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**Research Workshop in Condensed Matter,
Atomic and Molecular Physics
(22 June - 11 September 1992)**

**Working Party on
"NOISES IN MESOSCOPIC SYSTEMS"
(27 July - 7 August 1992)**

"The Main Ideas in Mesoscopical Physics"

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The main ideas in mesoscopical physics

Three types of systems:

1. Macroscopical systems
2. Microscopical systems.
3. Mesoscopical systems

1. Total average of properties
2. Detailed description
3. Memory about individual properties

Mesoscopical systems

The averaging on impurities conserves the phase memory in dirty metals

at $\underline{T=0}$: Schrödinger Equation describes all properties.
The solutions know about initial conditions

Temperature and inelastic processes destroy the phase memory

Characteristic lenght:

$$L_T = \left(\frac{\hbar D}{T}\right)^{1/2} \quad \text{or} \quad L_\varphi = \sqrt{D\tau_\varphi}$$

D is diffusion coefficient

at $T=0$ all systems are mesoscopical

$$E_c = \frac{\hbar D}{L^2}$$

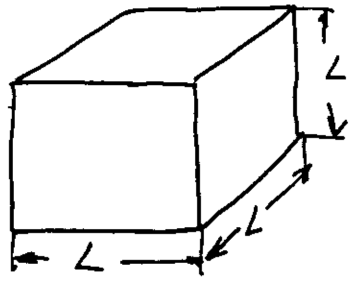
1. Universal fluctuations

Boltzman conductivity

$$\sigma = e^2 D \nu \qquad g = \sigma \frac{\hbar}{e^2} L^{d-2}$$

ν is the density of states

$$\frac{\hbar}{e^2} \simeq 3{,}6\ \mathrm{k}\Omega$$



Thouless formulae

$$g = \frac{\delta E}{\Delta} = \frac{\hbar}{\Delta \tau_{dif.}} = \frac{E_c}{\Delta}$$

$$E_c = \frac{\hbar}{\tau_{dif}} = \frac{D\hbar}{L^2}$$

Δ is the energy spacing

$$\Delta = \frac{1}{\nu L^d} \qquad \Delta_0 = \frac{1}{\langle \nu \rangle L^d}$$

Fluctuations

$$\frac{\langle \delta g^2 \rangle^{1/2}}{\langle g \rangle} \sim \frac{\delta D}{D} + \frac{\delta \nu}{\nu} \sim \frac{\delta \nu}{\nu} \simeq \frac{\Delta_0}{E_c} \sim \frac{1}{\langle g \rangle}$$

$$\delta \nu^{-1} \simeq \frac{\hbar}{\tau_{dif.}} = E_c \gg \Delta_0$$

$$\boxed{\langle \delta g^2 \rangle^{1/2} \simeq 1} \qquad \delta \nu \ll \nu$$

(Altshuler; Stone)

$g = \frac{E_c}{\Delta} = N_T$ is the number of electron states in the energy strip of the width E_c

In metals $g \gg 1$, $N_T \gg 1$

Metal - isolator transition : $g_c \simeq 1$

$$\underline{N_T^c \simeq 1}$$

Universal fluctuations mean

$$\underline{\langle \delta N_T^2 \rangle \sim 1}$$ (Altshuler and Shklovski)

According to the theory of energy levels statistics (Wigner, Dyson)

$$\langle \delta N^2 (E_c) \rangle = \frac{8}{\pi^2 \beta} \ln \frac{E_c}{\Delta}$$

β depends on the symmetry:

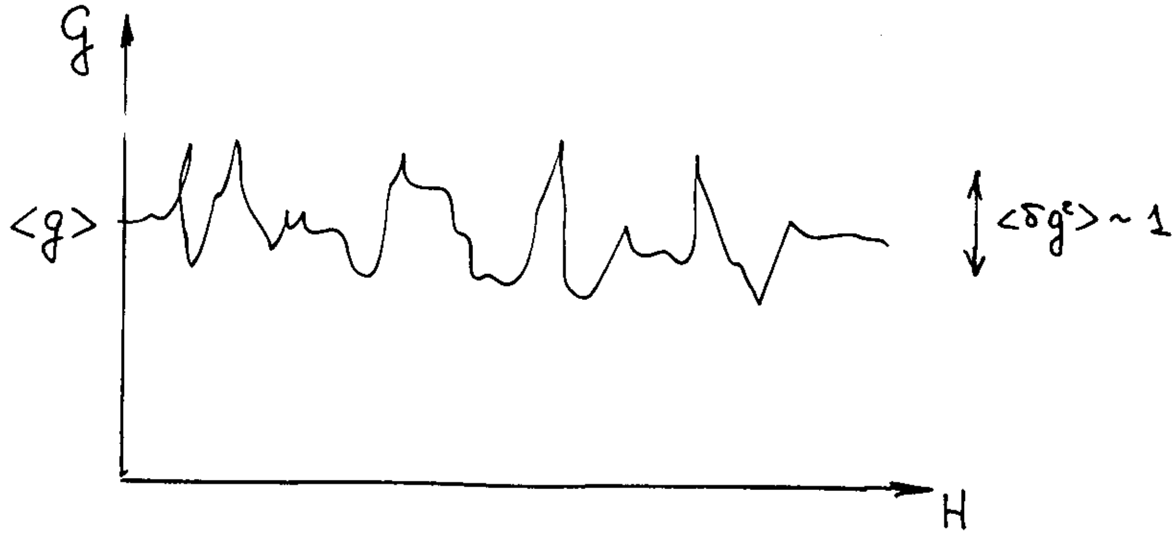
$\beta = 1$ for T - invariant system without spin - orbit interaction

$\beta = 2$ when T - invariance is broken

$\beta = 3$ for a T - invariant system with s-o. interaction

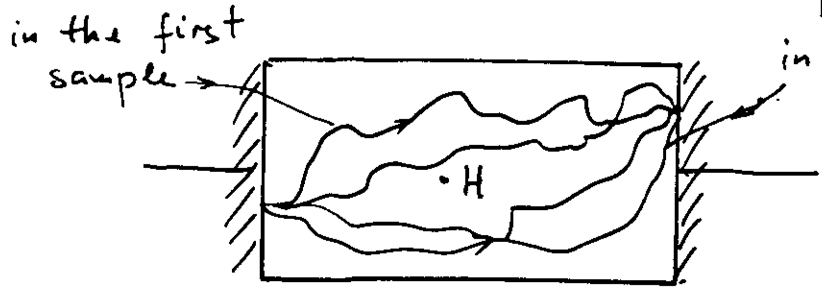
$\ln \frac{E_c}{\Delta}$ reflects the difference between a closed and open systems

Magnetofingerprints



What is the reason?

The reason is the quantum interference



$$\Delta\varphi = 2\pi \frac{\Phi(L)}{\Phi_0} \sim 2\pi$$

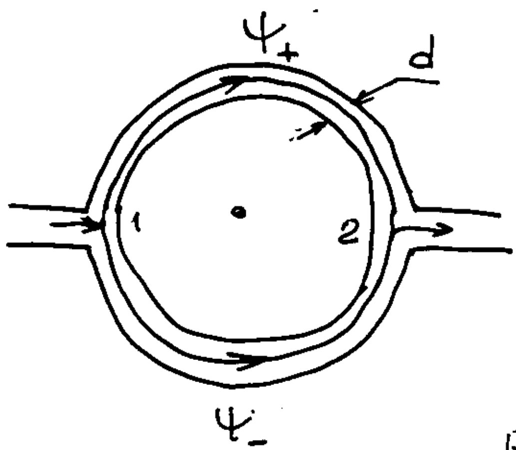
$$\Phi_0 = \frac{hc}{e} \qquad H_L \simeq \frac{\Phi_0}{L^2}$$

at $T \neq 0$ $\qquad \Delta\varphi \simeq 2\pi \frac{\Phi(L_T)}{\Phi_0}$ $\qquad \Phi_0 \simeq 10^{-7}$ g·cm²

$$H_T \simeq \frac{\Phi_0 T}{D}$$

at $T \simeq 1K$, $D \simeq 10^2 \frac{cm^2}{s}$

$H_T \simeq 10^2$ gauss



One dimensional case

$d \ll L_T \qquad \varphi_+ = \frac{e}{\hbar}\int_{1+}^{2+} A\,dl$

$$|\Psi|^2 = |\Psi_+ + \Psi_-|^2 =$$

$$= |\Psi_+|^2 + |\Psi_-|^2 + 2\,\mathrm{Re}\,\Psi_+^*\Psi_- \cos\frac{2\pi\Phi}{\Phi_0}$$

periodic with $\Phi_0 = \frac{hc}{e}$

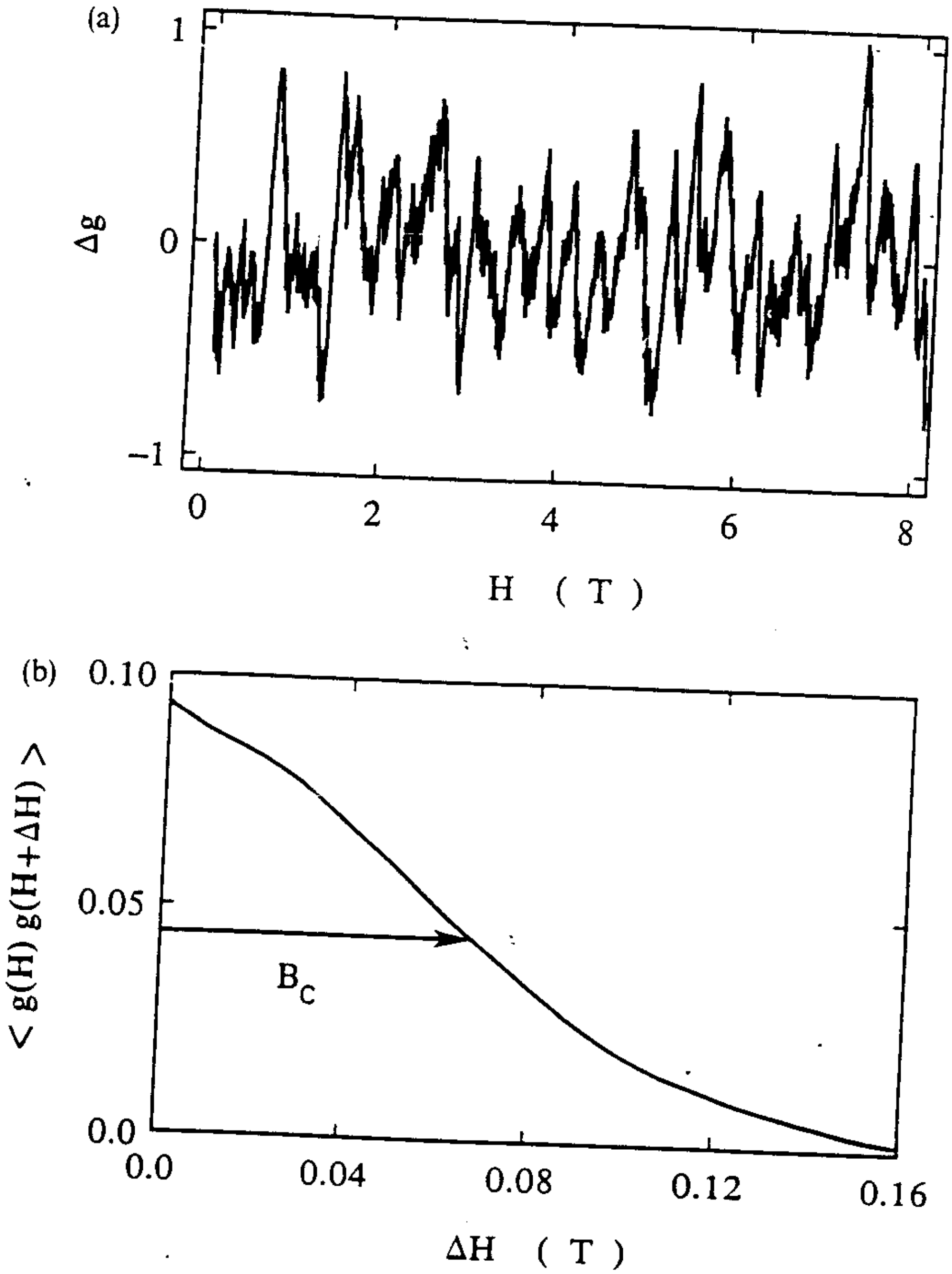


Fig. 3. (a) The reproducible random fluctuations in the magnetoresistance of a Au wire. (b) The auto-correlation of the data in (a) illustrating the correlation field scale.

(Washburn, Webb)

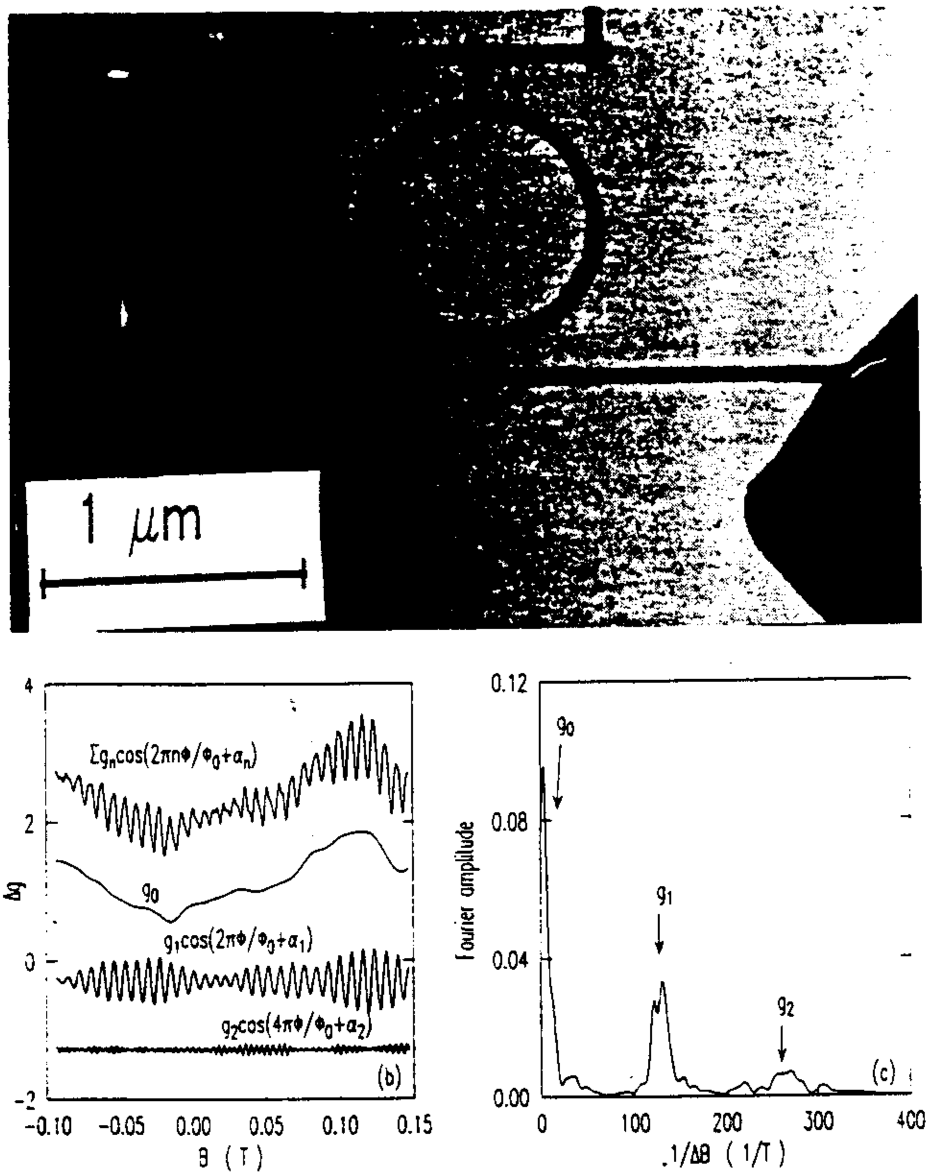


Figure 4. (a) An electron microscope photograph of a loop of gold that was 820 nm in diameter formed of 40 nm wide, 37 nm thick wires. (b) The dimensionless conductance $g(H)$ from the loop contains obvious periodic oscillations. The oscillation period is precisely that expected for adding a unit of flux h/e to the area encircled by the loop. (c) The Fourier spectrum (the square-root of the power spectrum) contains 3 peaks corresponding to a slow random fluctuation ($1/\Delta H \lesssim 30/T$) and h/e oscillation and a fainter $h/2e$ oscillation. By selecting a region of the Fourier spectrum (of appropriate width, say $\simeq 50/T$ for g_0) and transforming back, one may easily view the respective components of the conductance individually.

at $T \neq 0$ $\qquad N_T = \dfrac{\max\{T, E_c)}{\Delta}$

$$\frac{\langle \delta g^2 \rangle^{1/2}}{\langle g \rangle} \simeq \frac{1}{N_T(T)} \rightarrow \langle \delta g^2 \rangle^{1/2} \simeq \left(\frac{L_T}{L}\right)^2$$

The fluctuations $\langle \delta X^2 \rangle$ is

$$\frac{\langle \delta X^2 \rangle^{1/2}}{\langle X \rangle} \simeq \frac{\langle \delta g^2 \rangle^{1/2}}{\langle g \rangle} \simeq \frac{1}{N_T(T)}$$

Exceptions occur when $\langle X \rangle$ is zero or very small due to symmetry.

The main property is the absence of any symmetry

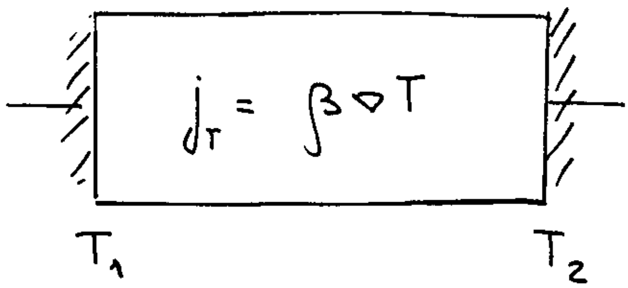
1. The spatial symmetry is broken due to arbitrary distribution of impurities

It is possible to observe all phenomena if they are not forbidden by energy conservation law only.

Example: 1. the generation of second (even) garmonic of AC electric field in centrosymmetrical crystals.

2. Nonlinear V-I characteristics of metals.

2. There is the special symmetry in metals: electron-hole symmetry near Fermi level.



$$\langle \beta \rangle \simeq \frac{\sigma}{e}\,\frac{T}{\mu} \qquad\qquad J_T = B\,(T_1 - T_2)\,\frac{1}{e}$$

$$\bar{B} = \beta L^{d-2}\,\frac{\hbar}{e^2}$$

$$\langle \delta \bar{B}^2 \rangle^{1/2} \simeq \langle \delta g^2 \rangle^{1/2} A \qquad\qquad \langle \bar{B} \rangle = \langle g \rangle \frac{T}{\mu}$$

A is asymmetry near Fermi level.

$A = \frac{T}{\mu}$ in pure metals.

In fluctuations $A \simeq \frac{T}{E_c}$ at $T \ll E_c$

$$\frac{\langle \delta \bar{B}^2 \rangle^{1/2}}{\langle \bar{B} \rangle} \simeq \frac{\langle \delta g^2 \rangle^{1/2}}{\langle g \rangle}\,\frac{\mu}{E_c} \gtrsim 1$$

(Anisovitch, Altshuler, Aronov)

3. <u>Sensibility to the moving of impurity</u>

(Altshuler and Spivak; Lee, Stone and Feng)

r_1 —— r_2 $\qquad \Delta r \sim 1/k_F$

$$\langle \delta g_1^2 \rangle \simeq \frac{1}{\langle g \rangle} \neq \frac{1}{N_{im}}$$

If m impurities move in an uncorrelated fashion

$$\langle \delta g^2 \rangle \simeq \begin{cases} \dfrac{m}{\langle g \rangle} < 1 \\ \text{or } 1 \quad \text{if } m > \langle g \rangle \end{cases}$$

$1/f$ noise $\longrightarrow$

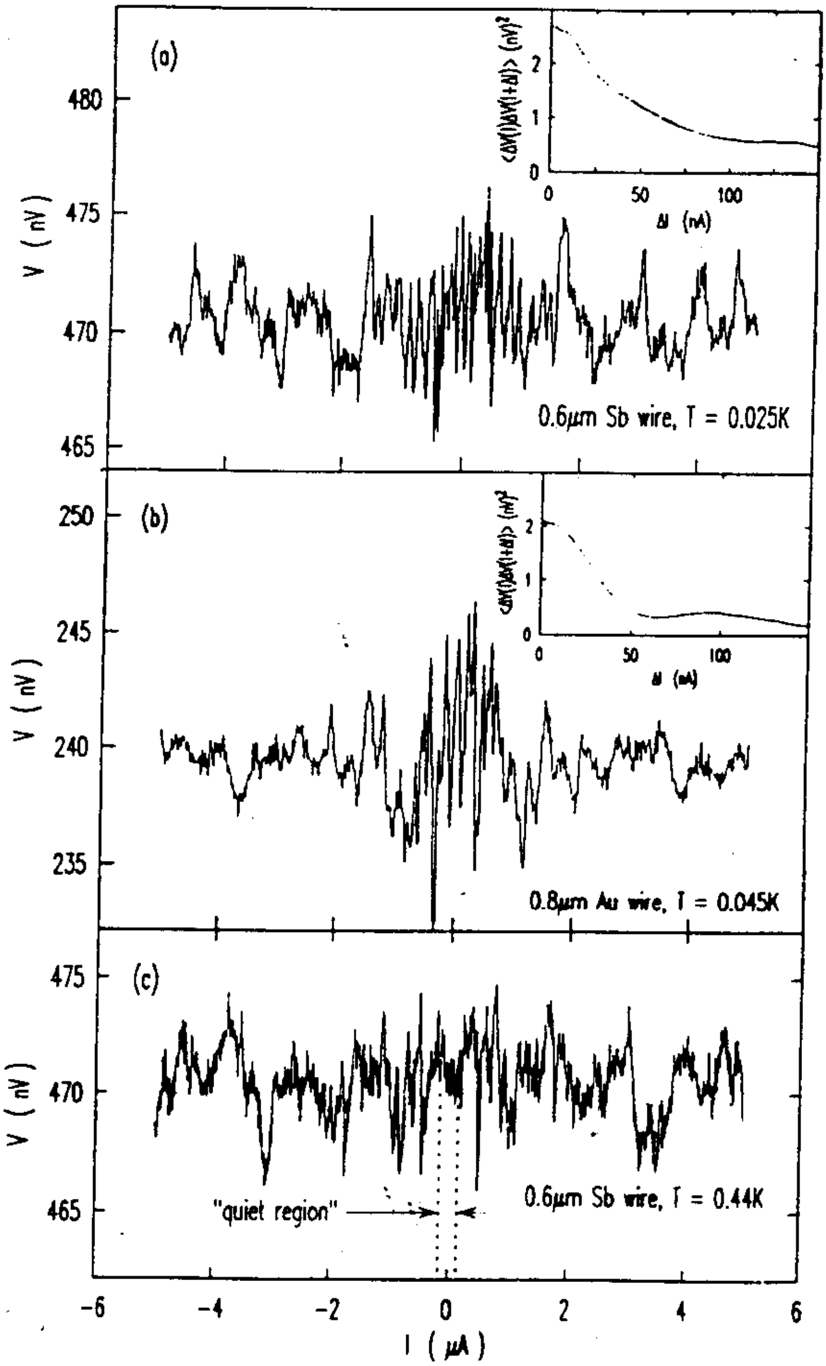
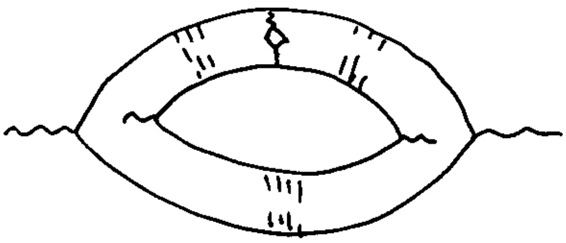


Figure 13. The fluctuations in the voltage (with an AC sampling current of 10 nA) as a function the DC current bias in a 0.6 μm Sb (a) wire at $T = 0.010$ K, and a 0.8 μm Au wire (b) at $T = 0.045$ K. The auto-correlations of the data are inset. Similar data for the Sb wire at $T = 0.45$ K (c) contain a quiet region near $|I| = 0$ [65].



 $= \frac{u^2}{N_i} \mathrm{Re}\left[1 - e^{i\vec{p}\,\delta\vec{R}}\right]$

$\delta\vec{R}$ is displacement of impurity

$$\langle \delta g_1^2 \rangle \simeq \frac{1}{\langle g \rangle} C(k_F \delta R) \qquad d = 3$$

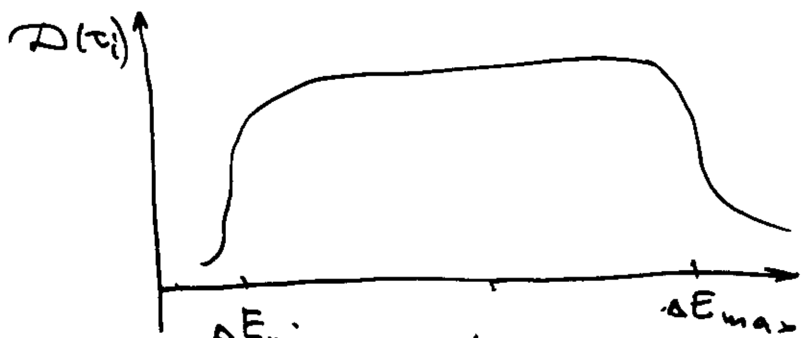
$$C(x) = 1 - \frac{\sin^2 \frac{x}{2}}{\left(\frac{x}{2}\right)^2}$$

$$S_v(\omega) = \int_0^\infty \langle \delta V(t_0)\, \delta V(t_0 + t) \rangle \cos \omega t$$

$$S_v(\omega) = A \int D(\tau_i) \frac{\tau_i \, d\tau_i}{1 + \omega^2 \tau_i^2}$$

$D(\tau)$ is the distibution function of τ_i

If $\tau_i = \tau_0 \exp\left\{\frac{\Delta E}{T}\right\}$ and



$S \propto \frac{1}{\omega}$

$$S_v(\omega) \propto \frac{1}{\omega}$$

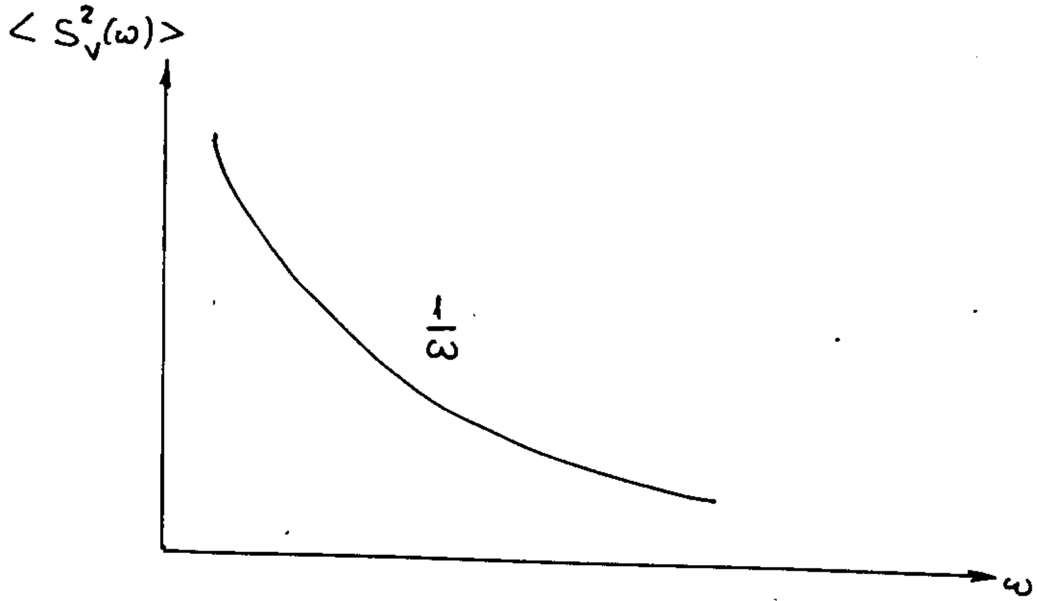
$$\frac{1}{\tau_0} e^{-\frac{\Delta E_{max}}{T}} < \omega < \frac{1}{\tau_0} e^{-\frac{\Delta E_{min}}{T}}$$

at $E_{max} \sim 1\,eV$ $\quad \tau_0 \sim 10^{-12}\,s$ $\quad T \simeq 300K$

$\omega_{min} \approx 10^{-6}\,s^{-1}$

$1/f$ noise extends to a very low frequency

$\underline{\omega_{min} \simeq 10^{-6}\,s^{-1}}$



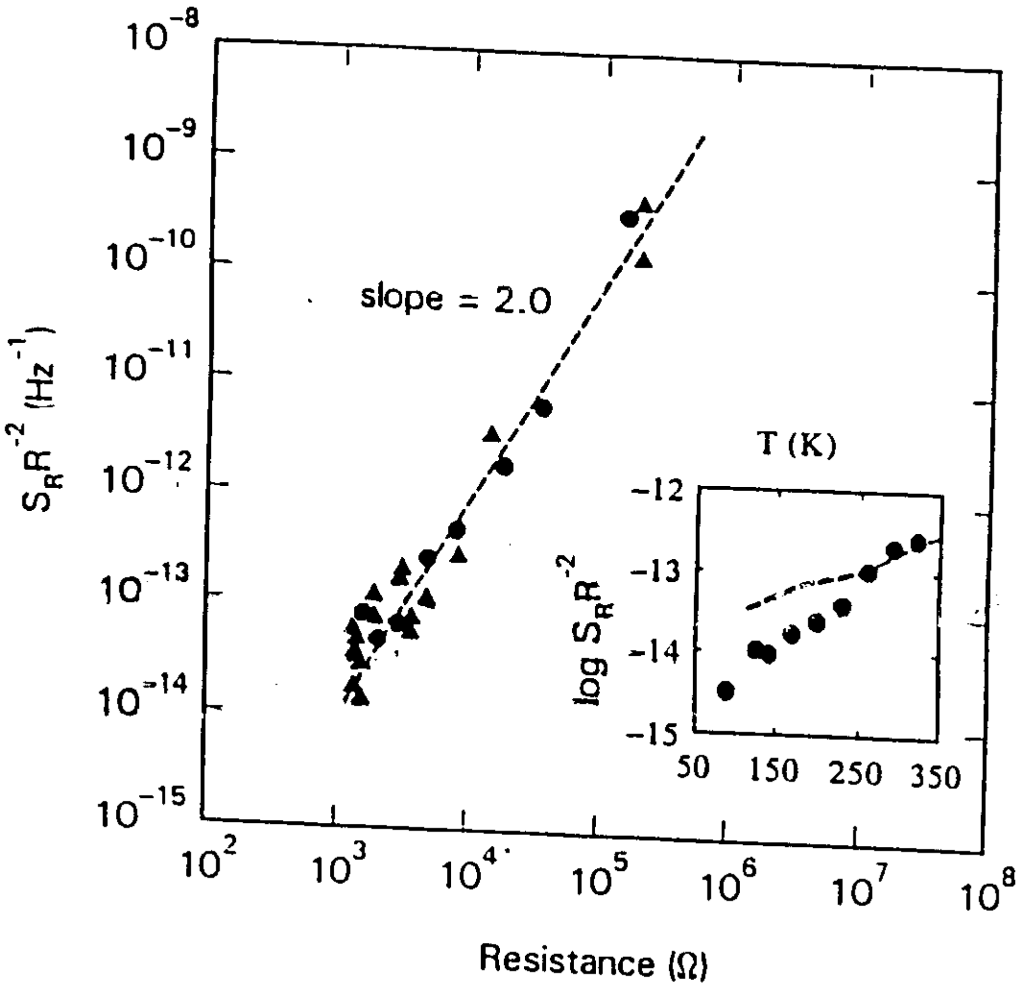


Fig. 9 Magnitude of the noise of a thin Au film as a function of resistance, *R*, which is approximately the sheet resistance. The resistance was varied by ion milling. The different symbols denote data for different samples. After Koch et al. (1988).

$d = 2$

$S_R \propto R^4 \rightarrow$

$\langle \delta G^2 \rangle = \text{const} \{R\}$

(USF)

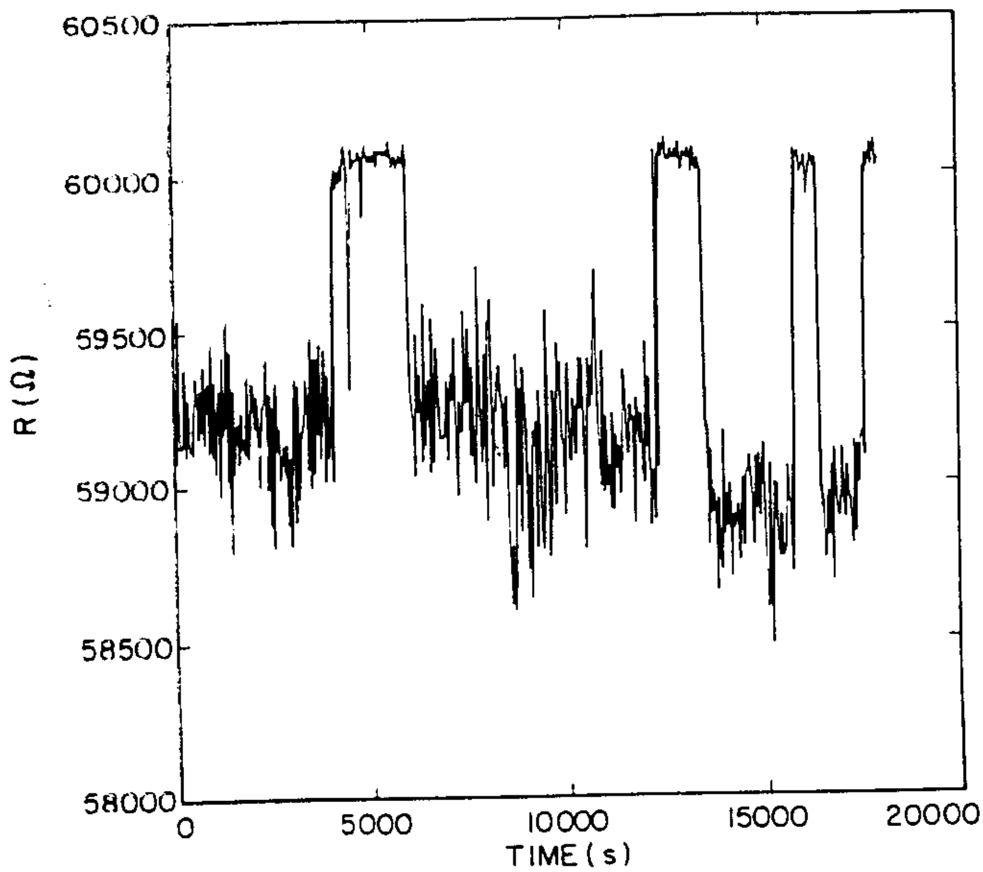


Fig. 5. Resistance as a function of time for a 10 Å thick Pt film at 250 mK. After Meisenheimer et al. (1987b).

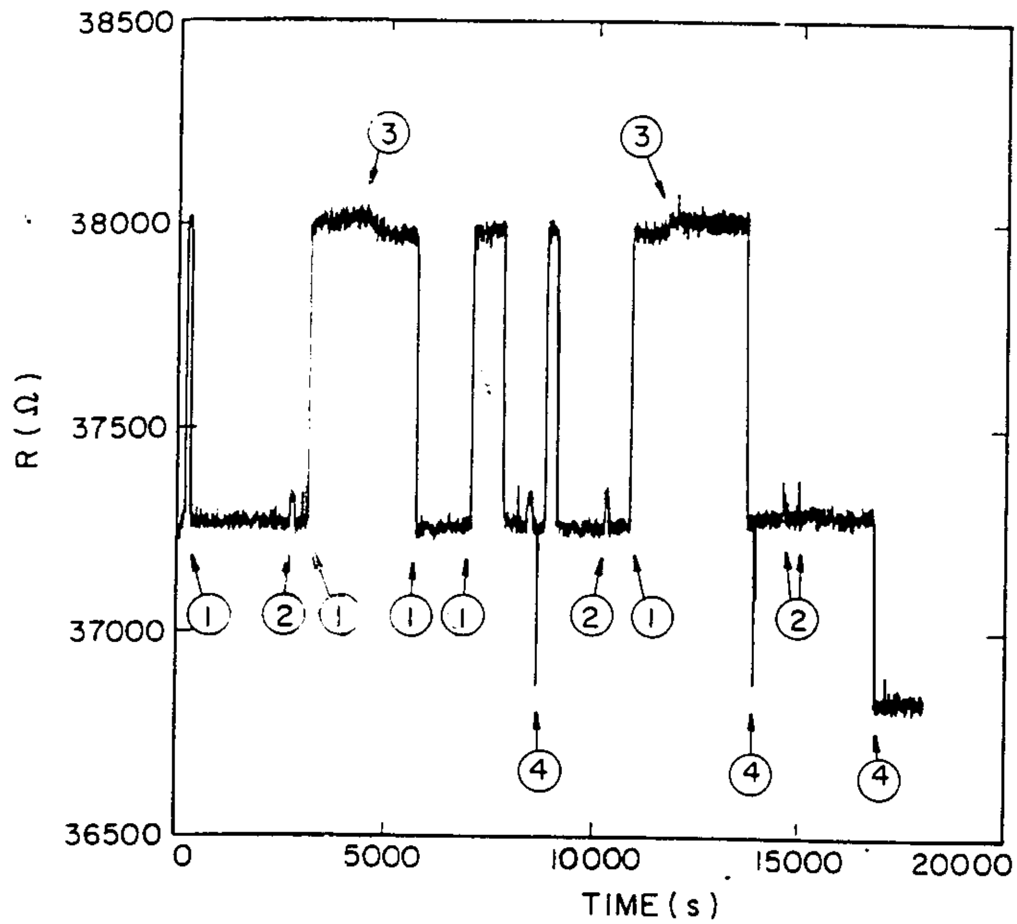
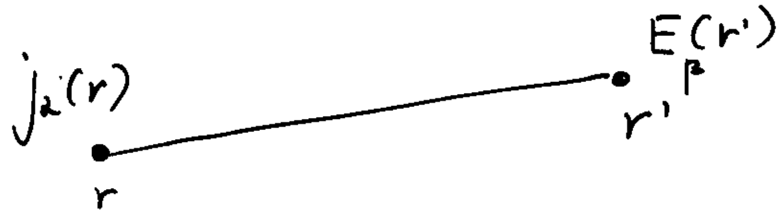


Fig. 2. Resistance of a thin Bi film as a function of time at 70 mK. The sample was 10 μm long and 4 μm wide. Different fluctuators are indicated by the arrows. The numbers labeling the ar are used to indicate what appear to be repetitions of the same fluctuator. After Meisenhe et al. (1987b).

Local properties.



$$\langle j_\alpha(r) \rangle = \int dr' \, K_{\alpha\beta}(r-r') \, E_\beta(r')$$

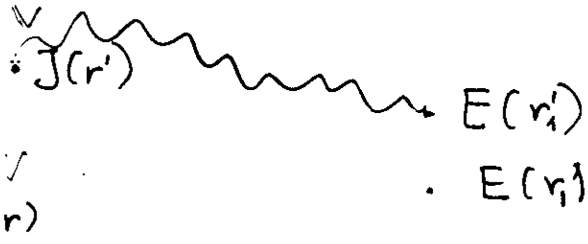
$$K_{\alpha\beta}(r-r') \propto \delta_{\alpha\beta} \, e^{-\frac{|r-r'|}{\ell}}$$

ℓ is mean free path.

But why does the correlation between current and electric field decay on the mean free path ℓ, if we have the phase memory?

Electric current - field correlations

(Aronov, Spivak and Zyzin)



$$J_\alpha(r) = \int dr_1\; K_{\alpha\beta}(r, r_1)\, E_\beta(r_1)$$

$$\langle J_\alpha(r)\rangle = \int dr_1\; \langle K_{\alpha\beta}(r - r_1)\rangle\, E_\beta(r_1)$$

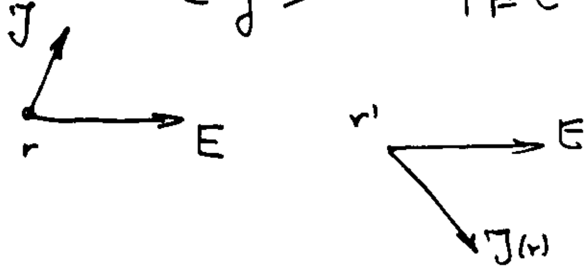
$$K(r - r_1) \propto \frac{1}{|r - r_1|^2}\, e^{-|r - r_1|/\ell}$$

$$\langle J_\alpha(r)\, J_\beta(r')\rangle = \int dr_1\, dr_1'\; \overbrace{\langle K_{\alpha\gamma}(r, r_1)\, K_{\beta\delta}(r', r_1')\rangle}^{\Lambda_{\alpha\gamma\beta\delta}} \times E_\gamma(r_1)\, E_\delta(r_1')$$

$$\Lambda_{\alpha\gamma\beta\delta} \propto e^{-\frac{|r - r'|}{\ell}} \cdot \frac{e^{-\frac{|r - r_1|}{L_T}} \times e^{-|r' - r_1'|/L_T}}{|r - r_1|^2\, |r' - r_1'|^2}\, e^{-\frac{|r_1 - r_1'|}{\ell}}$$

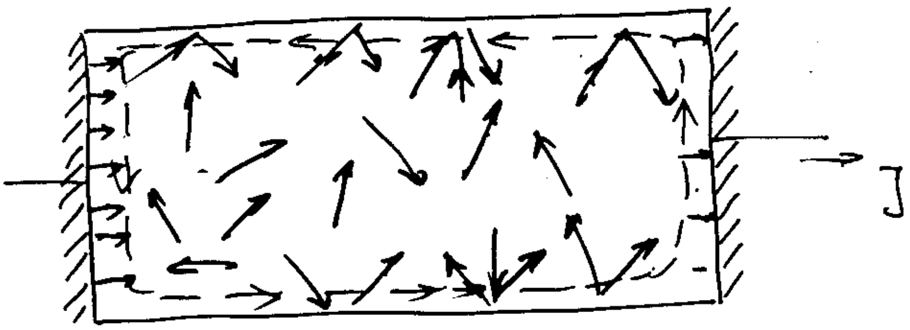
$$\langle J^2(r)\rangle \simeq \frac{e^4}{\hbar}\, \frac{L_T P_F^2}{\ell}\, E^2$$ if E is homogeneus on the scale L_T

$$\frac{\langle J^2(r)\rangle^{1/2}}{\langle J\rangle} = \frac{1}{P_F \ell}\left(\frac{L_T}{\ell}\right)^{1/2} > 1$$
can be



There is the current in each point perpendicular to $\vec{E}$

There are a short distance correlations besides a long distance correlations



$$\langle m^2 \rangle^{1/2} \simeq \alpha \frac{e^2}{\hbar c} E \qquad T=0$$

18

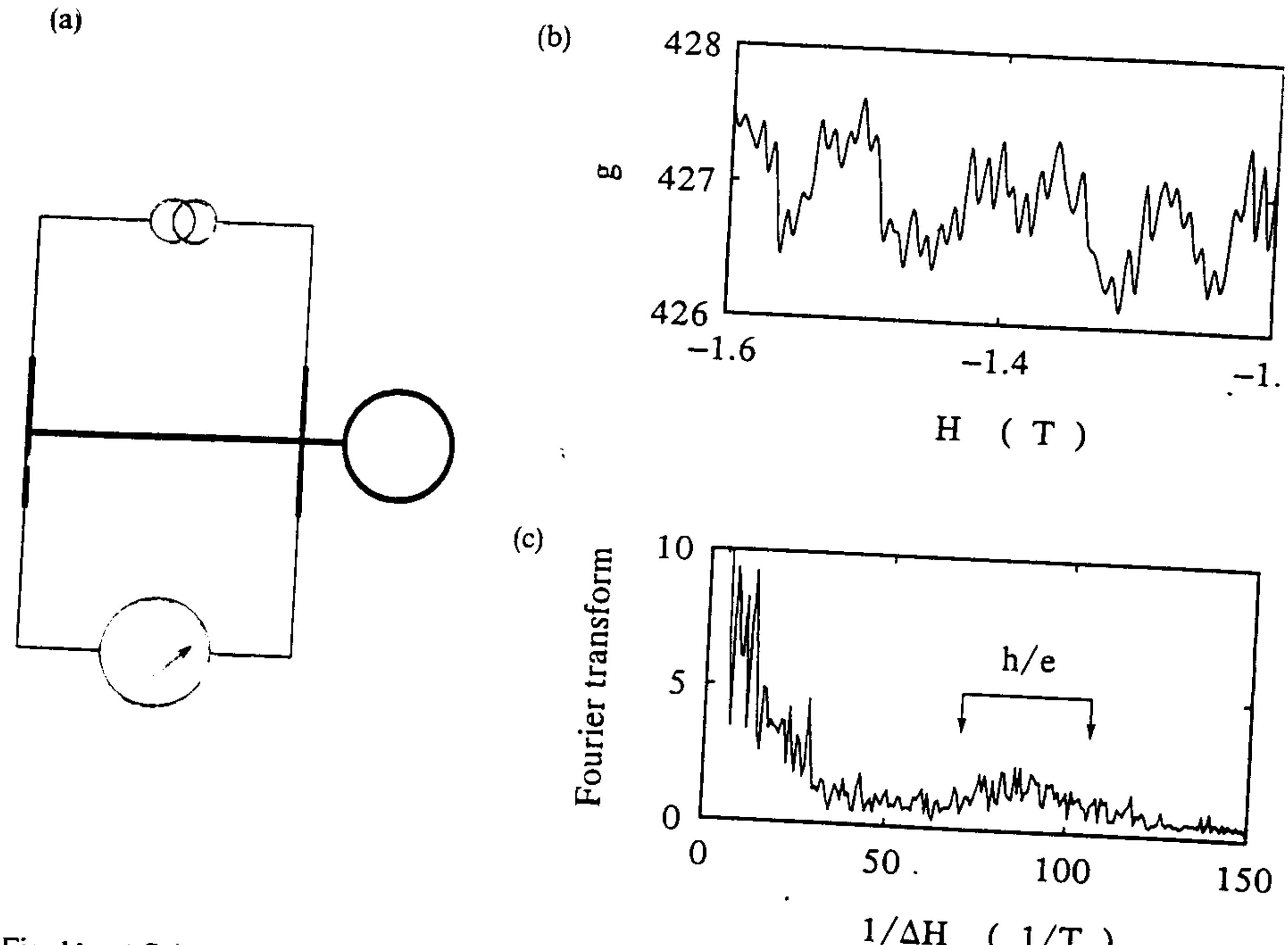
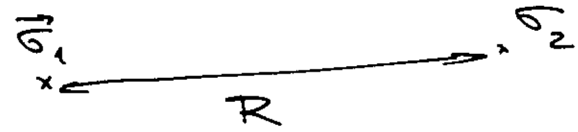


Fig. 11. (a) Schematic drawing of a four-probe wire with a loop dangling outside of the path of the net current. (b) The conductance of the wire, and the Fourier transform (c) of the conductance which include periodic h/e oscillations from the trajectories that encircle the loop.

Local properties of dirty metals.

$\vec{S}-\vec{S}$ interaction

Long correlations due to phase memory at $T=0$



1. In pure metals RKKI interaction

$$J(R) \propto \frac{\cos 2k_F R}{R^3}$$

Usual logic $\longrightarrow$ $\langle J(R) \rangle \propto e^{-R/\ell}$

ℓ is mean free path. It is not right.

$$J_L(R) \propto \frac{1}{R^3} \cos(2k_F R + \delta_L)$$

δ_L is the phase due to scattering

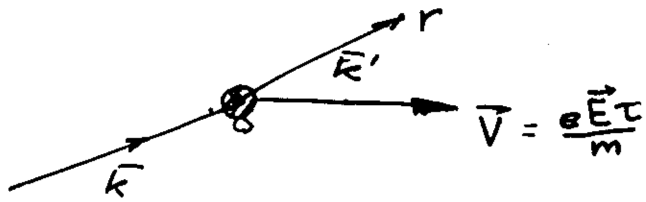
$$\langle J^2(R) \rangle^{1/2} \propto \frac{1}{R^3} e^{-R/L_T}$$

The long range correlation is broken on L_T, L_{so} only

(Spivak and Zyzin;
Bulaevski et al)

Mesoscopic spin polarization

Physical picture (Aronov, Lyanda-Geller, Nazarov; Zyzin and Serota).



$$\psi_k(r) \propto e^{i\underline{k}\underline{r}} + \left(\frac{i}{kr}\right)^{1/2} \hat{f}(\theta)\, e^{ikr}$$

$$\hat{f}(\theta) = a + ib\,\vec{\sigma}\,[\underline{k} \times \underline{k}'] \qquad \underline{d=2}$$

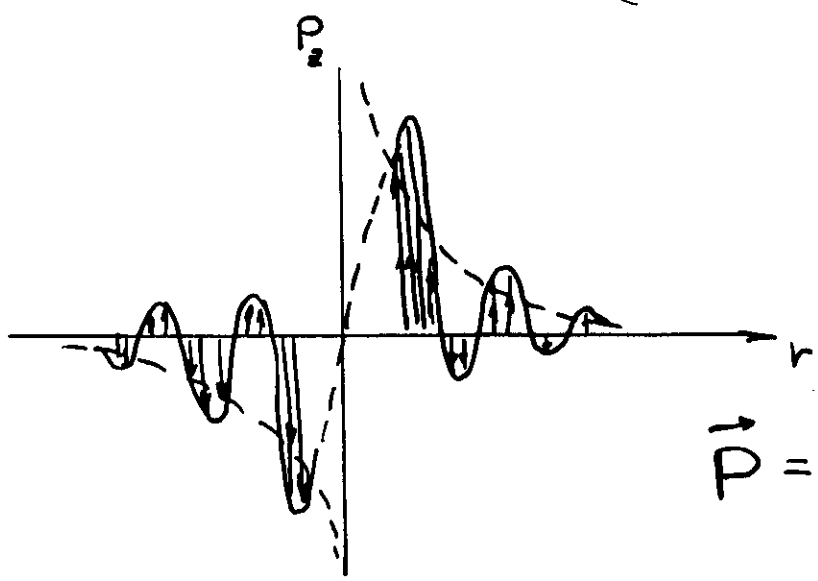
Distribution of spin polarization is

$$P_z(r) = \frac{1}{v_F} \int (dk)\, (\vec{V}\cdot\vec{k})\, Sp\,\sigma_z(r)\, \delta(k-k_F)\, |\psi_k(r)|^2$$

$\vec{V}$ is the transport velocity

$$P_z = -\frac{2mk_F b}{\sqrt{\pi}} [\vec{V} \times \vec{n}]_z \frac{1-\sqrt{2}\cos k_F r}{(\pi r)^2} \qquad \underline{\text{at } k_F r \gg 1}$$

$\underline{d=2}$



$$\vec{P} = c[\vec{V} \times \vec{r}]$$

1. If the spin-orbit scattering is weak $\frac{\hbar}{T\tau_{so}} \ll 1$, but $\frac{\tau_0}{\tau_{so}}$ is arbitrary and $T\tau_0 \gg 1$

$$\int dr \langle \sigma_\perp^2 \rangle \simeq g \frac{\int dr E^2(r) L_T^2}{48\pi\hbar D^2} \ln\left(1+\frac{4\tau_0}{3\tau_{so}}\right) \qquad d=2$$

$\mathcal{P}$ is Joule heat

$$\int dr \langle \sigma_\perp^2 \rangle = \sigma_0 \frac{\int dr E^2(r) L_T^2}{48\pi D^2 \hbar L_0} \left(\sqrt{1+\frac{4\tau_0}{3\tau_{so}}} - 1\right) \qquad d=$$

g is conductance, σ_0 is conductivity

$L_0 = \sqrt{D\tau_0}$ is inelastic lenght,

$L_T^2 = D/T$, τ_{so} is spin-orbit scattering time.

2. If $\tau_{so} \ll \frac{1}{T}$ the spin polarization does not depend on τ_{so}

$$\int dr \langle \sigma_z^2 \rangle = g \frac{L_T^2}{48\pi\hbar D^2} \int dr E^2(r) \cdot \ln T\tau_0 \qquad d=2$$

$E(r)$ is the local real electric field in sample

Anisotropy

Spin-orbit relaxation time can have strong anisotropy.

a. Impurity spin-orbit scattering

$$\frac{1}{\tau_{so}^{(\alpha)}} = \pi \nu b^2 \overline{[\underline{p} \times \underline{p}']_\alpha^2}$$

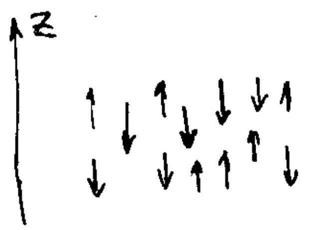
b. Spin-orbit scattering in crystals without inversion symmetry

$$\frac{1}{\tau_{so}^{\alpha}} = \langle \overline{\Omega_\alpha^2} \tau \rangle$$

In two dimensional case

a) $$\frac{1}{\tau_{so}^{x}} = \frac{1}{\tau_{so}^{y}} = 0 \qquad \frac{1}{\tau_{so}^{z}} \neq 0$$

$$\langle S_x^2 \rangle = \langle S_y^2 \rangle = 0 \qquad \langle S_z^2 \rangle \neq 0$$



Spins are polarized parallel z.

<u>Nonequilibrium Ising glass</u>

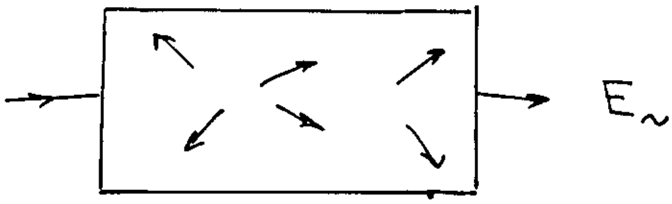
b). $$\frac{1}{\tau_{so}^{x}} = \frac{1}{\tau_{so}^{y}} = \frac{1}{2\tau_{so}^{z}}$$

Weak spin-orbit scattering : anisotropy is two

Strong spin-orbit scattering : anisotropy is zero

Nonequilibrium isotropic spin glass

How is it possible to observe the spin polazization?



$$\sigma(r) \propto \underset{\sim}{E}(r) e^{i\omega t}$$

Due to hyperfine interaction

$$\mathcal{H}_i = a \langle \sigma(r,t) \rangle$$ — thermodynamic averag of local spin density

Intensity of NER

$$I \propto \mathrm{Jm}\, \chi_N(\omega) \int dr \, |\langle \sigma(r) \rangle|^2 \propto V \, \overline{\langle \sigma(r) \rangle^2}$$

Intensity of resonance is proportional to the correlator of spin densities in the one point which is not zero.

!

1. UCF is result of quantum mechanic

$$\langle \Delta g^2 \rangle^{1/2} \simeq 1.$$

It is natural restriction on the size of microscopical device

2. $\langle \delta N^2 \rangle^{1/2} \simeq 1$ N is the number electron levels in the energy strip with width

$$E_c = \frac{\hbar D}{L^2}.$$

Near Anderson transition $N_c = 1$

It means that $\boxed{\langle \delta N^2 \rangle^{1/2} \simeq N_c \simeq 1}$

3. The high sensitibility to the changing of position of single impurity

If $\delta R \simeq \lambda$

$$\langle \delta g^2 \rangle_{single} \simeq \frac{1}{\langle g \rangle}$$

4. It gives the possibility to explain in some cases $1/f$ - noise

5. Ergodic hypotese the ageraging on magnetic field is equivalent to the averaging on the samples.

6. All spatial symmetries are broken.

It is possible to observe any phenomenon if it is not forbidden by energy conservation law (absolute freedom)

$$\frac{\langle \Delta X^2 \rangle^{1/2}}{\langle X \rangle} \simeq \frac{\langle \Delta g^2 \rangle^{1/2}}{\langle g \rangle} \simeq \frac{1}{\langle g \rangle}$$

2

Conclusion

1. In dirty metals there are conditions when properties of sample depend on the concrete distribution of impurities
2. This is quantum phenomena due to interference between different acts of scattering
3. There are a lot of effects in which it is possible to observe quantum interference effects (Magnetofingerprints, Aharonov-Bohm effect, Macroscopical forbidden effects and ets)
4. There is the long range correlation between local currents and electric field
5. The current introduces the spin polarization and creates nonequalibrium spin glass.