frac{1}{n} = \nu_i^{-1}$

- ADD $2p$ FLUX QUANTA TO EACH ELECTRON.
- CORRELATIONS DO NOT CHANGE.

  THIS NEW STATE IS ALSO INCOMPRESSIBLE.
- SMEAR THE FLUX. MEAN FIELD.
- FILLING FACTOR $\nu_f = \frac{n}{2pn+1}$

  BECAUSE $\nu_f^{-1} = \frac{1}{n} + 2p$

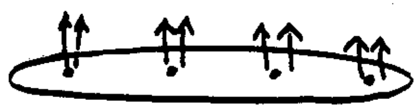
FQHE OF ELECTRONS AT $\frac{n}{2pn+1}$ CAN BE VIEWED AS IQHE OF COMPOSITE FERMIONS.

INTUITIVE PICTURE OF THE FQHE

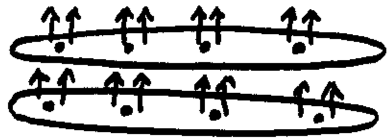
$$\boxed{\Phi_n \xrightarrow{(2p)\phi_0} \chi_{\frac{n}{2pn+1}}}$$

EXAMPLES $(p=1)$ CONSIDER $\upuparrows$

$\chi_{\frac{1}{3}}$:



$\chi_{\frac{2}{5}}$:



$\chi_{\frac{3}{7}}$:

FQHE OF ELECTRONS IS LIKE IQHE OF COMPOSITE FERMIONS

New Hierarchy (Jain & Goldman)

Three basic elements.

1. $\underline{\underline{D}}$ Attach two flux quanta

   $$\boxed{\nu} \xrightarrow{D} \boxed{\nu/(2\nu+1)}$$

2. $\underline{\underline{C}}$ Particle-hole symmetry

   $$\boxed{\nu} \xrightarrow{C} \boxed{1-\nu}$$

3. $\underline{\underline{L}}$ Landau level addition

   $$\boxed{\nu} \xrightarrow{L} \boxed{1+\nu}$$

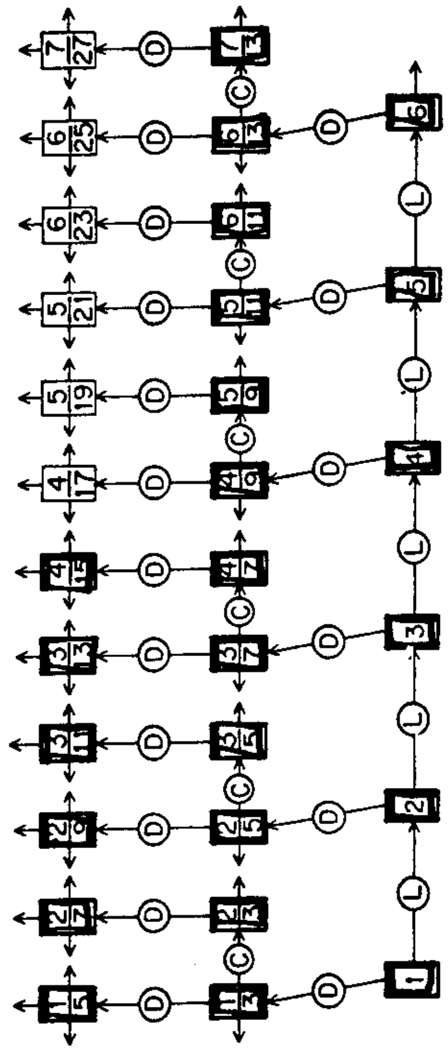
All odd-denominator rationals are obtained starting from integers. Each is obtained only once.

D weakens the state significantly.

C produces a state of comparable strength.

L weakens the state enormously. Ignore.

JAIN
GOLDMAN



The experimentally observed states are obtained quickly.
Strong evidence in favor of this theory.

TRIAL STATES

USE THE ANALOGY OF LAUGHLIN'S STATES

$$\Phi_1 \xrightarrow{(2p)\phi_0} \chi_{\frac{1}{2p+1}}$$

WRITE LAUGHLIN STATE AT $\nu = \frac{1}{2p+1}$ AS

$$\chi_{\frac{1}{2p+1}} = \prod_{j<k} (z_j - z_k)^{2p+1} \exp\left[-\frac{1}{4}\sum_i |z_i|^2\right]$$

$$= \prod_{j<k} (z_j - z_k)^{2p} \Phi_1$$

$$= D^p \Phi_1 \qquad \left[D \equiv \prod_{j<k} (z_j - z_k)^2\right]$$

MULTIPLICATION BY D ADDS TWO FLUX QUANTA (VORTICES, REALLY)

$$\boxed{\chi_{\frac{n}{2np+1}} = [P]\, D^p\, \Phi_n}$$ JASTROW-SLATER FORM

EXAMPLES

$\chi_{1/3} = P D \Phi_1$ $\qquad$ $\chi_{1/5} = D^2 \Phi_1$

$\chi_{2/5} = P D \Phi_2$ $\qquad$ $\chi_{2/9} = D^2 \Phi_2$

$\chi_{3/7} = P D \Phi_3$ $\qquad$ $\chi_{3/13} = D^2 \Phi_3$

$\vdots$ $\qquad$ $\vdots$

filling factor	N	$(\text{overlap})^2$	$\frac{E_{tr}-E_0}{E_0}$
2/5	4	1	0
	6	0.99964	0.0034%
	8	0.99928	0.0055%
2/9	4	0.99988	0.0021%

Dev and Jain

QUASIPARTICLES

$$\chi_{\frac{n}{2np+1}} = [P]\, D^{p}\, \Phi_{n}$$

$$\boxed{\chi^{qp}_{\frac{n}{2np+1}} = [P]\, D^{p}\, \Phi^{qp}_{n}}$$

FOR EXAMPLE

— $\chi^{qh}_{1/3} = [P] D\, \Phi^{qh}_{1} = [P] \prod_{j<k} (z_j - z_k)^2 \times$ 

IDENTICAL TO LAUGHLIN'S TRIAL STATE $\chi^{qh}_{1/3} = \prod_{j} (z_j - z_0)\, \chi_{1/3}$

— $\chi^{qe}_{1/3} = [P] D\, \Phi^{qe}_{1} = [P] \prod_{j<k} (z_j - z_k)^2 \times$ 

NEW.

— CLOSE CORRESPONDENCE BETWEEN IQHE AND FQHE BOTH IN THE GROUND STATE AND IN THE LOW LYING EXCITATIONS

$$|\text{new}\rangle = P D \Phi^{qe}_{1}$$

$$|\text{old}\rangle = \left(\prod_j \frac{\partial}{\partial z_j}\right) \chi_{1/3}$$

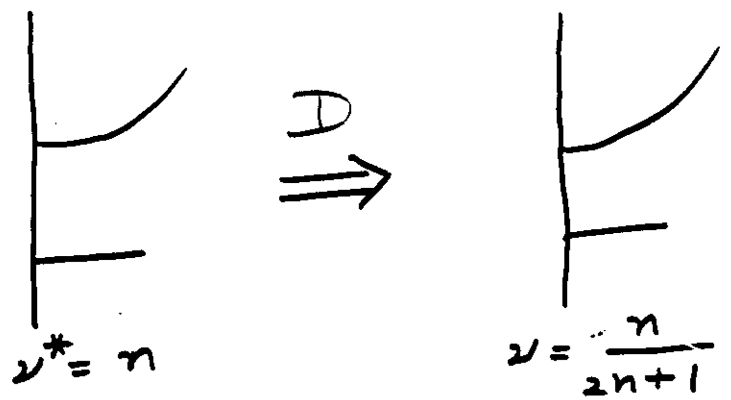
N	$\langle \text{true} \vert \text{new} \rangle^2$	$\langle \text{true} \vert \text{old} \rangle^2$
4	0.9938	0.9974
5	0.9860	0.9934
6	0.9882	0.9771
7	0.9659	0.9314
8	0.9353	0.8770

Dev and Jain

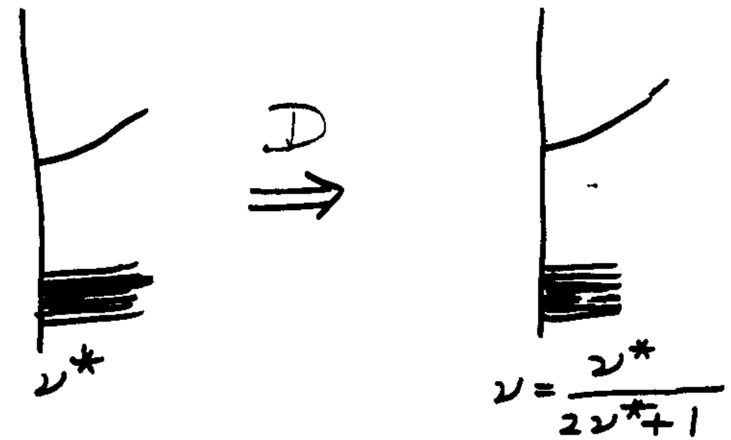
DISC GEOMETRY IS USED

Generalization to arbitrary ν

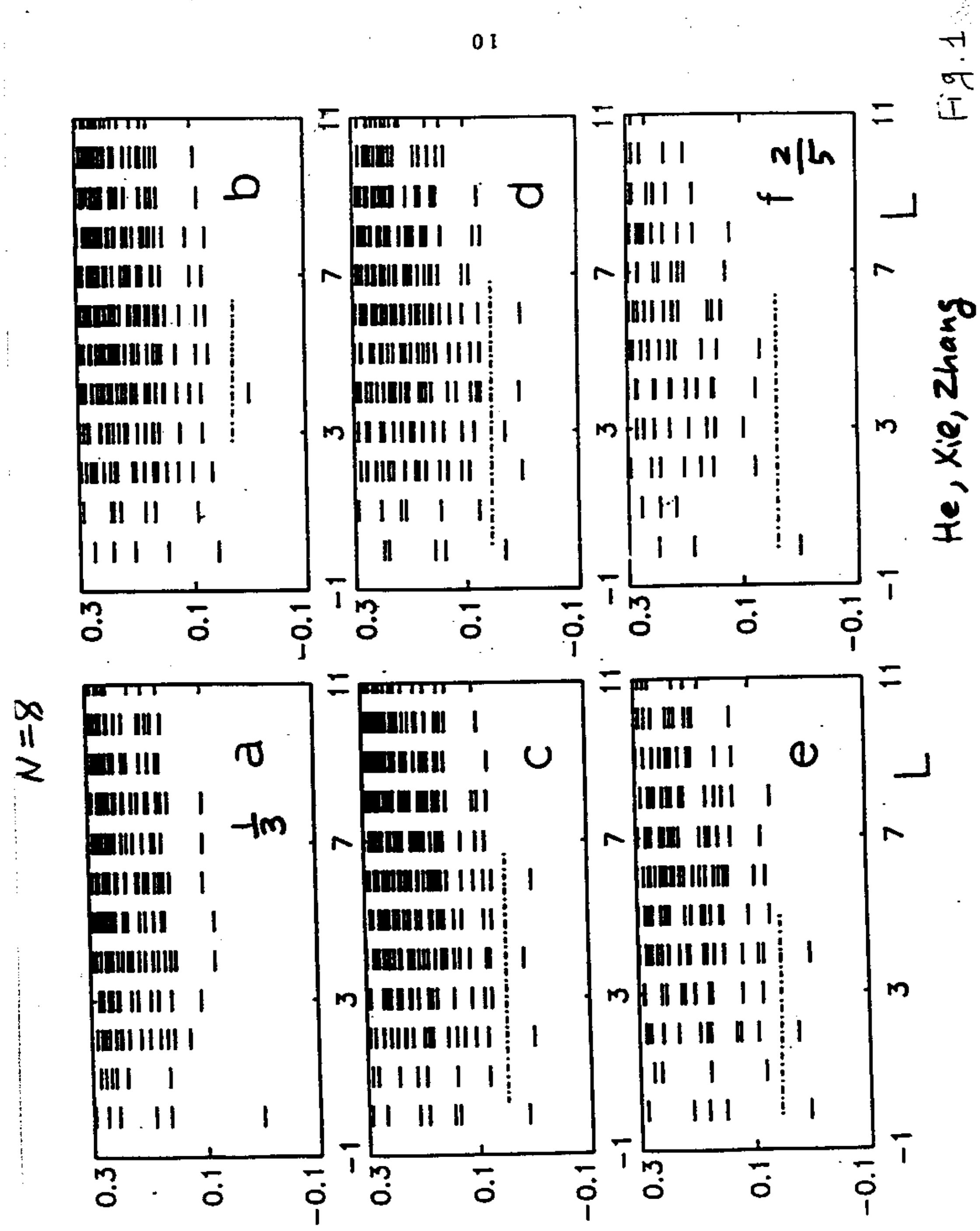
$\nu^* = n$ (filled LL's)



$\nu^* \neq n$



Conservation of number of states and their quantum numbers.



Composite Fermion Theory

Consider $\frac{1}{3} \le \nu \le \frac{2}{5}$

The low energy spectrum at ν lookes like that at ν^*, where

$$\nu = \frac{\nu^*}{2\nu^*+1}$$

As ν goes from $\frac{1}{3} \to \frac{2}{5}$

ν^* goes from $1 \to 2$

Take a system of 8 electrons.

ν	$\frac{1}{3}$					$\frac{2}{5}$
V^0_ν	319,770	203,490	125,970	75,582	43,758	24,310
$V^0_{\nu^*}$	1	9	28	35	15	1
I^*_ν	1	9	28	35	15	1

* He, Xie, Zhang

V^0_ν = # of degenerate states for non-interacting electrons at ν

$V^0_{\nu^*}$ = # of degenerate states for non-interacting electrons at ν^*

I_ν = # of approximately degeneratat states for interacting electrons at ν

GENERALIZED TRIAL STATES AT ARBITRARY FILLING

(JAIN, KIVELSON, TRIVEDI)

IN THE PRESENCE OF DISORDER Φ_{ν^*} IS UNIQUELY DETERMINED AT ARBITRARY FILLING

$$\boxed{\chi_\nu = D\,\Phi_{\nu^*}} \qquad (p=1)$$

$$\nu = \frac{\nu^*}{2\nu^*+1}$$

LAW OF CORRESPONDING STATES

AS ν^*: $1 \to 2 \to 3 \to 4 \to$

ν: $\frac{1}{3} \to \frac{2}{5} \to \frac{3}{7} \to \frac{4}{9} \to$

$$\xi^{FQHE}_{loc}(\nu) = \xi^{IQHE}_{loc}(\nu^*)$$

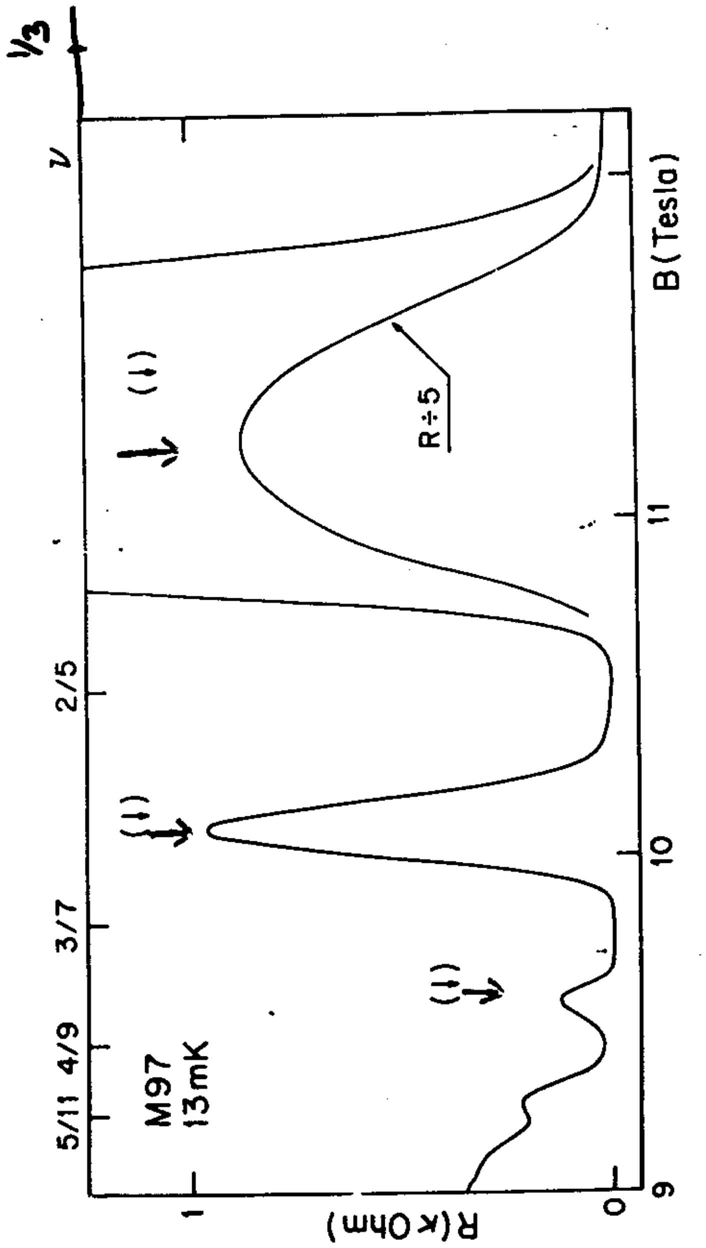
- POSITIONS OF ρ_{xx} PEAKS IN FQHE PREDICTED

ν^*_∞: $1 - \boxed{\frac{3}{2}} - 2 - \boxed{\frac{5}{2}} - 3 - \boxed{\frac{7}{2}} - \dots$

ν_∞: $\frac{1}{3} - \boxed{\frac{3}{8}} - \frac{2}{5} - \boxed{\frac{5}{12}} - \frac{3}{7} - \boxed{\frac{7}{16}} - \dots$

- SCALING IN THE FQHE

- OBSERVATION OF SAME SCALING EXPONENT FOR $1 \to 2$ (WEI ET AL) AND $\frac{1}{3} \to \frac{2}{5}$ (ENGEL ET AL) EXPLAINED



Goldman, Jain, Shayegan
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Experiments on Delocalization and Universality in the Integral Quantum Hall Effect

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We find that the transport coefficients ρ_{xy} and ρ_{xx} of two-dimensional electrons in InGaAs/InP show a characteristic power-law dependence on T. In the range 0.1 K $\lesssim T \lesssim$ 4.2 K, the maximum of $d\rho_{xy}/dB$ diverges like $\sim T^{-\kappa}$ with $\kappa = 0.42 \pm 0.04$, for Landau levels $N = 0\downarrow$, $1\uparrow$, and $1\downarrow$, and the half-width ΔB for ρ_{xx} vanishes as $\Delta B \sim T^{\kappa}$. These results confirm the prediction of the scaling theory that the characteristic power-law behavior in the transport coefficients is a universal feature of delocalization in the integral quantum Hall effect.

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$\Delta \sim T^{K}$

$K = 0.42 \pm .004$

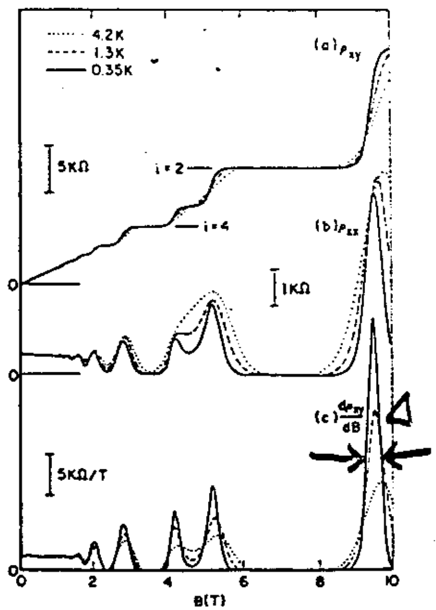


FIG. 1. Quantum transport coefficients (a) ρ_{xy} and (b) ρ_{xx} as functions of B at three temperatures, $T = 4.2$, 1.3, and 0.35 K. (c) The corresponding $d\rho_{xy}/dB$. The sample is an $In_{0.53}Ga_{0.47}As/InP$ heterostructure with a two-dimensional electron density $n_{2D} = 3.3 \times 10^{11}$ cm^{-2} and mobility $\mu = 34000$ cm^2/V s at $T = 0.8$ K.

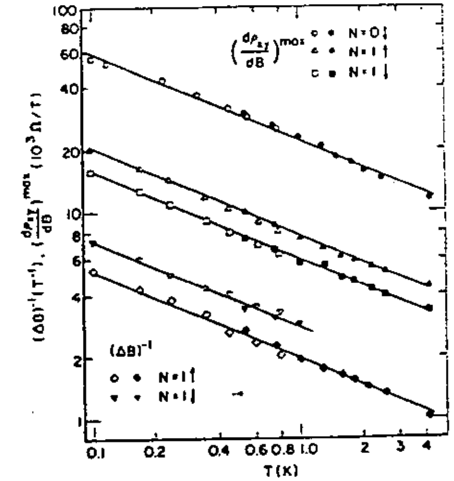
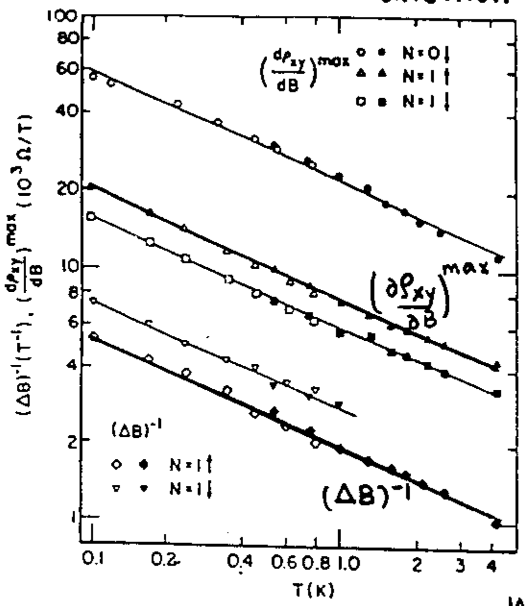


FIG. 2. The upper portion shows the T dependence of $(d\rho_{xy}/dB)^{max}$ for Landau levels $N = 0\downarrow$, $1\uparrow$, and $1\downarrow$; the lower portion shows the T dependence of $1/\Delta B$ for the $N = 1\uparrow$ and $1\downarrow$ Landau levels. The open symbols are data taken in a dilution refrigerator, whereas the filled symbols are data taken in a ^3He system. The slope of the straight lines gives $(d\rho_{xy}/dB)^{max} \sim T^{-\kappa}$ and $\Delta B \sim T^{\kappa}$ with $\kappa = 0.42 \pm 0.04$. The typical uncertainty in T is ~ 0.02 K at 0.4 K.

$1 \to 2$

Transition between Integer plateaus



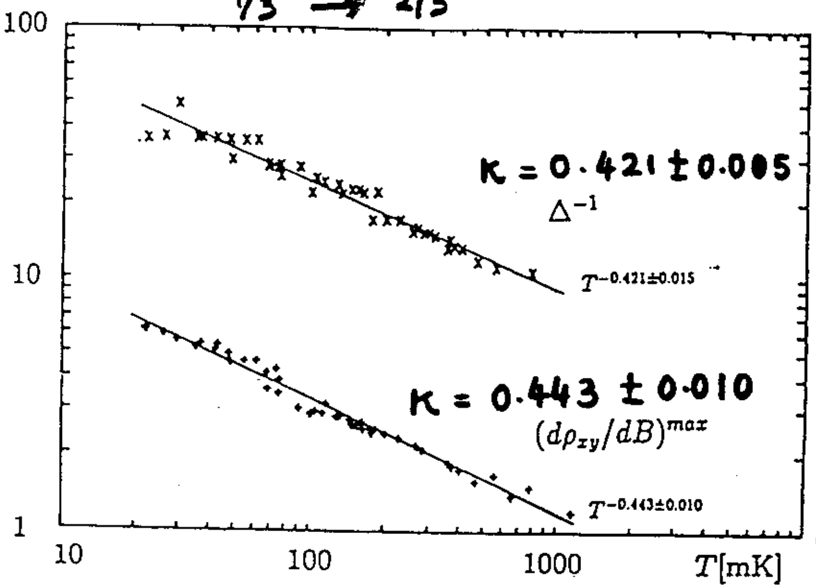
$K = 0.42 \pm 0.04$

(indept. of LL index)

Wei, Tsui, Paalanen & Pruisken PRL 61 1294 ('88)

Transition between fractional plateaus.

$1/3 \to 2/5$



Engel, Wei, Tsui, Shayegan Proc. EP2DS VIII 1989

The trial wave functions are valid representations of the FQHE states even without the projection. Because:

(i) They are largely in the lowest Landau level,

(ii) They are adiabatically connected to the projected wave functions.

$$\frac{\text{K.E.}}{\text{N}} = (t + \tfrac{1}{2})\hbar\omega_c$$

filling factor	wave function	t
2	Φ_2	0.5
2/5	$D\Phi_2$	0.04
2/9	$DD\Phi_2$	0.02

Trivedi and Jain

Trugman Kivelson interaction

$$H = \sum_j \frac{1}{2m}\left(\vec{p}_j + \frac{e}{c}\vec{A}_j\right)^2 + \frac{1}{2}\sum_{i<j} V(r_{ij})$$

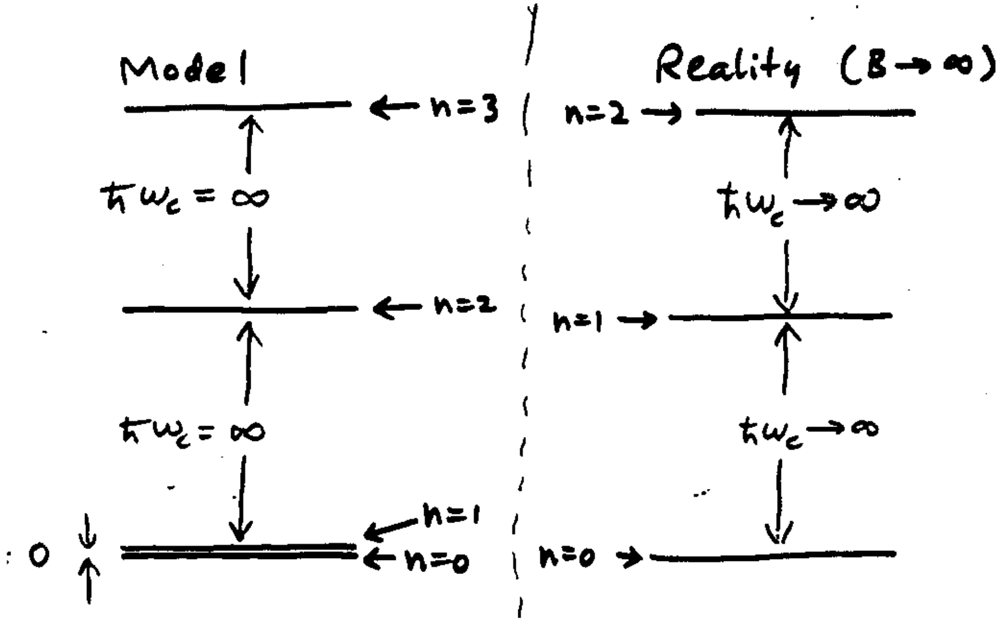
$$V(r_{ij}) = \nabla^2 \delta(r_{ij})$$

$$\int \psi^* \nabla^2 \delta(r_{ij}) \psi = \int \delta(r_{ij}) \nabla^2 |\psi|^2$$

$$\begin{cases} \neq 0 & \text{if } \psi \sim r_{ij} \text{ as } r_{ij} \to 0 \\ = 0 & \text{if } \psi \sim r_{ij}^3 \text{ as } r_{ij} \to 0 \end{cases}$$

Keeping only the lowest Landau level, at $\nu = \frac{1}{3}$, Laughlin's state is the unique zero energy state.

Keep two lowest LL's. $\chi_{2/5} = \prod_{j<k}(z_j - z_k)^2 \Phi_2$ has zero interaction energy, but not an eigenstate unless the LL's are degenerate. In that case $\chi_{2/5}$ is the unique ground state.



Rezayi and MacDonald

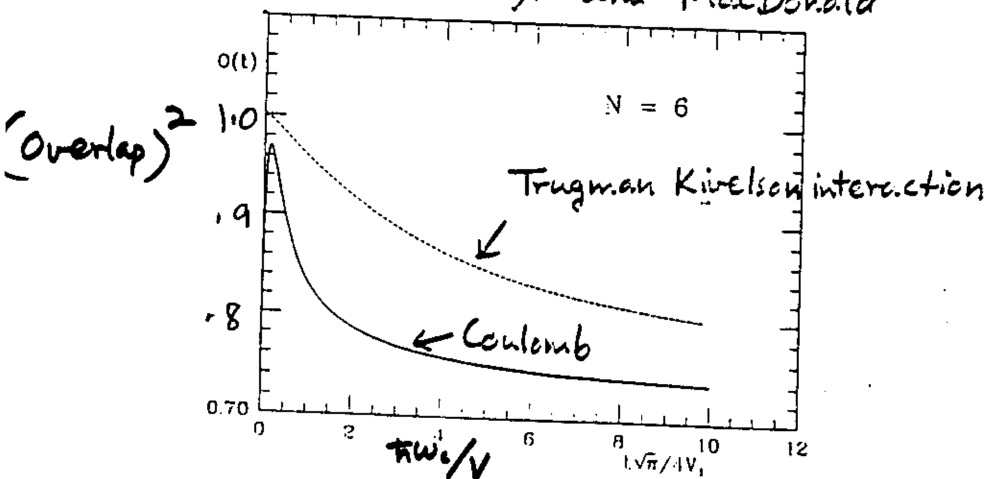


FIG. 1. Overlap with Jain's states for the ground state of the hard-core (dashed line) and Coulomb (solid line) models as a function of Landau-level separation.

<u>No Phase Transition !</u>

The physics of the formation of the composite fermions is quite clear for the unprojected states.

The trial functions

$$\prod_{j<k} (z_j - z_k)^{2p} \Phi_n$$

vanish as r_{jk}^{2p+1} as $r_{jk} \to 0$

and are efficient in keeping electrons apart and producing low interaction energy.

Thus, excellent short distance correlations can be built manifestly at the cost of a slight hybridization with higher LL's.

Composite fermion theory

(i) Idea — formation of Composite fermions

explains — gap at $\frac{p}{q}$, q odd ; principal sequences

(ii) Microscopic implementation — LOW ENERGY STATES ARE :

$$\chi^{FQHE}_{\nu,\alpha} = [\mathcal{P}] \prod_{j<k} (z_j - z_k)^{2m} \, \Phi^{IQHE}_{\nu^*,\alpha}$$

$$\nu = \frac{\nu^*}{2m\nu^* + 1}$$

(iii) Compares well with real and numerical experiments.

SUMMARY

$$\boxed{\chi_{\nu,\alpha} = P \underbrace{\prod_{j<k} (z_j - z_k)^{2p}}_{\text{TURNS EACH ELECTRON INTO A COMPOSITE FERMION}} \; \underbrace{\Phi_{\nu^*,\alpha}}_{\text{WEAKLY INTERACTING ELECTRONS}} \qquad \nu = \frac{\nu^*}{2p\nu^* + 1}}$$

- THE ESSENTIAL NON-PERTURBATIVE EFFECT OF REPULSIVE INTERACTIONS IS TO GENERATE COMPOSITE FERMIONS. THE RESIDUAL INTERACTIONS BETWEEN COMPOSITE FERMIONS CAN OFTEN BE NEGLECTED, ESPECIALLY WHEN THERE IS A GAP IN THE EXCITATION SPECTRUM.
- THE STRONGLY CORRELATED ELECTRON SYSTEM AT ν IS SIMILAR TO WEAKLY INTERACTING COMPOSITE FERMIONS AT ν^*. THERE IS A ONE-TO-ONE CORRESPONDENCE BETWEEN THE LOW LYING STATES OF THE TWO SYSTEMS.

> THE PHYSICS OF FQHE LIES IN THE FORMATION OF COMPOSITE FERMIONS THROUGH THE FACTOR $\prod_{j<k} (z_j - z_k)^{2p}$.