INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

H4.SMR/638-3

**College on Medical Physics:
Imaging and Radiation Protection**

31 August - 18 September 1992

Cavity Theory

**F.H. Attix
University of Wisconsin-Madison
Dept. of Medical Physics
1530 Medical Sciences Center
Wisconsin, Madison
U.S.A.**

Lecture #3 at the College on Medical Physics
ICTP, Trieste, September 3, 1992

Cavity Theory

BRAGG–GRAY THEORY

If a fluence Φ of identical charged particles of kinetic energy T passes through an interface between two different media, g and w, as shown in Fig. 10.1*a*, then one can write for the absorbed dose on the g side of the boundary

$$D_g = \Phi\left[\left(\frac{dT}{\rho dx}\right)_{c,g}\right]_T \qquad (10.1)$$

and on the w side,

$$D_w = \Phi\left[\left(\frac{dT}{\rho dx}\right)_{c,w}\right]_T \qquad (10.2)$$

where $[(dT/\rho dx)_{c,g}]_T$ and $[(dT/\rho dx)_{c,w}]_T$ are the mass collision stopping powers of the two media, evaluated at energy T. Usually we may omit the brackets and subscript T, evaluation at an appropriate energy T being implied.

Assuming that the value of Φ is continuous across the interface (i.e., ignoring backscattering) one can write for the ratio of absorbed doses in the two media adjacent to their boundary

$$\frac{D_w}{D_g} = \frac{(dT/\rho dx)_{c,w}}{(dT/\rho dx)_{c,g}} \qquad (10.3)$$

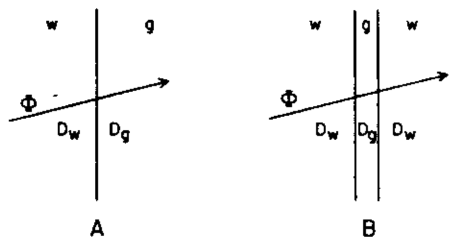

FIGURE 10.1. (*A*) A fluence Φ of charged particles is shown crossing an interface between media w and g. Assuming Φ to be continuous across the boundary, the dose ratio D_w/D_g equals the corresponding ratio of mass collision stopping powers. (*B*) A fluence Φ of charged particles passes through a thin layer of medium g sandwiched between regions containing medium w. Assuming Φ to be continuous across layer g and both interfaces, the dose ratio D_w/D_g is again equal to the corresponding ratio of mass collision stopping powers.

W. H. Bragg (1910) and L. H. Gray (1929, 1936) applied this equation to the problem of relating the absorbed dose in a probe inserted in a medium to that in the medium itself. Gray in particular identified the probe as a gas-filled cavity, whence the name "cavity theory". The simplest such theory is called the Bragg–Gray (B–G) theory, and its mathematical statement, referred to as the *Bragg-Gray relation*, will be developed next.

Suppose that a region of otherwise homogeneous medium w, undergoing irradiation, contains a thin layer or "cavity" filled with another medium g, as in Fig. 10.1*b*. *The thickness of the g-layer is assumed to be so small in comparison with the range of the charged particles striking it that its presence does not perturb the charged-particle field.* This assumption is often referred to as a "Bragg–Gray condition". It depends on the scattering properties of w and g being sufficiently similar that the mean path length (g/cm^2) followed by particles in traversing the thin g-layer is practically identical to its value if g were replaced by a layer of w having the same mass thickness. Similarity of backscattering at w–g, g–w, and w–w interfaces is also implied.

For heavy charged particles (either primary, or secondary to a neutron field), which undergo little scattering, this B–G condition is not seriously challenged so long as the cavity is very small in comparison with the range of the particles. However, for electrons even such a small cavity may be significantly perturbing unless the medium g is sufficiently close to w in atomic number.

Bragg–Gray cavity theory can be applied whether the field of charged particles enters from outside the vicinity of the cavity, as in the case of a beam of high-energy charged particles, or is generated in medium w through interactions by indirectly ionizing radiation. In the latter case it is also assumed that no such interactions occur in g. All charged particles in the B–G theory must originate elsewhere than in the cavity. Moreover charged particles entering the cavity are assumed not to stop in it.

A second B–G condition, incorporating these ideas, can be written as follows: *The absorbed dose in the cavity is assumed to be deposited entirely by the charged particles crossing it.* This condition tends to be more difficult to satisfy for neutron fields than for photons, especially if the cavity gas is hydrogenous, thus having a large neutron-interaction cross section. The heavy secondary charged particles (protons, α-particles, and recoiling nuclei) also generally have shorter ranges than the secondary electrons that result from interactions by photons of quantum energies comparable to the neutron kinetic energies. Thus we see that the first B–G condition is the more difficult of the two to satisfy for photons and electrons, while the second B–G condition is the more difficult to satisfy for neutrons.

Under the terms of the two B-G conditions, the ratio of absorbed doses in the adjacent medium w to that in the cavity g is given by Eq. (10.3) for each monoenergetic component of the spectrum of charged particles crossing g. For a differential energy distribution Φ_T (particles per cm^2 MeV) the appropriate average mass collision stopping power in the cavity medium g is

$$_m\bar{S}_g \equiv \frac{\int_0^{T_{max}} \Phi_T \left(\frac{dT}{\rho dx}\right)_{c,g} dT}{\int_0^{T_{max}} \Phi_T \, dT}$$

$$= \frac{1}{\Phi} \int_0^{T_{max}} \Phi_T \left(\frac{dT}{\rho dx}\right)_{c,g} dT = \frac{D_g}{\Phi} \tag{10.4}$$

and likewise, for a thin layer of wall material w that may be inserted in place of g,

$$_m\bar{S}_w \equiv \frac{\int_0^{T_{max}} \Phi_T \left(\frac{dT}{\rho dx}\right)_{c,w} dT}{\int_0^{T_{max}} \Phi_T \, dT}$$

$$= \frac{1}{\Phi} \int_0^{T_{max}} \Phi_T \left(\frac{dT}{\rho dx}\right)_{c,w} dT = \frac{D_w}{\Phi} \tag{10.5}$$

Combining Eqs. (10.4) and (10.5) gives for the ratio of absorbed dose in w to that in g, which is the B-G relation in terms of absorbed dose in the cavity:

$$\boxed{\frac{D_w}{D_g} = \frac{_m\bar{S}_w}{_m\bar{S}_g} \equiv {_m\bar{S}_g^w}} \tag{10.6}$$

If the medium g occupying the cavity is a gas in which a charge Q (of either sign) is produced by the radiation, D_g can be expressed (in grays) in terms of that charge as

$$D_g = \frac{Q}{m}\left(\frac{\overline{W}}{e}\right)_g \tag{10.7}$$

where Q is expressed in coulombs, m is the mass (kg) of gas in which Q is produced, and $(\overline{W}/e)_g$ is the mean energy spent per unit charge produced (J/C; see Chapter 2, Section V.B, and Chapter 12, Section VI). By substituting Eq. (10.7) into Eq. (10.6), we obtain the B-G relation expressed in terms of cavity ionization:

$$\boxed{D_w = \frac{Q}{m}\left(\frac{\overline{W}}{e}\right)_g \cdot {_m\bar{S}_g^w}} \tag{10.8}$$

This equation allows one to calculate the absorbed dose in the medium immediately surrounding a B-G cavity, on the basis of the charge produced in the cavity gas, provided that the appropriate values of m, $(\overline{W}/e)_g$, and $_m\bar{S}_g^w$ are known.

Note that Q is generally greater than the charge Q' *collected* from the ion chamber, because of ionic recombination, requiring a correction.

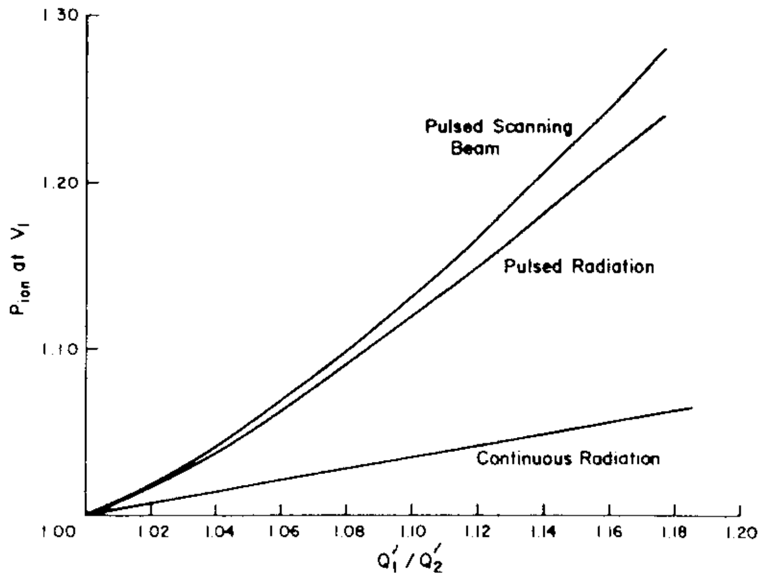

FIGURE 12.25. Ionization-recombination correction factors (P_{ion}) = Q/Q_1' for continuous radiation, pulsed radiation, and pulsed scanning electron beams. P_{ion}-values apply to the chamber potential P_1 when Q_1'/Q_2' is the collected charge ratio for potentials P_1/P_2 = 2. (AAPM, 1983. Reproduced with permission from R. J. Schulz and The American Institute of Physics.)

B-G theory also may be applied to solid- or liquid-filled "cavities" g, using Eq. (10.6) to calculate D_w from a value of D_g measured in some way. For example, medium g might be a thin plastic film that gradually darkens as a known function of absorbed dose. Thus D_g could be determined after an exposure by means of a densitometer measurement. However, it is relatively difficult to satisfy the B-G conditions with condensed cavity media, since the cavity thickness must be only ~0.001 times as great as for a gas-filled cavity at 1 atm to obtain a comparable mass thickness of g. Thus a 1-mm gas-filled cavity is comparable to a 1-μm layer of a condensed medium.

So long as $_m\bar{S}_g^w$ is evaluated for the charged-particle spectrum Φ_T that crosses the cavity, as in Eqs. (10.4)-(10.6), the B-G relation requires neither charged-particle equilibrium (CPE) nor a homogeneous field of radiation. However, the charged-particle fluence Φ_T must be the same in the cavity and in the medium w at the place where D_w is to be determined.

SPENCER CAVITY THEORY

By the 1950s experiments had shown that the B-G cavity theory did not accurately predict the ionization in air-filled cavities, especially with walls of high atomic number. At the National Bureau of Standards, the preliminary results of Attix, De La Vergne, and Ritz (1958) suggested to Spencer that δ-ray production had to be taken into account (Spencer and Attix, 1955).

In examining the inadequacy of the B-G theory it should be remembered that the stopping-power ratio in Eq. (10.8) is evaluated under the assumption of the CSDA, upon which collision stopping powers are based. Actually δ rays (energetic electrons) are produced in knock-on electron–electron collisions, and these δ rays join the flux of electrons crossing the cavity. Their presence enhances the equilibrium spectrum at the lower electron energies, since the kinetic energy of an electron undergoing a knock-on collision is immediately shared with the electron it hits.

Spencer's goal in modifying the B-G cavity theory was not only to incorporate the δ-ray effect, but to do it in such a way that the observed variation of ionization density with cavity size could be accounted for, at least for cavities small enough to satisfy the B-G conditions.

The cavity, containing medium g (typically air), is characterized with respect to size by a parameter Δ, which was somewhat arbitrarily taken to be the mean energy of electrons having projected ranges just large enough to cross the cavity.

The equilibrium spectrum, $\Phi_T^{e,\delta}$, of electrons (including δ-rays) generated in the surrounding medium is arbitrarily divided into two components in Spencer's schematization:

a. The "fast" group: electrons that have energies $T \geq \Delta$, and that can therefore transport energy. In particular they have enough energy to cross the cavity if they strike it.

b. The "slow" group: electrons with $T < \Delta$. These are assumed to have *zero range*, i.e., to drop their energy "on the spot" where their kinetic energy falls below Δ. Hence they are assumed not to be able to enter the cavity, nor to transport energy.

Table 10.2 gives values of D_g/D_w calculated by Spencer for air cavities having Δ-values from 2.5 to 82 keV, in media of $Z = 6$ to 82 containing distributed monoenergetic electron sources of $T_0 = 1308$, 654, and 327 keV. For comparison the final column (also calculated by Spencer) provides the corresponding B-G-theory values, which can be seen to agree most closely with the Spencer theory for the larger cavity sizes. The difference from unity generally increases with decreasing cavity size, because of the influence of more and more δ-rays.

TABLE 10.2 Values of D_g/D_w Calculated for Air Cavities by Spencer[a] from Spencer Cavity Theory, vs. Bragg–Gray Theory

			D_g/D_w						
			Spencer						
Wall Medium	T_0 (keV)	Δ (keV) = Range[b] (cm) =	2.5 0.015	5.1 0.051	10.2 0.19	20.4 0.64	40.9 2.2	81.8 7.2	Bragg-Gray
C	1308		1.001	1.002	1.003	1.004	1.004	1.005	1.005
	654		0.990	0.991	0.992	0.992	0.993	0.994	0.994
	327		0.985	0.986	0.987	0.988	0.988	0.989	0.989
Al	1308		1.162	1.151	1.141	1.134	1.128	1.123	1.117
	654		1.169	1.155	1.145	1.137	1.131	1.126	1.125
	327		1.175	1.161	1.151	1.143	1.136	1.130	1.134
Cu	1308		1.456	1.412	1.381	1.359	1.340	1.327	1.312
	654		1.468	1.421	1.388	1.363	1.345	1.329	1.327
	327		1.485	1.436	1.400	1.375	1.354	1.337	1.353
Sn	1308		1.786	1.694	1.634	1.592	1.559	1.535	1.508
	654		1.822	1.723	1.659	1.613	1.580	1.551	1.547
	327		1.861	1.756	1.687	1.640	1.602	1.571	1.595
Pb	1308		—	2.054	1.940	1.865	1.811	1.770	1.730
	654		—	2.104	1.985	1.904	1.848	1.801	1.796
	327		—	2.161	2.030	1.946	1.881	1.832	1.876

[a] Personal communication. These data replace those given in Table II of Spencer and Attix (1955), which did not take account of the polarization effect (see Chapter 8, Section III.E).
[b] In air.

BURLIN CAVITY THEORY

Burlin (1966, 1968) recognized the need for a γ-ray cavity theory that would bridge the gap between small cavities for which the B-G or Spencer theory could be applied, and very large cavities for which the wall influence is negligible.

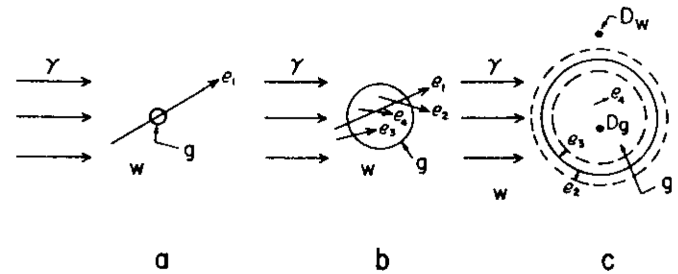

FIGURE 10.5. The cavity-size transition in Burlin theory (see text).

Figure 10.5 illustrates this cavity-size transition. A region of homogeneous medium w is shown uniformly irradiated by γ-rays. Three cavities containing medium g are considered: small (satisfying the B–G conditions), intermediate, and large compared to the ranges of the secondary electrons present. Using the terminology of Caswell (1966), the absorbed dose in the small cavity (Fig. 10.5*a*) is delivered almost entirely by "crossers", i.e., secondary electrons completely traversing the cavity, such as e_1. The average absorbed dose in the intermediate-sized cavity in Fig. 10.5*b* is delivered partly by crossers, but also by "starters" like e_2 that originate in the cavity and stop in the wall, "stoppers" (e_3) starting in the wall and terminating in the cavity, and "insiders" (e_4) that start and stop within the cavity. Note that the dose in this case will in general not be uniform throughout the cavity, but may depend on the distance inward from the wall.

If a cavity is made large enough so that the maximum-range stoppers from the wall can affect the dose in only a negligibly thin layer of cavity medium (the thickness of which is exaggerated in Fig. 10.5*c*), then the average dose in the cavity is practically all delivered by the insiders, e_4, which are generated by γ-ray interactions in the cavity medium g itself.

The Burlin cavity relation can be written in its simplest form as follows:

$$\frac{\overline{D}_g}{D_w} = d \cdot {}_m\overline{S}_w^g + (1 - d)\left(\frac{\overline{\mu_{en}}}{\rho}\right)_w^g \tag{10.41}$$

where d is a parameter related to the cavity size that approaches unity for small cavities and zero for large ones, thus providing the proper values of Eq. (10.41) for the limiting cases; $\overline{D}_g$ is the average absorbed dose in the cavity medium g; $D_w = (K_c)_w$ is the absorbed dose in medium w under CPE conditions (i.e., not within electron range of the cavity); ${}_m\overline{S}_w^g$ is the mean ratio of mass collision stopping powers for g and w, obtained either on the basis of the B-G or Spencer theory; and $(\overline{\mu_{en}/\rho})_w^g$ is the mean ratio of the mass energy-absorption coefficients for g and w.

Burlin expressed d as the average value of Φ_w/Φ_w^e in the cavity (see Fig. 10.6):

$$d \equiv \frac{\overline{\Phi}_w}{\Phi_w^e} = \frac{\int_0^L \Phi_w^e\, e^{-\beta l}\, dl}{\int_0^L \Phi_w^e\, dl} = \frac{1 - e^{-\beta L}}{\beta L} \tag{10.42}$$

where l is the distance (cm) of any point in the cavity from the wall, along a mean chord of length L, which is taken as being equal to *four times the cavity volume V divided by its surface area S*, for convex cavities and diffuse (i.e., isotropic) electron fields.

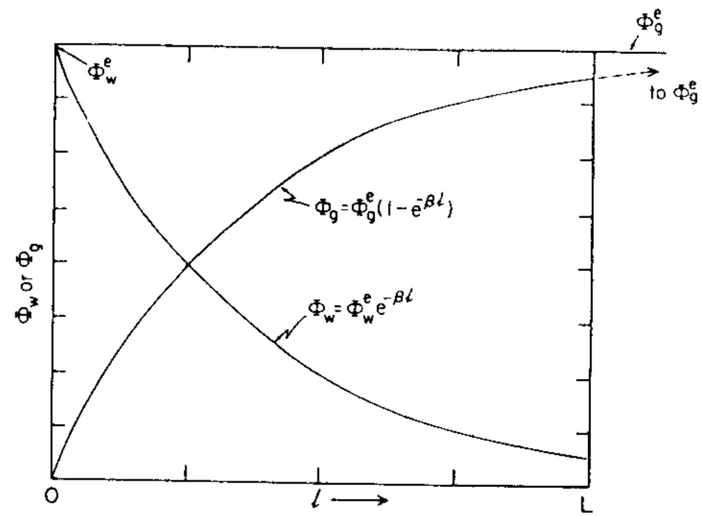

FIGURE 10.6. Illustration of the exponential-decay and -buildup assumption in the Burlin cavity theory. The equilibrium wall fluence of electrons, Φ_w^e, is shown decaying exponentially as they progress into a homogeneous cavity for which the wall w and cavity g media are assumed to be identical. The electrons under consideration are only those flowing from left to right. The buildup of the cavity-generated electron fluence Φ_g follows a complementary exponential, asymptotically approaching its equilibrium value $\Phi_g^e = \Phi_w^e$.

In applications involving air-filled cavities Burlin (1966) evaluated β (cm^{-1}) from an empirical formula due to Loevinger:

$$\beta = \frac{16\rho}{(T_{max} - 0.036)^{1.4}} \tag{10.46}$$

where ρ is the air density (g/cm^3) and T_{max} is the maximum value of the starting energies T_0 of the β-rays in MeV.

The Burlin theory has been found to estimate the average dose in cavities fairly well over a wide range of sizes. It is particularly useful in relation to condensed-state dosimeters, which typically have dimensions that are comparable to the ranges of the electrons present. Ogunleye et al. (1980) measured the dose in stacks of LiF thermoluminescence dosimeters (TLDs), each 0.1 g/cm^2 thick, sandwiched between equilibrium-thickness walls of various media and irradiated perpendicularly by ^{60}Co γ-rays, as shown in Fig. 10.7. The data were normalized to the homogeneous case where the wall medium also consisted of solid LiF.

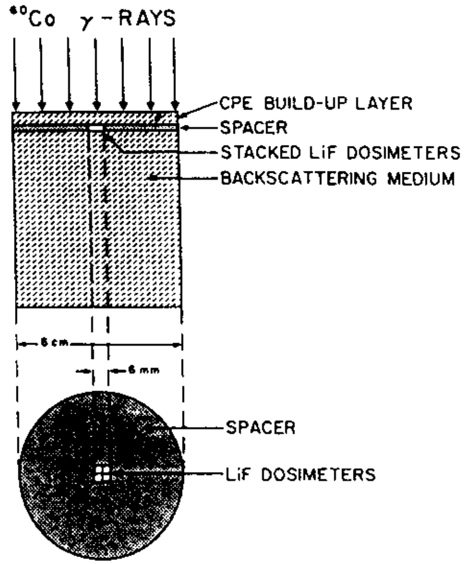

FIGURE 10.7. ^{60}Co γ-ray experiment to test the Burlin theory as applied to LiF TLD chips, each 0.38 × 3.18 × 3.18 mm^3, ρ = 2.64 g/cm^3, stacked four per layer in 1, 2, 3, 5, and 7 layers. The CPE buildup layer and backscattering medium were both made of the same wall material, either LiF, polystyrene, Al, Cu, or Pb. The spacer was adjusted to equal the TLD stack thickness, and for the results presented here was made of LiF to produce a semi-infinite one-dimensional cavity. (After Ogunleye, et al., 1980. Reproduced with permission of The Institute of Physics, U.K.)

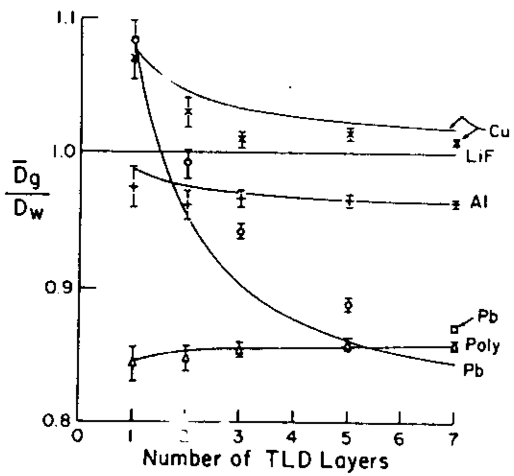

FIGURE 10.9. Comparison of the Burlin theory (solid curves) with the experiment referred to in Figs. 10.7 and 10.8. The application of the theory in this case, as described in the text, differs from that of Ogunleye et al. (1980).

Simple Dosimeter Model in Terms of Cavity Theory

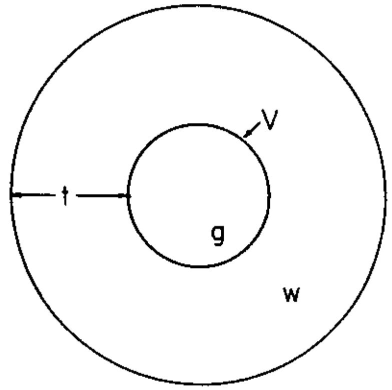

FIGURE 11.1. Schematic representation of a dosimeter as a sensitive volume V containing medium g, surrounded by a wall of medium w and thickness t.

A *dosimeter* can be defined generally as any device that is capable of providing a reading r that is a measure of the absorbed dose D_g deposited in its *sensitive volume* V by ionizing radiation. If the dose is not homogeneous throughout the sensitive volume, then r is a measure of some kind of mean value $\overline{D}_g$. Ideally r is proportional to D_g, and each volume element of V has equal influence on the value of r, in which case $\overline{D}_g$ is simply the average dose throughout V.

A dosimeter can generally be considered as consisting of a sensitive volume V filled with medium g, surrounded by a wall (or envelope, package, container, capsule, buffer layer, etc.) of another medium w having a thickness $t \geq 0$, as shown in Fig. 11.1. Its resemblance to a cavity and its surroundings is more than coincidental. A simple dosimeter can be treated in terms of cavity theory, the sensitive volume being identified as the "cavity", which may contain a gaseous, liquid, or solid medium g, depending on the type of dosimeter. Cavity theory provides one of the most useful means of interpretation of dosimeter readings, as will be seen in the following sections.

The dosimeter wall can serve a number of functions simultaneously, including:

- being a source of secondary charged particles that contribute to the dose in V, and provide charged-particle equilibrium (CPE) or transient charged-particle equilibrium (TCPE) in some cases,
- shielding V from charged particles that originate outside the wall,
- protecting V from "hostile" influences such as mechanical damage, dirt, humidity, light, electrostatic or RF fields, etc., that may alter the reading,
- serving as a container for a medium g that is a gas, liquid, or powder, and
- containing radiation filters to modify the energy dependence of the dosimeter.

GENERAL GUIDELINES ON THE INTERPRETATION OF DOSIMETER MEASUREMENTS

Ordinarily one is not interested in measuring the absorbed dose in a dosimeter's sensitive volume as an end in itself, but rather as a means of determining the dose (or a related quantity) for another medium in which direct measurements are not feasible. Interpretation of a dosimeter reading in terms of the desired quantity is the central problem in dosimetry, usually exceeding in difficulty the actual measurement. In some cases the dosimeter can be calibrated directly in terms of the desired quantity (e.g., exposure, or tissue dose), but such a calibration is generally energy-dependent unless the dosimeter closely simulates the reference material.

IMPORTANCE OF CPE OR TCPE

It may be recalled from Eqs. (2.13), (4.6), and (4.11) that under CPE and TCPE conditions, respectively,

$$D \overset{\text{CPE}}{=} K_c = \Psi \left(\frac{\mu_{en}}{\rho}\right) \tag{11.1}$$

and

$$D \overset{\text{TCPE}}{=} K_c(1 + \mu'\bar{x}) \equiv K_c\beta = \Psi \left(\frac{\mu_{en}}{\rho}\right) \beta \tag{11.2}$$

for photons. For neutrons, referring also to Eq. (2.8), one has the corresponding relationships:

$$D \overset{\text{CPE}}{=} K = \Phi F_n \tag{11.3}$$

Consider now a dosimeter with a wall of medium w, thick enough to exclude all charged particles generated elsewhere, and at least as thick as the maximum range of secondary charged particles generated in it by the photon or neutron field. The dosimeter reading r provides us with a measure of the dose D_g in the dosimeter's sensitive volume. If the latter volume is small enough to satisfy the B-G condition of nonperturbation of the charged-particle field, and assuming that the wall is uniformly irradiated, CPE exists in the wall near the cavity. B-G or Spencer cavity theory can be used to determine the dose D_w there from that (D_g) in the sensitive volume. Then Eq. (11.1) or (11.3) permits the calculation of Ψ or Φ for the primary field from the value of D_w. More importantly, the dose D_x in any other medium x replacing the dosimeter and given an identical irradiation under CPE conditions can be gotten from

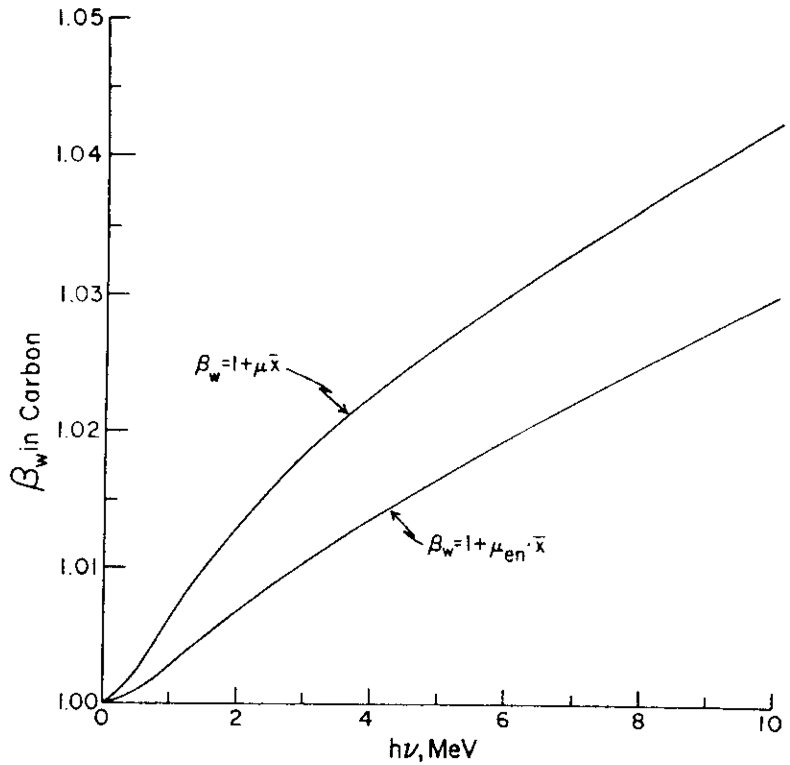

FIGURE 13.4. β_w in graphite, as a function of photon energy, from calculations of $\bar{x}$ by Roesch (1968) (see Attix, 1979). This $\bar{x}$ may be adequately approximated by 0.54 $\Re_{CSDA}$ for the average secondary-electron energy $\bar{T} = E_\gamma(\sigma_{en}/\sigma)$. The lower curve gives values of $\beta_w \cong 1 + \mu_{en}\bar{x}$; the upper curve is $\beta_w \cong 1 + \mu\bar{x}$. For chamber-wall applications, or at shallow depths in a phantom penetrated by a broad γ-ray beam, values of β_w approximate the lower limit. At greater depths in the phantom, β_w lies between the limiting values, tending to increase with increasing depth and with decreasing beam diameter. These results apply also to β_w for other low-Z media.

$$D_x \overset{\text{CPE}}{=} D_w \frac{\overline{(\mu_{en}/\rho)_x}}{\overline{(\mu_{en}/\rho)_w}} \quad \text{for photons} \tag{11.5}$$

or

$$D_x \overset{\text{CPE}}{=} D_w \frac{(\overline{F}_n)_x}{(\overline{F}_n)_w} \quad \text{for neutrons} \tag{11.6}$$

The exposure X (C/kg) for photons can in turn be derived from the absorbed dose D_{air} (for x = air) through this relation (from Eq. 4.8):

$$X \overset{\text{CPE}}{=} D_{air}\left(\frac{e}{\overline{W}}\right)_{air} = \frac{D_{air}}{33.97} \tag{11.7}$$

For higher-energy radiation ($h\nu \gtrsim 1$ MeV or $T_n \gtrsim 10$ MeV), where CPE fails but TCPE takes its place in dosimeters with walls of sufficient thickness, Eqs. (11.2) and (11.4) replace Eqs. (11.1) and (11.3), respectively. Relating D_w to Ψ or Φ then requires evaluation of the ratio $\beta = D/K_c$ for each case. However, the value of β is generally not much greater than unity (see Fig. 13.4) for radiation energies up to a few tens of MeV, and it is not strongly dependent on atomic number. Thus for media w and x not differing very greatly in Z, Eqs. (11.5) and (11.6) are still approximately valid. Equation (11.7) may be extended to higher energies, where TCPE exists, by dividing D_{air} by β.

If the dosimeter in question has too large a sensitive volume for the application of B-G theory, Burlin theory [Eq. (10.41)] can be substituted to calculate the equilibrium dose D_w in the wall medium at the point of interest, from the value of $\overline{D}_g$ given by the dosimeter reading. Then all of the preceding equations (11.1)–(11.7) are still operational. Note however that the sensitive-volume-size parameter d is required to be known in this case, and it may not follow exactly the simple forms suggested by Burlin and others, leading to uncertainty in dosimetry interpretation.

ADVANTAGES OF MEDIA MATCHING

There are clear advantages in matching a dosimeter to the medium of interest x, and also matching the media composing the wall (w) and sensitive volume (g) of the dosimeter to each other. The most obvious matching parameter is atomic composition, but the density state (gaseous vs. condensed) also influences the collision-stopping-power ratio of w to g for electrons by the polarization effect. More adaptable guidelines for media matching will be discussed in following subsections.

a. $w \cong g$

If the wall and sensitive-volume media of the dosimeter are identical with respect to composition and density, then $D_w = \overline{D}_g$ for all homogeneous irradiations.

To the extent that w and g are at least made similar to each other with respect to composition and density state (i.e., gaseous vs. condensed), the influence of cavity theory is kept minimal. Consequently the requirement for information about the radiation energy spectrum to allow accurate evaluation of the terms in, for example, the Burlin cavity relation (10.41) is lessened, allowing the use of convenient approximations without significant loss of accuracy in determining D_w from $\overline{D}_g$.

b. $w = g \cong x$

If a variety of homogeneous dosimeters ($w = g$) were available, it would be advantageous to choose one made of a material that matched the medium of interest, x, as closely as possible. If they were identical, then the dosimeter would be truly representative of that medium with respect to radiation interactions, and $D_x = D_w$ ($=\overline{D}_g$) in Eqs. (11.5) and (11.6). To the degree that the dosimeter simulates the medium x, the calculation of D_x through the application of one of those equations is again simplified by reducing the need for spectral information.

Unfortunately, dosimeters are only available in a finite variety, and there are other selection constraints besides composition that limit even further the choice of a dosimeter for a particular application. Cavity theories can be thought of as means of obtaining needed flexibility in matching dosimeters to media of interest, since they allow w to differ from g. For example, it may be easier to match only the wall w to the medium x, relying on cavity theory to calculate D_w from $\overline{D}_g$. Trying to match g to x is generally made more difficult by the additional dosimetric requirements imposed on the medium in the sensitive volume.

3. MEDIA MATCHING OF w AND g IN PHOTON DOSIMETERS

Since it is often infeasible in trying to devise a homogeneous dosimeter to make w and g actually similar in atomic composition, it will be helpful to point out the important parameters involved. The Burlin cavity relation (10.41) is useful in this connection:

$$\frac{\overline{D}_g}{D_w} = d \cdot {}_m\overline{S}^g_w + (1-d)\overline{\left(\frac{\mu_{en}}{\rho}\right)}^g_w$$

It can be seen that the average dose $\overline{D}_g$ in the dosimeter's sensitive volume will be equal to the equilibrium dose D_w in the wall medium if

$${}_m\overline{S}^g_w = \overline{\left(\frac{\mu_{en}}{\rho}\right)}^g_w = 1 \tag{11.8}$$

independent of the value of d, which varies with the size of the dosimeter's sensitive volume.

In other words, the matching criteria between the media w and g call for the respective matching of their mass collision stopping powers and their mass energy-absorption coefficients. When those parameters are each the same for the wall as for the medium in the sensitive volume V, the need to evaluate d in the Burlin equation (10.41) is eliminated, providing a substantial simplification.

Moreover, since $\overline{D}_g$ then remains equal to D_w, which is the CPE dose value in the wall medium at the point of interest, Eq. (11.5) may be used to calculate the dose in medium x from the observed value of D_w measured by the dosimeter.

The requirements given in Eq. (11.8) are still quite stringent and difficult to achieve, especially for a material w that is not identical to g in atomic composition. A more flexible and practicable matching relationship between media w and g is the following:

$$ {}_m\overline{S}_w^g = \left(\frac{\overline{\mu_{en}}}{\rho}\right)_w^g = n \tag{11.9} $$

where n is some constant, no longer required to be unity. In other words, the ratio of mass collision stopping powers for g/w is only required to be equal to the corresponding ratio of mass energy absorption coefficients. Under these conditions the Burlin equation simplifies to

$$ \frac{\overline{D}_g}{D_w} = dn + (1 - d)n = n \tag{11.10} $$

irrespective of the value of d, as was the case for Eq. (11.8). However, now we see that $\overline{D}_g$ is n times as large as D_w.

To understand how the value of D_g depends on n, we write the following Burlin equations for two dosimeters containing the same sensitive volume medium g, and given the same photon irradiation. One is enclosed in wall w_1 and obeys Eq. (11.8), while the other is enclosed in w_2 and obeys Eq. (11.9):

$$ \frac{\overline{D}_{g1}}{D_{w1}} = d \cdot {}_m\overline{S}_{w1}^g + (1 - d)\left(\frac{\overline{\mu_{en}}}{\rho}\right)_{w1}^g = 1 \tag{11.11} $$

$$ \frac{\overline{D}_{g2}}{D_{w2}} = d \cdot {}_m\overline{S}_{w2}^g + (1 - d)\left(\frac{\overline{\mu_{en}}}{\rho}\right)_{w2}^g = n \tag{11.12} $$

But D_{w1} and D_{w2} are equilibrium absorbed doses in media w_1 and w_2, and are related by

$$ \frac{D_{w1}}{D_{w2}} \overset{CPE}{=} \left(\frac{\overline{\mu_{en}}}{\rho}\right)_{w2}^{w1} = n \tag{11.13} $$

the last equality having been derived from Eqs. (11.8) and (11.9).

Comparison with Eq. (11.12) shows that

$$ \overline{D}_{g2} = D_{w1} $$

and Eq. (11.11) then provides the equality

$$ \overline{D}_{g1} = \overline{D}_{g2} \tag{11.14} $$

This proves that the dose in the dosimeter's sensitive volume is independent of the value of n so long as Eq. (11.9) is satisfied. This is because, under these conditions, the equilibrium dose in the wall is inversely proportional to n, thus maintaining $\overline{D}_g$ constant. Thus the reading of the dosimeter gives a value of $\overline{D}_g$ that is the same as if the wall were perfectly matched to g.

The practical case to which this approach is relevant occurs where photons interact only by the Compton effect in g and w. Then μ_{en}/ρ is nearly proportional to the number of electrons per gram, $N_A Z/A$. To a first approximation so is the mass collision stopping power of the secondary electrons. Consequently Eq. (11.9) is approximately satisfied, with $n \cong (Z/A)_g/(Z/A)_w$.

Example 11.1. A dilute aqueous chemical dosimeter (assume = water) is enclosed in an equilibrium-thickness capsule of polystyrene and exposed to ^{60}Co γ-rays. Calculate the approximate ratio of $\overline{D}_g$ in this dosimeter to the dose (D_{water}) under CPE conditions in water, assuming $d = 1$ and $d = 0$.

Solution: $(\mu_{en}/\rho)_{water} = 0.0296$ cm^2/g; $(\mu_{en}/\rho)_{poly} = 0.0288$ cm^2/g. The average starting electron energy from the Compton effect is

$$ \overline{T}_0 = \frac{\sigma_{tr}}{\sigma} \cdot 1.25 \text{ MeV} = 0.588 \text{ MeV} $$

The average equilibrium-spectrum electron energy is approximately

$$ \overline{T} \cong \frac{\overline{T}_0}{2} = 0.3 \text{ MeV} $$

Thus

$$ \left(\frac{dT}{\rho dx}\right)_{water, 0.3 MeV} = 2.355 \text{ MeV cm}^2/\text{g} $$

$$ \left(\frac{dT}{\rho dx}\right)_{poly, 0.3 MeV} = 2.305 \text{ MeV cm}^2/\text{g} $$

$$\frac{\overline{D}_g}{D_{\text{poly}}} = d\,\frac{2.355}{2.305} + (1 - d)\,\frac{0.0296}{0.0288}$$

$$= 1.022\,d + 1.028\,(1 - d)$$

$$\frac{\overline{D}_g}{D_{\text{water}}} = \frac{\overline{D}_g}{D_{\text{poly}}} \cdot \frac{D_{\text{poly}}}{D_{\text{water}}} = \frac{\overline{D}_g}{D_{\text{poly}}} \left(\frac{\mu_{en}}{\rho}\right)^{\text{poly}}_{\text{water}}$$

$$= \left[d\,\frac{2.355}{2.305} + (1 - d)\,\frac{0.0296}{0.0288}\right] \frac{0.0288}{0.0296}$$

$$= d\,\frac{2.355}{2.305} \cdot \frac{0.0288}{0.0296} + (1 - d)$$

$$= 0.994d + (1 - d) = 1 - 0.006d$$

Thus we see that in this case $\overline{D}_g/D_{\text{water}}$ is equal to 0.994 when $d = 1$, rising to 1.000 for $d = 0$.

In this example polystyrene walls are seen to provide a close ($\leq 0.6\%$) match to the water in the dosimeter's sensitive volume.