



INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE



UNITED NATIONS INDUSTRIAL DEVELOPMENT ORGANIZATION

INTERNATIONAL CENTRE FOR SCIENCE AND HIGH TECHNOLOGY
c/o INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS 34100 TRIESTE (ITALY) VIA GRIGNANO, 9 (ADRIATICO PALACE) P.O. BOX 586 TELEPHONE 040/234512 TELEFAX 040/234513 TELEX 460444 APH I

SMR. 628 - 28

**Research workshop in Condensed Matter,
Atomic and Molecular Physics
(22 June - 11 September 1992)**

**Working Party on:
"Energy Transfer in Interactions with
Surfaces and Adsorbates"
(31 August - 11 September 1992)**

**"Multiphonons excitation in
HAS from surfaces"**

V. CELLI
University of Virginia
Department of Physics
Charlottesville, VA 22901
U.S.A.

These are preliminary lecture notes, intended only for distribution to participants.

He scattering for $10 \text{ meV} < E_i < 100 \text{ meV}$

**The Trajectory Approximation
and its Improvements**

**The Transition from Quantum to
Classical Scattering**

**One-Phonon and Multiphonon
Processes**

D. Himes - Virginia

Bortolani, Franchini, Santoro - Modena

Toennies, Wöll, Zhang - MPI Göttingen

**Old Wisdom: Structureless
Multiphonon Background**

New Wisdom: Multiphonon Features

PRL 66 (1991) 3160 ; Surf. Sci. 242 (1991) 518

JR Manson, Phys. Rev. B43 (1991) 3524

2
ITA ANALYSIS IN THE ONE-PHONON APPROX.

PEAKS IN TDF SPECTRA $\rightarrow$ SURFACE PHONONS

SCATTERED INTENSITY $\rightarrow$ SURFACE-PROJECTED
PHONON DENSITY OF STATES

(ALMOST) ONLY TOP SURFACE LAYER SAMPLED

$$U_2 \frac{\partial V}{\partial z} + i \vec{Q}_{//} \cdot \vec{u}_{//} V \text{ MATRIX ELEMENT}$$

(WITH BOND CHARGES?)

V IS A SUM OF PAIRWISE POTENTIALS

DISTORTED WAVE BORN APPROX.
(DWBA)

ONE-PHONON INTENSITY $\sim \begin{cases} n_B \\ n_B + 1 \end{cases}$ ANNI.
CR.

MUST BE MULTIPLIED BY

$$e^{-2W}$$

WITH $2W \sim T$

DEBYE-WALLER FACTOR

NEED TO UNDERSTAND MULTIPHONON PROCESSES TO

- SUBTRACT "STRUCTURELESS MULTIPHONON BACKGR

- USE He ENERGIES $\gtrsim 40$ meV

- DESCRIBE Ne, Ar, NO ... SCATTERING

OF EXCHANGED PHONONS = $2W$

$$P_n = e^{-2W} \frac{(2W)^n}{n!}$$

$$2W \approx \frac{24 \mu T (D + E_i \cos^2 \theta_i)}{k_B T_D^2} = \langle (\vec{q} \cdot \vec{u})^2 \rangle$$

$T_D \approx$ bulk Debye temperature

D = Surface well depth

E_i = Incident energy μ = mass ratio

WHAT HAPPENS FOR $W \gtrsim 1$?

Trajectory Approximation (TA)

$$N(\Delta \vec{k}, \Delta E) = \int dt d^2 R_{\parallel} e^{i(\Delta E t - \Delta \vec{k}_{\perp} \cdot \vec{R}_{\parallel})} e^{\Gamma(\vec{R}_{\parallel}, t) - 2W}$$

where $\Gamma(\vec{R}_{\parallel}, t)$ is linearly related to the correlation function

$$\langle u_{\alpha}(\vec{R}_{\parallel}, t) u_{\beta}(\vec{0}, 0) \rangle$$

and $2W = \Gamma(\vec{0}, 0)$ is the DW factor.

For 1-d hard-wall collision:

$$\Gamma(\vec{R}_{\parallel}, t) = \langle u_z(\vec{R}_{\parallel}, t) u_z(\vec{0}, 0) \rangle q_z^2$$

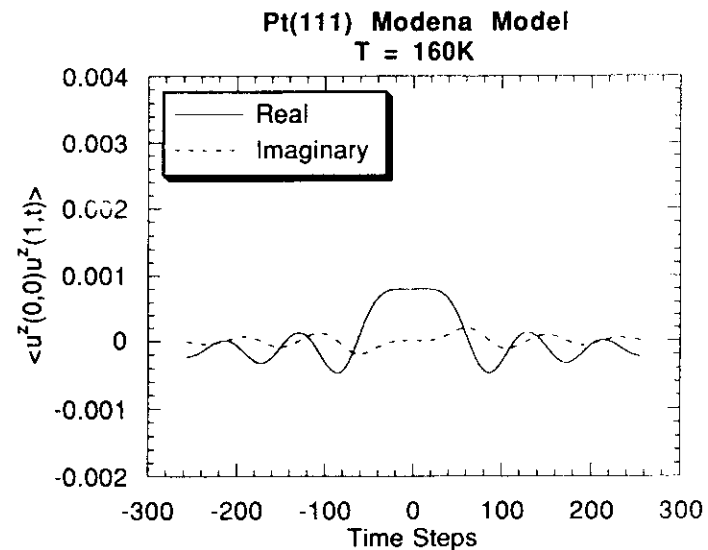
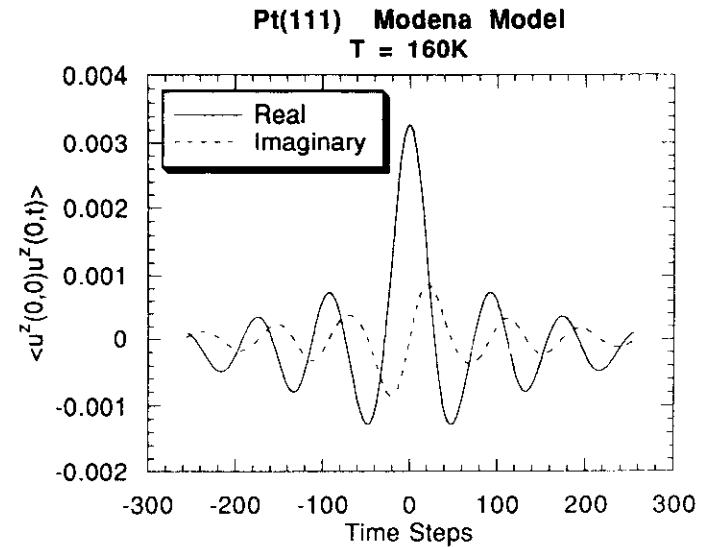
where q_z is the $\perp$ momentum transfer

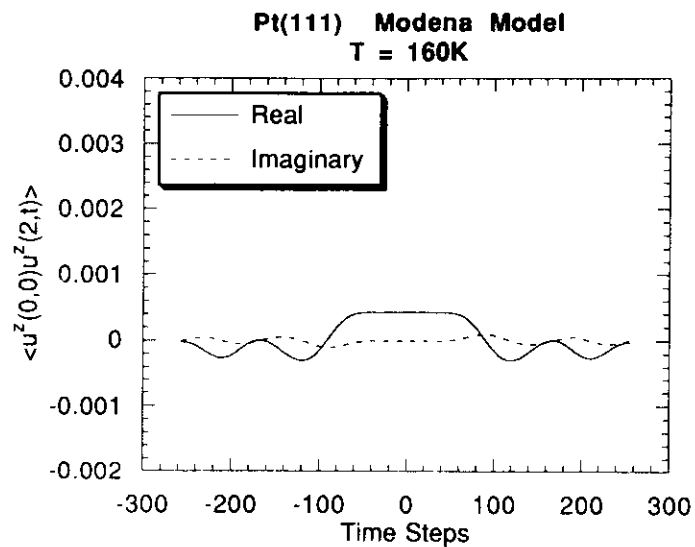
$$(q_z u_z \Rightarrow \sum_{\vec{r}} \int d\tau \vec{F}(\vec{r}(\tau) - \vec{R}_{\parallel}) \cdot \vec{u}(\vec{R}_{\parallel}, t))$$

generally

$\vec{F}$ is the He-surface atom force along the trajectory $\vec{r}(t)$

Typical correlation functions





6

TIME OF FLIGHT

Kinematics - Scan Curves

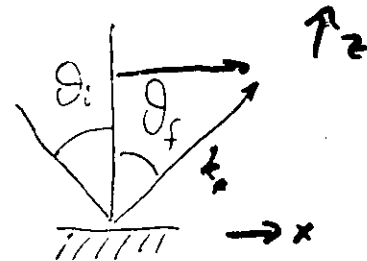
Conservation laws:

Energy $\hbar\omega_f = \hbar(\omega_i + \omega) = E_i + \Delta E$

Parallel Momentum $K_{fx} = k_{ix} + \Delta K_x$

Geometry:

$$K_{fx} = k_f \sin \theta_f$$



Algebra:

$$k_f = \frac{K_{fx}}{\sin \theta_f} = \frac{(K_i + \Delta K)_x}{\sin \theta_f}$$

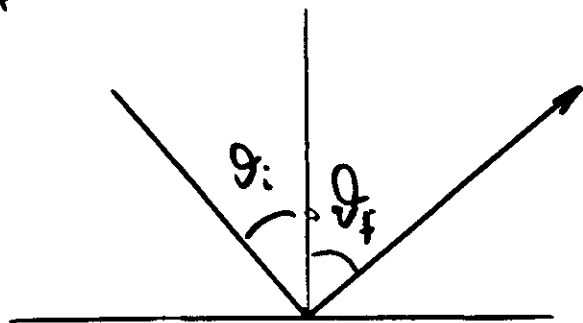
$$\frac{\omega_f}{\omega_i} = \frac{k_f^2}{k_i^2} = \frac{(K_i + \Delta K)_x^2}{k_i^2 \sin^2 \theta_f}$$

x-comp.

$$\boxed{\frac{\Delta E}{\hbar\omega_i} = \frac{(K_i + \Delta K)_x^2}{k_i^2 \sin^2 \theta_f} - 1}$$

a parabola

$$\theta_i + \theta_f = 90^\circ$$



8

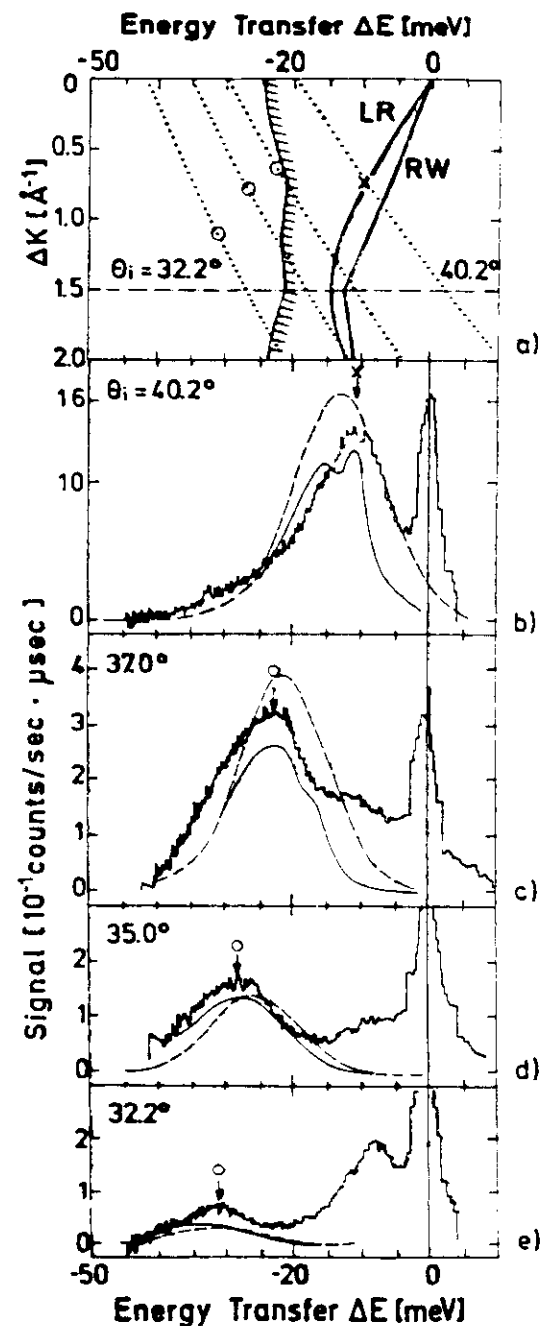
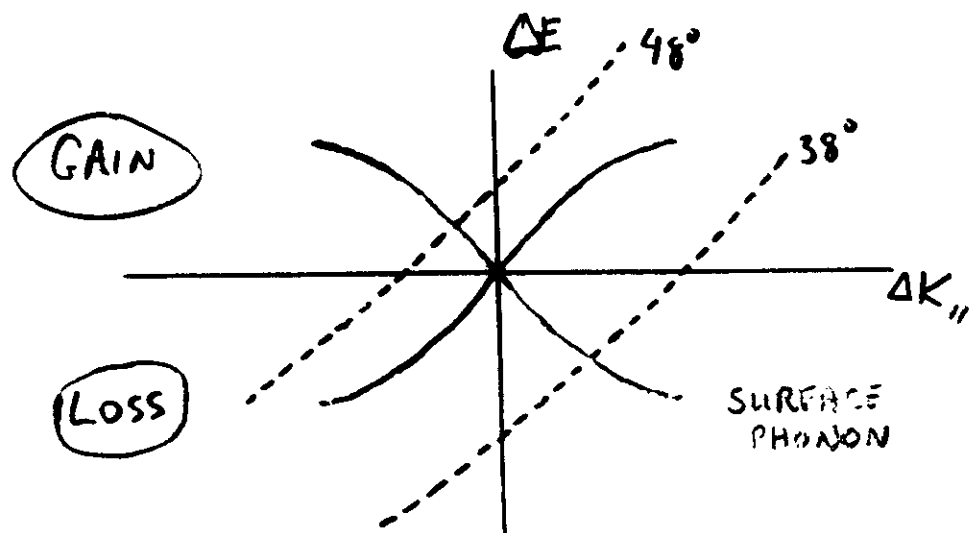
Pt (111)
Göttingen data
PRL '91

9

ON A GIVEN TOF RUN, THE ENERGY TRANSFER ΔE IS RELATED TO THE MOMENTUM TRANSFER ΔK

$$\text{BY } \Delta E = \hbar \omega = E_i \left[\frac{(\sin \theta_i + K_{||} / k_i)^2}{\sin^2 \theta_f} - 1 \right]$$

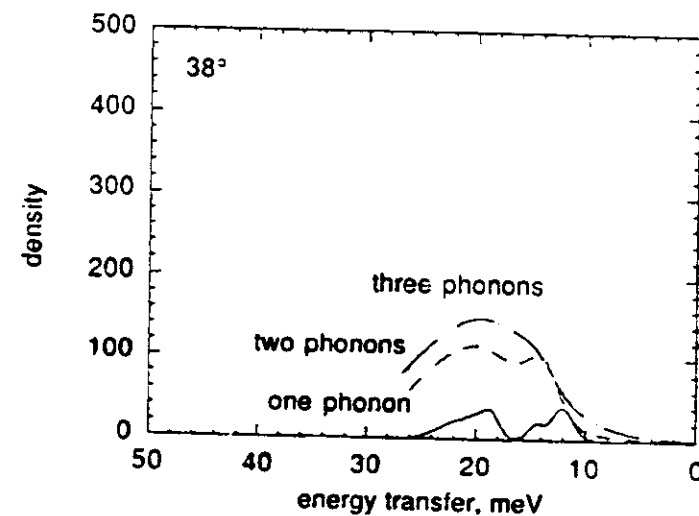
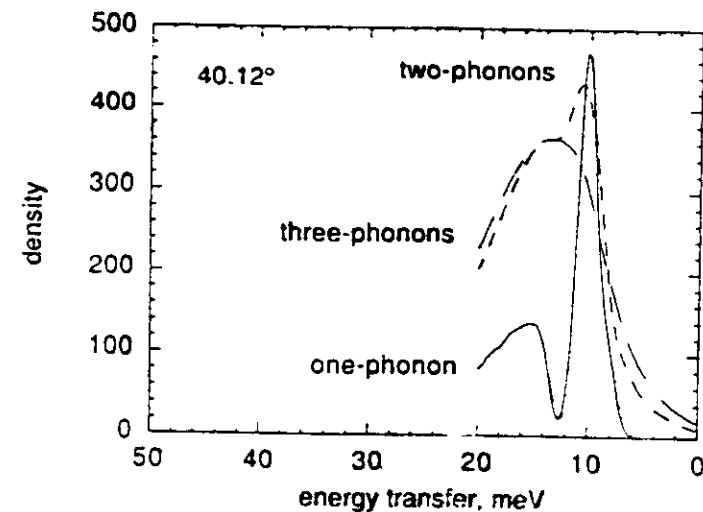
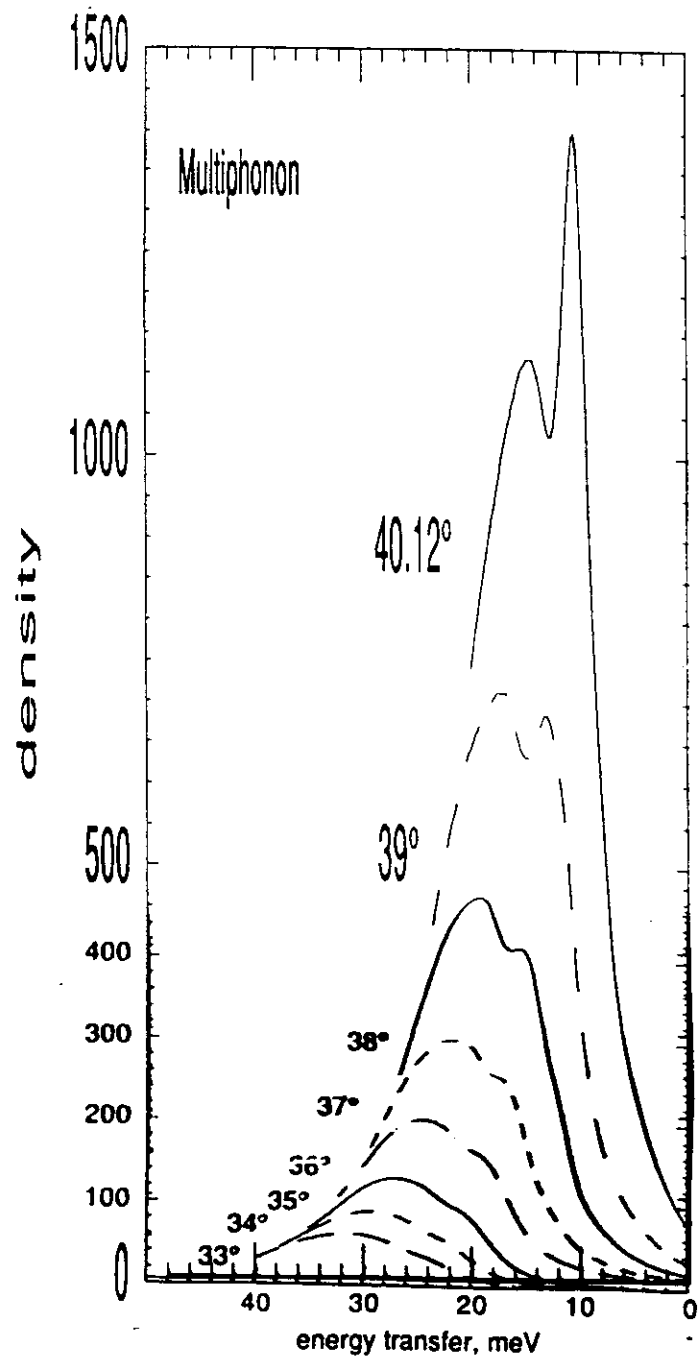
A SCAN CURVE IS A PLOT OF ΔE VS. $\Delta K_{||}$



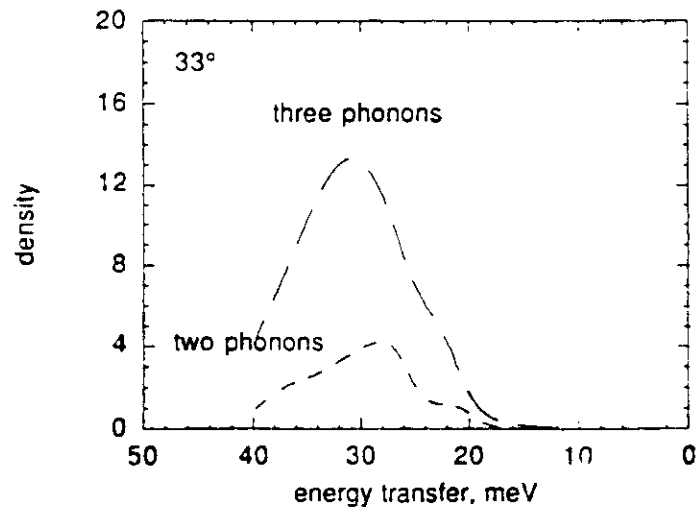
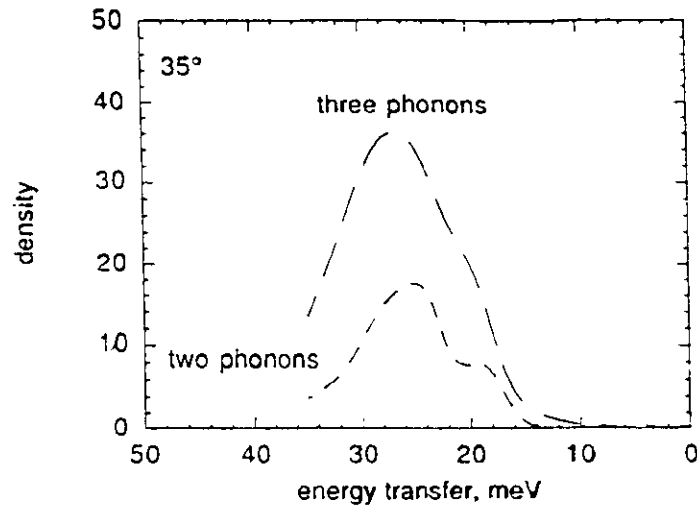
Pt(111)
Himes

Pt(111)
Himes

$$\beta = 1.83 \text{ \AA}^{-1} \quad Q_c = 0.58 \text{ \AA}^{-1}$$



kt(III)
times



BRAGO-NEWNS:

$$N(\Delta \vec{k}_{||}, \Delta E) \approx \frac{1}{T^{3/2}} e^{-\frac{(\Delta E - \bar{\Delta E})^2}{4k_B T \bar{\Delta E}}} e^{-\frac{k^2 v^2 (\Delta k_{||})^2}{2k_B T \bar{\Delta E}}}$$

WITH BAULE FORMULA FOR AVERAGE ENERGY TRANSFER

$$\bar{\Delta E} = -\frac{4\mu}{(1+\mu)^2} [E_i \cos^2 \theta_i + D]$$

- IT IS A GAUSSIAN IN ΔE , $\Delta \vec{k}_{||}$

- IT IS CENTERED AT $\bar{\Delta E} = (\bar{\Delta E})_{\text{Baule}}$

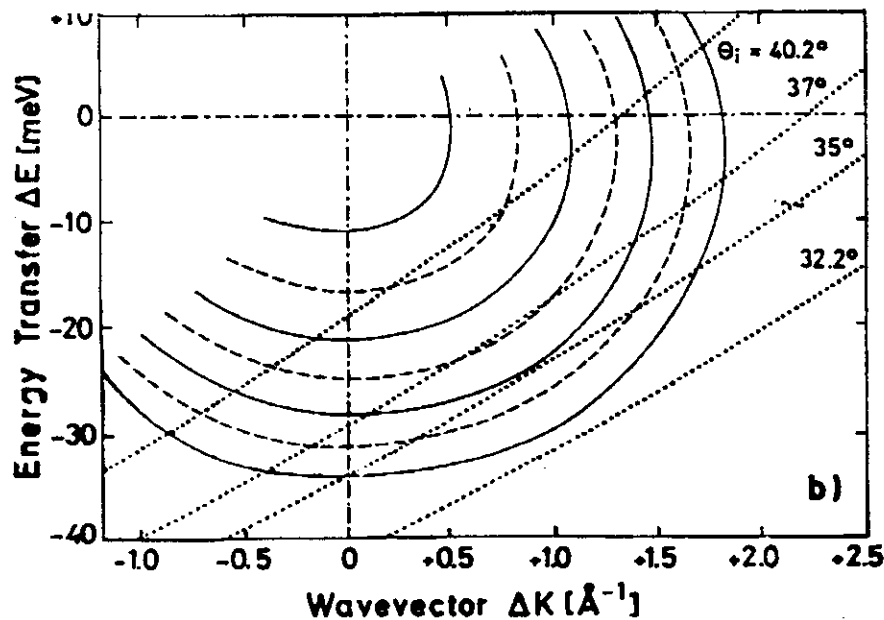
- IT IS CENTERED AT $\bar{\Delta k}_{||} = 0$

(CONSISTENT WITH BAULE FORMULA FOR $\bar{\Delta E}$ AND HARD CUBE MODEL)

- IT ASSUMES THAT THE SURFACE PHONON SPECTRUM IS DOMINATED BY A BRANCH

$$\omega = v k_{||}$$

He/Pt (111) at $E_i = 69$ meV



THE CALCULATIONS REPORTED SO F
USED THE CLASSICAL SPECULAR TRAJECT
DEFECTS OF STA

- IT GIVES AN AVERAGE ENERGY LOSS ALWAYS

INDEPENDENT OF SURFACE TEMPERATURE

- IN THE CLASSICAL LIMIT, IT GIVES A GAUSSIAN
CENTERED AT THE AVERAGE ENERGY LOSS

• INCLUDE He RECOIL WITH

EXPONENTIATED DISTORTED WAVE BORN

$$\Delta E(T) = 4\mu [E_i \cos^2 \theta_i + D - k_B T]$$

$$N(\Delta K_{||}, \Delta E) \sim \frac{1}{T^{3/2}} e^{-\frac{(\Delta E - \Delta E(T))^2}{4k_B T |\Delta E(0)|}} e^{-\frac{\frac{1}{2} v^2 \Delta K^2}{2k_B T \Delta E}}$$

IT GIVES A GAUSSIAN CENTERED AT $\Delta E(T)$,

Exponentiated

Where the Distorted Wave Born Approximation gives

$$\sum_j \frac{n(\omega(Q, j))}{\omega(Q, j)} |e(Q, j) \cdot F(k_f, k_i)|^2 e^{-i\omega(Q, j)t}$$

the Trajectory Approximation gives:

$$\sum_j \frac{n(\omega(Q, j))}{\omega(Q, j)} |e(Q, j) \cdot F(Q, j)|^2 e^{-i\omega(Q, j)t}$$

The explicit expressions for F in the two cases are

$$F(k_f, k_i) = \int dz \chi(k_{fz}|z)^* F(Q|z) \chi(k_{iz}|z)$$

$$F(Q, j) = \int_{-\infty}^{\infty} dt F(Q|z(t)) e^{-i(V \cdot Q + \omega(Q, j))t}$$

The wave functions χ are normalized to unit incoming current. Semiclassically $F(k_f, k_i)$ is

$$\int_0^{\infty} dz \frac{e^{i \int^z [k_{iz}(z') - k_{fz}(z')] dz'}}{\frac{\hbar}{m} \sqrt{k_{iz}(z) k_{fz}(z)}} F(z) +$$

$$\int_{-\infty}^0 dz \frac{e^{i \int^z [k_{fz}(z') - k_{iz}(z')] dz'}}{\frac{\hbar}{m} \sqrt{k_{iz}(z) k_{fz}(z)}} F(z)$$

which resembles $F(Q, j)$, as can be seen by replacing

dt with $\frac{dz}{v}$.

RATIONALE FOR THE EDWBA

$$e^{\Gamma} = 1 + \Gamma + \dots$$

↑
ONE-PHONON TERM $\Rightarrow$ DWBA

PRESCRIPTION

$$\text{DWBA} \xrightarrow{(F.T.)^{-1}} \Gamma(\vec{R}_n, t) \rightarrow e^{\Gamma} \xrightarrow{F.T.} N(\Delta \vec{k}_n, \Delta E)$$

THIS IS A TRAJECTORY APPROXIMATION, IN

THE SENSE THAT

$$N(\Delta \vec{k}_n, \Delta E) = \int d^2 R dt e^{\Gamma(\vec{R}_n, t) - 2W} e^{i(\Delta E t - \Delta \vec{k}_n \cdot \vec{r})}$$

WHERE $\Gamma(\vec{R}_n, t)$ DOES NOT DEPEND ON $\Delta E, \Delta \vec{k}$

THERE ARE RELATED APPROXIMATIONS IN

WHICH Γ (AND $2W$) DO DEPEND ON $\Delta \vec{k}_n, \Delta E$

THE SIMPLEST IS THE FICOVA. FOR A VIBRATION.

(EVEN SPACIAL TORSION.)

"RIPPLING WALL"

$$V(\Delta E) = \int dt e^{\Gamma(t, \Delta E) - 2W(\Delta E)} e^{-i\Delta E t}$$

$$\Gamma(t, \Delta E) = \langle u_z(t) u_z(0) \rangle [p_i + p_f(\Delta E)]$$

$$p_f^2(\Delta E) = p_i^2 + 2m\Delta E/\hbar^2$$

THE EDWBA HAS $\int d\omega' [p_i + p_f(\omega')] u_z^2(\omega') e^{-i\omega' t}$

INSTEAD OF $[p_i + p_f(\Delta E)] u_z^2(t)$

ADVANTAGES OF RWA

$$\overline{\Delta E}(T) = 4\mu [\bar{E}_i \cos^2 \theta_i + D - k_B T] \quad \left(\begin{array}{l} \text{same} \\ \text{as EDWBA} \end{array} \right)$$

CLASSICALLY, IT DOES NOT GIVE A GAUSSIAN
ENTERED AT $\overline{\Delta E}(T)$, BUT AN ASYMMETRIC
DISTRIBUTION PEAKED AT $\overline{\Delta E}(0)$.

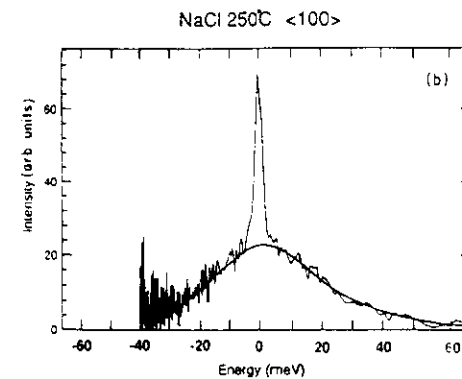
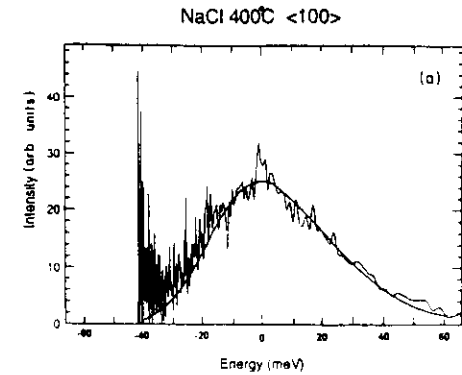


FIG. 1. The scattered intensity as a function of energy exchange for He on a NaCl(001) surface in the <100> direction. The incident beam wave vector is $9.2 \text{ \AA}^{-1}$, and the angle of incidence and the detector angle are both 45° , measured from the surface normal. The solid curve is the calculation. (a) Surface temperature of 673 K. (b) Surface temperature of 523 K.

The cutoff parameter is $Q_c = 5 \text{ \AA}^{-1}$ and the range parameter is $R = 6 \text{ \AA}^{-1}$ which is a reasonable value.

CONCLUSIONS:

- IN He TDF SPECTRA, MULTIPHONON PROCESSES HAVE A CHARACTERISTIC SIGNATURE
- WE UNDERSTAND BETTER THE TRANSITION FROM QUANTUM TO CLASSICAL BEHAVIOR FOR He AND Ne SCATTERING
- (HOPEFULLY) THIS UNDERSTANDING CAN BE EXPORTED TO STICKING AND REACTIVE SCATTERING.

