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Quantities for Describing Ionizing Radiation

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FUNDAMENTALS OF RADIATION DOSIMETRY - I

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Quantities for Describing Ionizing Radiation

Simple Description of Radiation Fields by Nonstochastic Quantities

1. FLUENCE

Referring to Fig. 1.1, let N_e be the expectation value of the number of rays striking a finite sphere surrounding point P during a time interval extending from an arbitrary starting time t_0 to a later time t. If the sphere is reduced to an infinitesimal at P with a great-circle area of da, we may define a quantity called the *fluence*, Φ, as the quotient of the differential of N_e by da:

$$\Phi = \frac{dN_e}{da} \tag{1.5}$$

which is usually expressed in units of m^{-2} or cm^{-2}.

2. FLUX DENSITY (OR FLUENCE RATE)

Φ may be defined by (1.5) for all values of t through the interval from $t = t_0$ (for which $\Phi = 0$) to $t = t_{max}$ (for which $\Phi = \Phi_{max}$). Then at any time t within the interval we may define the *flux density* or *fluence rate* at P as

$$\varphi = \frac{d\Phi}{dt} = \frac{d}{dt}\left(\frac{dN_e}{da}\right) \tag{1.6}$$

where $d\Phi$ is the increment of fluence during the infinitesimal time interval dt at time t, and the usual units of flux density are $m^{-2}\,s^{-1}$ or $cm^{-2}\,s^{-1}$.

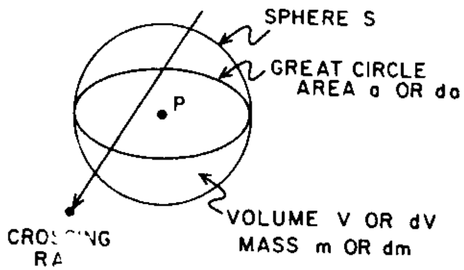

FIGURE 1.1. **Characterizing the radiation field at a point P in terms of the radiation traversing the spherical surface S.**

It should be noted that φ and Φ express the sum of rays incident from all directions, and irrespective of their quantum or kinetic energies, thereby providing a bare minimum of useful information about the field. However, different types of rays are usually not lumped together; that is, photons, neutrons, and different kinds of charged particles are measured and accounted for separately as far as possible, since their interactions with matter are fundamentally different.

3. ENERGY FLUENCE

The simplest field-descriptive quantity which takes into account the energies of the individual rays is the *energy fluence* Ψ, for which the energies of all the rays are summed.

Let R be the expectation value of the total energy (exclusive of rest-mass energy) carried by all the N_e rays striking a finite sphere surrounding point P (see Fig. 1.1) during a time interval extending from an arbitrary starting time t_0 to a later time t*. If the sphere is reduced to an infinitesimal at P with a great-circle area of da, we may define a quantity called the *energy fluence*, Ψ, as the quotient of the differential of R by da:

$$\Psi = \frac{dR}{da} \tag{1.9}$$

which is usually expressed in units of $J\,m^{-2}$ or $erg\,cm^{-2}$.

For the special case where only a single energy E of rays is present, Eqs. (1.5) and (1.9) are related by

$$R = EN_e \tag{1.9a}$$

and

$$\Psi = E\Phi \tag{1.9b}$$

Individual particle and photon energies are ordinarily given in MeV or keV, which is the kinetic energy acquired by a singly charged particle in falling through a potential difference of one million or one thousand volts, respectively. Energies in MeV can be converted into ergs and joules through the following statements of equivalence:

$$\begin{aligned} 1\ \text{MeV} &= 1.602 \times 10^{-6}\ \text{erg} = 1.602 \times 10^{-13}\ \text{J} \\ 1\ \text{erg} &= 10^{-7}\ \text{J} = 6.24 \times 10^{5}\ \text{MeV} \\ 1\ \text{J} &= 6.24 \times 10^{12}\ \text{MeV} = 10^{7}\ \text{erg} \end{aligned} \tag{1.10}$$

*ICRU (1980) calls R the *radiant energy*, and defines it as "the energy of particles (excluding rest energy) emitted, transferred, or received."

4. ENERGY FLUX DENSITY (OR ENERGY FLUENCE RATE)

Ψ may be defined by Eq. (1.9) for all values of t throughout the interval from $t = t_0$ (for which $\Psi = 0$) to $t = t_{max}$ (for which $\Psi = \Psi_{max}$). Then at any time t within the interval we may define the *energy flux density* or *energy fluence rate* at P as:

$$\psi = \frac{d\Psi}{dt} = \frac{d}{dt}\left(\frac{dR}{da}\right) \tag{1.11}$$

where $d\Psi$ is the increment of energy fluence during the infinitesimal time interval dt at time t, and the usual units of energy flux density are J m^{-2} s^{-1} or erg cm^{-2} s^{-1}.

For monoenergetic rays of energy E the energy flux density ψ may be related to the flux density φ by an equation similar to (1.9b):

$$\psi = E\varphi \tag{1.13a}$$

KERMA

This nonstochastic quantity is relevant only for fields of indirectly ionizing radiations (photons or neutrons) or for any ionizing radiation source distributed within the absorbing medium.

Definition

The kerma K can be defined in terms of the related stochastic quantity *energy transferred*, ϵ_{tr} (Attix, 1979, 1983) and the *radiant energy* R (ICRU, 1980). The energy transferred in a volume V is:

$$\epsilon_{tr} = (R_{in})_u - (R_{out})_u^{nonr} + \sum Q \tag{2.1}$$

where $(R_{in})_u$ = radiant energy of uncharged particles entering V,
$(R_{out})_u^{nonr}$ = radiant energy of uncharged particles leaving V, *except* that which originate.. :om radiative losses of kinetic energy by charged particles while in V, and
ΣQ = net energy derived from rest mass in V ($m \rightarrow E$ positive, $E \rightarrow m$ negative).

By radiative losses, we mean conversion of charged-particle kinetic energy to photon energy, through either bremsstrahlung x-ray production or in-flight annihilation of positrons. In the latter case only the kinetic energy possessed by the positron at the instant of annihilation (which is carried away by the resulting photons along with 1.022 MeV of rest-mass energy) is classified as radiative energy loss.

Upon consideration of Fq. (2.1) it will be seen that energy transferred is just the kinetic energy received by charged particles in the specified finite volume V, regardless of where or how they in turn spend that energy.

We may now define the kerma K at point of interest P in V as

$$K = \frac{d(\epsilon_{tr})_e}{dm} \equiv \frac{d\epsilon_{tr}}{dm} \tag{2.2}$$

where $(\epsilon_{tr})_e$ is the expectation value of the energy transferred in the finite volume V during some time interval, $d(\epsilon_{tr})_e$ is that for the infinitesimal volume dv at the internal point P, and dm is the mass in dv. Since the argument of any legitimate differential quotient may always be taken to be nonstochastic, the symbol $d(\epsilon_{tr})_e$ may be simplified to $d\epsilon_{tr}$ as indicated in Eq. (2.2).

The average value of the kerma throughout a volume containing a mass m is simply the expectation value of the energy transferred divided by the mass, or $(\epsilon_{tr})_e/m$.

Kerma can be expressed in units of erg/g, rad, or J/kg. The latter unit is also called the *gray* (Gy) in honor of L. H. Gray, a pioneer in radiological physics. The rad is still commonly employed for kerma and absorbed dose at the time of this writing, but J/kg is to be preferred as part of a general shift to the International System of units. Fortunately all these units are simply related by

$$1 \text{ Gy} = 1 \text{ J/kg} = 10^2 \text{ rad} = 10^4 \text{ erg/g} \tag{2.3}$$

Relation of Kerma to Energy Fluence for Photons

For monoenergetic photons the kerma at a point P is related to the energy fluence there by the *mass energy-transfer coefficient* $(\mu_{tr}/\rho)_{E,Z}$, which is characteristic of the photon energy E and the atomic number Z of the matter at P:

$$K = \Psi \cdot \left(\frac{\mu_{tr}}{\rho}\right)_{E,Z} \tag{2.4}$$

Here μ_{tr} is called the *linear energy-transfer coefficient* in units of m^{-1} or cm^{-1}, and ρ is the density in kg/m^3 or g/cm^3. Ψ is the energy fluence at P in J/m^2 (preferred) or erg/cm^2. K is the kerma at P, expressed in J/kg (preferred) or in erg/g, respectively, either of which can be converted into rads, if desired, by Eq. (2.3).

Relation of Kerma to Fluence for Neutrons

Equation (2.4) could be applied to neutrons as well as x- and γ-ray photons, but this is not customary. Usually neutron fields are described in terms of flux density and fluence, instead of *energy* flux density and *energy* fluence as is usually the case with photons. Thus for consistency a quantity called the *kerma factor* F_n is tabulated for neutrons instead of the mass energy-transfer coefficient:

$$(F_n)_{E,Z} = \left(\frac{\mu_{tr}}{\rho}\right)_{E,Z} \cdot E \tag{2.6}$$

If $(\mu_{tr}/\rho)_{E,Z}$ is given in units of cm^2/g, the neutron energy E in this relation is commonly expressed in g-rad/neutron in place of MeV/neutron.

Thus, instead of Eq. (2.4), for monoenergetic neutrons one uses the following relation:

$$K = \Phi \cdot (F_n)_{E,Z} \quad \text{(rad)} \tag{2.8}$$

Components of Kerma

The kerma for x- or γ-rays consists of the energy transferred to electrons and positrons per unit mass of medium. The kinetic energy of a fast electron may be spent in two ways:

1. Coulomb-force interactions with atomic electrons of the absorbing material, resulting in the local dissipation of the energy as ionization and excitation in or near the electron track. These are called *collision* interactions.
2. Radiative interactions with the Coulomb force field of atomic nuclei, in which x-ray photons (bremsstrahlung, or "braking radiation") are emitted as the electron decelerates. These x-ray photons are relatively penetrating compared to electrons and they carry their quantum energy far away from the charged-particle track.

Since the kerma includes kinetic energy received by the charged particles whether it is destined to be spent by the electrons in collision or radiative-type interactions, we can subdivide K into two parts according to whether the energy is spent nearby in creating excitation and ionization (K_c) or is carried away by photons (K_r):

$$K = K_c + K_r = (1-g)K + gK \tag{2.10}$$

where g is the fraction of the electron energy lost to photons, and the subscripts refer to "collision" and "radiative" interactions, respectively.

For the case of neutrons as the indirectly ionizing radiation, the resulting charged particles are protons and heavier recoiling nuclei, for which K_r is vanishingly small. Thus $K = K_c$ for neutrons, and we need not consider the partition of K in that case.

It will be convenient in discussing the concept of charged-particle equilibrium (CPE) if we now define the *collision kerma* (K_c) in a manner corresponding to that employed for K in Eqs. (2.1) and (2.2).

Let ϵ_{tr}^n be the related stochastic quantity called the *net energy transferred*, which can be defined for a volume V as

$$\epsilon_{tr}^n = \epsilon_{tr} - R_u' \tag{2.11}$$

where R_u' is the radiant energy emitted as radiative losses by the charged particles which themselves originated in V, regardless of where the radiative loss events occur. ϵ_{tr} and K include energy that goes to radiative losses, while ϵ_{tr}^n and K_c do not.

Now we can define K_c at a point of interest P as

$$K_c = \frac{d\epsilon_{tr}^n}{dm} \tag{2.12}$$

where ϵ_{tr}^n is now the expectation value of the net energy transferred in the finite volume V during some time interval, $d\epsilon_{tr}^n$ is that for the infinitesimal volume dv at point P, and dm is the mass in dv.

For monoenergetic photons K_c is related to the energy fluence Ψ by another energy- and material-dependent coefficient $(\mu_{en}/\rho)_{E,Z}$ called the *mass energy-absorption coefficient*, so that the equation corresponding to Eq. (2.4) becomes

$$K_c = \Psi\left(\frac{\mu_{en}}{\rho}\right)_{E,Z} \tag{2.13}$$

where the units are as given for Eq. (2.4).

The value of $(\mu_{en}/\rho)_{E,Z}$ at a point P is not only characteristic of the atomic number Z of the material present there [as is the case for $(\mu_{tr}/\rho)_{E,Z}$], but is also dependent to some degree upon the material present along the tracks of the electrons which originate at P. This is because radiative energy losses by electrons are greater in higher-Z materials, for which K_r is larger and K_c correspondingly less.

$(\mu_{en}/\rho)_{E,Z}$ is close to $(\mu_{tr}/\rho)_{E,Z}$ in value for low Z and E where radiative losses are small; Table 2.1 lists the percentage by which $(\mu_{en}/\rho)_{E,Z}$ is less than $(\mu_{tr}/\rho)_{E,Z}$ (and K_c less than K) for a few sample cases.

TABLE 2.1

γ-ray Energy (MeV)	100 $(\mu_{tr} - \mu_{en})/\mu_{tr}$		
	$Z = 6$	29	82
0.1	0	0	0
1.0	0	1.1	4.8
10	3.5	13.3	26

ABSORBED DOSE

The absorbed dose is relevant to all types of ionizing radiation fields, whether directly or indirectly ionizing, as well as to any ionizing radiation source distributed within the absorbing medium.

Definition

The absorbed dose D can best be defined in terms of the related stochastic quantity *energy imparted* ϵ (ICRU, 1980). The energy imparted by ionizing radiation to matter of mass m in a finite volume V is defined as

$$\epsilon = (R_{in})_u - (R_{out})_u + (R_{in})_c - (R_{out})_c + \sum Q \tag{2.17}$$

where $(R_{in})_u$ and ΣQ are defined the same as for Eq. (2.1), $(R_{out})_u$ is the radiant energy of all the uncharged radiation leaving V, $(R_{in})_c$ is the radiant energy of the charged particles entering V, and $(R_{out})_c$ is the radiant energy of the charged particles leaving V.

We can now define the absorbed dose D at any point P in V as

$$D = \frac{d\epsilon}{dm} \tag{2.18}$$

where ϵ is now the expectation value of the energy imparted in the finite volume V during some time interval, $d\epsilon$ is that for an infinitesimal volume dv at point P, and dm is the mass in dv.

Thus the absorbed dose D is the expectation value of the energy imparted to matter per unit mass at a point. The dimensions and units of absorbed dose are the same as those used for K. The average value $\bar{D}$ of the absorbed dose throughout a volume containing mass m is $(\epsilon)_\epsilon/m$. $(\epsilon)_\epsilon = \bar{D}m$ is also called the *integral dose*, expressed in units of g rad or joules.

It should be recognized that D represents the energy per unit mass which remains in the matter at P to produce any effects attributable to the radiation. Some kinds of effects are proportional to D, while others depend on D in a more complicated way. Nevertheless, if $D = 0$ there can be no radiation effect. Consequently, the absorbed dose is the most important quantity in radiological physics.

It is not possible to write an equation relating the absorbed dose directly to the fluence or energy fluence of a field of indirectly ionizing radiation, as was done for the kerma in Eqs. (2.4) and (2.8) and for collision kerma in Eq. (2.13). The absorbed dose is not directly related to such a field, being deposited by the resulting secondary charged-particles.

COMPARATIVE EXAMPLE OF ENERGY IMPARTED, ENERGY TRANSFERRED, AND NET ENERGY TRANSFERRED

To see how these quantities can be applied, consider Fig. 2.1*a*. Photon $h\nu_1$ is shown entering volume V, and undergoing a Compton interaction which produces scattered photon $h\nu_2$ and an electron with kinetic energy T. The electron is assumed to produce one bremsstrahlung x-ray ($h\nu_3$) before leaving V with remaining energy T'. It then produces another x-ray ($h\nu_4$). In this example the energy imparted, energy transferred, and net energy transferred in V are, respectively,

$$\epsilon = h\nu_1 - (h\nu_2 + h\nu_3 + T') + 0$$

$$\epsilon_{tr} = h\nu_1 - h\nu_2 + 0 = T$$

$$\epsilon_{tr}^{n} = h\nu_1 - h\nu_2 - (h\nu_3 + h\nu_4) + 0$$

$$= T - (h\nu_3 + h\nu_4)$$

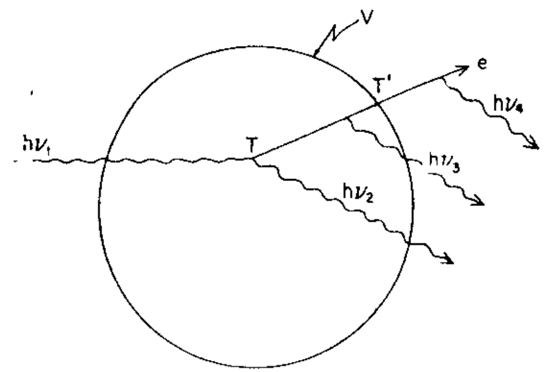

FIGURE 2.1*a*. Illustration of the concepts of energy imparted, energy transferred, and net energy transferred for the case of a Compton interaction followed by bremsstrahlung emission (Attix, 1983).

EXPOSURE

Exposure is the third of the important fundamental nonstochastic quantities with which we are concerned in radiological physics. It is historically the oldest of the three, and in earlier times (before 1962) was known as "exposure dose"; still earlier (before 1956), it had no name but was merely the quantity that was measured in terms of the roentgen (R) unit, which had been defined by the ICRU in 1928. By convention exposure is defined only for x-ray and γ-ray photons.

Definition

Exposure is symbolized by X, and is defined by the ICRU (1980) as "the quotient of dQ by dm, where the value of dQ is the absolute value of the total charge of the ions of one sign produced in air when all the electrons (negatrons and positrons) liberated by photons in air of mass dm are completely stopped in air." Thus

$$X = \frac{dQ}{dm} \tag{2.20}$$

In a note of clarification the ICRU also points out that "the ionization arising from the absorption of bremsstrahlung emitted by the electrons is not to be included in dQ."

The exposure X is the ionization equivalent of the collision kerma K_c in air, for x- and γ-rays.

Definition of $\overline{W}$

We must define precisely what is meant by "ionization equivalent" in the above statement of exposure. Here we must introduce a conversion factor symbolized by $\overline{W}$, *the mean energy expended in a gas per ion pair formed.*

$\overline{W}$ is usually expressed in units of eV per ion pair, and the best current value for x and γ rays in dry air is 33.97 eV/i.p. (Boutillon and Perroche, 1985). By dividing $\overline{W}$ by the charge of the electron in coulombs (noting that only the ions of *either* sign, not both, are counted in the definition of exposure) and converting the energy from electron volts to joules, one obtains $\overline{W}$ in a form that is more convenient for relating $(K_c)_{air}$ and X:

$$\frac{\overline{W}_{air}}{e} = \frac{33.97 \text{ eV/i.p. (or electron)}}{1.602 \times 10^{-19} \text{C/electron}} \times 1.602 \times 10^{-19} \text{J/eV} = 33.97 \text{ J/C} \tag{2.22}$$

We see that the conversion constants cancel each other so as to give $\overline{W}/e$ in J/C the same *numerical* value as $\overline{W}$ has in eV/i.p., which is a convenience. Moreover $\overline{W}$ may be regarded as a *constant* for each gas, independent of photon energy, for x- and γ-ray energies above a few keV.

Relation of Exposure to Energy Fluence

It is now possible to describe specifically what is meant by "ionization equivalent" in the statement of exposure at the end of Section III.A. Referring to Eq. (2.13), we can write that the exposure at a point due to an energy fluence Ψ of monoenergetic photons of energy E is given by

$$X = \Psi \cdot \left(\frac{\mu_{en}}{\rho}\right)_{E,air} \left(\frac{e}{\overline{W}}\right)_{air} = (K_c)_{air}\left(\frac{e}{\overline{W}}\right)_{air} = (K_c)_{air}/33.97 \tag{2.23}$$

where Ψ is most conveniently expressed in J/m^2,
$(\mu_{en}/\rho)_{E,air}$ is in m^2/kg,
K_c is in J/kg,
$(e/\overline{W})_{air}$ = (1/33.97) C/J, and
X is the exposure in C/kg.

NOTE: $K = K_c/(1-g)$
$= 8.76(-3)X/(1-g)$
where X is in R.

The roentgen (R) is the customary and more commonly encountered unit of exposure. It is defined as the exposure that produces, in air, one esu of charge of either sign per 0.001293 g of air (i.e., the mass contained in 1 cm^3 at 760 Torr, 0°C) irradiated by the photons. Thus

$$1 \text{ R} = \frac{1 \text{ esu}}{0.001293\text{g}} \times \frac{1\text{C}}{2.998 \times 10^9 \text{ esu}} \times \frac{10^3\text{g}}{1 \text{ kg}} = 2.580 \times 10^{-4} \text{ C/kg} \tag{2.24}$$

serves as a conversion factor from R to C/kg. That is

$$X\text{ (C/kg)} = 2.58 \times 10^{-4}\, X\text{ (R)}$$
$$X\text{ (R)} = 3876\, X\text{ (C/kg)} \tag{2.25}$$

Significance of Exposure

Exposure (and its rate) provides a convenient and useful means of characterizing an x- or γ-ray field, for the following reasons:

1. The energy fluence Ψ is proportional to the exposure X for any given photon energy [see Eq. (2.23)] or spectrum [Eq. (2.26)].
2. The mixture of elements in air is sufficiently similar in "effective atomic number" to that in soft biological tissue (i.e., muscle) to make air an approximately "tissue-equivalent" material with respect to x- or γ-ray energy absorption. Thus if one is interested in the effects of such radiations in tissue, air may be substituted as a reference medium in a measuring instrument.
3. Because of the approximate tissue equivalence of air noted in item 2, the value of the collision kerma K_c in muscle, per unit of exposure X, is nearly independent of photon energy. This follows from the fact that for a given energy fluence Ψ of photons of energy E, the exposure X is proportional to $(\mu_{en}/\rho)_{E,air}$, while K_c in muscle is proportional to $(\mu_{en}/\rho)_{E,musc}$ [see Eqs. (2.13) and (2.23), and $(\mu_{en}/\rho)_{E,musc}/(\mu_{en}/\rho)_{E,air}$ is nearly constant (1.07 ± 3% total spread) vs. E over the range 4 keV–10 MeV, as shown in Fig. 2.2*a*. That figure also shows corresponding ratios of $(\mu_{en}/\rho)_{E,Z}$ for water/air.
relative to air. Water/air (1.07 ± 5%) is nearly as constant as muscle/air, but both acrylic plastic and compact bone show larger differences from air, especially below 0.1 MeV, due to the *photoelectric effect*, which will be discussed in Chapter 7.
4. One can characterize an x-ray field at a point by means of a statement of exposure or exposure rate *regardless of whether there is air actually located at the point in question*. The statement that "the exposure at point P is X" simply means that the photon energy fluence Ψ [or its spectrum Ψ′(E)] at the point is such that Eq. (2.23) [or (2.26)] would give the stated value of X. Similar remarks apply also to the kerma K or collision kerma K_c, except that the reference medium is not necessarily air, and must therefore be specified.

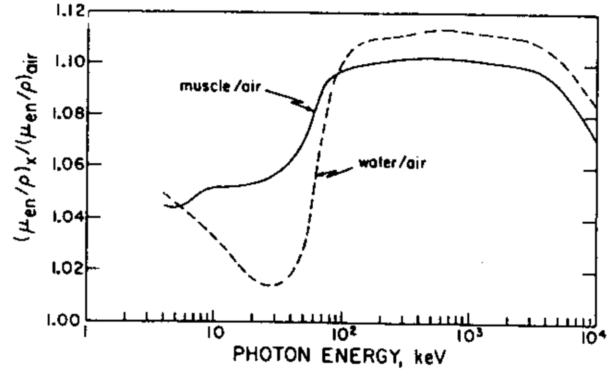

FIGURE 2.2*a*. Ratio of mass energy-absorption coefficients for muscle and water relative to air. [Based on data of Hubble, as given by Evans (1968) for $h\nu$ > 0.15 MeV, and by Greening, (1972) for $h\nu \leq$ 0.15 MeV.]

CHARGED-PARTICLE EQUILIBRIUM

Charged particle equilibrium (CPE) exists for the volume v if each charged particle of a given type and energy leaving v is replaced by an identical particle of the same energy entering, in terms of expectation values

This is further demonstrated in Fig. 4.3 for the simplified case of straight charged-particle tracks, all emitted at angle θ with respect to the monodirectional primary rays. Consider first the track of charged particle e_1, generated by the total absorption of an indirectly ionizing ray at a point P_1 just inside the boundary of v. Particle e_1 crosses v and carries out of that volume a kinetic energy of, say $\frac{2}{3}$ of its original energy. A second identical interaction occurring at point P_2 generates charged particle e_2, which enters v with $\frac{2}{3}$ of its original energy, and leaves with $\frac{1}{3}$ of that energy. Likewise a third identical interaction at P_3 generates charged particle e_3, which enters v with $\frac{1}{3}$ of its original energy, and expends all of that energy in v. Thus CPE exists for the nonstochastic limit, and the total kinetic energy spent in v by the three particles equals that which e_1 alone would have spent if its entire track had remained inside of v.

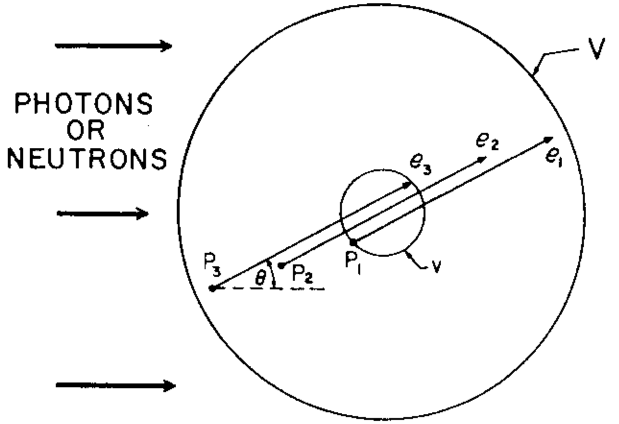

FIGURE 4.3. Charged-particle equilibrium conditions for an external source. The volume V contains a homogeneous medium, uniformly irradiated throughout by indirectly ionizing radiation (i.e., attenuation of the latter is assumed to be negligible). Secondary charged particles are thus produced uniformly throughout V, not necessarily isotropically, but with the same directional and energy distribution everywhere. If the minimum distance separating the boundaries of V and smaller internal volume v is greater than the maximum range of charged particles present, CPE exists in v. (Also see text.)

Reducing v to the infinitesimal volume dv, containing mass dm about a point of interest P, we can write :

$$\bar{\epsilon} = \bar{\epsilon}^n_{tr} ; \quad \frac{d\bar{\epsilon}}{dm} = \frac{d\bar{\epsilon}^n_{tr}}{dm} ; \text{ and hence } \boxed{D \overset{CPE}{=} K_c} \quad (4.6)$$

Eq. (4.6) is a very important relationship, as it equates the measurable quantity D with the calculable quantity K_c ($= \Psi \cdot \mu_{en}/\rho$).

Moreover, if the same photon energy fluence Ψ is present in media A and B having two different average energy absorption coefficients $(\overline{\mu_{en}/\rho})_A$ and $(\overline{\mu_{en}/\rho})_B$, the ratio of absorbed doses under CPE conditions in the two media will be given by

$$\boxed{\frac{D_A}{D_B} \overset{CPE}{=} \frac{(K_c)_A}{(K_c)_B} = \frac{(\overline{\mu_{en}/\rho})_A}{(\overline{\mu_{en}/\rho})_B}} \quad (4.7a)$$

where $(\overline{\mu_{en}/\rho})_{A,B}$ can be calculated for the photon fluence spectrum $\Psi'(E)$ from a formula corresponding to Eq. (2.5a). Likewise for the same neutron fluence $\Phi'(E)$ present in the two media,

$$\boxed{\frac{D_A}{D_B} \overset{CPE}{=} \frac{K_A}{K_B} = \frac{(\bar{F}_n)_A}{(\bar{F}_n)_B}} \quad (4.7b)$$

where the average kerma factors $(\bar{F}_n)_{A,B}$ can be calculated from Eq. (2.9a).

Note that D_A can differ from D_B in Eqs. (4.7a, b) either because the atomic compositions of A and B are different, or because the radiation spectra present are not identical.

CPE IN THE MEASUREMENT OF EXPOSURE

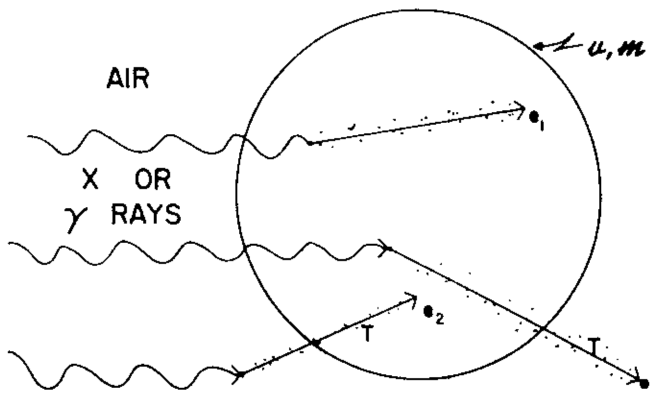

FIGURE 4.5. The role of CPE in the measurement of exposure X. The average exposure in the finite air volume v equals the total charge of either sign released in air by all electrons (e_1) that originate in v, divided by the air mass m in v. If CPE exists, each electron carrying an energy (say, T) out of v is compensated by another electron (e_2) carrying the same energy in. Thus the same ionization occurs in v as if all electrons e_1 remained there. The measurement of that charge divided by m is thus equivalent to a measurement of the average exposure in v. Radiative losses are assumed to escape from v, and any ionization they produce is not to be included in X.

RELATING ABSORBED DOSE TO EXPOSURE FOR x- AND γ-RAYS

It is sometimes useful to know how much absorbed dose would be deposited at some point in air as a result of an exposure X. The relationship is indeterminate in the absence of CPE,* since

$$\underset{\text{J/kg}}{D_{air}} \overset{\text{CPE}}{=} \underset{\text{J/kg}}{(K_c)_{air}} = \underset{\text{C/kg}}{X} \cdot \underset{33.97\ \text{J/C}}{\left(\frac{\overline{W}}{e}\right)_{air}} \tag{4.8}$$

where the first equality is valid only if CPE exists at the point in question.

If D_{air} is expressed in rads and X in roentgens, Eqs. (2.3) and (2.24) can be used for converting units to rewrite Eq. (4.8) as

$$0.01 D_{air} \overset{\text{CPE}}{=} 0.01\,(K_c)_{air} = 2.58 \times 10^{-4} \times 33.97X \tag{4.9}$$

or

$$D_{air} \overset{\text{CPE}}{=} (K_c)_{air} = 0.876X \tag{4.10}$$

where $(K_c)_{air}$ and D_{air} are in rads, and X in roentgens. It should be emphasized that Eq. (4.10) is valid only where X is the exposure at the point of interest in air, under CPE conditions.

CAUSES OF CPE FAILURE IN A FIELD OF INDIRECTLY IONIZING RADIATION

There are four basic causes for CPE failure in an indirectly ionizing field :

a. Inhomogeneity of atomic composition within volume V.
b. Inhomogeneity of density in V.
c. Non-uniformity of the field of indirectly ionizing radiation in V.
d. Presence of a non-homogeneous electric or magnetic field in V.

Some practical situations where CPE failure occurs are the following:

Proximity to a Source

If the volume V in Fig. 4.3 is too close to the source of the indirectly ionizing radiation, then the energy fluence will be significantly nonuniform within V, being larger on the side nearest the source, say on the left. Thus there will be more particles (e_3) produced at points like P_3 than particles e_1 at P_1, and more particles will enter v than leave it. CPE consequently fails for v.

High Energy Radiation

As the energy of indirectly ionizing radiation increases, the penetrating power of the secondary charged particles increases more rapidly than the penetrating power of the primary radiation. Table 4.1 expresses this for both γ-rays and neutrons, and shows that, for example, a 7% attenuation of γ-rays would occur in a water layer equal in thickness (≅ 5 cm) to the maximum range of secondary electrons produced by 10-MeV γ-rays. The neutron effect is much smaller (1%) at that energy, assuming hydrogen-recoil proton secondaries.

As a result of this phenomenon, the same type of CPE failure occurs as described above. That is, in Fig. 4.3, the number of charged particles generated at point P_3 is greater than at P_1, because of the attenuation of the indirectly ionizing radiation in penetrating from the depth of P_3 to that of P_1 in the medium. The degree of CPE failure becomes progressively larger for higher energies, as the table indicates.

Because of this kind of CPE failure, and the usual dependence of x- and γ-ray exposure measurements on the existence of CPE as noted in Section IV, exposure measurements have been conventionally assumed to be infeasible for photon energies above about 3 MeV. This limitation is sometimes erroneously interpreted as a failure of the definition of exposure itself; hence the exposure would simply not be defined for high-energy photons, or indeed for any other situation where CPE cannot be achieved. This is not the case however; only the *measurement* of exposure usually depends upon CPE. Moreover, even that constraint has a "loophole": If some other known relationship between D_{air} and $(K_c)_{air}$ can be attained under achievable conditions, and substituted for the simple equality that exists for CPE, exposure can still be measured, at least in principle (Attix, 1979). Such a relationship does exist for a situation known as TCPE, which will be considered in the next section.

TABLE 4.1. Approximate Attenuation[a] of Gamma Rays and Neutrons within a Layer of Water Equal to the Maximum Range of Secondary Charged Particles

Primary Radiation Energy (MeV)	Gamma-Ray Attenuation (%) in Maximum Electron Range	Neutron Attenuation (%) in Maximum Proton Range
0.1	0	0
1.0	1	0
10	7	1
30	15	4

[a] For "broad-beam" geometry, see Chapter 3, employing μ_{en} as an effective attenuation coefficient.

TRANSIENT CHARGED-PARTICLE EQUILIBRIUM (TCPE)

TCPE is said to exist at all points within a region in which D is proportional to K_c, the constant of proportionality being greater than unity. This relationship is illustrated in Figs. 4.7*a* and *b*. In both cases a broad* "clean" beam of indirectly ionizing radiation (i.e., unaccompanied by charged particles) is shown falling perpendicularly on a slab of material whose surface is supposed to be coincident with the ordinate axis of the figure. In Fig. 4.7*a* the kerma at the surface is shown as K_0, attenuating exponentially with depth as indicated by the K-curve. We assume in this case that radiative losses by the secondary charged particles are nil ($K_r \cong 0$), which would be strictly true only for incident neutrons. However, in carbon, water, air, and other low-Z media $K_r = K - K_c$ remains less than 1% of K for photons up to 3 MeV. Figure 4.7*b* shows the corresponding situation where K_r is significant and the radiative-loss photons are allowed to escape from the phantom.

The absorbed-dose curve is shown rising with increasing depth near the surface as the population of charged particles flowing toward the right is augmented by more and more interactions of indirectly ionizing rays. The dose curve reaches a maximum

*The beam diameter must be at least twice the maximum range of secondary charged particles, and points of interest must be distant from the edge of the beam by at least that range.

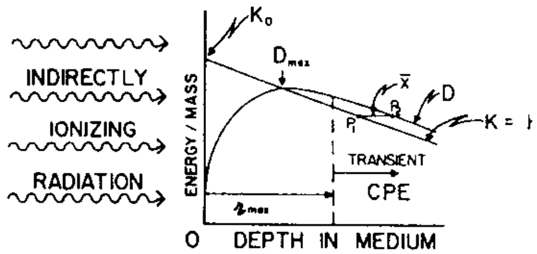

FIGURE 4.7a. Illustrating transient CPE for high-energy indirectly ionizing radiation incident from the left on a slab of material. Radiative losses (e.g., bremsstrahlung) are assumed to be absent, so $K_r = 0$ and $K = K_c$.

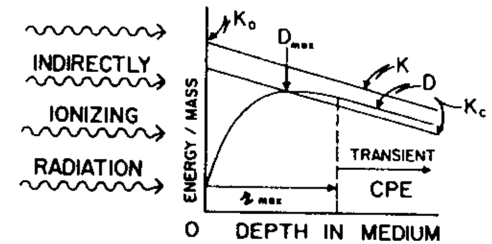

FIGURE 4.7b. Same as Fig. 4.7*a*, except that radiative losses are significant, so $K = K_c + K_r = K_c\,(\mu_{tr}/\mu_{en})$, and the resulting photons are assumed to escape from the phantom.

(D_{max}) at the depth where the rising slope due to buildup of charged particles is balanced by the descending slope due to attenuation of the indirectly ionizing radiation. For a "clean" beam of indirectly ionizing radiation D_{max} occurs at approximately the same depth as where the D-curve crosses the K_c-curve.* However, the presence of charged-particle "contamination" in the beam is often observed to shift the depth of D_{max} closer to the surface, where it no longer approximates the depth at which $D = K_c$ (Biggs and Ling, 1979). Thus one should not assume that $D = K_c$ at D_{max}.

At a somewhat greater depth r_{max}, equal to the maximum distance the secondary charged particles starting at the surface can penetrate in the direction of the incident rays, the D-curve becomes parallel to the K_c- and K-curves, although all may gradually change slope together with depth. D therefore becomes proportional to K_c, and we say that TCPE exists. Roesch (1958) suggested a relationship between the D- and K-curves for TCPE conditions, but he assumed that no radiative interactions occurred, and ignored scattered photons. In terms of present terminology we can write that:

$$\begin{aligned} D &\overset{\text{TCPE}}{=} K_c e^{\mu'\bar{x}} \\ &\overset{\text{TCPE}}{=} K_c \left(1 + \mu'\bar{x} + \frac{(\mu'\bar{x})^2}{2!} + \ldots\right) \\ &\overset{\text{TCPE}}{=} K_c\,(1 + \mu'\bar{x}) \end{aligned} \tag{4.11}$$

where D and K_c are for the same given depth, at which TCPE is required, μ' is the common slope of the D, K, and K_c curves at that depth; and $\bar{x}$ is the mean distance the secondary charged particles carry their kinetic energy in the direction of the primary rays while depositing it as dose. $\bar{x}$ is shown in Fig. 4.7*a* as the distance separating the depths of the points P_1 and P_2 where K_c and D have equal values. μ' and $\bar{x}$ of course must be expressed in consistent reciprocal units so their product is dimensionless.

The "TCPE" above the equal signs in Eq. (4.11) indicates that these equalities are valid only where transient CPE exists. The final relation in Eq. (4.11) should actually be an approximation, since only the first two terms of the series are employed. However, the higher-order terms are truly negligible in practical cases.

The above discussion applies equally well to Fig. 4.7*a* and *b*, where radiative losses are or are not negligible, respectively. The D-curve continues to bear the same relationship to the K_c-curve, but where $K_r \neq 0$ the K_c-curve moves down below the K-curve by the amount $K_r = [(\mu_{tr} - \mu_{en})/\mu_{tr}]K$. (We assume here that the radiative-loss photons escape from the medium.)

In conclusion, with respect to Eq. (4.11), this relationship in principle allows the relating of D and K_c where transient CPE conditions exist for high-energy indirectly ionizing radiations. However, a knowledge of $\bar{x}$ and the effective attenuation coefficient μ' is required for each case, so Eq. (4.11) is not as readily applicable as the simple equality of D and K_c that exists under CPE conditions.