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**" ELECTRONIC STRUCTURE OF DISORDERED SURFACES  
AND ITS APPLICATIONS "**

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**These are preliminary lecture notes, intended only for distribution to participants.**

# ELECTRONIC STRUCTURE OF DISORDERED SURFACES AND ITS APPLICATIONS.

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MOTIVATION: surface disorder occurs frequently

- disordered/incomplete overlayers on substrates:  
(Ag,Pd) on Ag(110); gases on TM surfaces (catalysis)
- surface disordering: Cu<sub>50</sub> Au<sub>50</sub> (001)
- surfaces of TM alloys with non-uniform composition:  
(Ni,Pt) or (Cu,Ni) low-index surfaces
- semiconductor interfaces/superlattices: CdTe/(Cd,Hg)Te

Electronic structure is key to microscopic understanding

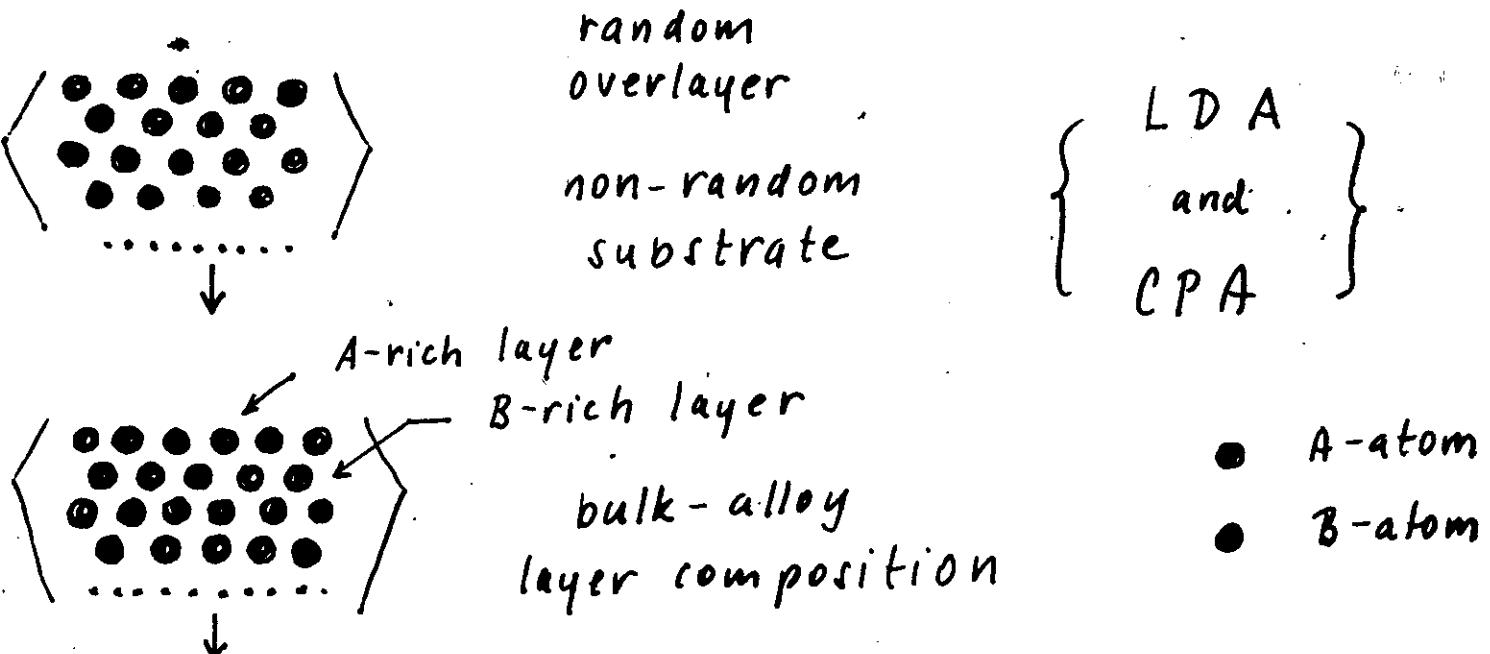
AIM: develop electronic theory of systems with  
disordered surfaces/interfaces which

- starts from first principles . . .
- semiinfinite sample geometry
- describes reliably concentration trends

To this end we combine

- TB-LMTO to describe electron states
- SGF method within the PL concept
- CPA to include disorder (inhomogeneous CPA)
- LSDA to account for electron correlations
- Empty spheres to describe vacuum

# STUDIED SYSTEMS



Basic approximation (in addition to LDA & CPA):  
Sample inhomogeneity in conjunction with LDA and CPA implies that each layer has different properties and layers are coupled each other  $\rightarrow \infty$  set of equations

Surface-bulk approximation: only first  $N$ -top layers have different properties to be found selfconsistently, remaining layers have bulk properties

Typically concentrations / potentials differ from bulk ones in few top layers for metallic systems.

To facilitate simultaneous treatment of disorder and surface we transform from original ( $\beta$ ) to new ( $\beta$ ) LMTO representation

ORIGINAL  $\rightarrow \beta_{RL}$

NEW  $\rightarrow \beta_{RL} \equiv \beta_L$

$$G(z) = (z - H)^{-1}$$

$$g^\beta(z) = (P^\beta(z) - S^\beta)^{-1}$$

$$H = C + \Delta^{1/2} S^\delta \Delta^{1/2}$$

$$P^\beta(z) = (z - C) / \{ \Delta + (\beta - \beta)(z - C) \}$$

$$S^\delta = S^0 (1 - \beta S^0)^{-1}$$

$$S^\beta = S^0 (1 - \beta S^0)^{-1}$$

- off diagonal disorder

- site diagonal disorder

{ KKR-CPA like }

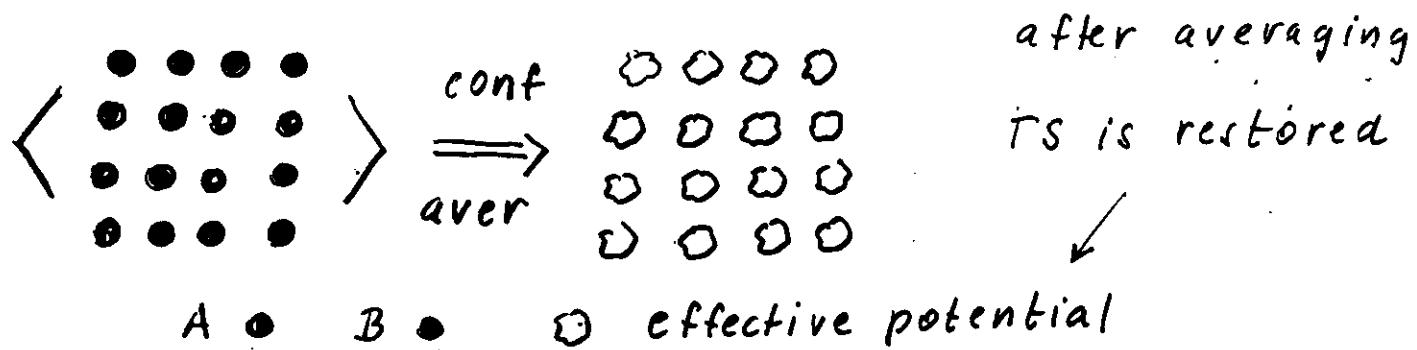
- large spatial extent of  $S^\delta$

- shortest possible spatial extent of  $S^\beta$

Basic trick: we perform CPA averaging / introduce surface in  $\beta$  and final results transform back to physical  $\beta$  using scaling relation of  $G(z)$  and  $g^\beta(z)$

$$G_{RL,R'L'}(z) = \lambda_{RL}^\beta(z) \delta_{RR'} \delta_{LL'} + \mu_{RL}^\beta(z) g_{RL,R'L'}^\beta(z) \mu_{R'L'}^\beta(z)$$

# COHERENT POTENTIAL APPROXIMATION



$$c_A \begin{pmatrix} \circ & \circ & \circ \\ \circ & \bullet & \circ \\ \circ & \circ & \circ \end{pmatrix} + c_B \begin{pmatrix} \circ & \circ & \circ \\ \circ & \bullet & \circ \\ \circ & \circ & \circ \end{pmatrix} = \begin{pmatrix} \circ & \circ & \circ \\ \circ & \circ & \circ \\ \circ & \circ & \circ \end{pmatrix}$$

CPA  
equation

CPA concept applied to each layer in question ( $i=1, 2, \dots, N$ )

$$\mathcal{P}_i^{\beta}(z) = \langle \mathcal{P}_i^{\beta}(z) \rangle + [\mathcal{P}_i^{\beta,A}(z) - \langle \mathcal{P}_i^{\beta}(z) \rangle] \phi_{ii}^{\beta}(z) / [\mathcal{P}_i^{\beta,B}(z) - \langle \mathcal{P}_i^{\beta}(z) \rangle]$$

$$\phi_{ii}^{\beta}(z) = \frac{1}{N_{ii}} \sum_{k_{ii}}^{SBZ} f_{ii}^{\beta}(k_{ii}, z) / \{ \mathcal{P}_j^{\beta}(z) \} \quad \begin{matrix} \text{- layer GF} \\ \text{central quantity} \end{matrix}$$

$\mathcal{P}_i^{\beta}(z)$  - coherent potential function (non-random)  
determined from above coupled set of Eqs.  
(coupled via  $\phi_{ii}^{\beta}(z)$ )

Properties of  $\mathcal{P}_i^{\beta}(z)$ :

- site diagonal, complex function
- non diagonal with respect to orbital index  $L = l_m$ : due to lowering symmetry at the surface (bulk!)

## PRINCIPAL LAYERS (PL)

Localized nature of  $S^\beta$  allows to define naturally PL which is very useful tool to determine layer GF (TB-LMTO ?)

Def.: solid is divided into PL (p) such that only nn PL are coupled by  $S^\beta$  (site off-diagonal part of problem,  $P^\beta$ -diagonal one)

$$S_{pq}^\beta = S_{01}^\beta \delta_{p,q+1} + S_{10}^\beta \delta_{p,q-1}, \quad S_{pp}^\beta = S_{00}^\beta \\ (\text{bulk } S^\beta \text{ are assumed})$$

Thus only  $S_{00}^\beta(k_u)$ ,  $S_{01}^\beta(k_u) = [S_{10}^\beta(k_u)]^*$  fully describe our system [and  $S^{\beta,\lambda\lambda}$ ,  $S^{\beta,\lambda 0} = [S^{\lambda,0\lambda}]^*$  for overlayer]

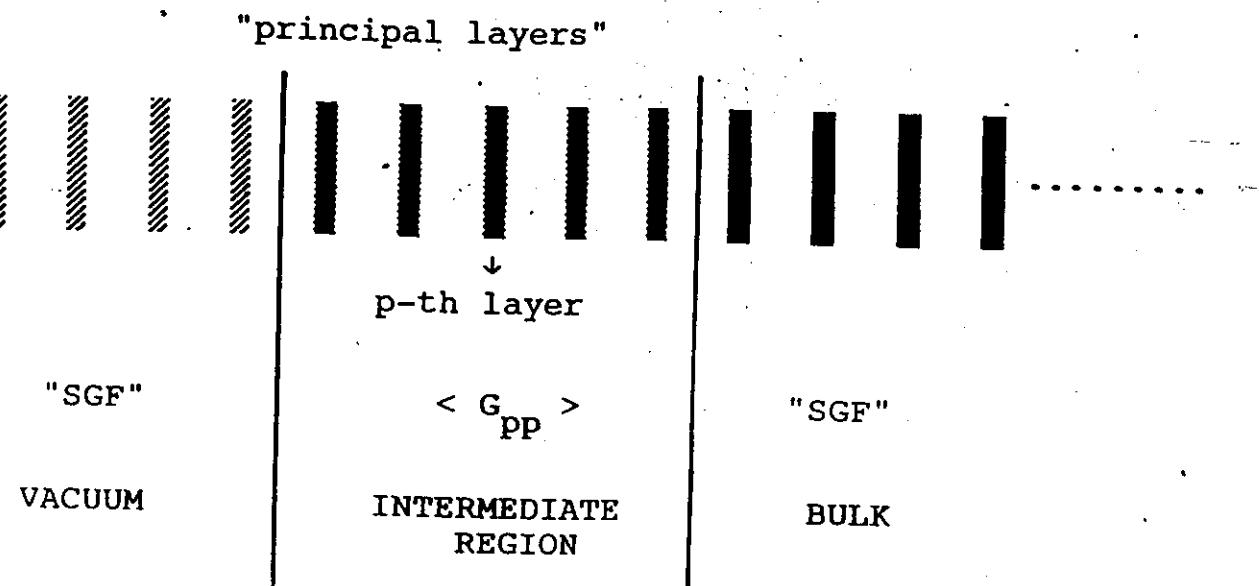
PL includes 1 to few atomic layers depending on lattice type, sample face & spatial extent of  $\delta^\beta$

Example: fcc (001), fcc (111), bcc (110)  $\rightarrow$  PL = 1 at. layer  
fcc (110), bcc (100)  $\rightarrow$  PL = 2 at. layers

fcc - 1<sup>st</sup> nn, bcc - 2<sup>nd</sup> nn : most-localized MTO repres.

Remark: if p-th PL includes more atomic layers;  
 $\phi_{ii}^\beta$  in the CPA set is a superdiagonal element of  $\phi_{nn}^\beta$

# Description of the semi-infinite geometry



$P_{-1} - S_{00}$	$-S_{01}$					
$-S_{10}$	$P_0 - S_{00}$	$-S_{01}$				
	$-S_{10}$	$P_1 - S_{00}$	$-S_{01}$			
		$-S_{10}$	$P_2 - S_{00}$	$-S_{01}$		
			$-S_{10}$	$P_3 - S_{00}$	$-S_{01}$	
				$-S_{10}$	$P_4 - S_{00}$	$-S_{01}$
					$-S_{10}$	$P_5 - S_{00}$
						$P_6 - S_{00}$

←

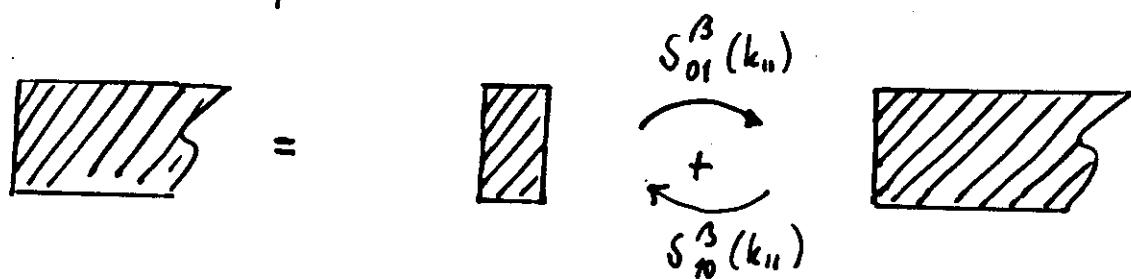
semi-infinite vacuum      intermediate region      semi-infinite bulk alloy

Remark :  $P_5 = P_6 = \dots P^{\text{bulk}}$ ,  $P_0 = P_{-1} = \dots P^{\text{vac}}$

# SURFACE GREEN's FUNCTION (SGF)

We determine SGF employing removal invariance principle (RIP) avoiding knowledge of bulk GF and necessity of  $k_{\perp}$ -integration of corresponding Dyson equation (cleavage perturbation method)

SGF by definition relates to ideal substrate, i.e. all layers have identical properties



$$g^{B,\sigma}(k_{\parallel}, z) = \left\{ g^{B,b}(z) - S_{00}^B(k_{\parallel}) - S_{01}^B(k_{\parallel}) g^{B,\sigma}(k_{\parallel}, z) S_{10}^B(k_{\parallel}) \right\}^{-1}$$

Remarks :

- (i) selfconsistent equation for  $g^{B,\sigma}(k_{\parallel}, z)$  is solved iteratively in complex energy plane followed by deconvolution to real axis at the end.
- (ii) simple and effective alternative to layer-doubling (RDT) and RS-MST techniques
- (iii) serves as an exact identity to simplify expressions for interfaces, superlattices, etc.

## Summary :

Originally infinite problem in PL-indices is reduced to a finite one using surface GF to describe the semiinfinite bulk / vacuum

$P_1 - S_{00}$ $- S_{10} \mathcal{G}^v S_{01}$	$-S_{01}$	0	0
$-S_{10}$	$P_2 - S_{00}$	$-S_{01}$	0
0	$-S_{10}$	$P_3 - S_{00}$	$-S_{01}$
0	0	$-S_{10}$	$P_4 - S_{00}$ $- S_{01} \mathcal{G}^b S_{10}$

surface Green's functions  
 $\mathcal{G}^v$  and  $\mathcal{G}^b$  are found from  
removal invariance principle

$$\mathcal{G}^b(k_{||}, z) = [P^b(z) - S_{00}(k_{||}) - S_{01}(k_{||}) \cdot \\ \cdot \mathcal{G}^b(k_{||}, z) S_{10}(k_{||})]^{-1}$$

$$\mathcal{G}^v(k_{||}, z) = [P^v(z) - S_{00}(k_{||}) - S_{10}(k_{||}) \cdot \\ \cdot \mathcal{G}^v(k_{||}, z) S_{01}(k_{||})]^{-1}$$

LAYER-RESOLVED  $k_{\parallel}$ -INTEGRATED GF:

$$\phi_p(z) = \frac{1}{N_{\parallel}} \sum_{k_{\parallel}} \langle g(k_{\parallel}, z) \rangle_{pp} = f(\{\phi_p(z)\})$$

(i) depends on all  $\phi_p$  relating them via a set of coupled layer-CPA equations

(ii) enters also LDA part thus making necessary link of both CPA and LDA

LAYER-RESOLVED LDA-POTENTIAL  $V_p^{\alpha}(r)$  :  $\alpha = A, B$

$$V_p^{\alpha}(r) = -\frac{Z^{\alpha}}{r} + V_p^{\alpha, H}(\tilde{\rho}(r)) + V_p^{\alpha, ex}(\tilde{\rho}(r)) + \sum_L \sum_q M_{pq}^{SL} \bar{Q}_q^L$$

$\tilde{\rho}^{\alpha}$  - spherical part of  $\rho^{\alpha}(r)$  monopole  $\propto$  dipole  
 $\bar{Q}_p^L$  - net charge in a given layer terms of multipole expansion of  $\rho^{\alpha}(r)$

$$\bar{Q}_p^L \propto \sum_{\alpha} c_p^{\alpha} \left\{ \int Y_L(r) \left( \frac{r}{s^{\alpha}} \right) \rho_p^{\alpha}(r) dr - Z^{\alpha} \delta_{L,0} \right\} \quad \begin{array}{ll} l=0 & \text{mono} \\ l=1 & \text{dipole} \end{array}$$

$$\bar{Q}_p^{l=0} \text{ from } \phi_p^{LL}(z) \quad \bar{Q}_p^{l=1} \text{ from } \phi_p^{L,L'}(z)$$

$M_{pq}^{SL}$  are intra- and inter-layer Madelung constants describing 1st interaction of mono/dipo charges

ITERATION LOOP:

$$V_p^{\alpha, \text{inp}}(r) \xrightarrow{\text{CPA}} \bar{Q}_p^L \xrightarrow{\text{LDA}} V_p^{\alpha, \text{out}}(r)$$

## WORK FUNCTION:

i) Clean Ag (001)

theory 4.78 eV (present)

4.74 eV (FLAPW: Surf. Sci. 243, 317 (1991))

4.95 eV (SEG: PRB 37, 6682 (1988))

experiment: 4.2 - 4.64 eV (surface roughness)

Model: 'sandpaper'-like  $\Rightarrow \sim 20\%$  vacancies  
4.45 eV

Dipole barrier:  $8Ag + 10ES (9Ag + 9ES)$  } 7.1 - 7.2 eV  
 $M_{pq}^{s_1 s_2 l=1} = 0$

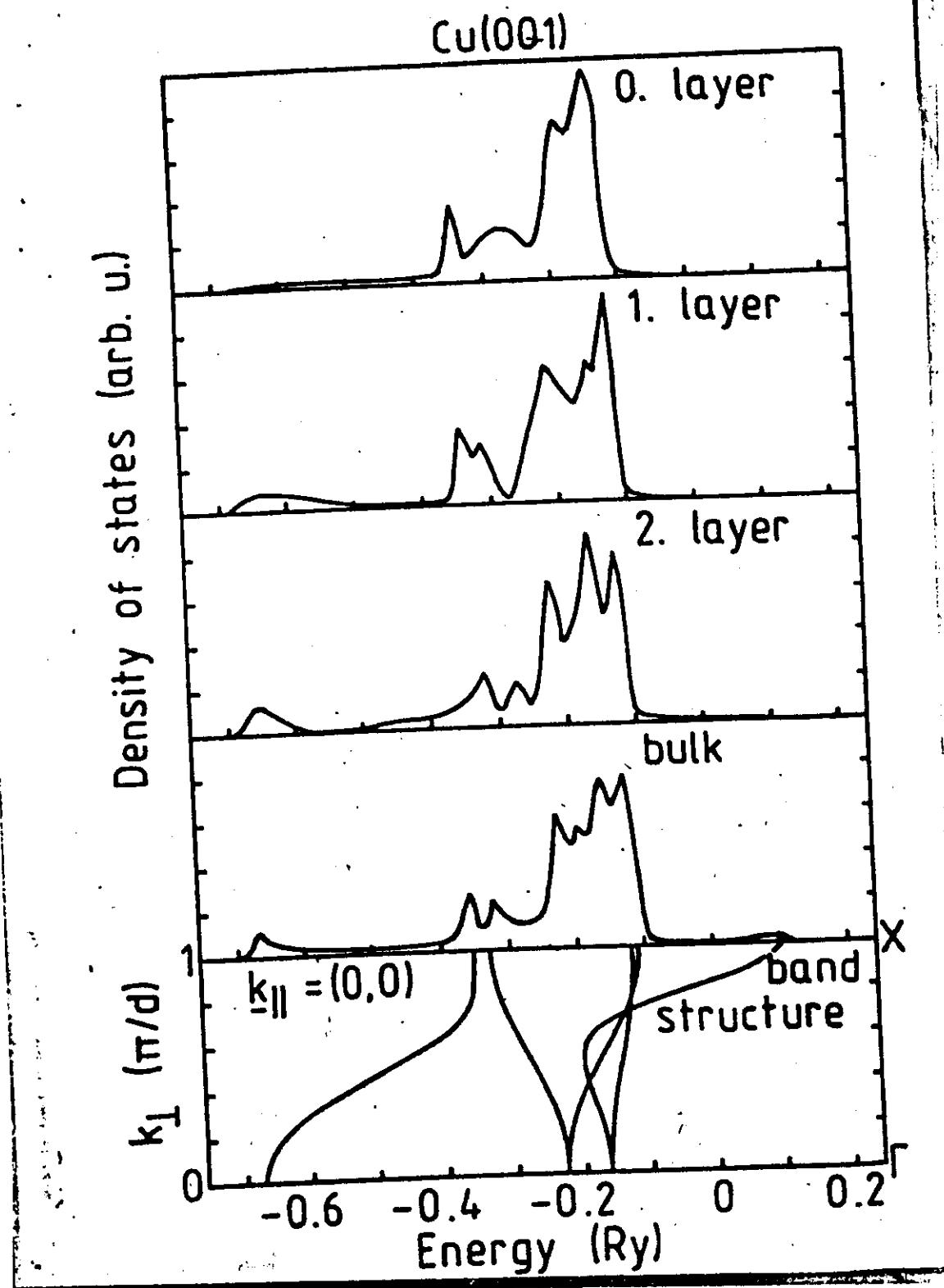
ii) Clean Pd (001)

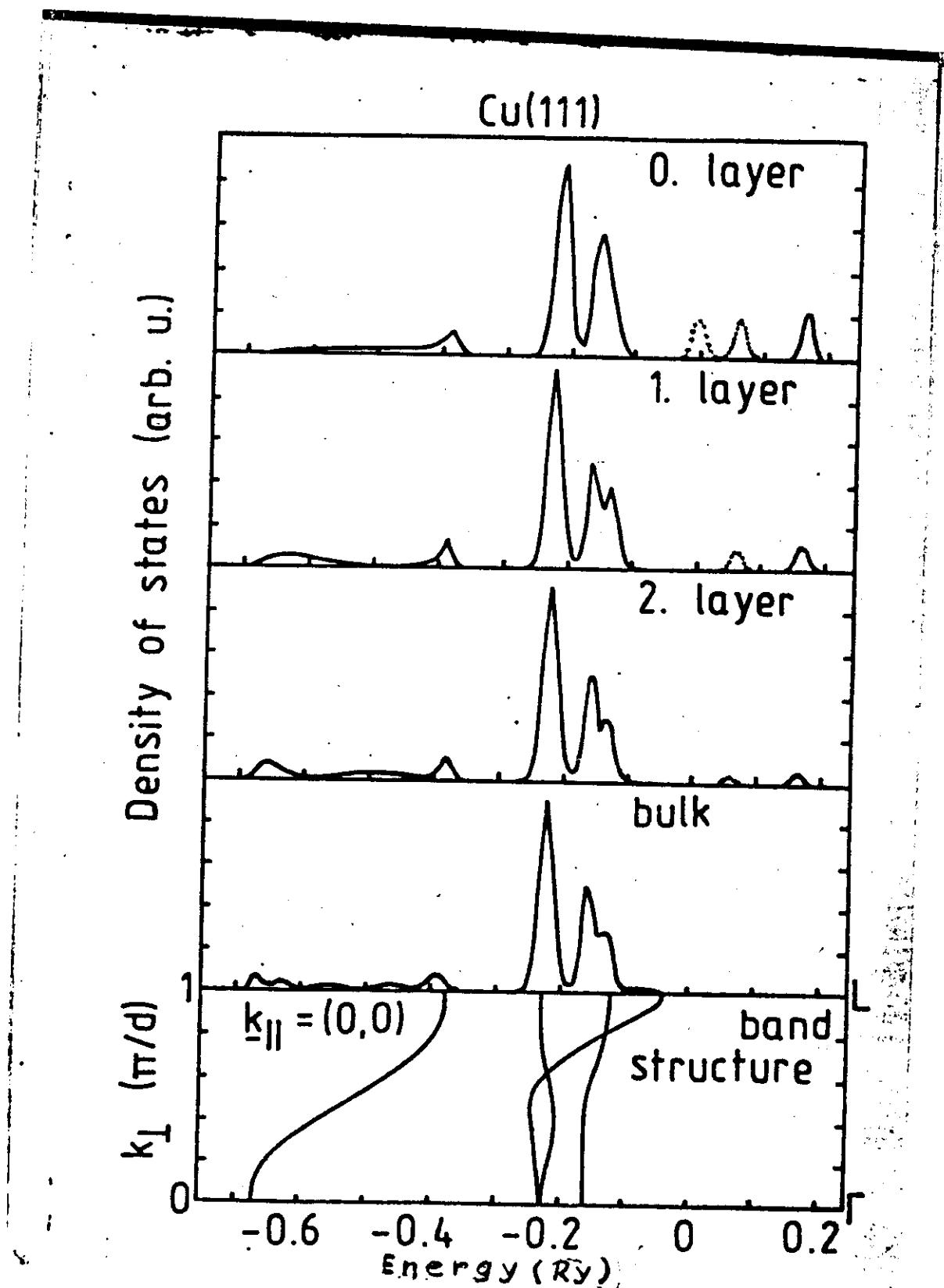
theory equilibrium: 5.83 eV expanded: 5.61 eV  
(Ag)

experiment: 5.8 eV

iii) (Ag, Pd) overlayer on Ag (001)

monotonic increase from 4.78 eV (clean Ag (001)) to  
5.67 eV (Pd monolayer)  $\rightarrow$  expanded Pd (001)

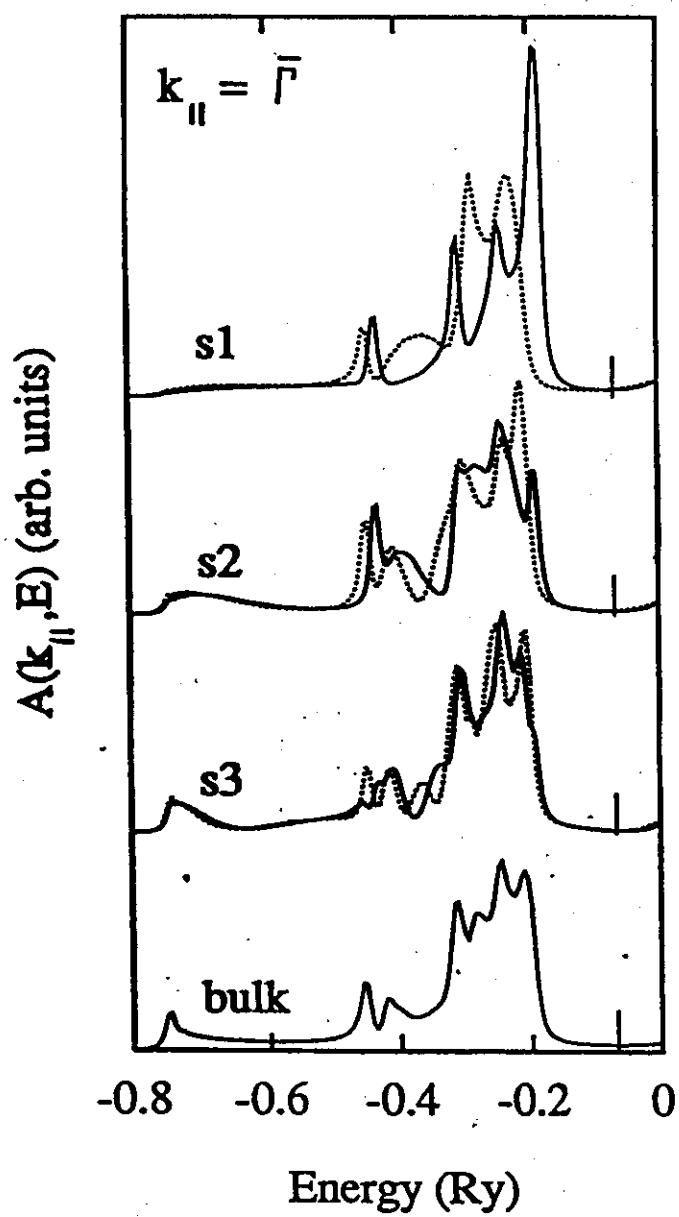




scf calculations

nscf calculations: bulk potentials up to  
the surface

clean Cu(001)

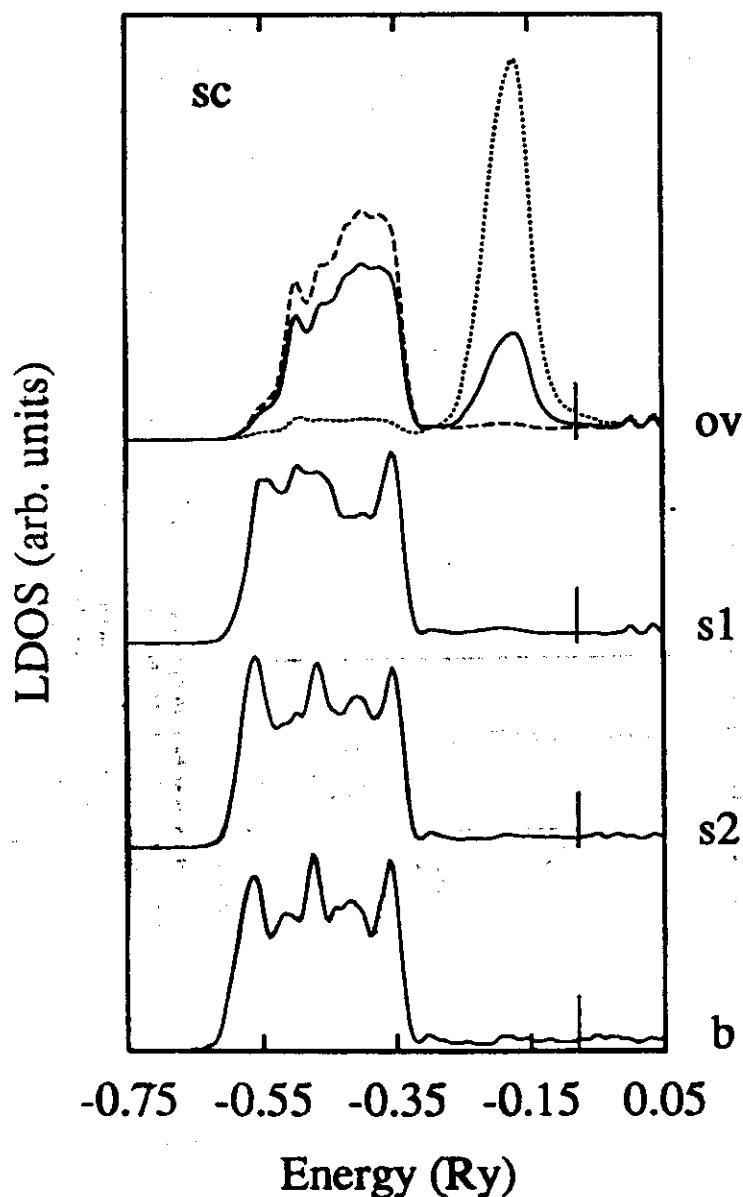


Intermediate region: 2 ES + overlayer + 2 substrate layers  
SBZ integration : 21 special  $k_{\parallel}$ -points

### Layer-resolved DOS

— total DOS  
---- Ag LDOS  
..... Pd LDOS

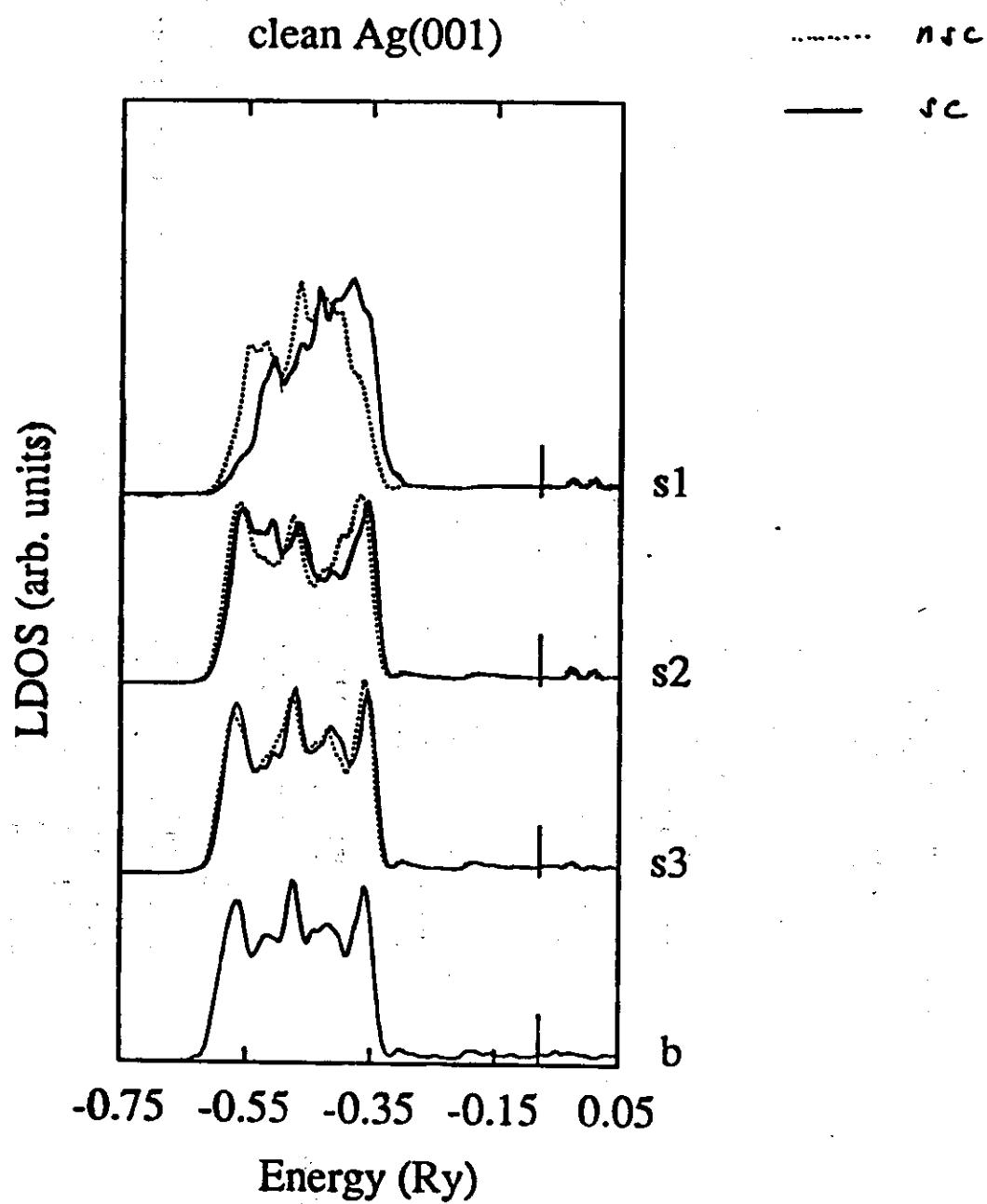
$\text{Ag}_{75} \text{Pd}_{25}$  on  $\text{Ag}(001)$

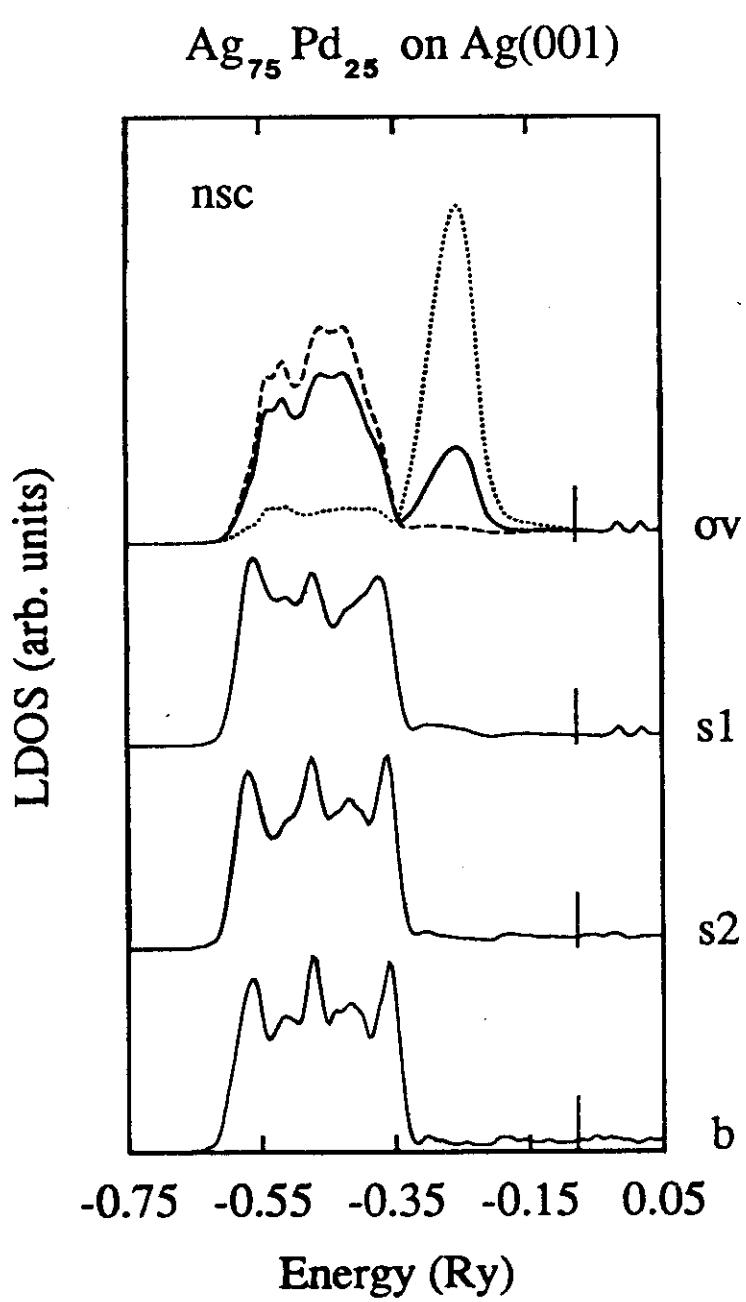


Intermediate region: 2 ES + 3 substrate layers

SBZ integration: 21 special  $k_{\parallel}$ -points

### Layer-resolved DOS





## ORDERING ON SURFACES: Cu(001) c(2x2)-Pd

LEED exp:  $\approx 50\%$  Pd coverage, c(2x2) pattern found

Two possibilities:

- (i) CuPd surface alloy
- (ii) Pd overlayer  
(limited to top sample layer)

Remark: similar case is Au coverage on Cu(001)

Dynamical LEED calculation  $\rightarrow$  surface alloy

ARPES  $\rightarrow$  surface alloy (Pd-induced features  
similar as bulk CuPd)

New approach : PRL 69 (1992) 308

- we study the stability of disordered phases - Cu<sub>50</sub>Pd<sub>50</sub> and Pd<sub>50</sub>Vac<sub>50</sub> overlayers on Cu(001) towards the formation of ordered phase  
(analogy to magnetic instabilities of  $\chi(k)$  - e.g. bcc paramagnetic Cr  $\rightarrow$  antiferromagnetic Cr).
- ordering is studied in the frame of effective 2D Ising model constructed from corresponding ab-initio electronic structure within the GPM

$$H = \sum_{R+R'} V_{RR'}^A \eta_R \eta_{R'}^A, \quad V_{RR'}^A = V_{RR'}^{AA} + V_{RR'}^{BB} - V_{RR'}^{AB} - V_{RR'}^{BA}$$

$$V_{RR'}^{AA} = -\frac{1}{\pi} \operatorname{Im} \operatorname{tr} \int \left\{ t_R^{(2)} \bar{f}_{RR'}(z) t_{R'}^{(2)} \bar{f}_{RR'}(z) \right\} dE \quad z = E + i\delta \quad x, \alpha' = A, B$$

$t_R^k(z)$  - overlayer CPA  $\tau$ -matrix

$$\bar{f}_{RR'}(z) = \frac{1}{N_{k_\parallel}} \sum_{k_\parallel}^{SBZ} \bar{f}(k_\parallel z) e^{ik_\parallel(R-R')} \quad (R, R' \in \text{overlayer})$$

- we look for extrema of  $V(k_\parallel) = \sum_R V_{0R} e^{ik_\parallel R}$

(i)  $\max \{-V(k_\parallel)\} = 0 \rightarrow \text{no ordering}$

(ii)  $\max \{-V(k_\parallel)\}$  at  $k_\parallel^0 = (0, 1)$   $\rightarrow c(2 \times 2)$  ordering

( $k_\parallel^0$  - 2D analogy of Lifshitz special points)

Remark:  $V_{0R}$  up to 11-th shell were determined

Values of  $V_{0R}$  for first few shells:

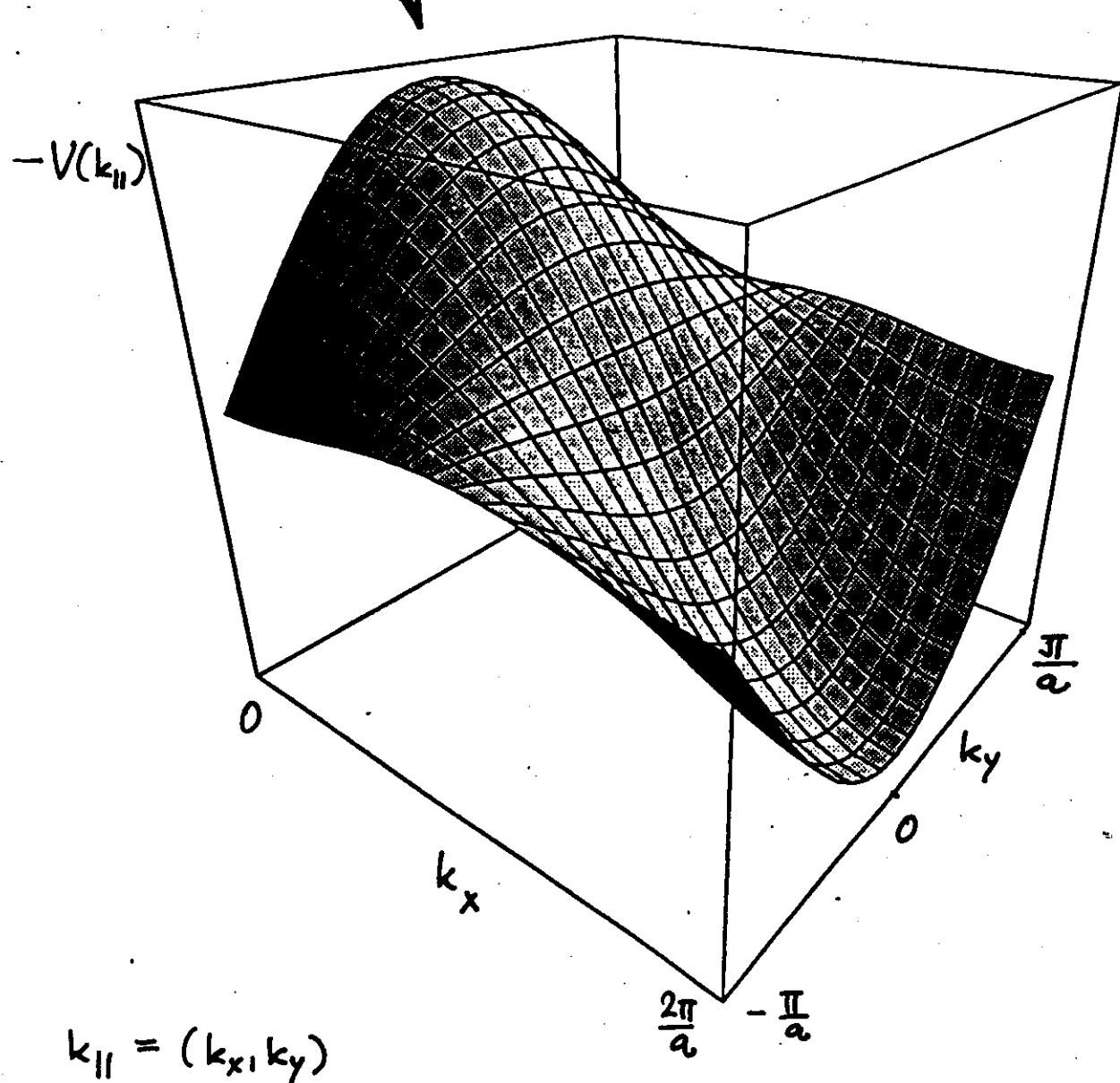
$$V_{0R} = 6.51, -0.59, -0.36, -0.18 \text{ [mRy]} \dots \text{ CuPd}$$

$$V_{0R} = -19.79, 1.38, -0.39, -0.03 \text{ [mRy]} \dots \text{ PaVac}$$

Remark: the same tendency to ordering (1st NN dominating); not the type of superstructure

- Pd<sub>50</sub> Vac<sub>50</sub> overlayer on fcc (001) Cu

maximum at  $k_{||} = (0,0)$  : ORDERED STRUCTURE  
UNSTABLE

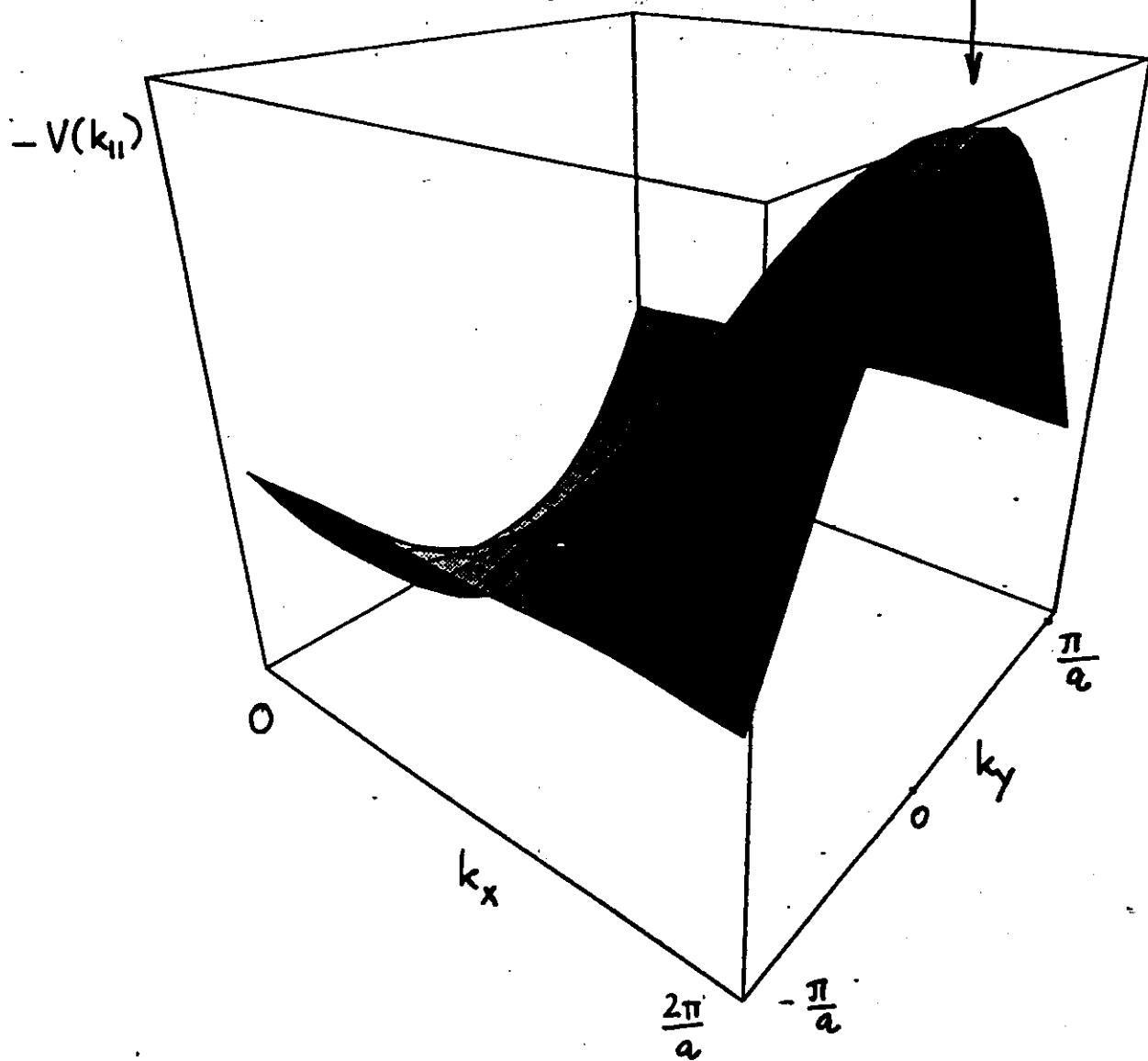


$$k_{||} = (k_x, k_y)$$

$\text{Cu}_{50}\text{Pd}_{50}$  overlayer on fcc (001) Cu

maximum at  $k_{||} = \frac{2\pi}{a}(1,0)$

STABLE ORDERED  
STRUCTURE



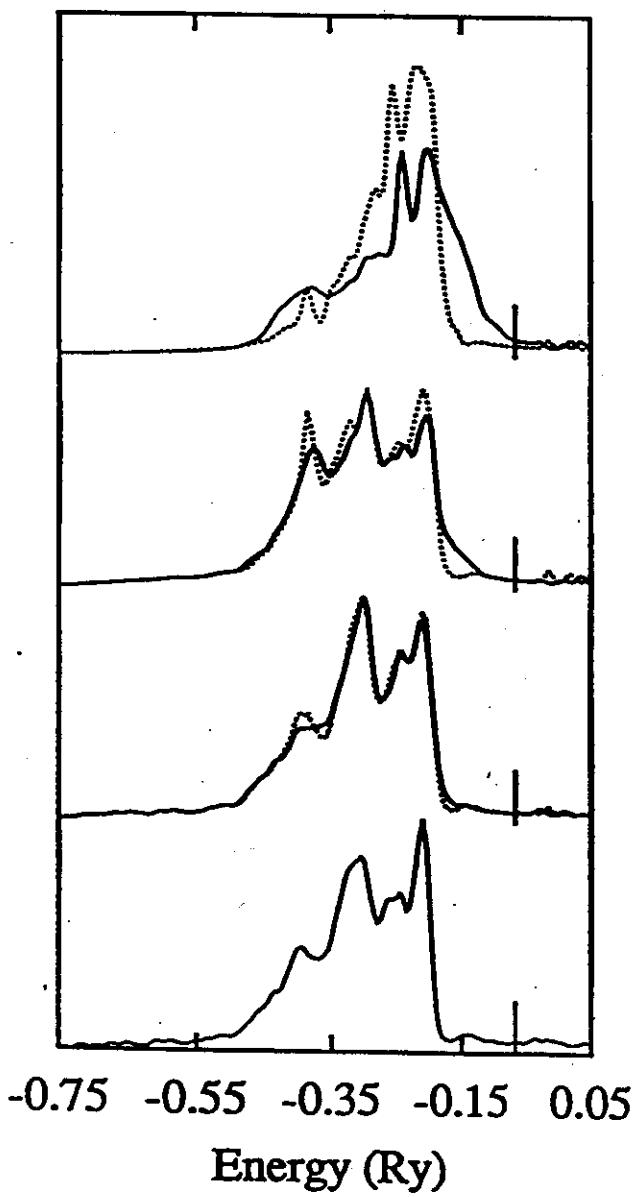
$$k_{||} = (k_x, k_y)$$

Total DOS for Cu(001) covered by overlayer

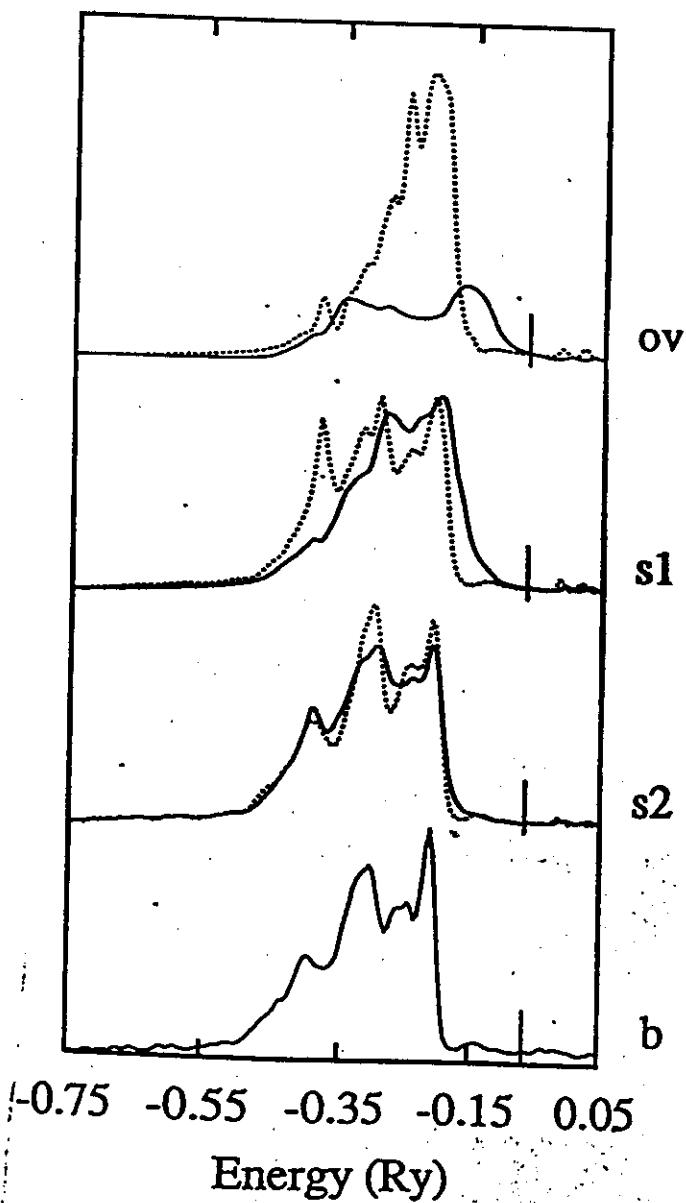
..... Total DOS for clean Cu(001)

$\text{Cu}_{50}\text{Pd}_{50}$  on Cu(001)

LDOs (arb. units)



$\text{Pd}_{50}\text{Vac}_{50}$  on Cu(001)



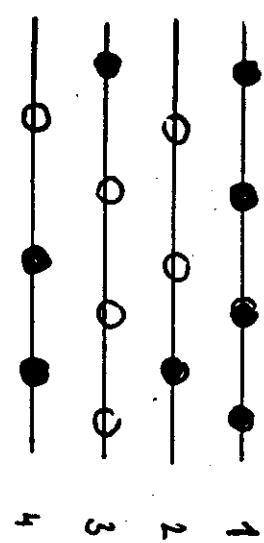
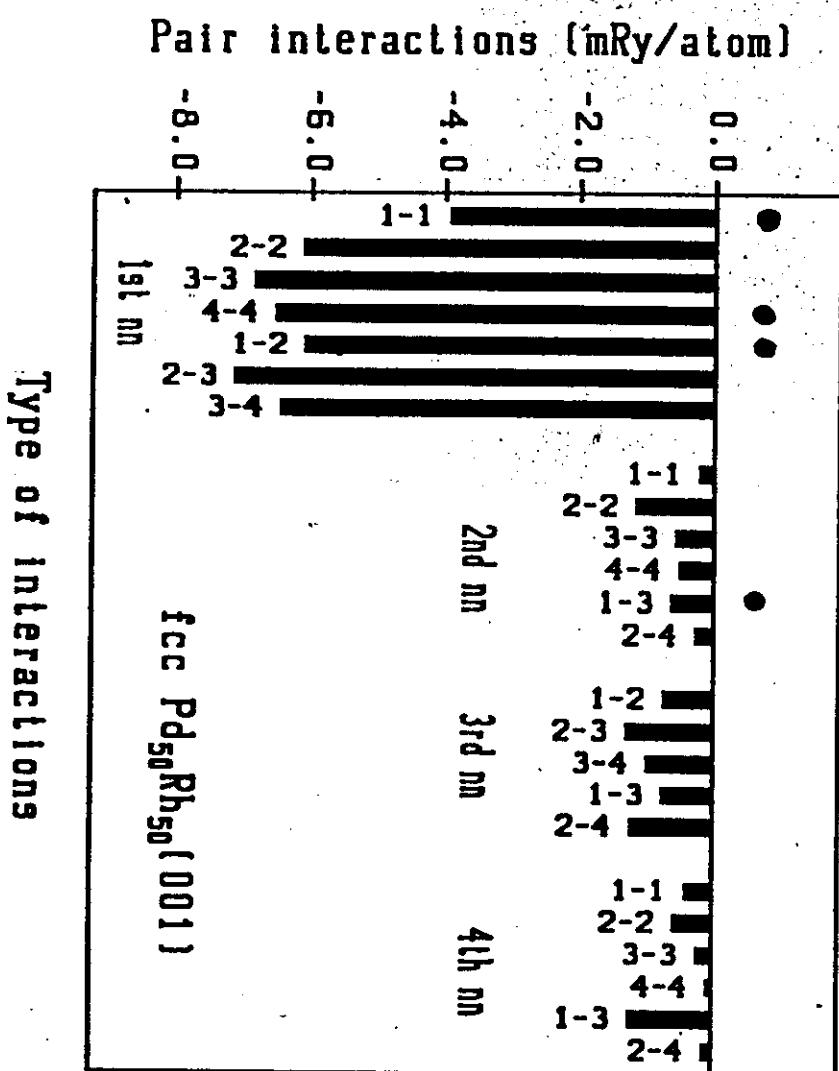
## MONTE-CARLO SIMULATIONS OF SEGREGATION

We study segregation in  $Cu_{1-x}Ni_x(001)$  alloys based on ab-initio Ising model derived for a homogeneous reference medium — scf bulk alloy

Details:

- computational cell consists of 8 fcc(001) alloy planes each containing  $15 \times 17$  atoms with periodic boundary condition in layers // surface
- we start from a random configuration and interchange randomly two atoms. Their energy difference  $\Delta E$ :
  - (i)  $\Delta E < 0$  — favorable, represents a new configuration
  - (ii)  $\Delta E > 0$  — represents a new configuration only if  $\exp\{-\Delta E/kT\} > \hat{a}$ ,  $\hat{a}$  — random number  $\in (0,1)$
- this step is repeated and the system approaches the thermodynamical equilibrium if the number of interchanges is sufficiently large
- during MC steps, two bottom layers of slab have fixed bulk concentration

homogeneous profile

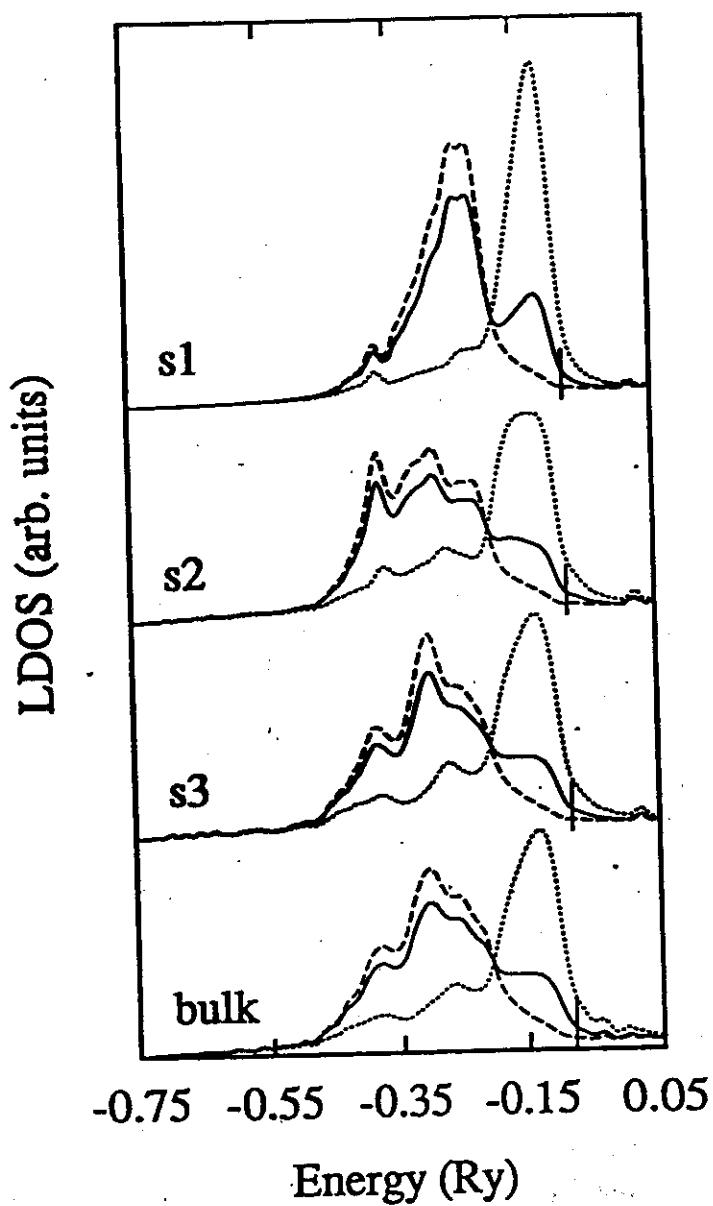


total DOS

Cu - LDOS

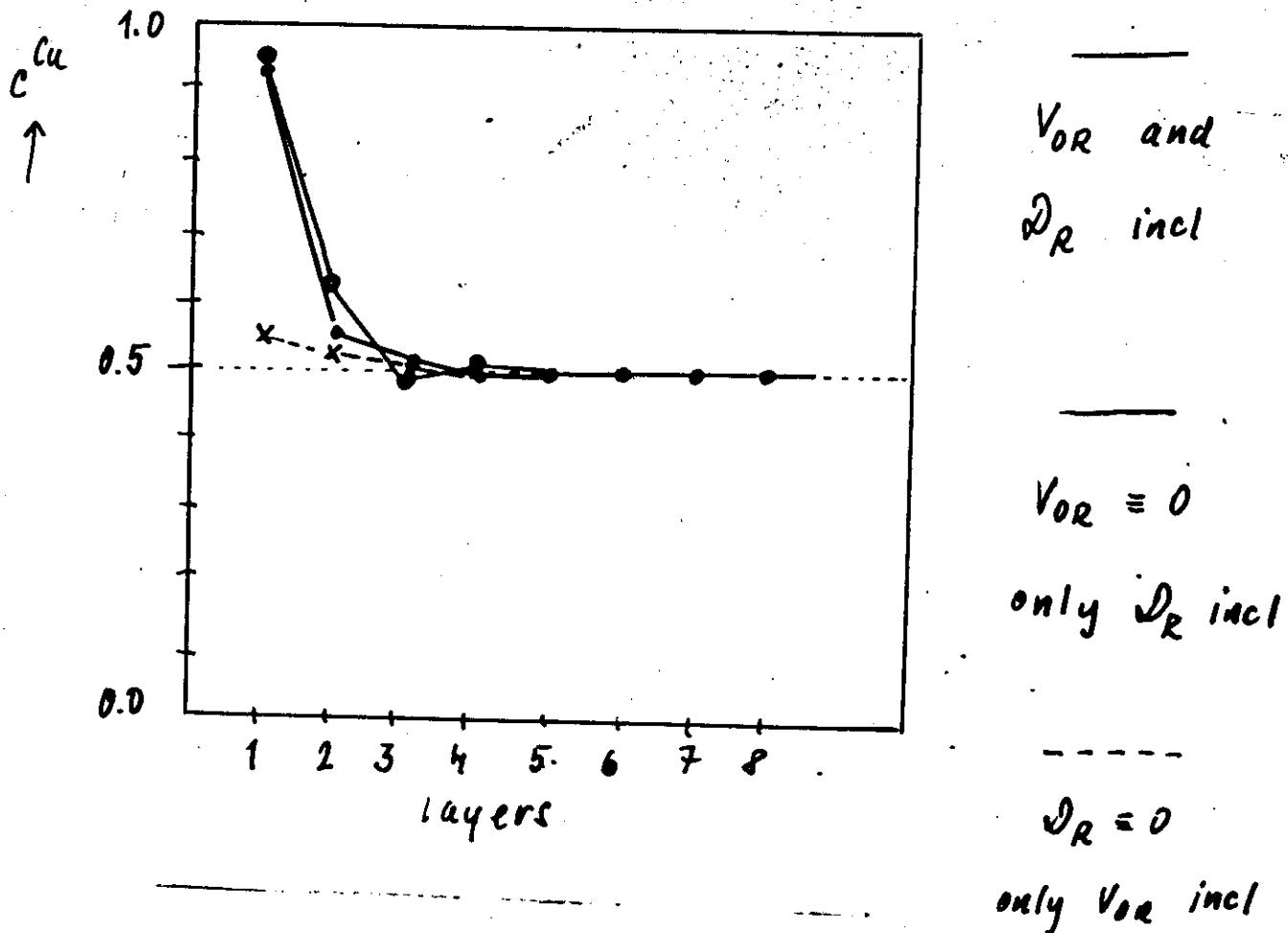
Ni - LDOS

fcc - Cu<sub>75</sub>Ni<sub>25</sub>(001)



Results of MC simulations : fcc - Cu<sub>50</sub> Ni<sub>50</sub> (001)

T = 800 K



Remarks :

- (i) Values of on-site terms of Ising Hamiltonian are decisive for segregation
- (ii) Good agreement with experiment over all concentration range

## SUMMARY

We have developed self theory of electronic states of disordered surfaces within the LDA

- use of screened structure constants allows to include semiinfinite geometry simply and effectively
- use of inhomogeneous CPA allows a unified treatment of clean surfaces, various stages of adsorption (low coverage  $\rightarrow$  films) and surface segregation
- use of multipole expansion of charge density at the surface gives proper description of dipole barrier
- in conjunction with GPM forms a theoretical background for study of ordering and segregation phenomena at surfaces from first principles

## OUTLOOK

- surface magnetism: magnetic overlays, surfaces of mag. allo.
- relativistic effects: 4d and 5d metals, Dirac equation
- magnetic GPM: interplay of chemical & magnetic interactions

