INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

UNITED NATIONS INDUSTRIAL DEVELOPMENT ORGANIZATION
INTERNATIONAL CENTRE FOR SCIENCE AND HIGH TECHNOLOGY
c/o INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS 34100 TRIESTE (ITALY) VIA GRIGNANO, 9 (ADRIATICO PALACE) P.O. BOX 586 TELEPHONE 040-224572 TELEFAX 040-224575 TELEX 460449 APH I

SMR. 628 - 2

Research Workshop in Condensed Matter,
Atomic and Molecular Physics
(22 June - 11 September 1992)

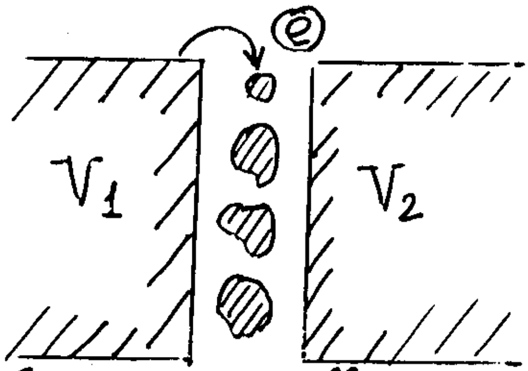
Working Party on
"NOISES IN MESOSCOPIC SYSTEMS"
(27 July - 7 August 1992)

" Coulomb Blockade of an Electron Tunnelling"

R. SHEKHTER
Institute of Theoretical Physics
Chalmers University of Technology
S-412 96 Goteborg
Sweden

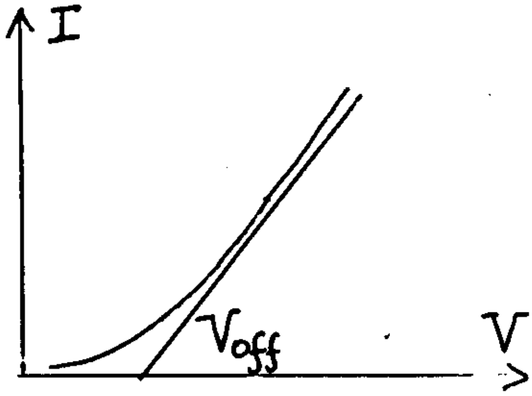
These are preliminary lecture notes, intended only for distribution to participants.

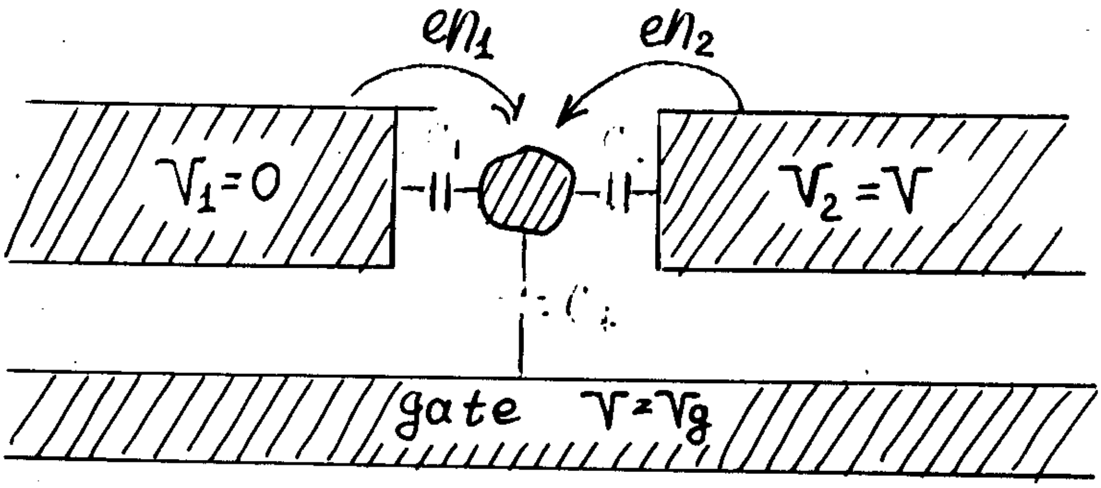
Coulomb Blockade of an Electron Tunnelling



I. Giaever, H.R. Zeller
PRL, 20, 1504, (1968)

$U = \frac{e^2}{C}$

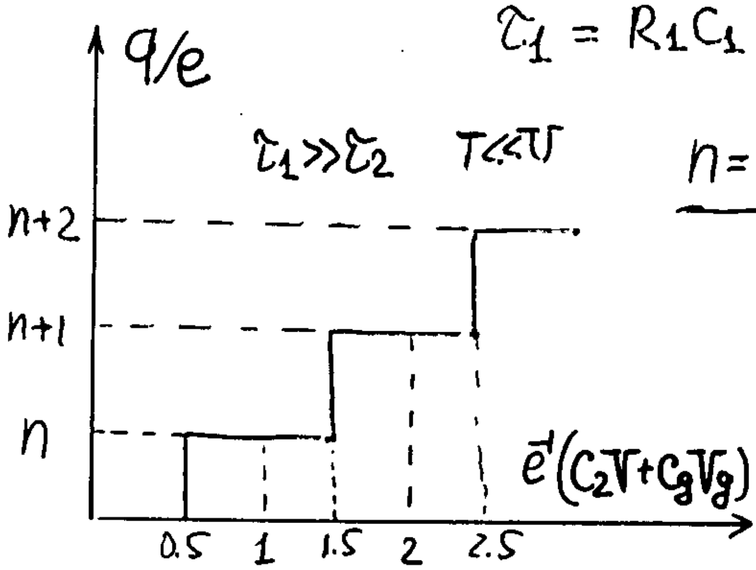




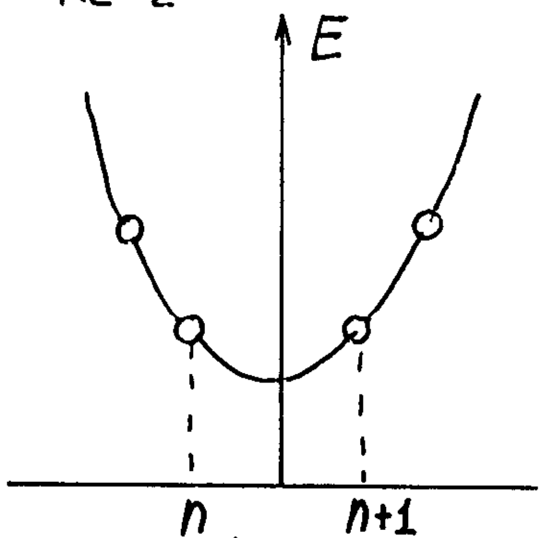
$$E(n_1 n_2) = -eV_1 n_1 - eV_2 n_2 + U(n_1+n_2)^2 + \frac{en}{C}\left[C_1V_1 + C_2V_2 + C_gV_g\right]$$

$\tau_1 = R_1C_1 \qquad \tau_2 = R_2C_2$

$n = n_1 + n_2$

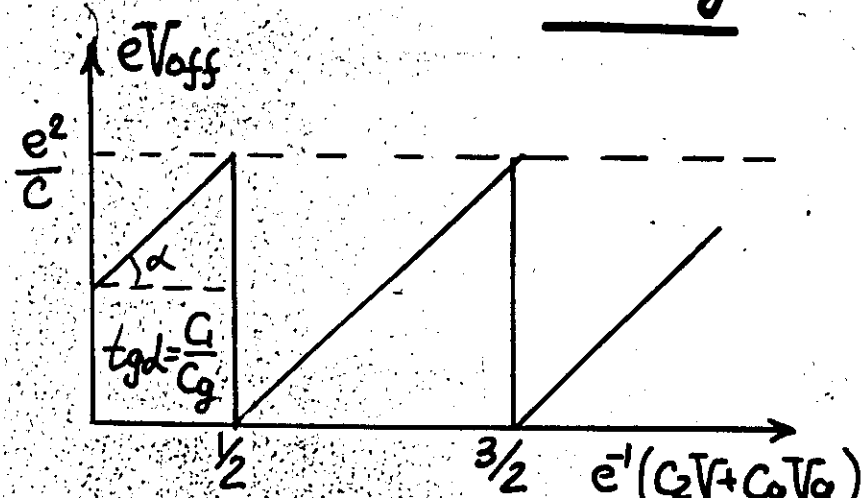


J. Lambe, R.C. Jaclevic
PRL, 22, 1371 (1969)

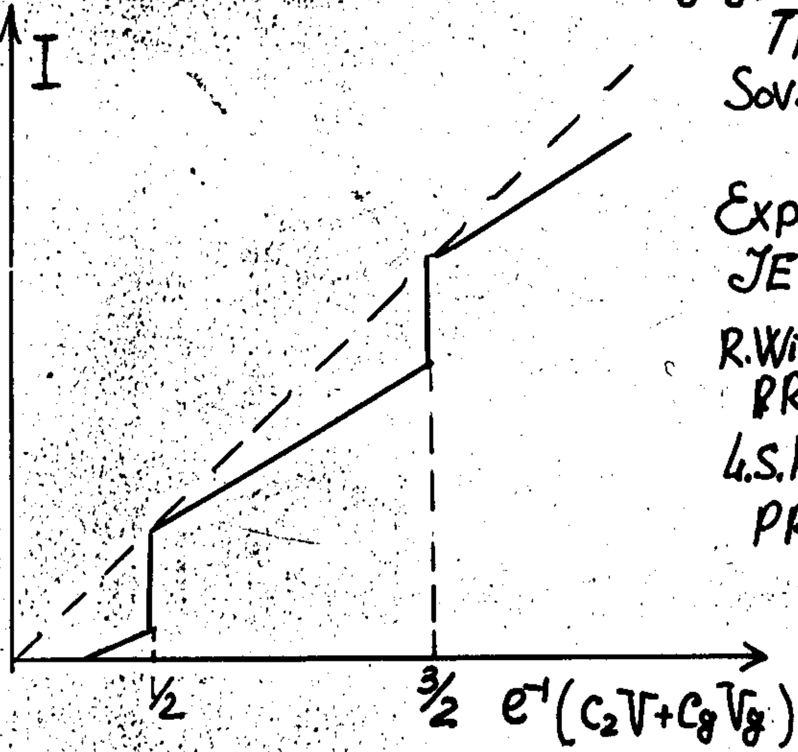


R.I. Shekhter
Sov. Phys. JETP, 36, 747, (19

Coulomb Oscillations of Electrical Conductivity.



$$V_{off} = \frac{e}{C}\left[\frac{1}{2} + \frac{q}{e} + C_2V + C_gV_g\right]$$

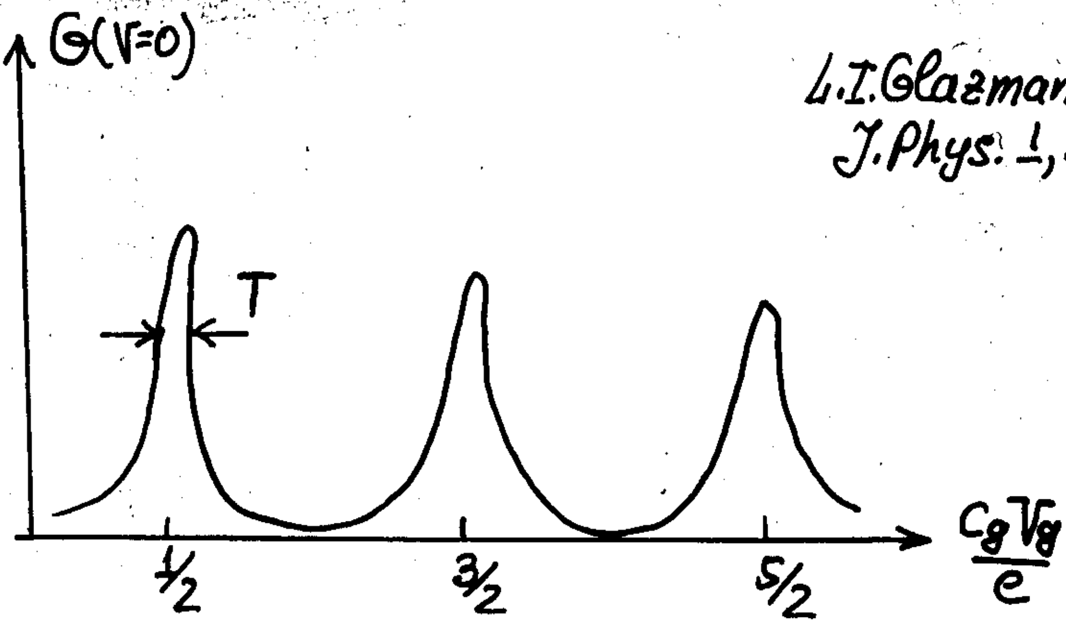


Theory: I.O. Kulik, R.I. Shekhter Sov. Phys. JETP, 41, 306, (1975)

Exp. L.S. Kuzmin, K.K. Likharev JETP Lett. 45, 495, (1987)

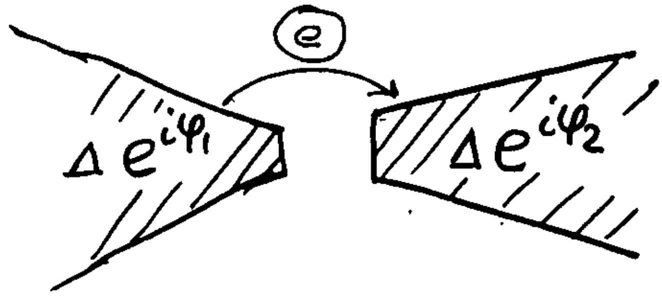
R. Wilkins, E. Ben Jacob, R.C. Jaclevicl, PRL, 63, 801, (1989)

L.S. Kuzmin, P. Delsing, T. Claeson, PRL 62, 2539, (1989)



L.I. Glazman, R.I. Shekhter J. Phys. 1, 5811, (1989)

Proximity effect in superconductors



Coupling Energy

$$E_J = -E_0 \cos(\varphi_1 - \varphi_2)\,; \quad E_0 = \frac{1}{2}\frac{G}{G_0}\Delta\, \mathrm{th}\frac{\Delta}{2T}$$

G - normal state conductance $\quad G_0 = \frac{4e^2}{h}$

Josephson current

$$I_J = I_c \sin(\varphi_1 - \varphi_2)\,; \quad I_c = \frac{2e}{\hbar} E_0$$

Wave function of the condensate

$$\Psi_{BSC} = \prod_K (u_K + e^{i\varphi} v_K a^+_{K\uparrow} a^+_{-K\downarrow})|0\rangle$$

$$u_K^2 = \frac{1}{2}\left(1 + \frac{\xi_K}{\varepsilon_K}\right); \quad v_K^2 = \frac{1}{2}\left(1 - \frac{\xi_K}{\varepsilon_K}\right)$$

$$\xi_K = \frac{\hbar^2 K^2}{2m} - \mu; \quad \varepsilon_K = \sqrt{\xi_K^2 + \Delta^2}$$

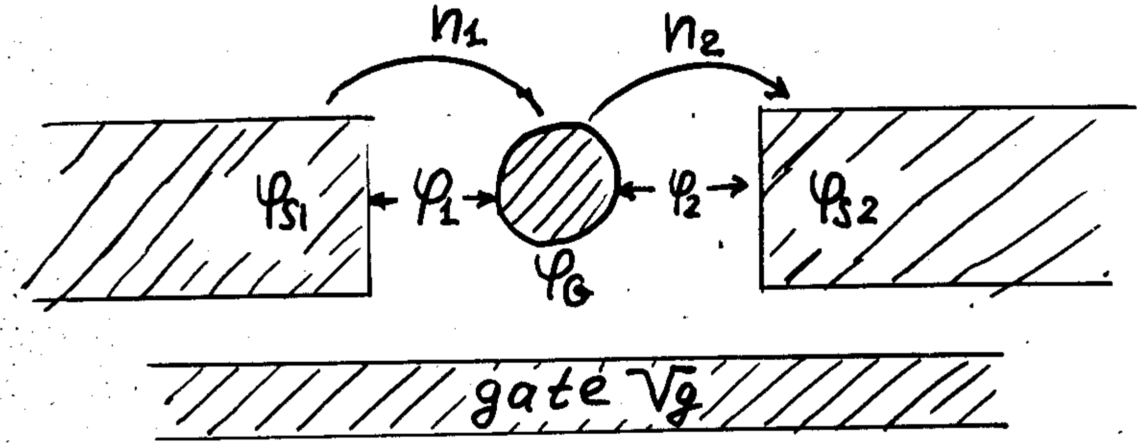
$$N = \sum_{K\alpha} a^+_{K\alpha} a_{K\alpha}, \quad \boxed{-N\Psi_{BSC} \equiv 2i\frac{\partial}{\partial\varphi}\Psi_{BSC}}$$

$$[\varphi, N] = 2i$$

$$\delta N \cdot \delta\varphi = 2\pi$$

P.W. Anderson (1962)

Josephson Tunneling in a Small-area Junctions



$$E(n_1,n_2)=U\left(n_1-n_2+e^{-1}C_gV_g\right)^2$$

$$\varphi_1=\varphi_G-\varphi_{S1} \qquad \varphi_2=\varphi_{S2}-\varphi_G \qquad \varphi_1+\varphi_2=\varphi_{S2}-\varphi_{S1}$$

$$n_1=-2i\frac{\partial}{\partial\varphi_1} \qquad n_2=2i\frac{\partial}{\partial\varphi_2}$$

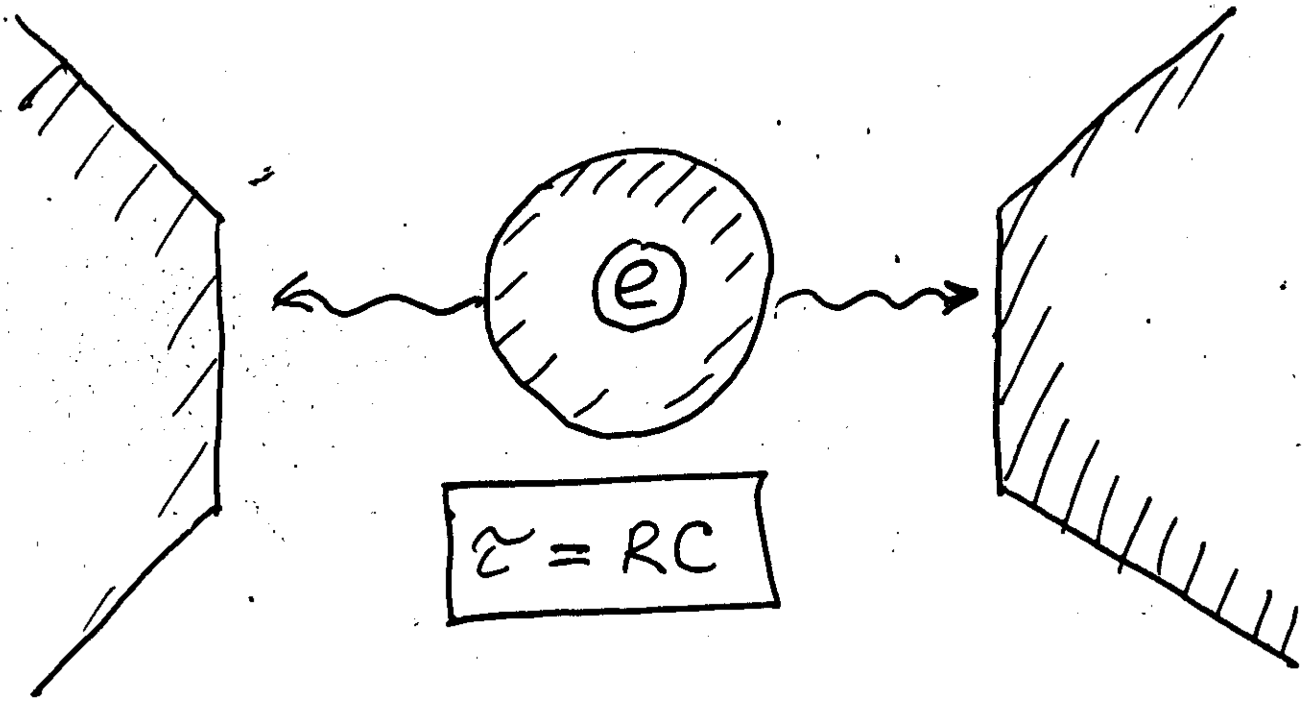
$$\boxed{H_c=U\left[2i\left(\frac{\partial}{\partial\varphi_2}-\frac{\partial}{\partial\varphi_1}\right)+e^{-1}C_gV_g\right]^2}$$

A.B. Zorin, K.K. Likharev. J. Low. Temp. Phys. 59, 347, (1985)

$$H=H_c-E_J(\varphi_1,\varphi_2)$$

Quatum fluctuation of the phase

But: $\varphi_1+\varphi_2\equiv\varphi_{S2}-\varphi_{S1}$ – Does not fluctuate!

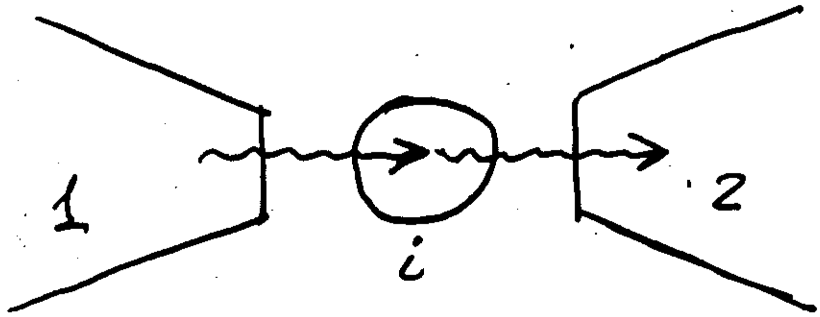


Condition for the Coulomb Blockade

$$\frac{\hbar}{\tau} \ll U = \frac{e^2}{2C}$$

$$\frac{G}{G_0} \ll 1 \ , \quad G_0 = \frac{4e^2}{\hbar}$$

Tunneling through the grain



Double electron transition

$$T_{12} \simeq T_{1i} T_{i2}$$

Normal-state conductance

$$G \simeq \frac{G_1 G_2}{G_0} \max\left\{ \left(\frac{T}{U}\right)^2, \frac{\Delta E}{U} \right\}$$

D.V. Averin, A.A. Odintsov, Phys. Lett. A, 140, 251 (1989)

L.I Glazman, K.A. Matveev, JETP Lett. 51, 484, (1990)

Formulation of the Problem

$$H = H_G + H_{S1} + H_{S2} + H_T^{(1)} + H_T^{(2)}$$

$$H_T^{(i)} = \sum_{p_i q} \left(T_{p_i q}\, a^+_{p_i\sigma} a_{q\sigma} + h.c. \right)$$

$$H_G = \sum_{q\sigma} \xi_q\, a^+_{q\sigma} a_{q\sigma} + U(n-n^*)^2$$

$i=1,2 \quad H_{Si} = \sum_{p_i\sigma} \varepsilon_{p_i}\, \alpha^+_{p_i\sigma} \alpha_{p_i\sigma} \; ; \quad \varepsilon_p = \sqrt{\xi_p^2 + \Delta^2}$

$$a_{p\sigma} = u_p \alpha_{p\sigma} + \mathrm{Sign}(\sigma)\, v_p^* \alpha^+_{-p-\sigma}$$

$$a^+_{p\sigma} = u_p \alpha^+_{p\sigma} + \mathrm{Sign}(\sigma)\, v_p\, \alpha_{-p-\sigma}$$

$$u_p^2 = \frac{1}{2}\left(1 + \frac{\xi_p}{\varepsilon_p}\right); \quad |u_p|^2 + |v_p|^2 = 1 \; ;$$

$$(u_p v_p)_i = -\frac{\Delta}{\varepsilon_p} e^{i\varphi_i}$$

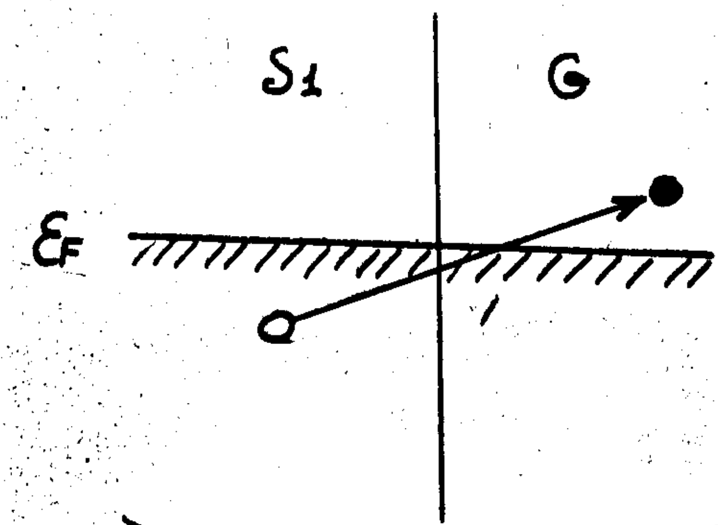
Electric Current Calculation

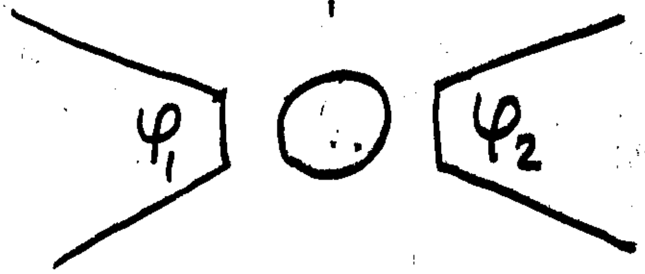
$$T \ll \Delta \ll U$$

$$I = e\left\langle \frac{dN_1}{dt} \right\rangle = \frac{ie}{\hbar}\left\langle [H_T^{(1)}, N_1] \right\rangle$$

$$I = \frac{2e}{\hbar}\, \mathrm{Im}\left\{ \sum_{pq\sigma} T_{pq} \left\langle a^+_{p\sigma} a_{q\sigma} \right\rangle \right\}$$

$$\langle \ldots \rangle = \sum_n W(n)\, Sp\{\ldots\}_n \; ; \quad W(n) = \frac{1}{Z} \exp -\beta U (n-n^*)$$





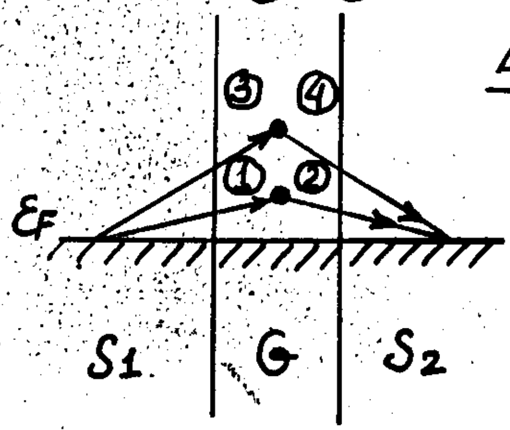
$$\boxed{I = I_c \cdot \sin(\varphi_1 - \varphi_2)}$$

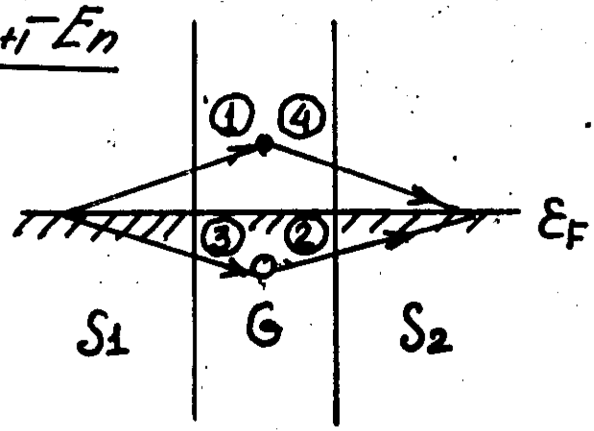
$I_c(V_g)$ is periodic function

Diagrams of the Cooper Pair Tunnelling.

Electron channels of tunnelling

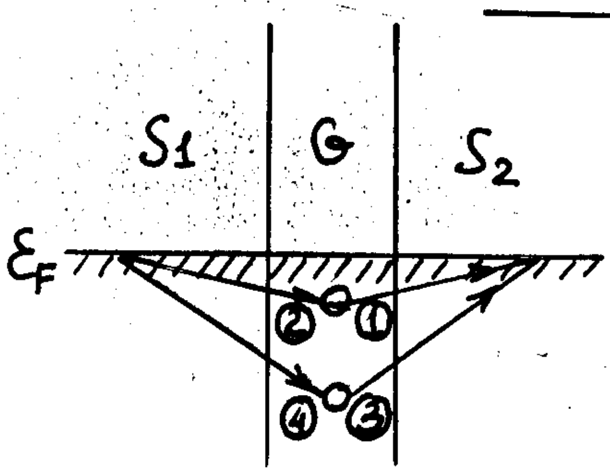
$$E^{(e)} = E_{n+1} - E_n$$

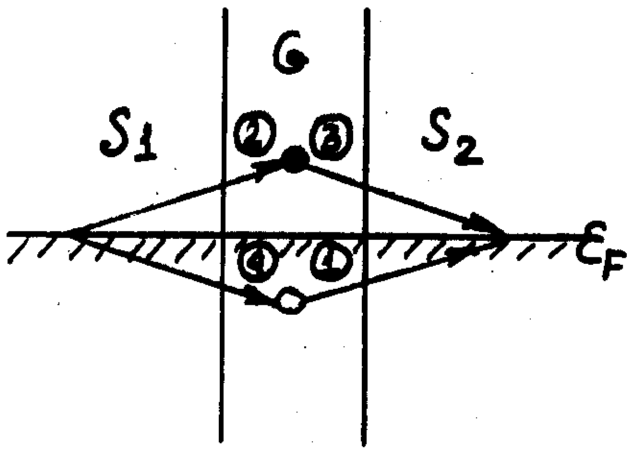




Hole channels of tunnelling

$$E^{(h)} = E_{n-1} - E_n$$





$$\boxed{\longrightarrow \;=\; \frac{T_{pq}}{E_1 - E_2}}$$

The Critical Josephson Current.

$$I = I_c \sin\varphi$$

$$I_c = I_c^{(e)} + I_c^{(h)}$$

$$I_c^{(e)} = 4e \sum_{\substack{p_1 p_2 \\ q_1 q_2 \sigma}} (T_{p_1 q_1} T_{q_1 p_2})(T_{p_1 q_2} T_{q_2 p_2})^* (uv)_{p_1} (uv)_{p_2} \times$$

$$\times \left\{ \frac{(1-n_{q_1\sigma})(1-n_{q_2\sigma})}{(E^{(e)}(V_g)+\xi_{q_1}+\varepsilon_{p_1})(E^{(e)}(V_g)+\xi_{q_2}+\varepsilon_{p_2})(\varepsilon_{p_1}+\varepsilon_{p_2})} - \right.$$

$$\left. - \frac{n_{q_2\sigma}(1-n_{q_1-\sigma})}{(E^{(e)}(V_g)+\xi_{q_1}+\varepsilon_{p_1})(E^{(e)}(V_g)+\xi_{q_1}+\varepsilon_{p_2})(\varepsilon_{p_1}+\varepsilon_{p_2}+\xi_{q_1}-\xi_{q_2})} \right\}$$

$$I_c^{(h)} = 4e \sum_{\substack{p_1 p_2 \\ q_1 q_2 \sigma}} (T_{p_1 q_1} T_{q_1 p_2})(T_{p_1 q_2} T_{q_2 p_2})^* (uv)_{p_1} (uv)_{p_2} \times$$

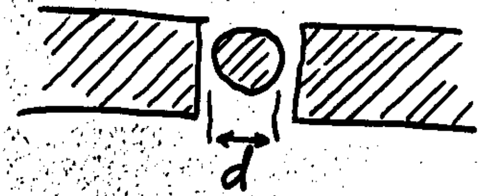
$$\times \left\{ \frac{n_{q_1\sigma} n_{q_2-\sigma}}{(E^{(h)}(V_g)-\xi_{q_1}+\varepsilon_{p_1})(E^{(h)}(V_g)-\xi_{q_2}+\varepsilon_{p_2})(\varepsilon_{p_1}+\varepsilon_{p_2})} - \right.$$

$$\left. - \frac{n_{q_1\sigma}(1-n_{q_2-\sigma})}{(E^{(h)}(V_g)-\xi_{q_1}+\varepsilon_{p_1})(E^{(h)}(V_g)-\xi_{q_1}+\varepsilon_{p_2})(\varepsilon_{p_2}+\varepsilon_{p_1}+\xi_{q_1}-\xi_{q_2})} \right\}$$

Charge degeneracy condition

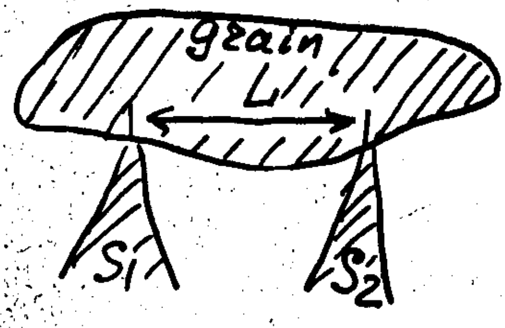
$$E^{(e)}(V_g) = 0 \quad \text{or} \quad E^{(h)}(V_g) = 0; \quad V_g = V_N = \frac{e}{C_g}(N + 1/2)$$

Mesoscopic Oscillations



$$(T_{p_1q_1}T_{q_1p_2})(T_{p_1q_2}T_{q_2p_2})^* \simeq e^{i(q_1-q_2)d}$$

Mesoscopic oscillations are absend in a semiconducting grain.



$$L \lesssim \lambda_F$$

I. $\Delta E_{space} \ll U$

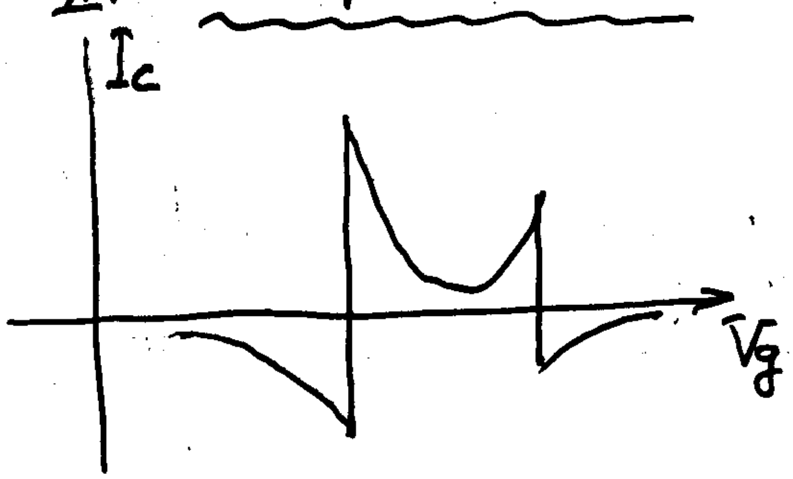
$$\boxed{I_c(V_g) = \frac{G_1G_2}{2G_0}\frac{\Delta}{e}\ln^2\left(\frac{U}{\Delta+\alpha e|V_g-V_N|}\right);\ \alpha=\frac{C_g}{C}}$$

II. $\Delta E_{space} \gg U$

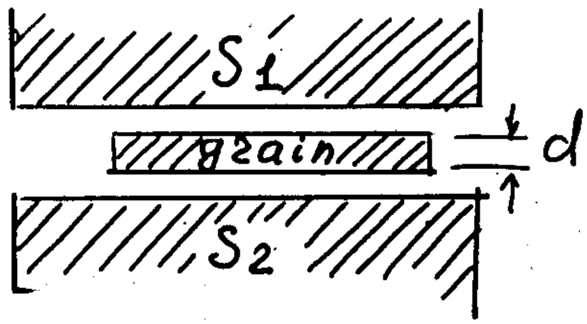
$$I_{c\,max} \simeq G_0\frac{\Gamma_1\Gamma_2}{\Delta}$$

$$\Gamma_1\Gamma_2 \simeq 2\pi\frac{(G(T))^2}{G_0\,G(0)}$$

$$G(T)=G(T\gg\Gamma_{1,2})$$
$$G(0)=G(T\ll\Gamma_{1,2})$$



Josephson Current as a Mesoscopic Effect.



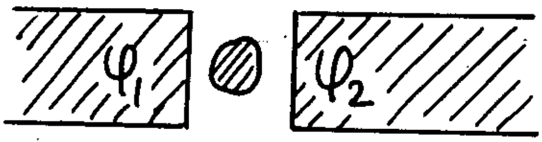
$$I_{c\,max} \equiv I_c(V_g = V_N) = 8\,\frac{G_1 G_2}{G_Q}\,\frac{\Delta}{e}\left(\frac{\delta E}{\Delta}\right)^2$$

$$\delta E = \frac{\hbar v_F}{d}$$

$$\boxed{\begin{gathered} I_{c\,max}(T) \simeq I_{c\,max}(0)\left(\frac{T}{\delta E}\right)^2 \exp\left(-2\pi\frac{T}{\delta E}\right); \\ T \gg \delta E \end{gathered}}$$

$$I_c(T) \simeq \exp\left(-d/\xi_N(T)\right);\quad \xi_N(T) = \frac{\hbar v_F}{2\pi T}.$$

Tunnelling Through The Superconducting Grain.



$$I = I_c \sin(\varphi_1 - \varphi_2)$$

$$I_c = I_c^{(e)} + I_c^{(h)}$$

$$I_c^{(e,h)} = 16 \sum_{\substack{p_1 p_2 \\ q_1 q_2}} |T_{p_1 q_1}|^2 \, |T_{p_2 q_2}|^2 \times$$

$$\times \frac{(u\vartheta)_{p_1} (u\vartheta)^*_{p_2} (u\vartheta)_{q_1} (u\vartheta)^*_{q_2}}{\left(E^{(e,h)}(V_g) + \varepsilon_{p_1} + \varepsilon_{q_1}\right)\left(E^{(e,h)}(V_g) + \varepsilon_{p_2} + \varepsilon_{q_2}\right)\left(\varepsilon_{p_1} + \varepsilon_{p_2} + \varepsilon_{q_1} + \varepsilon_{q_2}\right)}.$$

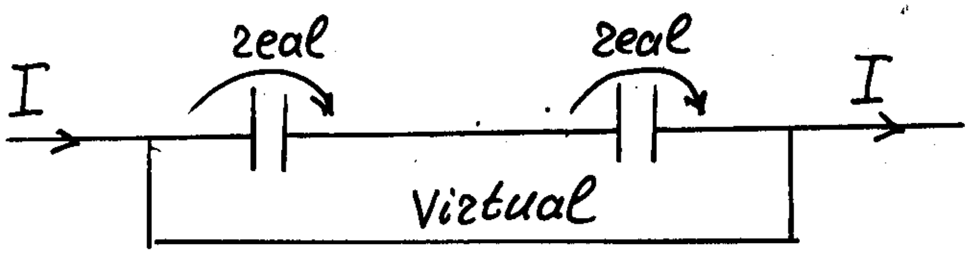
$$I_{c\,max} \equiv I_c (V_g = V_N) = \gamma \cdot \frac{G_1 G_2}{G_0} \frac{\Delta}{e}; \quad \gamma \simeq 1$$

$$\boxed{\begin{gathered} I_c(V_g) \simeq I_{c\,max} \cdot \left(\frac{\alpha \Delta}{e|V_g - V_N|}\right)^3 \\ \alpha e |V_g - V_N| \gg \Delta; \quad \alpha = \frac{C_g}{C} \end{gathered}}$$

$$I_{c\,max} \simeq \frac{G_1 G_2}{G_0 (G_1 + G_2)} I_{c0} \simeq \frac{G}{G_0} I_{c0}$$

The Possibility of an Experimental Observation.

$\frac{G}{G_0} \ll 1.$



Parameter Values from the experiment: P. Delsing, K.K. Likharev, L.S. Kuzmin, T. Claeson, PRL (1989), 63, N17, 1861.

$$\frac{G}{G_0} \simeq 2.25 \cdot 10^{-2}; \quad E_J^{real} \simeq 3 \cdot 10^{-24}\,J; \quad \Delta = 0.15\,meV; \quad U = 0.3\,me$$

$$E_J^{virtual} = \frac{4}{\pi} \frac{G}{G_0} E_J^{real}$$

$$KT < E_J^{virt.} \implies T < 10\,mK$$

$$I_c^{virt} = 70\,pA$$

VOLUME 65, NUMBER 3 PHYSICAL REVIEW LETTERS 16 JULY 1990

Single Cooper-Pair Tunneling in Small-Capacitance Junctions

L. J. Geerligs, V. F. Anderegg, J. Romijn,[a] and J. E. Mooij
Department of Applied Physics, Delft University of Technology, P.O. Box 5046, 2600 GA Delft, The Netherlands
(Received 2 April 1990)

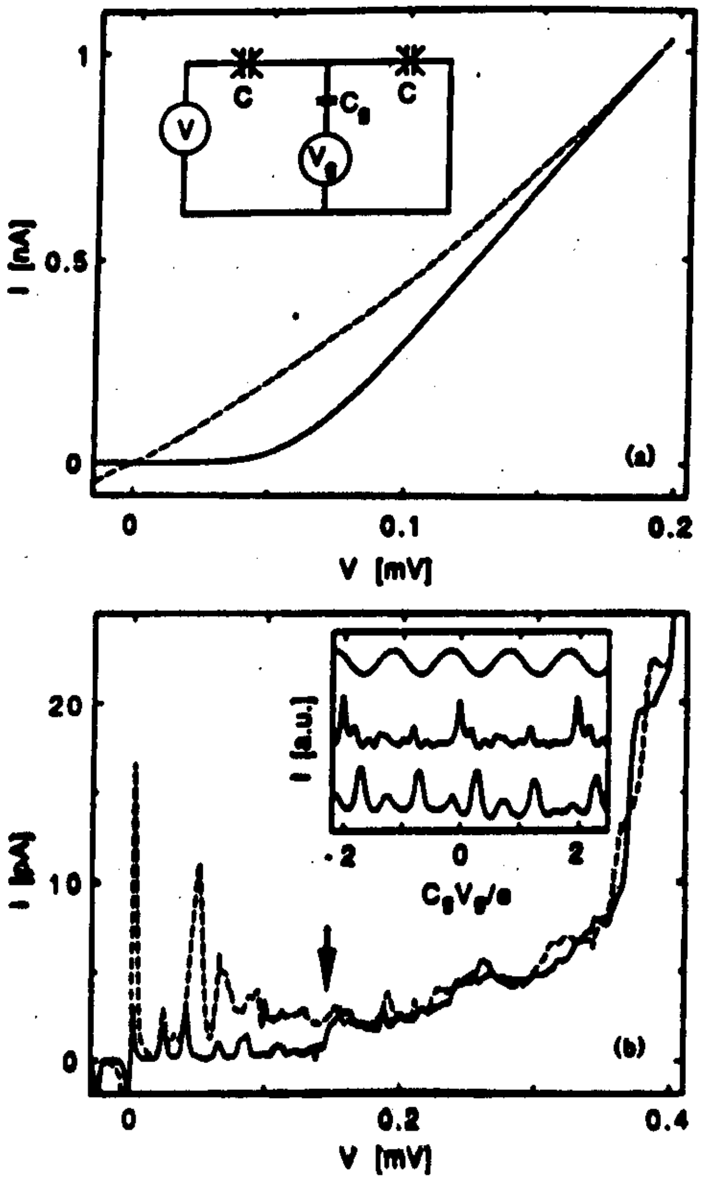


FIG. 1. I-V curves of a double junction with $R_n = 58$ kΩ ($E_J/E_C = 0.13$) for two different values of the gate voltage V_g at 10 mK. (a) In the normal state, realized by applying a magnetic field of 2 T. The Coulomb gap with a maximum value of about 70 μV (solid curve) can be suppressed with the gate voltage (dashed curve). Inset: Device and measurement layout. The junctions are denoted by crossed capacitor symbols, $C_g \approx 0.01C$. (b) In the superconducting state a Cooper-pair gap of about 150 μV arises (arrow). Coulomb gap and supercurrent are strongly dependent on gate voltage (compare solid and dashed curves). Inset: I-V_g curves for the normal state (top), the current peak at 20 μV (middle), and the supercurrent at $V = 0$ (bottom).