



INTERNATIONAL ATOMIC ENERGY AGENCY  
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION  
**INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS**  
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UNITED NATIONS INDUSTRIAL DEVELOPMENT ORGANIZATION

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**INTERNATIONAL CENTRE FOR SCIENCE AND HIGH TECHNOLOGY**

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**SMR. 628 - 6**

**Research Workshop in Condensed Matter,  
Atomic and Molecular Physics  
(22 June - 11 September 1992)**

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**Working Party on  
"NOISES IN MESOSCOPIC SYSTEMS"  
(27 July - 7 August 1992)**

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**"Co-tunneling and noises in the  
systems of tunnel junctions"**

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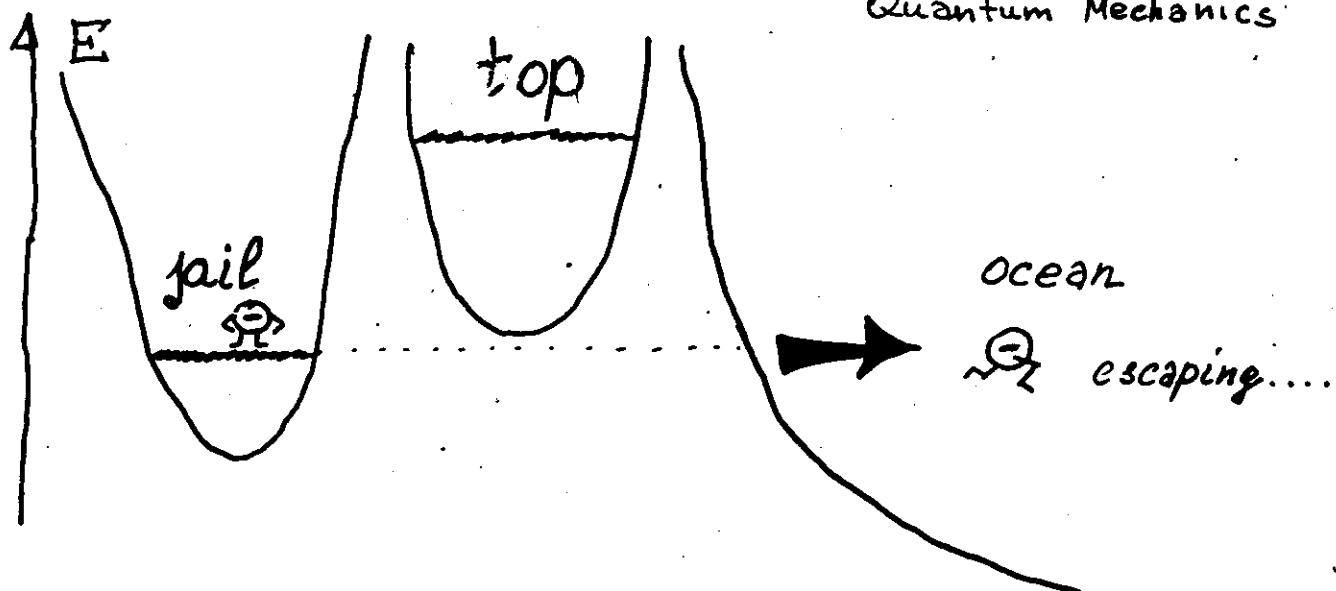
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D-7500 Karlsruhe 1  
GERMANY

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**These are preliminary lecture notes, intended only for distribution to participants.**

# 1. General principle of freedom: essence of

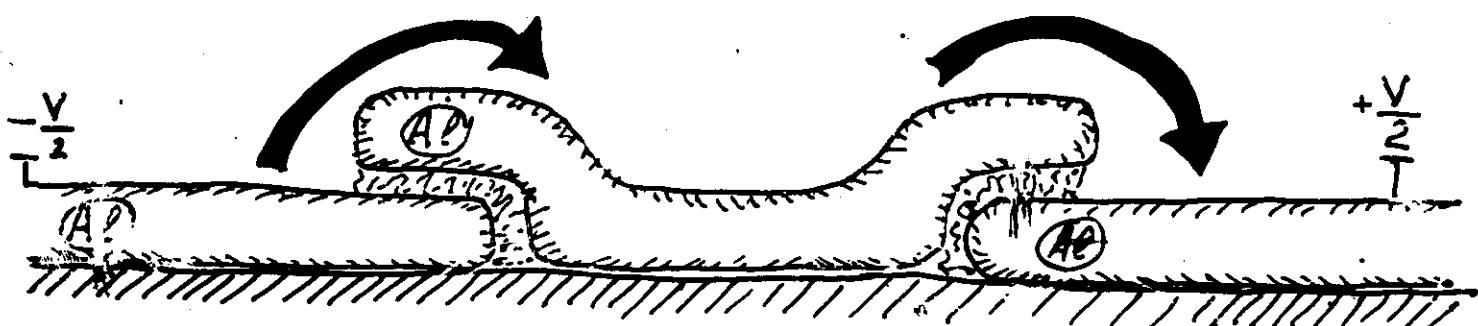
Quantum Mechanics



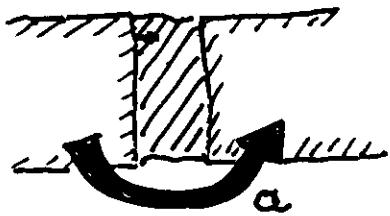
$$\Gamma = 2\pi \sum_{\text{ocean}} |A_m|^2 \delta(E_{\text{jail}} - E_{\text{ocean}})$$

$$A_m = \langle \text{jail} | \hat{H} | \text{top} \rangle \frac{1}{E_{\text{jail}} - E_{\text{top}}} \langle \text{top} | \hat{H} | \text{ocean} \rangle$$

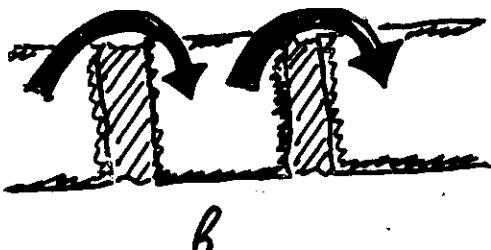
## 2. Systems of tunnel functions $\rightarrow$ poor man's manybody effects



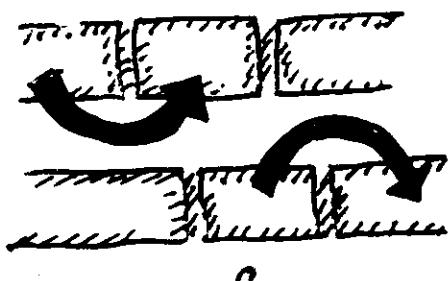
### 3. Types of Co-tunneling:



$$\Gamma_{\text{single}} = \frac{G V}{e^2} \approx \frac{G}{c} \quad (V \approx E_c \approx \frac{e^2}{c})$$

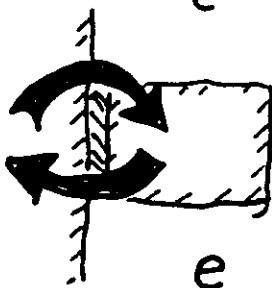


$$\Gamma_{\text{double}} = \Gamma^{(1)} \Gamma^{(2)} \frac{\frac{\hbar}{\Delta E}}{\text{time uncertainty}}$$

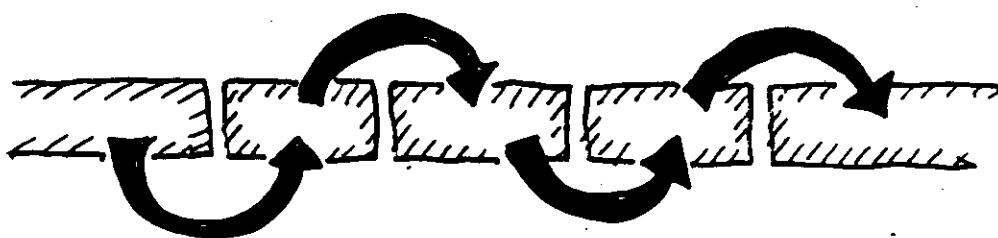


$$\approx \Gamma_{\text{single}} \cdot (R_K G)$$

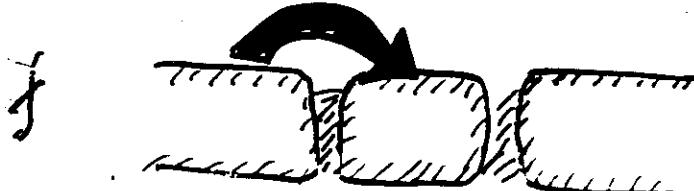
$$R_K = \frac{2\pi k}{e^2} \approx 6.5 \cdot 10^3 \text{ Ohm}$$



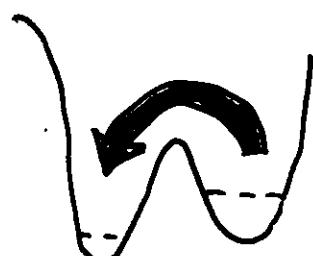
$$\text{Re}[G(\omega)] = (R_K G) \left( \frac{\hbar \omega}{E_c} \right)^6$$



$$\Gamma_N = \Gamma_{\text{single}} (R_K G)^{N-1}$$



$$\Gamma \approx \Gamma_{\text{fluct}} (R_K G)$$



fluctuation

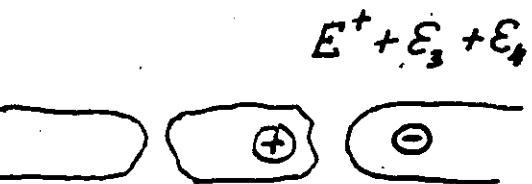
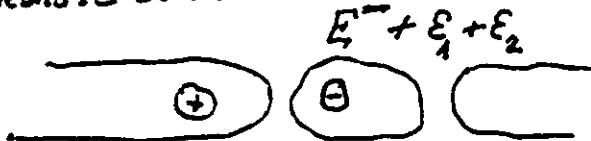
1.  $\approx$  co-tunneling of two electrons

H. V. L. Ustinov  
1989

initial state



two intermediate states



final state



$$\Delta m = T^{(1)} \left( \frac{1}{E_1^- + E_2^- + E^-} - \frac{1}{E_3^+ + E_4^+ + E^+} \right) T^{(2)}$$

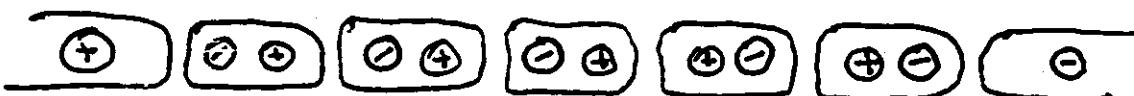
$$\Gamma = \frac{\hbar}{2\pi e^3} \left[ \left( 1 + \frac{2}{eV} \frac{E^+ E^-}{E^+ + E^- + eV} \right) \left( \sum_{i=+, -} \ln \left( 1 + eV/E_i \right) \right) - 2 \right] V$$

(divergency)

$$I(V) = \frac{k G_1 G_2}{4\pi e^2} \left( \frac{1}{E^+} + \frac{1}{E^-} \right)^2 \left[ (eV)^2 + (2\pi T)^2 \right] V$$

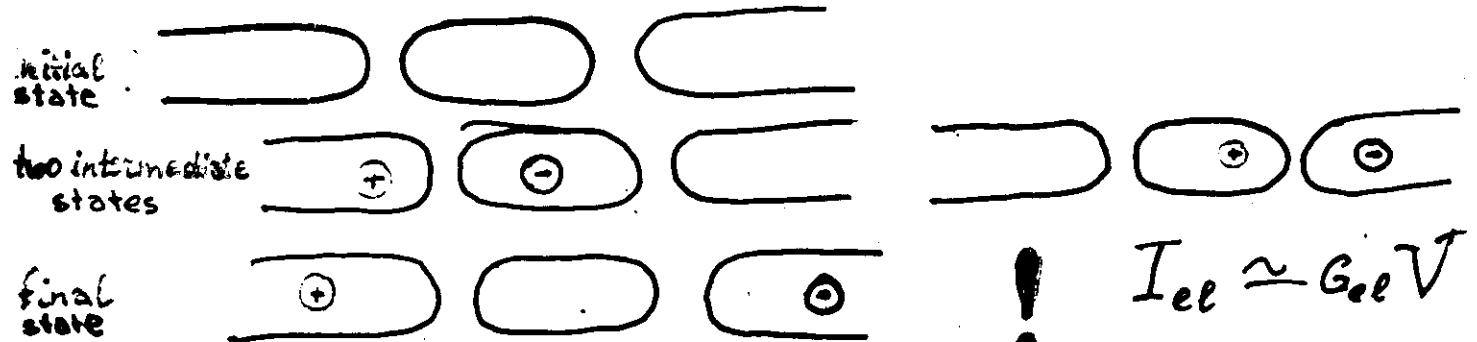
5. In long chains

reduction of phase volume



$$\Gamma = \frac{2\pi}{\hbar} \left( \frac{R_K G}{4\pi e^2} \right)^N \frac{N^N}{(2N-1)! N!} \left( \frac{eV}{E_c} \right)^{2N-1} E_c$$

## 5. Elastic g-MQT

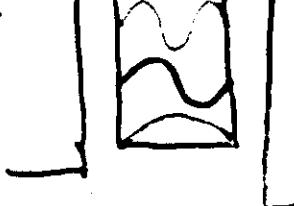


$$I_{el} \approx G_{el} V$$

Looks like an elastic scattering of one electron



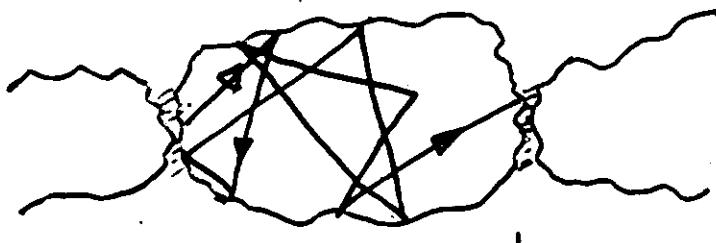
Example:  $\Rightarrow$



island = 1D well

$$\begin{aligned} T_1 &= T_2 \\ \text{or} \\ T_1 &= -T_2 \end{aligned}$$

## 6. Semiclassical methods



two cases:

$$1. \frac{\hbar}{E_c} \gg t_{trav}$$

universal

$$2. \frac{\hbar}{E_c} \ll t_{trav}$$

non-universal

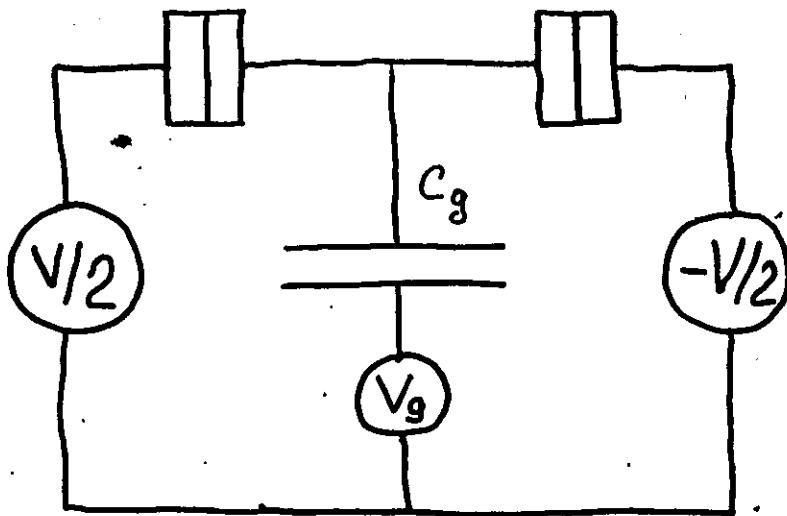
$$G_{el} = \frac{G_1 G_2 R_k}{8\pi^2} \delta \left( \frac{1}{E^+} + \frac{1}{E^-} \right)$$

$$G_{el} \approx G_1 G_2 R_k \left( \frac{\delta}{E_c} \right) \frac{1}{E_c t_{trav}}$$

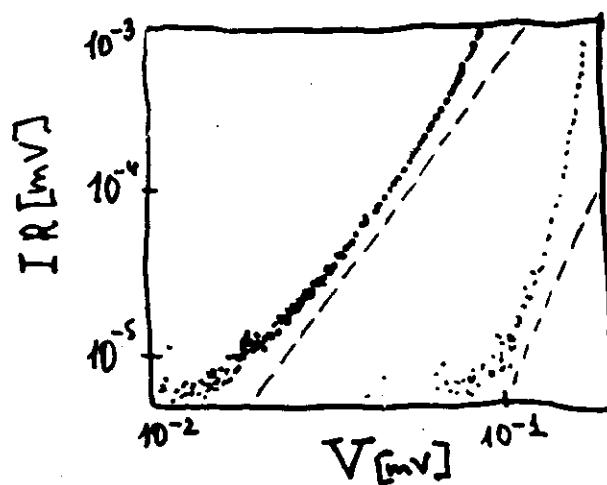
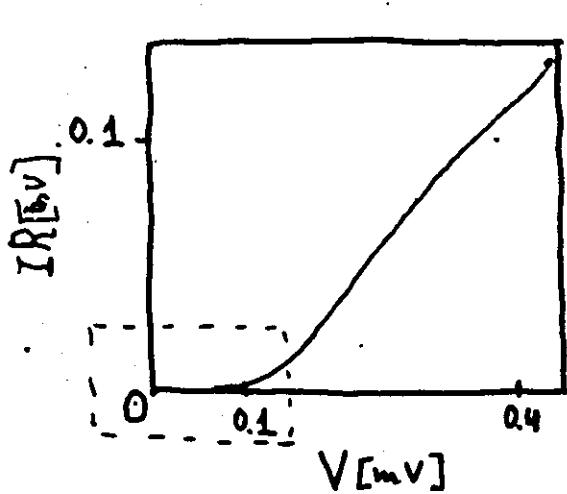
$\delta \rightarrow$  mean level spacing

Crossover between elastic and inelastic

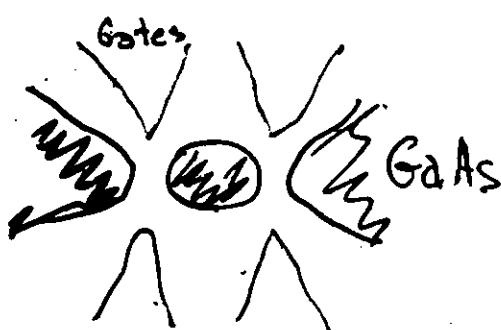
$$ev, T \approx \sqrt{S E_c} \ll E_c$$



Geertigs / verin  
Moerij 1990



$$\therefore 2 \text{ fm}, I \sim V^3 \quad \therefore 3 \text{ fm}, I \sim V^5$$

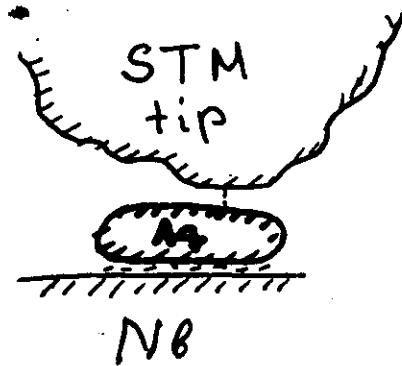


Glatti et al  
1991

10% accuracy  
of the formulas

Eiles Jensen  
Zimmerli Martinis  
1992

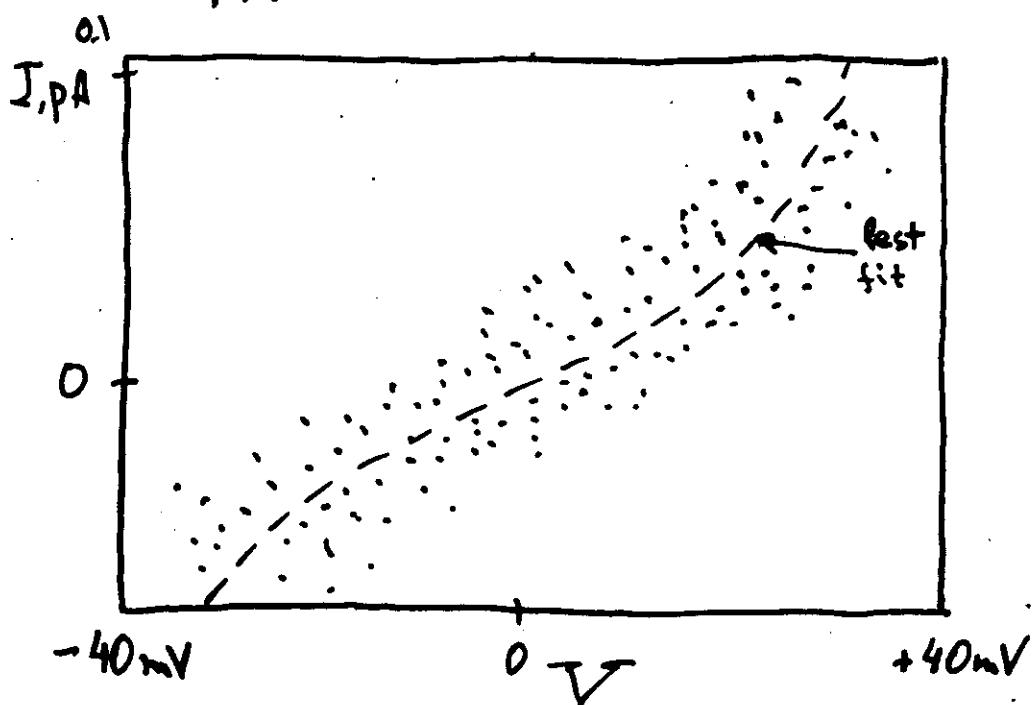
to raise  $S/E_c$  to reasonable values



M.Tinkham  
et al  
1992

estimated  
size  $10 \div 30 \text{ \AA}$

$$E_c \sim 0.5 \text{ V}$$

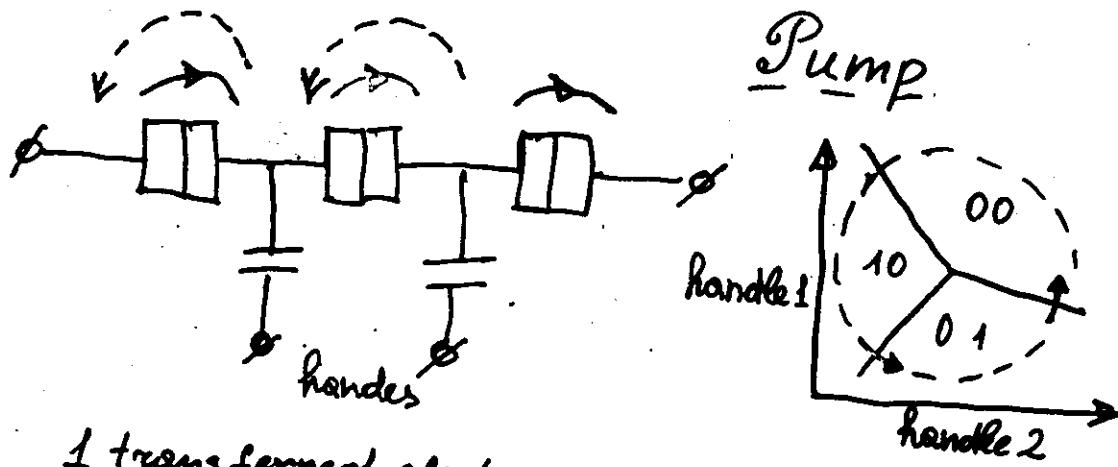


Despite of these fact,  
agreement with theory is unexpectedly reasonable  
(order of the value)

manipulation and trapping

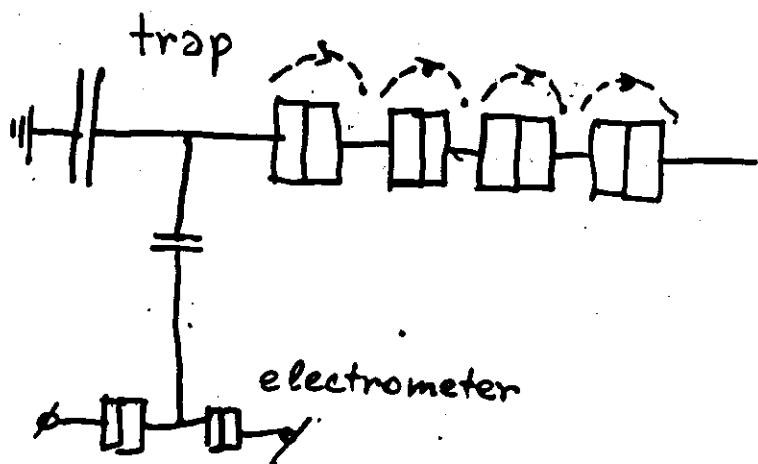
Everything is being done on purpose...

Purpose: to control charge transport (Applications?)



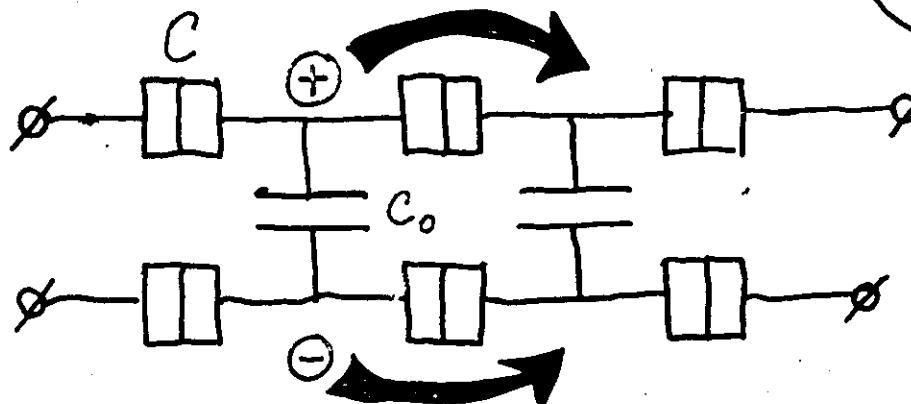
1 transferred electron per cycle  
Cotunneling: mistakes

## Trapping



Cotunneling: decay of metastable states

Cotunneling: a sort of noise



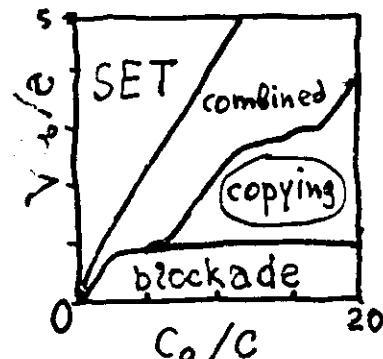
$$C_0 \gg C$$

electron + hole = pair.

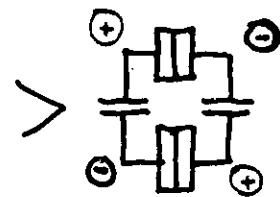
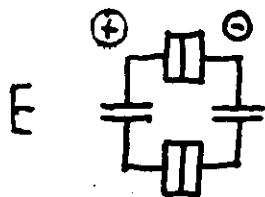
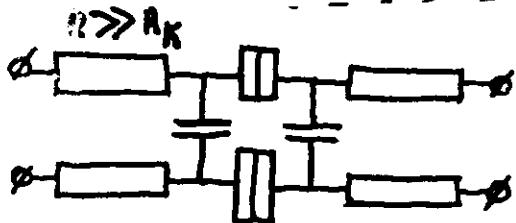
$C_0$ -tunneling moves it as a whole:

$$I_{up} = -I_{low}$$

in a region



Similar circuit:



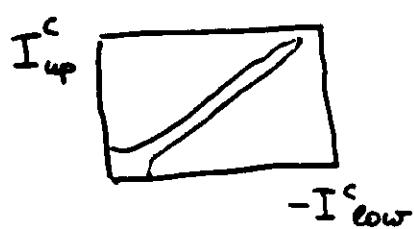
Beigannmüller  
Nazarov  
1991

## 11. Compound particles: building of

long Narrows:

long Superconductive arrays

long Super + Normal arrays



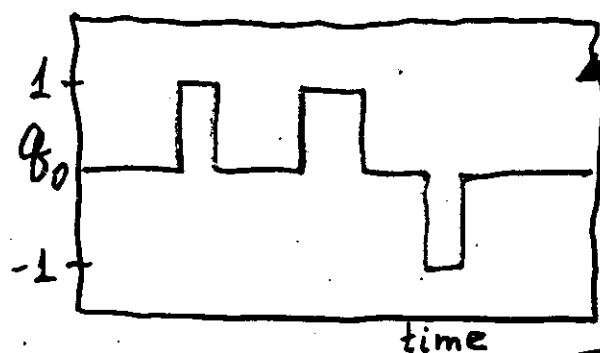
dissipative VS elastic motion  $\Leftrightarrow$  charge VS electron

Crossover: transport thru  
the array of 2D lakes

# Experimental - from freedom to anarchy

Electrons like freedom too much...  
Ecitec 1992

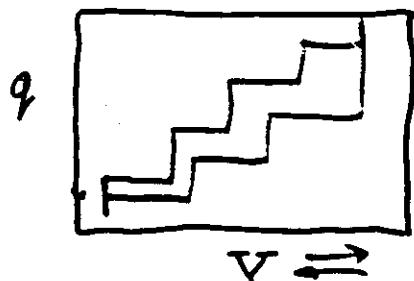
## 1 Trapping



- jumping between metastable states
1. rates are too high! ( $0.1 - 10\text{s}$ )
  2. No detailed balance!

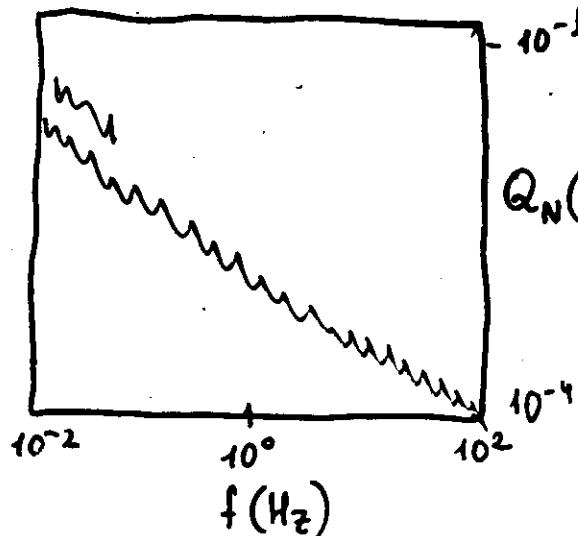
## 2. Hysteresis Fulton 1991

it's possible to measure delay sweeping voltage

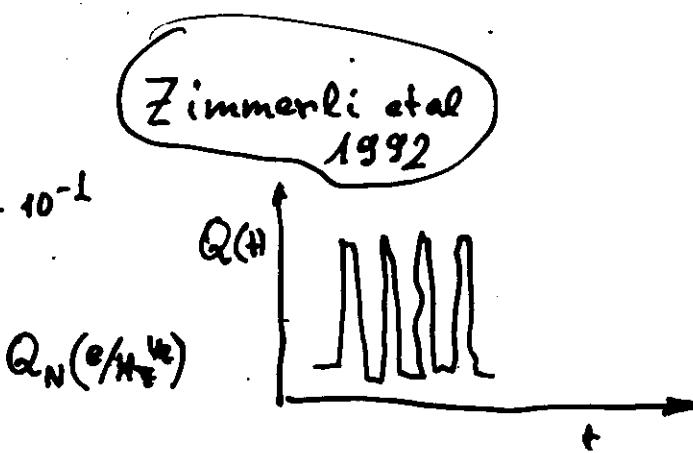


1. rates are too low! ( $0.1 - 10\text{s}$ )
2. Up to noticeable temperatures (thermal activation)

## 3. Equilibrium? 1/f noise



Zimmerli et al  
1992



Charge traps in the barriers



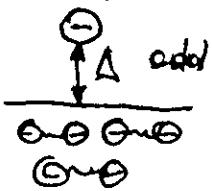
# Basics of superconductivity

## plain THEORY:

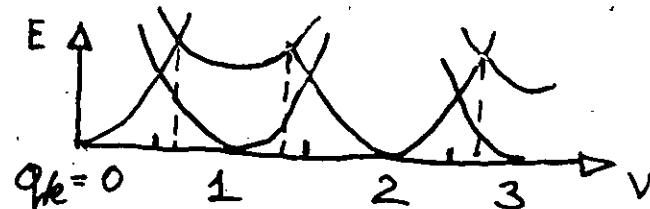
- Parity effect: uncoupled electron costs  $\Delta$

even

$\ominus\ominus\ominus\ominus$



Motived  
1989



- Transport properties: no co-tunneling at small voltages

price:  
 $2\Delta$



Averin  
Nozaryan  
1991

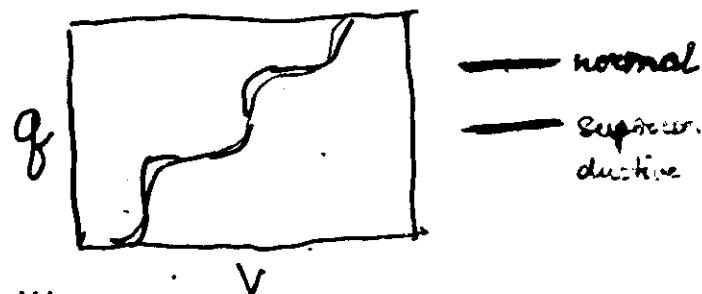
elastic contribution: small containing S

inelastic contribution: if odd number of electrons. Also small containing S

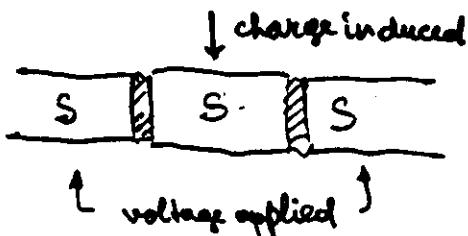
## EXPERIMENTS:

- No parity effect at equilibrium conditions

Esteve et al. 1991



- Parity effect under non-equilibrium condition



Small voltage:  $2e$  - periodicity  
larger voltages:  $e$  - periodicity

- Trapping: escape rates: no much difference between normal and superconductive

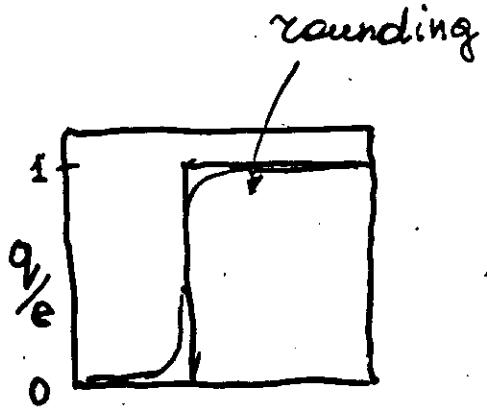
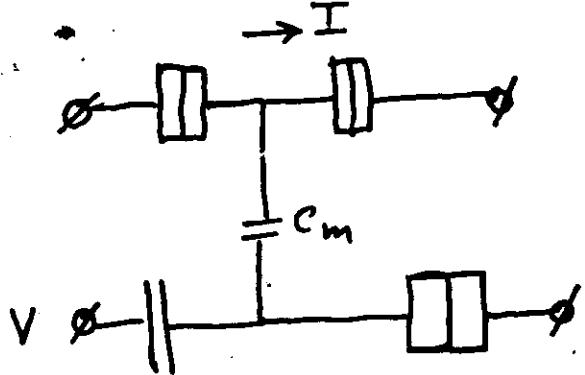
Esteve 1992

- faster escape

Fulton 1991

- slower escape

# feedback of measuring device



a) Equilibrium quantum effects

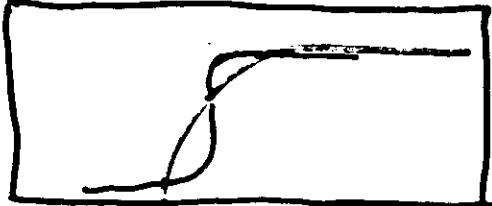
*Mattuck 1990*

resembles multi-channel Kondo problem

rounding  $\approx \exp(-\frac{4\alpha}{R_K G})$   $\rightarrow$  not relevant for experiments.

b) Feedback of electrometer: it shakes the box

classical  
rounding  
 $\sim C_m/c$



quantum limit (like NMR)  
narrower  
but  
longer tails

c)

Feedback of  $\alpha$ -tunneling electrometer: no shaking: no classical limit

rounding  $\sim (C_m/c)^2 I/E_c$

d)

Tempation:

Measurement

extra noise

forbidden transitions

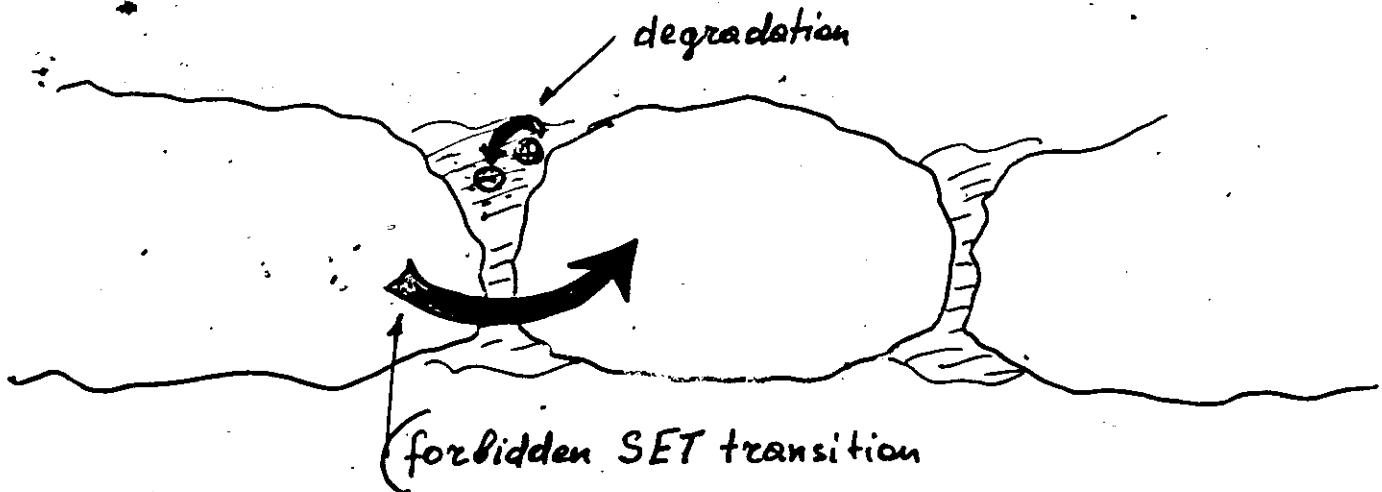
anarchy

but

Some experiments  
are in contradiction

working hypothesis.

## Degradation causes anarchy



1. An elementary degradation process disposes large Energy (volts)
2. it induces a voltage  $\nabla V$  at a certain function
3. if a forbidden SET pro. lacks energy  $E_{th}$

$P$  - probability to accompany degradation

$$P = \left( \frac{Gt}{24\pi e^2} \right) \left( \frac{\epsilon V}{E_{th}} \right)^2$$

Superconductivity  $\rightarrow$  no much changes

$$P_s = \left( \frac{Gt}{24\pi e^2} \right) (\epsilon V)^2 \int d\epsilon \frac{I(\epsilon)}{(E_{th} + \epsilon)^4}$$

Such a stimulation of forbidden groups  
may cause an anarchy observed.

