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Imaging and Radiation Protection

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Physics of Radiological Imaging

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PHYSICS OF RADIOLOGICAL IMAGING

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ARGUMENTS DISCUSSED IN THESE TWO LESSENS:

1. BASIC EXPERIMENTAL X RAY PHYSICS: SOURCES AND DETECTORS
2. RADIOGRAPHIC IMAGING
3. ABSORPTION COEFFICIENTS
4. STATISTICAL CONSIDERATIONS
5. GAMMA (SENSITIVITY), BACKGROUND
6. SPATIAL RESOLUTION, CONTRAST AND CONTRAST RATIO
7. RESPONSE FUNCTION OF THE ABSORPTIOMETRY
8. DUAL ENERGY ABSORPTIOMETRY AND K-EDGE FLUOROSCOPY
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10. IMAGING USI'G FILM AND FILM-SCREENS
11. NOISE, AND SIGNAL TO NOISE RATIO
12. IMAGE INTENSIFIERS AND THE NEW POSITION SENSITIVE DETECTORS
13. MODULATION TRANSFER FUNCTION AND WINER SPECTRA

1. BASIC EXPERIMENTAL X RAY PHYSICS: SOURCES AND DETECTORS

In the recent years is growing the interest of the other parts of thee physical research for the x ray physics which is a traditional field of experimental physics having a strong influence in the applied physics and mainly in the medical physics. In some sense there is a flow

1.1 Sources:

1.1.1 The X ray Tube

One of the early applications of the x rays, in the first months after their discover by , was the production of radiographic images. The most interesting feature of these image resides in the possibility to see inner parts of the imaged objects. Many of us remember the first radiograph the image of the hand of the Röentgen's wife with the ring.

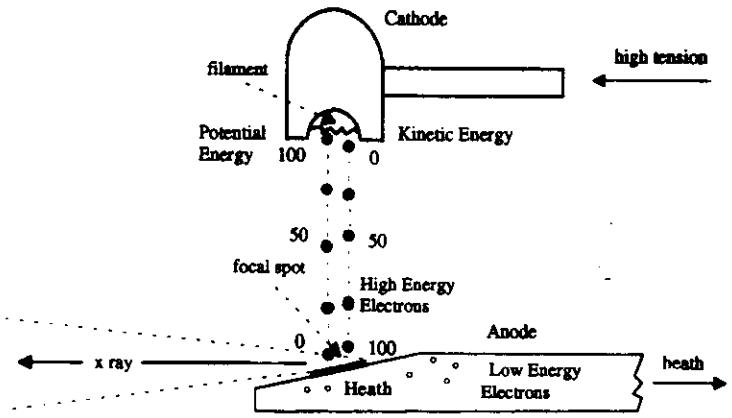
At that time the instrumentation in use consisted of a Crookes tube (the father of the modern x ray tube) and a photographic film. These two devices are still in use in the modern radiography.

An x ray tube is an high voltage vacuum tube fig.1 with an anode and a cathode; the electron are strongly accelerated from the cathode to the anode impinging on the latter with a kinetic energy of thousand electron-volt (eV), being accelerated in the vacuum by a strong electric field produced by a ΔV of tenths of KV.

$$\Delta V = \frac{1}{2} m_e v^2 \qquad (1)$$

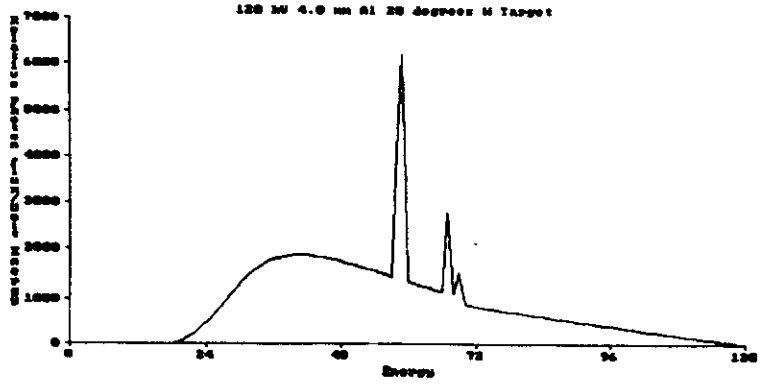
Where m_e is the electron mass; the electron impinging on the anode can be considered as monochromatic being their velocity at the start from the cathode negligible.

Entering in the anode the primary electrons give rice mainly to three types of interactions the scattering with the electrons of the target (ionization), the Rutherford scattering due to the coulumbian field of nuclei and the emission of photons during the decceleration of the primary electrons in the target (bremsstralhung). There are other interactions but, for an electron (i.e. a



charged particles characterized by a very large charge over mass ratio), their probabilities are very small.

As a consequence of above mentioned interactions the x ray spectrum produced is not monoenergetic, showing the characteristic shape of figure 2. The most prominent features of the spectrum are the occurrence of a maximum in all the curves, the existence of a high energy limit advancing in the direction of higher energies as the applied voltage increases and the presence in the spectrum of the characteristic x ray lines of the target. This high energy limit (known as the Duane-Hunt limit) is seen as directly proportional to the applied voltage, and is a direct consequence of the quantum nature of electromagnetic radiation, i.e., no photon can be emitted with an energy greater than that of the bombarding electrons. The maximum is related to the absorption of the photons emitted by the anode by the tube window, and by the



filters. The characteristic lines are a consequence of the electron-electron scattering (ionization).

Bremsstrahlung

Focusing our attention on the bremssrtalhung it is easy to note that this effect can be understood also from a classical point of view. In the development of electromagnetic theory it is established that an accelerated charge loses its energy progressively in the form of electromagnetic radiation, and an important situation arises when an electron is accelerated in the direction parallel to its motion. Let us consider a charge moving in the direction of x axis with an acceleration a and a point P (see figure 3) at a distance r having an angle θ with the x-axis, it is easy to observe that, in this point, the electric field **E** lies in the plane x-r and is perpendicular to r and the magnetic field **H** is perpendicular to it

$$|\mathbf{H}| = |\mathbf{E}| = \left(\frac{|\vec{a}|e}{rc^2}\right)\sin(\theta) \qquad (2)$$

The outward flow of energy associated with the acceleration is given by the Poynting vector **N**

$$\mathbf{N} = \frac{c}{4\pi}(\mathbf{E}\wedge\mathbf{H}) \quad |\mathbf{N}| = \frac{c}{4\pi}\left(\frac{|\vec{a}|e}{rc^2}\right)^2 \sin^2(\theta) \qquad (3)$$

The $\sin^2(\theta)$ term give rise to the familiar polar diagram for dipole radiation. It is well known that this equation is correct only when the velocity of the charge is small in comparison with c, i.e. $\beta = \frac{v}{c} << 1$, in our case the primary electrons enter in the target with a very high velocity $\beta \approx 1$ in this case the equation (3) must be substituted with

$$\mathbf{N} = \frac{c}{4\pi}\left(\frac{|e\,a}{rc^2}\right)^2 \frac{\sin^2(\theta)}{(1-\beta\cos(\theta))^6} \qquad (3')$$

For $\beta \approx 1$ the radiation is strongly forward peaked, but for longer slowing down time, i.e., in the case of emission of lower energy photons, the angular distribution is less forward peaked.

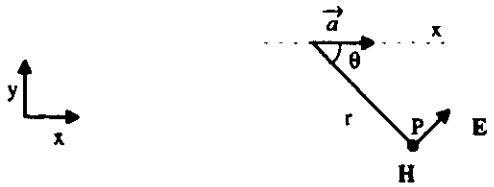


figure 3

It is possible to demonstrate that for thin target the distribution is characterized by minima in forward and backward directions and a gradual movement of the maximum of intensity toward smaller angles as the electron energy is increased. Also for electron opaque targets the distribution is similar, this is the reason of the fact that the electron beam in a x ray tube impinge on the anode surface with an angle of about 45°

Integrating the equation 3 over all angles we obtain the total intensity as a function of the acceleration

$$I = \int \mathbf{N} 2\pi r^2 \sin(\theta) d\theta = \frac{2(e\,a)^2}{3c^3} \qquad (4)$$

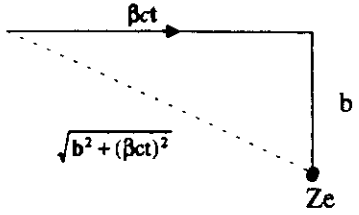


figure 4

That is, for a single charge, the Larmor formula.

The Larmor formula is valid for a charge with a $\beta<<1$, therefore, if we pass from the reference system of the nucleus (S) to that of the electron (S'), it can be used for calculating the energy irradiated by the electron during its interaction with the nucleus. In this case the electron is at rest at time zero and its velocity remain small, in comparison with c, during the interaction with the nucleus. Therefore, the acceleration to which is subjected the electron in S' is:

$$\left|\vec{a}\right| = \frac{\gamma Ze^2}{m_e(b^2+(\beta ct)^2}} \qquad (5)$$

Where Z is the atomic number of the target and γ is the factor for which the electric force must be multiplied passing from S to S'. The energy emitted in the system S', during the collision with the interaction parameters b is given by:

$$w(b) = \int_{-\infty}^{\infty} I'(t)dt' = \frac{2e^2e^2}{3c^3}\int_{-\infty}^{\infty} a^2(t)dt' = \left(\frac{\pi}{3}\right)\frac{\gamma(z^2Ze\,r_0)^2}{b^3} \qquad (6)$$

Where $r_0 = \frac{e^2}{mc^2}$ is the classical radius of the electron and b, as shown in figure 4, is the impact parameter. It worth to note that a interaction with a small impact parameter (fig.5 a) give rise to a photon in the high energies part of the spectrum, whereas a weak collision, i.e. a large impact parameter (fig.5 b), give rise to a photon in the lower part of the spectrum.

figure 5

Entering in the target per unit path (dx) the electron is subjected to $2\pi nbdbdx$ interactions with a impact parameters between b and b+db, being n the number of atoms for unit volume. Integrating equation for the impact parameters it is possible to show that:

$$\int_{b_{min}}^{b_{max}} w(b)2\pi\, nbdbdx = \frac{2\pi^2}{3}(z^2Zer_o)^2\gamma\, n\, dx\int_{b_{min}}^{b_{max}}\frac{db}{b^2} = \frac{2\pi^2}{3}(z^2Zer_o)^2\gamma\, n\, dx\left[\frac{1}{b_{min}}-\frac{1}{b_{max}}\right] \qquad (7)$$

The result of this equation is the energy irradiated by the electron for unit path $(\frac{dE}{dx})$, it is the integral of the emitted spectrum. For high energy x rays the $b_{min} << b_{max}$ therefore in equation 7 the term in square parenthesis can be approximated by $\frac{1}{b_{min}}$. It is easy to

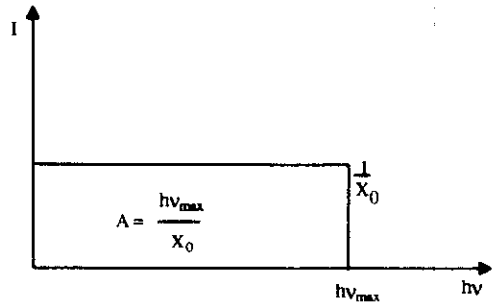


figure 6

demonstrate that the impact parameter is related to the time for which the electron is subjected to the acceleration a during its passage across the field produced by the nucleus.

$$\Delta t = \frac{2b}{\beta c};\quad \Delta t' = \frac{2b}{\gamma\beta c};\quad \text{therefeore}\quad \nu'_{max} = \frac{\gamma\beta c}{2b_{min}} \qquad (8)$$

It is evident the photon energy ($h\nu_{max}$) cannot exceed that of the electron E and is a reasonable hypothesis, suffrage by the experiments, that all impact parameters between the minimum and the maximum are equiprobable, therefore the spectrum is constant between zero and the maximum frequency. Under this hypothesis the energy irradiated in the energy interval $h\nu + h\nu + dE$ is given by:

$$\left(\frac{dE}{dxdh\nu}\right) = \frac{\left(-\frac{dE}{dx}\right)}{h\nu_{max}} = \frac{4\pi^2}{3hc}(z^2Zer_o)^2n \qquad (9)$$

Therefore, the number of photon irradiated in the unit energy interval (N) is given by:

$$N = \frac{1}{X_0 h\nu} \qquad \text{with}\ \frac{1}{X_0} = \frac{4\pi^2}{3hc}(z^2Zer_o)^2n \qquad (10)$$

Treating the problem with quantum mechanics it is possible to verify that the equation 10 is true apart from a slowing varying factor F(E,E') that can be neglected in many cases. In figure 5 it is shown a typical bremsstralhung spectrum from a thin foil

The high energy electrons in the target are subjected to a progressive reduction of its velocity, the above equations did not take into account of this. In other wards the above written equation are valid for the case of a thin target. It is easy to image that the high in a layer immediately below the surface the electrons have a energy lower that in the surface. These electrons would produce a bremsstralhung spectrum slightly different from that impinging on the surface. This effect can be described also using equation 7, which consider the fact that the energy emitted is directly proportional to the electron energy. It is possible rewrite this equation in a different manner, considering the relation between the impact parameter and the electron energy E_0:

$$\left(\frac{dE}{dx}\right)_{irr} = \frac{E}{X_0} \qquad (11)$$

The factor $\frac{1}{X_0}$ take the name of radiation path, it is a characteristic of the target. Integrating the equation 11, it is possible to show the meaning of this factor:

$$E = E_0 e^{-\frac{x}{X_0}} \qquad (12)$$

Therefore, the electrons entering in the target are subjected to an exponential reduction of its energy. Hence, the bremsstralhung spectrum in a thick target is given by the sum of the contributions of the single dx's inside the target; therefore in a particular energy interval around hν the intensity is given by:

$$N = \int_{E_0}^{E=h\nu} dN = \frac{1}{X_0}\frac{E_0 - E}{E} = \frac{1}{X_0}\frac{h\nu_{max} - h\nu}{h\nu} \qquad (13)$$

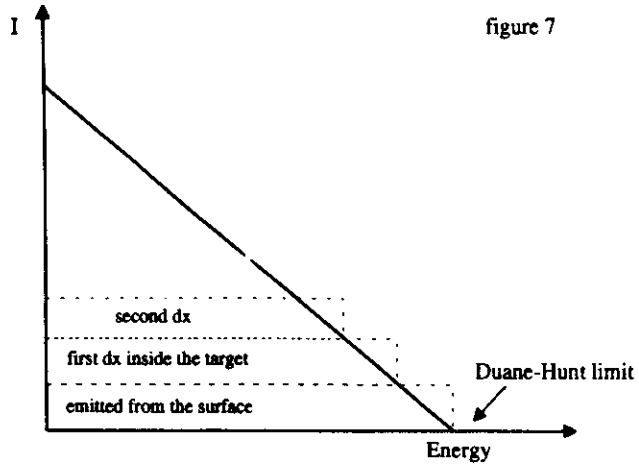


figure 7

Being the integral between the initial energy of the electrons (E_0) and the spectral energy E. It is obvious he fact that, the parts of the target in which the primary electrons have an energy lower than E do not give any contribution to the spectral windows with energy greater then E, figure 6. It worth remember that N is the number of photons emitted in the spectral window $h\nu = h\nu \pm dE$, in fact the intensity emitted in the same window is given by:

$$I = \frac{1}{X_0}(h\nu_{max} - h\nu) \qquad (14)$$

Finally, there are two points to be discussed the dependence of the intensity by the target Z and by the primary particles mass. The dependence from the target Z is linear because X_0 can be written as follow

$$\frac{1}{X_0} = \frac{4\pi^2}{3hc}(z^2 e r_o)^2 N \rho \left(\frac{Z}{A}\right) Z \qquad n = \rho\left(\frac{N}{A}\right) \qquad (15)$$

where ρ is the target density and N is the Avogadro number. The dependence of the bremsstralhung from the charged particle mass can be deduced substituting in the equation 5 the mass of the electron with that of the particle (M). In this case the particles in the nucleus field is subjected to a lower acceleration, therefore the emitted radiation is reduced by a factor $(\frac{m}{M})^2$, that for a proton is $(\frac{1}{1840})^2 \approx 3 \times 10^{-6}$.

Summary:

The bremsstralhung spectrum produced by an electron beam traversing a thin layer of material is continuous with equal intensity for unit energy (fig.6). However, if the beam, entering in the anode, traverse a thickness that is much more larger an its mean free path in the material (X_0), the spectrum show a linear decrease of the intensity with the photon energy (fig.7). In fact, the bremsstralhung produced in the layer immediately below the surface have proportionally less photon of high energy than the first one because some electrons have been partially decelerated and the mean energy of the beam is smaller. The electron impinging on the target are characterized by a relativistic velocity, consequently (equation 3) the emitted radiation is strongly forward peaked. Their velocity progressively decreases and the radiation become less forward peaked. In practice, an electron beam traversing a thick target produce a maximum of intensity in the foreword direction. In particular, the position of this maximum depends from some parameters, such as the energy of the electrons and the material. The manufacturer give the angle of the target, that is usually calculated taking as zero an angle of 90°.

Characteristic x ray lines

The second kind of interaction, i.e. the ionization process, is a electron-electron scattering that remove one electron from a atom of the target and, with the re-equilibrium of the atom, give rice to the production of the characteristic x ray lines of the elements of the anode. For example in the very common case of a tungsten anode the following x ray line will appear in the spectrum emitted by the tube:

Characteristic X Ray Lines of the Tungsten								
$K_{\alpha 2}$	$K_{\beta 1}$	$K_{\alpha 1}$	$K_{\alpha 2}$	$L_{\gamma 1}$	$L_{\beta 2}$	$L_{\beta 1}$	$L_{\alpha 1}$	$L_{\alpha 2}$
69.09	67.20	59.31	57.97	11.28	9.96	9.67	8.40	8.333

Obviously, if the electron beam energy is less than the binding energy of the electron in the particular subshell giving rise to the emission of a characteristic line, that line will not appear in the spectrum . In order to see the characteristic K-lines of the tungsten an high voltage of more than 70 KV is necessary.

Filters

The decrease of the intensity toward the low energies is due to the absorption of these photons by the window of the vacuum tube (inherent filtration). In many cases it is useful to place a thin sheet of an absorbing material after the window (added filtration). The absorption of the photons by the filter depends by the material constituting it, using an element of higher atomic number (or a more thick filter) the energy of the maximum of the tube spectrum will be shifted toward higher energies.

The absorption in a particular spectral window (hν) is given by the well known relation:

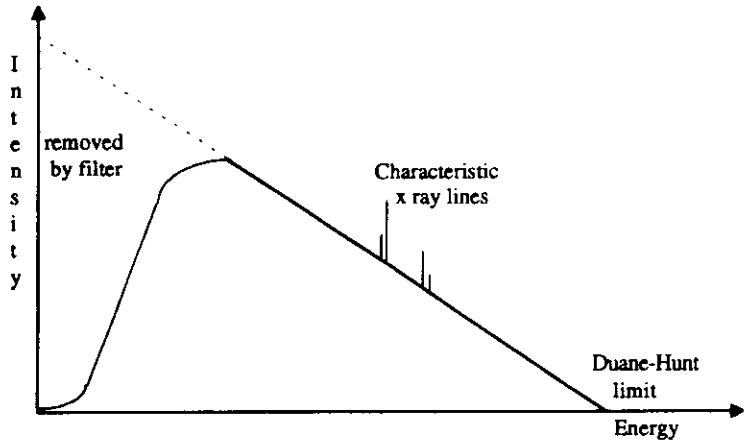
$$I = I_0 e^{(\frac{\mu}{\rho})\rho x} = I_0 e^{(\frac{\mu}{\rho})m} \qquad (16)$$

where $(\frac{\mu}{\rho})$ is the mass absorption coefficient of the filter and m is its the mass for unit surface. The mass absorption coefficient for the elements can be considered as the sum of three terms:

$$(\frac{\mu}{\rho}) = (\frac{\mu}{\rho})_{photo} + (\frac{\mu}{\rho})_{incoh.} + (\frac{\mu}{\rho})_{coh} \qquad (17)$$

where $(\frac{\mu}{\rho})_{photo}$ is the photoelectric mass absorption coefficient, $(\frac{\mu}{\rho})_{incoh}$ is the incoherent scattering (Compton) mass absorption coefficient and $(\frac{\mu}{\rho})_{coh}$ is the coherent scattering mass absorption coefficient. The $(\frac{\mu}{\rho})_{photo}$ decrease rapidly ($E^{-3.24}$) with energy, this interaction in fact is dominant in the energy range of the binding energies of the inner electrons of the atoms. The coefficient is characterized by sudden jumps in correspondence of the binding energies of the electrons, in the case of the binding energy of the K-shell it is common to speak of K-edge. Using a filter made with a correct element it is possible to place this K-edge in the energy interval of interest, in fact sometimes it is useful to use a couple of K-edge filters, in order to select an energy interval. In this case the spectrum emitted by the x ray tube encompass a small energy band.

The x ray spectrum after the filtration appear as that shown in figure 8.



1.1.2 Other x ray sources

There are others less common x ray sources of the tube. In particular, are available many radioisotopic x ray sources emitting x ray lines or a continuous spectrum. These sources are used for radiographic studies of large absorbing objects (γ-graphie), but are not valid alternative to the tube in the low and intermediate energy interval. For radiography are now available small and large particle accelerators as linear accelerator and synchrotrons. Particular interest is actually devoted to the **synchrotron light** that is a source of x rays of very interesting characteristics. As in the x ray tube the radiation is emitted during the acceleration of the electrons, in this case in the bending magnets of the accelerating machine. If E is the energy of the electrons, expressed in GeV, and R is the radius of curvature of the electron orbit, a stationary observer will receive a pulse of electromagnetic radiation at each revolution of the electron. The electron will radiate for a time of $\Delta t = 2(\frac{R}{c})(1-\beta^2)^{\frac{1}{2}}$, this pulse will be compressed by a factor of $(1-\beta^2)$ and its duration as seen by a stationary observer will thus be

$$\Delta t = \frac{R}{c}(1-\beta^2)^{\frac{1}{2}} = \frac{R}{c}(\frac{mc^2}{E})^3 \qquad (18)$$

The pulses will recur with a frequency of the order $\frac{c}{2\pi R}$ and so the radiation will consist of a fundamental at this frequency and a series of harmonics up to a limit of the order

$$\nu_c = \frac{3c}{2R}(\frac{E}{mc^2})^3 \qquad (19)$$

That is the critical frequency. This radiation is strongly forward directed due to the relativistic velocity of the electrons, in the plane of the electron orbit the divergence of the beam is $\Delta\theta = \frac{E}{mc^2}$ that in the case of a 1.5 GeV synchrotron is 0.3 mradians. The harmonics are so closely spaced that the spectrum appear as continuous with a characteristic shape.

$$\frac{dI}{d\theta} = \frac{\sqrt{3}}{2\pi}\alpha\frac{E}{mc^2}\frac{\Delta\nu}{\nu}\frac{I}{e}G(\frac{\nu}{\nu_c}) = 2.457\times10^{13}E(GeV)I(A)G(\frac{\nu}{\nu_c}) \qquad (20)$$

where α is the fine-structure constant, $\frac{\Delta\nu}{\nu}$ is the bandwidth and I is the storage ring current; G is a function equal to one at the maximum that is at ν_c.

The synchrotron radiation is polarized in the plane of the electron orbit and is elliptically polarized out of this plane.

1.2. Detectors.

There are many types of detectors for the energy interval of interest for the radiography (KeV's photons), unfortunately most of them are point detector that can be hardly used in the detection of an image. In the table are listed the principal class of x ray detectors with their principal characteristics such as their degree of energy and position sensitivity, energy resolution and spectral efficiency. It is evident that the detectors with a good energy resolution are characterized (unfortunately) by a low spatial resolution. Actually in the clinical use only few types of detector are in use for radiology: the films, the screen-film systems, the image intensifier, the photostimulable screens, the scintillators (in the CT-scanners and in the absorptiometric systems). The solid state detectors are the most used in the research.

Principal classes of x ray detectors

Type	energy resolution	spatial resolution	efficiency	price
x ray films	no	very good	poor	low
screen-film systems	no	very good	medium	low
fotostimulable screens	no	very good	high	high
image intensifiers	no	good	high	medium-high
scintillators	poor	no(poor)	high	medium
solid state	very good	no*	medium-high	high
gas-filled detectors	medium	good	low+	medium

* actually there are attempts to construct position sensitive solid state detectors especially for that applications in which the energy sensitivity is very important as in angiography

+ the efficiency of gass-filled detector is good in the energy range 1 to 20 KeV but decrease quickly with the energy; using high pressure gasses the efficiency is better.

In this particular moment the detectors play a fundamental role in the developments of the new digital radiography systems. In this sense some type of detectors can be classified as inherently analogic (films, screen-film systems) and others digital. The trend is to substitute the analogic actually in use with digital techniques; this process is started, but at this moment only a small fraction of the radiographic investigations are carried out with digital procedures. One limitation is in the high cost of digital systems that require a higher cost for the plant but

(potentially) a lower cost for the operation. A digital system, in fact, require a costly acquisition system and a computer for the elaboration and storage of data. The advantages for a radiologist are mainly in the better organization of the information (easier storage and retrieval) and in the possibility to elaborate (enhance, restore the image) and perform comparison between different image. For a medical physicist the principal advantage is in the fact that a digital image can be considered a quantitative result with which it is possible estimate parameters or to perform measures.

In these short note we will discuss mainly of the x ray films, that are actually largely the more used detector for radiograph, and only briefly of the others detectors.

1.2.1 The X Ray Film And The Screen-Film Systems.

A radiographic film can be considered as a continuous (analogic) x ray detector, the working principle of which is basically the same as for photography, i.e. the formation of a latent image through the interaction of the primary photons with the silver atoms. The subsequent development of the film, that is characterized by an amplification of the primary signal, produce a readable image. The efficiency of the detector is proportional to the linear absorption coefficient of the film and to the size of the silver bromine grains. A fast film is more efficient, i.e. with grains of larger size; in the development this kind of film give rice to large islands of silver that is possible to see with a microscope. This film is a low resolution one. The efficiency of a film is very small if compared with that typical of other x ray detectors therefore the doses of radiation needed for a radiographic examination are proportionally bigger. The real advantage in using films is in the possibility to image a large area, in fact a film should be considered an "area detector". Until the last decade very few alternatives it could be found, and in general characterized by poor resolutions if compared to that typical of an x ray film. The storage capacity of a film is immense. Resolution for very fine grained film is about 4 μm^2; therefore, a 14"x17" film contains about 4x 10^8 pixels; a figure that must be compared with the actual capacity of a digital system that is about 10^6 pixels. In this sense we are far to the possibility to substitute the film in the high resolution x ray imaging. Obviously if a resolution of 100 μm^2 is enough (or we want image a smaller area) we can find excellent alternative to the film.

The process with which the visible image is formed by exposing the film to the radiation coming from the source and partially absorbed by the object can be schematized as follow: (i) a high energy photon impinging on the film remove an electron from one (or some) silver atom(s), starting the local chemical reaction (reduction), so the number of silver grain that is transformed for unit film area is proportional to the photon flux (photons/sec) impinging on this area; (ii) during the development a reduced silver grain will form a small cluster of metallic silver with a controlled amplification process. At the end of this process it is possible to see a visible image.

A visible image can be physically represented by a two variable (x,y) continuous function; the physical meaning of this function is in general the point by point luminosity of the surface. It is well know that the luminosity depend from the external illumination, in order to avoid the problems arising from this fact it is useful to introduce a new quantity that can be more easily measured. The optical density (od), that is a measure of the physical sensation of luminosity, is related to the luminous flux emerging from one point of the film illuminated with an uniform light source by the relation:

$$od = \ln\left(\frac{\Phi_0}{\Phi}\right) \qquad (21)$$

where Φ and Φ_0 are the transmitted and incident light intensities respectively. In order to measure the optical density in one point of the film, it is possible to use a microdensitometer (figure 9). The film is sampled with a light pencil and the light transmitted is detected with a

suitable detector, for example a photomultiplier or a phodiode. The size of the light spot incident on the film can be reduced to a size of few microns.

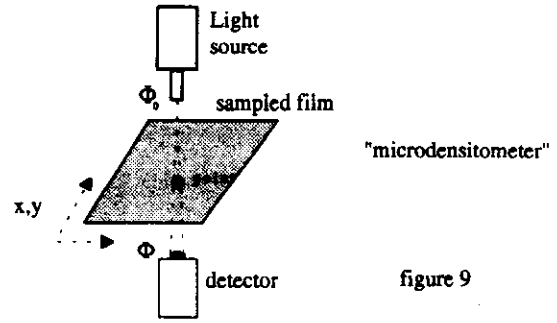


figure 9

An optical density equal to one means the in that point only one tenth of the impinging light is transmitted. Therefore, greater is the od more black is the point. In the hypothesis of silver grains all of the same area (σ) uniformly distributed on the emulsion, the optical density can be considered as a function of the number of silver atoms for unit surface (n) :

$$od = \sigma n \qquad (22)$$

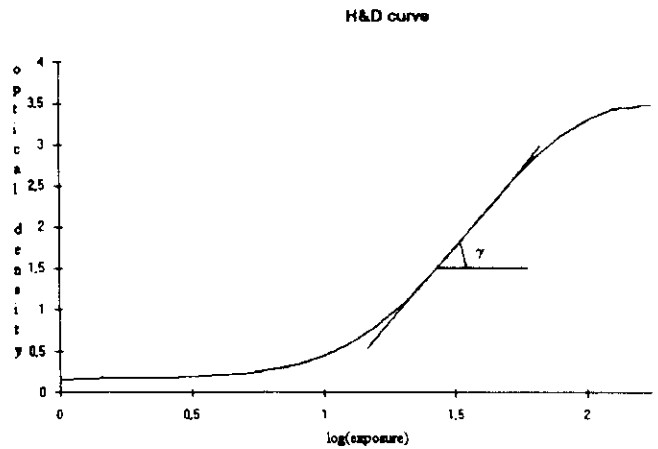
If initially the number of silver grains was n_0 and the probably that a impinging photon would interact with a silver atom (P) is constant, the number of silver grains reduced is proportional to the N, i.e. the number of photons for unit time and film area. Unfortunately n_0 is not a very large in comparison with the number of interactions occurring, considering also the fact that when a silver grain is hit it is not available for other interactions. Therefore $\frac{dn}{dN}$, that is the number of silver grain that are transformed by a increment dN of the impinging photons is proportional to the number of not reduced (transformed) grains:

$$\frac{dn}{dN} = k(n_o - n) \qquad (23)$$

Resolving this equation it is possible to shown that:

$$n = n_0(1 - e^{-kN}) \qquad od = d_0(1 - e^{-kN}) \qquad (24)$$

Where the second relation has been found using the equation (22). This is the response function of film to a monochromatic radiation. In the case of a polychromatic photon beam we must consider that the number of silver grains that is reduced in the passage of a single photon depends by the energy of this latter. It is obvious that for beam with a constant spectrum it is possible to use the equation 24. It is possible also to substitute N, the number of incident photon, with the exposure that is the total energy incident on the unit film surface. It is obvious that for a given time interval the exposure and the beam intensity (I) are equivalent, also for a polychromatic spectrum. This is not true in general because exposure is a extensive quantity and the intensity is intensive. If we plot the od vs the lg(I) we obtain the well known H&D (Hurter and Driffield) curves. In fig.10 a typical H&D curve, describing the



This response function is characterized by several factors (i) the gamma (γ), (ii) the "fog", (iii) the amplitude (or latitude). The gamma is the value of $\gamma = \frac{d(od)}{d\lg(I)}$, can be defined in a generic point of the curve or as the variation of the od for an increment od one order of magnitude of the primary beam intensity in the linear range. Usually the γ is about constant in an exposure range of two-three orders of magnitude. The fog is the shift toward higher values of sensitometric curve that can't be avoided, in fact in each film area some silver atoms are randomly transformed for many reasons, for example an unwanted pre-exposition of the film to light give rice to an higher value of the fog; it worth remember that the films must be kept in the refrigerator before the exposi The film in fact work as a counter, working on an integral mode, it means that it sum the exposures in the time. Therefore a pre or post-exposition of the film limits its performances, because some silver grains are already reduced.

The dynamic (linear) range of an x ray film is the exposure (intensity) range in which the sensitometric curve is linear, changing the film characteristics, size of the silver grains, superficial density of the grains etc., it is possible to extend and shift the sensitometric curve. The corresponding od range is almost fixed by the readability of the film, it is usually in the range 0.8-2.5 od units. Therefore the γ assume the meaning of a real measure of the performances of a film, in fact a smaller γ indicates a larger dynamic range. This range is usually very large in the modern x ray films, there are in addition many tricks to extend the γ's, for example using fluorescent screens.

The sensitometric curve can be determined experimentally using a step wedge, or a sensitometer i.e. a system which is capable to expose the film to light flashes of wanted intensity. Measuring the optical density in the film area exposed and plotting the values vs the given intensities one obtain the sensitometric curve and can determine the latitude and the γ.

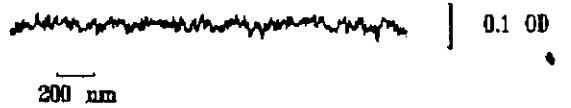
The major drawbacks of the films are in the low efficiency and in the low time resolution. For avoiding the first problem actually the film are not used by alone but with the auxil. of phosphors, i.e. of the radiographic screens. The concept is very simple the film, that is sensitive to the light, is enclosed in two screen, the x rays impinge on the screens that are more absorbing than the film, the light produced during the interaction of primary x rays with the screens expose the film giving rise to the latent image. The technique of the film-screen

screens expose the film giving rise to the latent image. The technique of the film-screen systems is actually the more used in radiography , in fact it offer a very good price/performances ratio because in comparison with the films these systems suffer of a small spatial resolution reduction but offer a reduction of dose for single exam that is due to the larger efficiencies of this detection system. In the film-screens systems because the detector is always the film , therefore it is not necessary discuss their characteristic. In particular, the latitude of these system is obviously larger than that of the film by alone.

Unfortunately, the second limitation of the film and film-screen systems, can't be eliminated with the use of screens, in fact the only possibility to use a film in a dynamic study is that to expose several films in a short interval of time. This can be done, and was done in the early angiographic systems, but with severe limitation and high costs. Practically the films and film-screen systems can be useful used only for static studies.

The last important characteristics of this class of detectors is its spatial resolution. It is not easy to answer to this question because there are several parameters that influence the response. The resolution of a detector is minimum distance between two points that can be distinctly seen. There are application, like autoradiography in which the resolution of the film is of the order of the microns, because the process of formation of the image allow in this case to reach this excellent resolution. It is not easy to discuss of the spatial resolution of a continuous detector, as a film, apart from the process of formation of the image. In this sense the potential spatial resolution of a film in a radiographic investigation is limited only by the process of formation of the image, as will be discussed below.

The correct answer to the question about spatial resolution is given by the measure of the Modulation Transfer Function (MTF) of the film (or film-screen system), this can be done studying with a microdensitometer the ability of the measure system to reproduce test objects. There are several of such test objects containing stripes of lead placed at decreasing distance in order to allow also an immediate visual determination of the limiting resolution.



A microdensitometer scan of a uniformly exposed processed photographic film. The x ray film exposed to light scanned by a 30 x 30 μm aperture. The density scale in OD units, the mean density being ~ 1.0

Strictly related to the problem of the resolution is that of the noise. Sampling a uniformly exposed area of a film with a microdensitometer it is possible to obtain result similar to that shown in figure 11. The reason of this fluctuations is in the noise that is present in the process of formation of latent image and in that of development of the film. The fluctuations are mainly due to the fluctuations of the number of photon impinging for unit film area and the non uniform distribution of silver grains in the emulsion, In the case of a photographic emulsion the fluctuations of the optical density for unit film area are normally distributed with

a variance proportional to the inverse of the smallest area observed (A), i.e. the spot of the microdensitometer. The variance of the sampled optical densities in a uniform film is given by:

$$\sigma^2(od) = \frac{k}{A} \qquad (25)$$

That relation take the name of Selwing law, it is the basis of the noise study in a radiographic film.

This is exactly what happen with the eye which sample an image with a characteristic aperture, if the typical aperture of the eye is to small it will observe fluctuations in the number of photons arriving from each unit area and the sensation would be of a noisy object. In the case of a film everything is function of the number of silver grains for unit film area; in a high definition film this number is very high and, if the exposure is optimal, for the some sampling aperture we observe a larger amount of silver with proportionally smaller fluctuations (in other words we have a better statistic). If the amount of silver for unit film area can't be increased, in order to have not a noise sensation we must increase the sampling area, and consequently the resolution will be worse. In this particular moment the detectors play a fundamental role in the developments of the new digital radiography systems. In this sense some type of detectors can be classified as inherently analogic (films, screen-film systems) and others digital. The trend is to substitute the analogic actually in use with digital techniques; this process is started, but at this moment only a small fraction of the radiographic investigations are carried out with digital procedures. One limitation is in the high cost of digital systems that require a higher cost for the plant but (potentially) a lower cost for the operation. A digital system, in fact, require a costly acquisition system and a computer for the elaboration and storage of data. The advantages for a radiologist are mainly in the better organization of the information (easier storage and retrieval) and in the possibility to elaborate (enhance, restore the image) and perform comparison between different image. For a medical physicist the principal advantage is in the fact that a digital image can be considered a quantitative result with which it is possible estimate parameters or to perform measures. In this particular moment the detectors play a fundamental role in the developments of the new digital radiography systems. In this sense some type of detectors can be classified as inherently analogic (films, screen-film systems) and others digital. The trend is to substitute the analogic actually in use with digital techniques; this process is started, but at this moment only a small fraction of the radiographic investigations are carried out with digital procedures. One limitation is in the high cost of digital systems that require a higher cost for the plant but (potentially) a lower cost for the operation. A digital system, in fact, require a costly acquisition system and a computer for the elaboration and storage of data. The advantages for a radiologist are mainly in the better organization of the information (easier storage and retrieval) and in the possibility to elaborate (enhance, restore the image) and perform comparison between different image. For a medical physicist the principal advantage is in the fact that a digital image can be considered a quantitative result with which it is possible estimate parameters or to perform measures. In this particular moment the detectors play a fundamental role in the developments of the new digital radiography systems. In this sense some type of detectors can be classified as inherently analogic (films, screen-film systems) and others digital. The trend is to substitute the analogic actually in use with digital techniques; this process is started, but at this moment only a small fraction of the radiographic investigations are carried out with digital procedures. One limitation is in the high cost of digital systems that require a higher cost for the plant but (potentially) a lower cost for the operation. A digital system, in fact, require a costly acquisition system and a computer for the elaboration and storage of data. The advantages for a radiologist are mainly in the better organization of the information (easier storage and retrieval) and in the possibility to elaborate (enhance, restore the image) and perform comparison between different image. For a medical

physicist the principal advantage is in the fact that a digital image can be considered a quantitative result with which it is possible estimate parameters or to perform measures.

1.2.2 Image Intensifier Television Systems.

Digitalizing a radiographic image:

There are several methods to digitalize the image contained in a radiographic film. The more sure is to use a microdensitometer, but this take time and is very expansive, other possibilities are in the use of a scanner or a TV-camera. In this case it is necessary to choose the reading condition carefully in order to avoid mistakes. In order to digitalize a film without lost of information we should consider that the latitude draw usually an exposure range of two or three order of magnitude and the precision should be of few percent. Therefore, you should convert the signal with at least a 12-bit ADC. For example using different exposure conditions you can have in the same film optical density values in the range 0.8-3 od, how many different gray levels you can differentiate in this range of od values? It depends from the noise, in fact this is the some question of this. If you can measure the radius of a cell with a precision of 1μm and the cell radius can be in the range between 5 μm and 105 μm how many cell size classes you can differentiate. The answer is 100.In any case in a photographic film we can observe, using the eye, several od levels but we can discriminate using a densitometer a much more larger number of levels. If we speak in terms of od with a noise level of 0.02 in a range of 3 we can discriminate 150 gray levels. It is better to speak of number of counts, like in tomography or in terms of optical density? The answer is, if we consider the ability of our eye to distinguish different gray levels in general it is assumed that 256 (8 bits) levels are much more than enough to produce a graphic image in a display. In addition our eye is characterized by a logarithmic response function, therefore the use of the optical density units match this characteristic. More in general is better to reason in terms of number of counts for unit area and time, because is more easy to make comparisons between different techniques.

If we acquire an x ray image with a 8 bits ADC we can observe that it is noisy and poor. Now if we observe the some image acquired with a system characterized by a wider dynamic range we are able to note that the image is less noisy. This is exactly what happens with a radiographic image, the some image taken with a low and high definition film and optimal exposure conditions appear of different quality. You should acquire with a wide dynamic in order to reduce the influence of noise in the image that you show to the eye. A film can achieve a dynamic of 10^5 that is very wide, if we speak in terms of exposure. The some wide range of exposure cannot be measured with an ordinary x-ray area detector, because it will probably saturate. In this sense a film is a much more powerful area detector. To give an example the ordinary CCD cameras are currently limited to 8 bits at video rates (30 frames/sec) only for particular applications is possible to use specialized cameras with wider dynamic (12 bits). These particular devices are necessary if you attempt to digitalized a film.

There are several sources of noise in the radiographic process of formation of the image; the primary source is the photon statistic, that in the tomographic systems is the most important one. In fact, for a given exposure level, the number of photon impinging on a small area of a film fluctuate following a Poisson distribution. In this case, being the variance equal to the number of photons, the relative fluctuations will decrease $\frac{\Delta N}{N} = \sqrt{N}$. The number of silver grains reduced will depend from the number of impinging photons and from the number of silver grains. The non uniform distribution of the number of grains for unit area, will amplified the fluctuation of the od, such as the fluctuation in the number of silver atom fixed in the development process for each reduced silver grain. Unfortunately, these noise sources convolute between them, i.e. are in cascade, therefore the final noise will be the product of all

these noises. This happen also in an electronic chain in which the noise generated in the detector is enlarged by the amplifier gain.

3 IMAGING WITH X RAYS

The basic assumption that one make in production of an image using the x ray is that the photon cross the imaged object following rectilinear lines. This means that the refractive index for the x rays is one, that is reasonably true in a first approximation. We assume in addition that traversing the imaged object only one part of the original number of photons are absorbed and that the removal of the photons from the beam traversing the object is due only to the interactions with the atoms belonging to this latter. With this hypothesis we can assume that the probability of interaction for unit length (the linear absorption coefficient, μ is constant along the target. In this case we can write the well known equation of absorption:

$$N = N_0 \, e^{-\mu x} \tag{26}$$

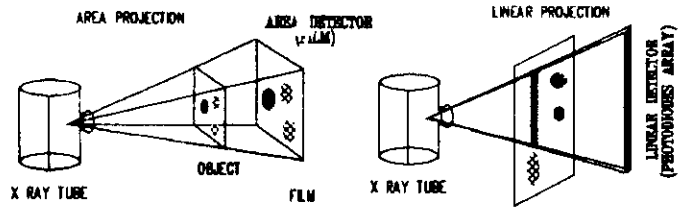
μ dx is the probability of an interaction in the path dx, such as dμ x is the increment of probability due to the a μ change; μ depends by the sample density ρ and elemental composition:

$$\mu x = \frac{\mu}{\rho}\rho x = \frac{\mu}{\rho} m \tag{26'}$$

where $\frac{\mu}{\rho}$ is the mass absorption coefficient and m is the mass for unit surface of the sample.

One useful concept to use is that of mean free path of the radiation in a target. If we know the composition (the matrix) and the density of the object being imaged we can calculate the mass absorption coefficient and also linear absorption coefficient. It is easy to deduct that the inverse of linear absorption coefficient is the mean free path of the radiation in the object. In fact:

$$\bar{x} = \frac{\int_0^\infty x \, e^{-\mu x} dx}{\int_0^\infty e^{-\mu x} dx} = \frac{1}{\mu} \tag{27}$$



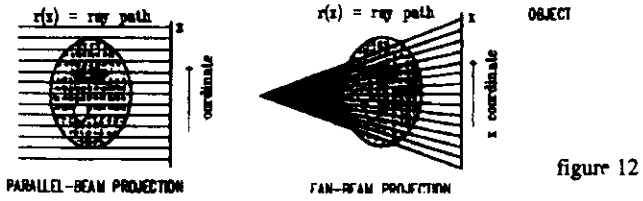
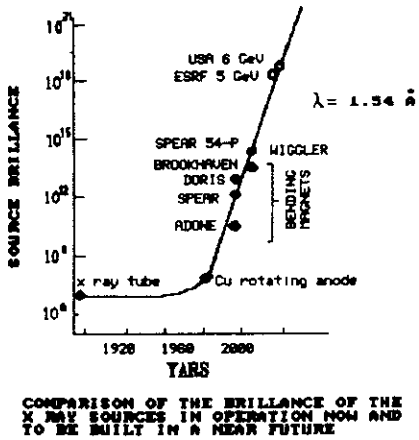


figure 12

in fact the probability that the path would be x is equal to $\frac{N}{N_0} = e^{-\mu x}$. It is useful to note that the thickness $\frac{1}{\mu}$ is also that at which the intensity of the beam is $\frac{1}{e}$ of the original intensity.

A very simple way to produce an x ray image is to perform a single projection obtaining a linear projection or an area projection (figure 12). Geometrical considerations, used also in the optic, lead to the conclusion that the source can be model as a point source if the distance between the source and the detector is much more larger than the diameter of the source. If this approximation can't be applied the source is extended. When we consider a point source approximation we speak about a fan beam, when the source is extended we speak of a parallel beam (see figure 12). An x ray tube can be considered in general a point source, being its emitting area ≈ 1 mm^2 for an ordinary x ray tube and less the 0.5 mm^2 for a microfocus

An interesting problem to consider is the divergence of the beam and its brightness. The brightness of a source is the intensity emitted by one small area dS of source in the unit solid angle in a give bandwidth. A small divergence source, like a laser, is in general characterized by an high brilliance, because the beam is concentrated in a small solid angle. This occur also for the Synchrotron Light (SL) that is characterized by an high brightness and a small divergence of the beam. An high brightness low divergence source can be considered, as in the case of SL a parallel source more then a point source. In figure 13 the brightness of a x ray tube is compared with that of actual and future SL sources; several orders of magnitude in brightness have been gained. The use of such x ray sources would be restricted to that problems in which an high brightness and small divergence of the beam are mandatory, as in the microanalyses or in microtomography in which a small object is analyzed with a very high spatial resolution. Using this source in the imaging of big objects is not in general useful.



COMPARISON OF THE BRILLANCE OF THE X RAY SOURCES IN OPERATION NOW AND TO BE BUILT IN A NEAR FUTURE

figure 13

One problem with the image obtained with a single projection is the formation of the penumbra when the source is extended; in fact, as clearly shown in figure 14 b, the projection of a point placed at a distance h' between the source and the detector (film) has a diameter ΔP given by:

$$\Delta P = \frac{(h-h')}{h} \tag{28}$$

Obviously the penumbra disappear when the object stick to the object and is very thin; unfortunately this approximation seldom can be applied in the case of medical radiography. An object imaged with a fan beam geometry is magnified h'/h times, this occur also in the tomographic techniques using fan beam. The use of a parallel beam can be done only using well collimated position sensitive detectors, in fact, the image produced with a parallel beam without collimator is completely degraded by the presence of the transverse rays (any point in a extended source can be seen as a point source). In tomography parallel beam geometry are in

use (with a narrow collimation of the detectors), whereas in radiography this case .s very rare.

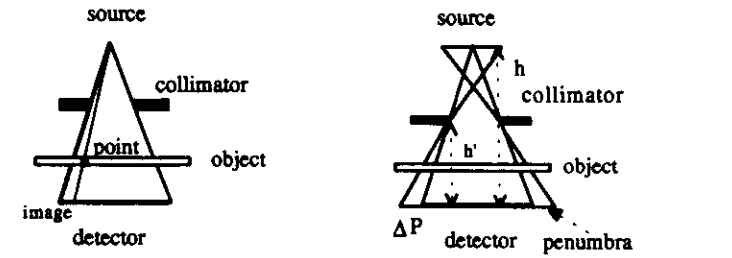


figure 14

In the case of a single projection (see fig.12) the process of formation of the image of a three dimensional object can be schematized as follow. Let $\mu(x,y,z,E)$ be the linear attenuation coefficient point by point in the object, it represent the capacity of the object to absorb photons of a given energy (E); let N(x,y) be the number of photons impinging on a point of the detector surface. Therefore, N(x,y) is given by

$$N(x,y) = N_0 \int_{\vec{r}} e^{\mu(x,y,z)l}\, dl \qquad (29)$$

In the above equation the integral is along the x ray path $\vec{r}$ (see figure 15), that in the case of a fan beam is a transverse line. At the end of this process the image we obtained is a two dimensional representation (N(x,y)) of a three dimensional object, it means that we have lost something. In particular, the integr. .long the line imply that we lose the information about the changes of μ along the path. In this simple image formation model we can consider that one x ray path $\vec{r}(x,y)$ correspond to one point in the area detector. Therefore the response function of a radiographic system can be written in a first approximatio

$$N(x,y) = N_0\, e^{-\Pi \vec{r}(x,y)} \qquad (30)$$

In the above equation we make the assumption that $\vec{r}$ is the path length inside the object and Π is a mean absorption coefficient along the ray path.

Unfortunately with a single projection we are not able to discriminate a variation due to a change in thickness or in density from another one due to change of mass absorption

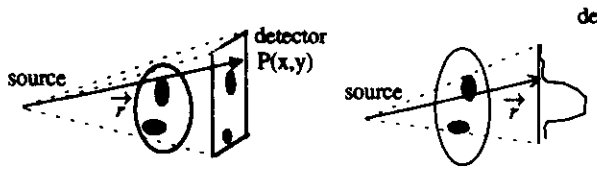


figure 15

coefficient. In fact a variation of density can mask a variation in thickness and composition and vice versa (figure 16). This is a severe restriction for the single projection radiography, only t

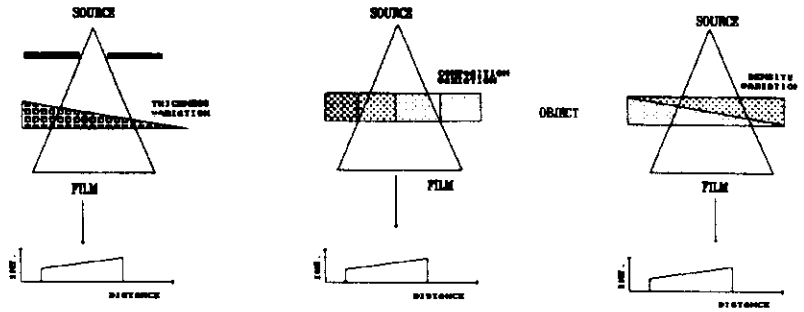


figure 17

he advent of the tomographic techniques have partially removed these limitations.

The mass absorption coefficient $\frac{\mu}{\rho}$ depends only from the elemental composition of examined sample and from the energy of the incident photons. This coefficient can be divided in three parts which are related to the major interactions that take place during the passage of a photon beam of energy less then 100 KeV through a sample (see fig.17, in these energy range the annihilation processes don't exist). That interactions are the photoelectric, Compton (or incoherent) and Rayleigh (or coherent) scattering. The dependence of these three parts of the mass absorption coefficient from the atomic number of the sample and the energy of the photons can be approximately written as follow:

$$\frac{\mu}{\rho} = 20.64\, E^{-3.28}\, \bar{Z}^{4.62} + \sigma_{KN}(E)\, \bar{Z} + 2.8\, E^{-2.02}\, \bar{Z}^{2.86} \qquad (31)$$

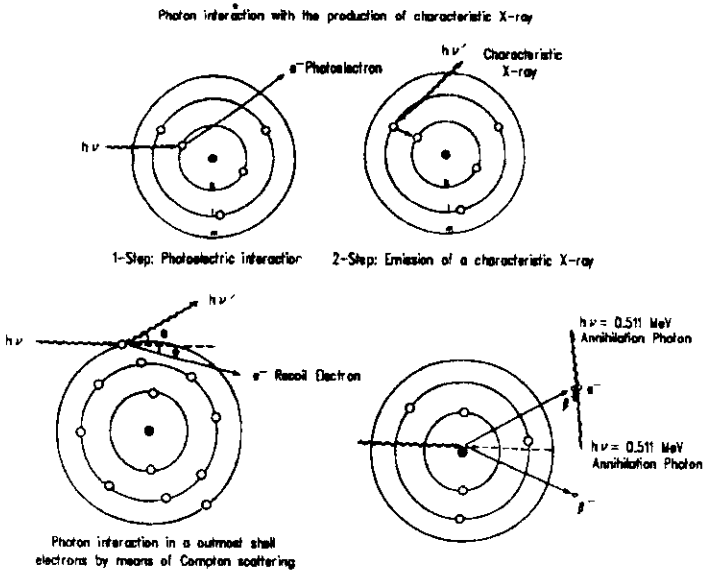
where $\sigma_{KN}(E)$ is the Klein-Nishina cross section and $\bar{Z}$ is the effective atomic number of the sample (Rutherford ed. al.. 1975). These formula shows as the dependence from energy and atomic number of the photoelectric part of the coefficient is stronger (3.24 and 4.62 respectively) that of coherent and incoherent. This mean that the decrease of this contribution is proportionally more rapid that of the other two terms. In this sense we can speak about one energy range in which the photoelectric scattering is dominant and an energy range, at higher energy, in which the incoherent scattering is dominant.

When we study the absorption by a sample of a polychromatic photon beam we should take into account the changes with energy of the mass absorption coefficients. The response function of an absorptiometric technique in this case change as follow:

$$N(x,y) = \int_0^{\infty} N_0(E)\, e^{-\Pi(E)\vec{r}}\, dE \qquad (32)$$

Where the integration is carried out on the whole source spectrum ($N_0(E)$). It is in general cumbersome to study the response function of absorptiometric technique considering the effect of polychromaticity of primary radiation, therefore it is considered an acceptable simplification to represent the primary beam with an effective energy, for example the mean energy:

$$\bar{E} = \frac{\int_0^{\infty} E\, N_0(E)\, dE}{\int_0^{\infty} N_0(E)\, dE} \qquad (33)$$



This simplify the formalism, but don't underline adequately effects that are in general related to the fact that the primary spectrum (e.g., the spectrum of x ray tube) and the detected spectrum (i.e., that impinging on the area detector) are different. This effect is referred to as beam hardening, in fact the detected spectrum is more rich of high energy photons than the primary one. In order to eliminate the influence of this effect we should use an energy dispersive detector (such as a solid state detector), but this is in practice impossible. It is useful to remark that the influence of this effect is very strong if we attempt to correlate absorption measurements made on samples with large changes in composition. This effect can be in general neglected in medical radiography because in this case the variation of μ are very small.

2.1 The Response Function of Absorptiometry

When we look at a radiographic image, that is a distribution in the x,y of the luminosity, some questions instinctively arise: (i) how I can define the minimum detectable luminosity variation, (ii) what is the luminosity threshold, i.e.. the minimum detectable signal, (iii) how is possible measure the spatial resolution, etc.

To start with the answers we should define what we measure; in the case of a radiographic film the measured quantity is the optical density, in that of a image intensifier-TV systems the a voltage, etc., but it is self evident that in any case the physical quantities that we measure is the intensity distribution (or the number of incident photons for each detector element or unit surface of the detector). To approach the not easy argument of the detectability of one defect in an image we must fix the limits of our considerations. We are interested to the detectability in a physical sense, therefore we will not speak about the ability of our eye to detect a particular defect in an image.

In order to fix some concepts it is useful to study the response function of an absorptiometric device, as a function of photon energy, sample thickness and composition. The extrapolation from a single absorptiometric system to a system using an area detector or a

position sensitive detector are straightforward. Obviously this is true for an ideal device, but not for a real one in which, passing from a well collimated single source-detector device to a multiple detector (or area detector system), many problems like the contribution of the scattering or the cross-talk between the detector elements became very important. In our consideration we will neglect these aspects. Let us consider to a parallel beam, in order to simply the formalism, and let start with the case of an uniform sample or a sample surrounding a defect (i.e., a zone with a different linear absorption coefficient).

If the object can model as an uniform sample, it is possible to measure its linear absorption coefficient knowing its thickness (L) (conversely knowing its mass for unit surface, we can calculated its mass absorption coefficient). If we measures the μ of two sample we can be curious to know what is the smallest variation of detected counts (N) that we can appreciate. The answer is immediate if we follow a simple statistical approach. We can consider that the difference $\Delta N = N' - N$ is statistically significant if:

$$N' > N + b\sqrt{\sigma^2(N') + \sigma^2(N)} \tag{34}$$

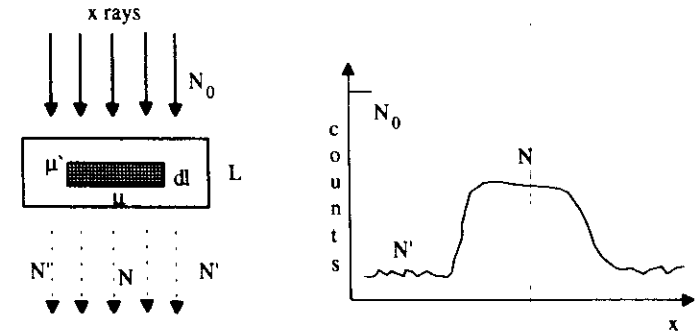
Where $\sigma^2(N)$ is the variance of N and b (= 1,2,3) can choose according to the uncertainty level satisfactory for our application. It is well know that the detected counts follow a Poisson density distribution, therefore a good estimation of the variance is N. Therefore, the equation 34 can be written also as:

$$N' > N + b\sqrt{N' + N} \quad \text{if } N' = N \quad N' > N + \sqrt{2N} \tag{34'}$$

The sensitivity to a linear absorption coefficient variation can be written:

$$\frac{dN}{d\mu} = -L\,N \tag{35}$$

Where L is the object thickness. The minimum variation of μ that we can detect is therefore given by:



$\Delta\mu = \mu - \mu'$
dl defect thickness
L object size

$$\Delta\mu = \frac{b\sqrt{N'+N}}{LN} = \frac{b\sqrt{2}}{L\sqrt{N}} = \frac{b\sqrt{2}\,e^{\frac{\mu L}{2}}}{L\sqrt{N_0}} \tag{36}$$

where N_0 is the number of photons incident on the object surface. It worth note that the minimum detectable $\Delta\mu$ decrease increasing the object size (L).

Let us consider the case of a small cube of side dl inside of an uniform sample (see figure 18), in order to discuss the case of a position sensitive detector (or an area detector). If we take a single linear projection of this object we will obtain a result very similar to that shown in figure 18. The difference $\Delta N = N - N'$ between the counts in a detector element inside the defect (N) and outside it (N') is the contrast and $\frac{\Delta N}{N}$ is the contrast ratio. Observing the detected profile you can note that the counts N(x) don't reproduce exactly the edge, this is due to the limited resolution of our measuring system. It is easy to image that with a poor contrast it is impossible to detect this edge.

The equation 36 can be still used also in this case with the correct meaning for N and N' and assuming that in a uniform part of the sample the variance of the detected count can be considered equal to N. Therefore, it is possible to know the minimum detectable contrast and the minimum detectable variation of μ. Assuming the thickness of the defect (dl) and of the object (L) as constants, $\Delta N = N - N'$ and $\frac{N}{N'}$ is given by:

$$\Delta N = N_0 e^{-\mu' L}(e^{-\Delta\mu dl} - 1) \quad ; \quad \frac{N}{N'} = e^{-\Delta\mu dl} \tag{37}$$

where μ' is the absorption coefficient of the object and $\Delta\mu = \mu - \mu'$

The sensitivity to a μ variation is given by:

$$\frac{d(N-N')}{d\mu} = -N'\, dl\, e^{-\Delta\mu dl} = -Ndl \tag{38}$$

Therefore the sensitivity depends directly from the defect size dl and it decreases with the object thickness (L), because decrease N.

It is useful note that the minimum detectable ΔN is given by:

$$\Delta N = b\sqrt{N + N'} \approx b\sqrt{2N} \tag{39}$$

so the minimum detectable $\Delta\mu$ can be written as:

$$\Delta N = \frac{d(\Delta N)}{d\mu}\Delta\mu = -Ndl\Delta\mu$$

$$\Delta\mu = \frac{b\sqrt{2}}{dl\sqrt{N}} \tag{40}$$

It worth note that the sensitivity the minimum detectable ΔN or $\Delta\mu$ depend by L. It mean in particular that they worsen if the some defect is inside a bigger object. This severe limitation con be overcome only using the tomography; if we take L=0 in the equation 38 and 40 we obtain two relations valid for tomography and we can immediately note that the contrast N is greater. In tomography the quantity that is reconstructed is the map of linear absorption coefficients $\mu(x,y)$ the equation 40 is still valid if we consider that each ray sum can be decomposed in m pixel of length dl (L = m dl), therefore the average detected counts are N_D = m N, substituting N_D in equation 40 we obtain the following relation:

$$\Delta\mu = \sigma(\mu) = b\frac{\sqrt{2m}}{dl\sqrt{N_D}} \tag{41}$$

This relation is very similar, apart the terms depending from the reconstruction algorithm, to that deducted by many authors. Our deduction clearly show that one of the advantages of tomography is in the decrement of the minimum detectable $\Delta\mu$ or in other words an increment of the contrast. Another advantage is that the dl in the tomography is chosen "a priori",

consequently we can more easily optimize the contrast, increasing or decreasing the exposure (and the dose) at the desired level.

A deduction that one can do from the above equations is that we can decrease the minimum detectable $\Delta\mu$ at will increasing the measuring (exposure) time. Can this be true? obviously not. What is inexact in our model to lead to these wrong conclusion? nothing of really unrecoverable, we have forget that our system, as all real measuring systems, is characterized by a dynamic range and by a background (N_B). In particular, as clearly shown in figure 19 there are two limits of total counts that you can detect for unit detector area and unit time, the background N_B counts and the N_S saturation counts. Our measuring system is in fact characterized by a background counting rate, that is the minimum amount of counts per unit time that are in any case detected, which origin can be scattering in the sample or noise in the detector, and by a saturation counting rate, which is the maximum number of count per unit time that the film or the detector can detect. If an higher flux of photons reach the detector the detected events fluctuate around the N_S value. The equation 40 is therefore valid only in the range $N_B << N << N_B$. The dynamic range of our measuring system is N_S - N_B, one advantage in using x ray films is that the dynamic range coincides with the od interval in which the response function od vs log(Ex) is linear, at the margins of this interval the od still increases or decreases with a smaller γ. In this sense the film is more reliable detector that others which at the margins of the dynamic range suddenly stop to work. Is frequent to see radiographic films that are overexposed or underexposed but that can be read, even with some difficulties. When we speak about the need of a wide dynamic range in the ray radiography, we implicitly refer to the fact that is not easy to match the dynamic of X-rays impinging on the detector with the detector dynamic range.

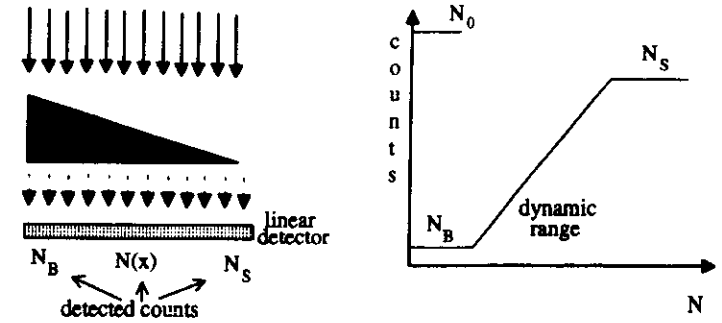


figure 19

Another interesting deduction that we can do from equation 40 is, considering only statistical fluctuation, the minimum detectable $\Delta\mu$ and the minimum detectable defect size (dl) are inversely proportional. To detect smaller defects we should increase the contrast, for example increasing the statistic or increasing the $\Delta\mu$, choosing an energy in which the μ jump is maximum. These are the basis for a optimization of a radiographic exam.

Let us suppose we can choose the object thickness or its μ (changing for example the energy of primary beam), we can know for example the optimal thickness of the sample? The answer can be deduct finding the minimum detectable $\Delta\mu$ as a function of the thickness (L). It can easily done deriving equation 36 and putting this derivative equal to zero, the result is very simple and attractive:

$$\frac{d(\Delta\mu)}{dL} = \frac{b\sqrt{2\,e^{\mu L}}}{\sqrt{N_0}}\left[\frac{2-\mu L}{2L^2}\right] = 0$$

$$L_{opt.} = \frac{2}{\mu} \qquad (42)$$

The some things can be done for the equation 40 deriving for the defect size dl and arriving to the conclusion that

$$dl_{opt.} = \frac{2}{\Delta\mu} \qquad (42')$$

fixing thus the minimum defect size that can be observed for a given $\Delta\mu$. This relation is very attractive because fix a inverse proportionality between, the maximum variation of μ that we can obtain, optimizing the primary beam energy, and the theoretical resolution of our system.

Anyway it is possible to ask to the response function of an absorptiometric system which is the optimal set of parameters in order to minimize for example the minimum detectable contrast. It is necessary underline that some of these parameters can't be changed in regular medical systems, such as the path length of the beam inside the body that is fixed by the body size. This is not true in general and the new revival of microradiographic and microtomographic techniques show how are the potentialities of the absorptiometric techniques when you can optimize simultaneously all the parameters. The procedure to find the optimal parameters set it is very easy in the case of a single energy system but can be more cumbersome for dual energy absorptiometers or for CT systems. The most interesting part of these studies are usually the optimization of the primary(-ies) beam energy. In order to correctly place the question we should remember there are many problems that you can solve using an absorptiometric system; in this sense you should define before the nature of the wanted information. Using an absorptiometric system it is possible to determine the density and the elemental composition of the examined sample. In particular, it is possible also to measure thickness (or better thickness variations), assuming that the sample is homogenous, this kind of measurement, frequent in industrial use, is quite infrequent in the medical applications. How it is possible to determine the optimal conditions in order to determine such quantities? The answer in essentially in the dependence of the μ from the quantity we want measure. In particular, it worth immediately recall that the mass absorption coefficient of a particular element ($(\frac{\mu}{\rho})_i$) is the sum of the three partial coefficients: photoelectric, incoherent (Compton) and coherent . The dependence from atomic number and energy of these coefficients is strongly different (see equation 31). In particular, the photoelectric coefficient is dominant in the low energy range, whereas the Compton one is dominant at higher energies. The dependence of the Compton coefficient from Z is about linear, therefore in the energy range in which this interaction is dominant the dependence of the μ from Z is more weak than in the energy range in which the photoelectric interaction is dominant. All these statements are well known to the people working with rays. We can conclude that in order to optimize the energy we should find the energy of the primary beam that maximize the $\Delta\mu$ between the two parts of our object we want discriminate. This task is very simple because if we know roughly the composition of the particular we want observe and that of the surrounding tissue we can choose that energy for which the $\Delta\mu$ is maximum. It worth stress that we should maximize the $\Delta\mu$ and not the $\Delta(\frac{\mu}{\rho})$ because several times the major contribution to the change is due to density variations more than Z ones. Unfortunately, working with radiography we can't do this simple procedure because we must take into account also other significant parameters, such as the object size. A statement that it is easy to do is the following: if you want measure Z-variations you should use lower energy in order to

increase the contribution of photoelectric part of mass absorption coefficient, if you want measure density variations is better to use higher energy in order to increase the Compton contribution. In practice you should found an useful compromise between the energy that optimize the counting rate and the energy that maximize the $\Delta\mu$, taking the object size as fixed.

Dual Energy Absorptiometry

The observed sample can be model in many cases as a two component mixture, such as in the case of measurement of Bone Mineral Content (BMC). In this case hold a simplified model in which the central region of one object is composed of a mixture of two components: (i) a completely mineralized tissue having a higher Z and (ii) the soft tissue. This central region is surrounded by soft tissue. The problem is more or less the some as before in which a small defect was surrounded by an uniform object. In this case the problem is to find the weight fraction of bone in the central region. This is a typical problem that can be solved with an absorptiometric measure. It is convenient to approximate the μ of the central region as follow.

$$\mu = \mu_{st}\, w_{st} + \mu_{bm}\, w_{bm} \qquad (43)$$

where μ_{st} and μ_{bm} are the absorption coefficient of soft tissue and mineralized tissue respectively and the w's are their weight fractions. It is necessary to impose the condition that $w_{st} + w_{bm} = 1$

Now we perform two independent transmission measures using different primary energies, for example at energies in which the photoelectric and Compton contribution respectively dominate. Thus it is possible to write two independent absorptiometeric relations:

$$\ln(\frac{I}{I_0}) = (\mu_{st}\, w_{st} + \mu_{bm}\, w_{bm})L \qquad (44)$$

$$\ln(\frac{I'}{I'_0}) = (\mu'_{st}\, w_{st} + \mu'_{bm}\, w_{bm})L$$

Where I_0 and I are the incident and transmitted intensities in the first measure (with a lower energy beam) an I'_0 and I' are the corresponding intensities in the second measure. Solving the system of two equations for w_{bm} we obtain:

$$w_{bm} = \frac{\mu'_{st} - k\mu_{bm}}{k\Delta\mu - \Delta\mu'} \qquad (45)$$

$$\text{where } k = \frac{\ln(\frac{I_0}{I})}{\ln(\frac{I'_0}{I'})},\ \Delta\mu = \mu_{bm} - \mu_{st} \text{ and } \Delta\mu' = \mu'_{bm} - \mu'_{st}$$

It is evident that the precision with which it is possible to measure w_{bm} depends from the statistical error on the measure of the intensities at the two energy. It is easy to observe that at the higher energy we have an higher counting rate, because the μ's are lower, but the intensity ratio is smaller, whereas at the lower energy happen the opposite. In practice, we detect the Bone Mineral with the lower energy and the higher energy measurement is used in order to correct for the bone thickness. Therefore, we can optimize the lower energy in order to measure the w_{bm} and the higher energy to measure the bone size.

The real situation is more accurately depicted in figure 20, where the bone with a thickness dl is surrounded by soft tissues. In this case a more precise measurement can be done performing a linear transverse scan of the bone section. In this case we can use as reference, instead of the I_0 and I'_0, the intensities measured in that part of section in which the bone is absent I_{st} and I'_{st}, for example at the two side of bone section (see figure 20). In this case the system of equations become:

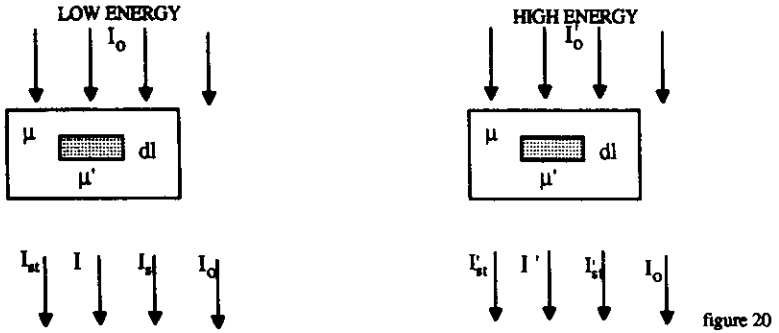


figure 20

$$\ln(\frac{I}{I_{st}}) = (\mu_{st}\, w_{st} + \mu_{bm}\, w_{bm})dl \qquad (44')$$

$$\ln(\frac{I'}{I'_{st}}) = (\mu'_{st}\, w_{st} + \mu'_{bm}\, w_{bm})dl$$

This is what it is done in practice, for example to measure the Bone Mineral of a small size bone, as that of the ulna in the leg, we can perform a scan at an energy of about 30 keV and another at 60 keV. In the case of spine, which is placed inside the trunk with an optical path of about 20 cm and an absorption that is very high higher energies are used in order to optimize the measurement.

It is possible to make the some consideration in the case of two white spectra obtained with two different KV and suitable filtration of the beam. This is done for dual energy tomography, using two mean energy, one that fall in the energy interval in which the photoelectric contribution is dominant and the other in an interval dominated by the Compton. This is actually an little more than an attractive prospective because the construction of a dual energy tomograph is a not easy job.

2.3 K-Edge Fluoroscopy.

Another very interesting possibility, that is used in practice in the radiographic exams, is the use of a contrast medium, i.e. the injection in the observed object of a substance having an higher absorption than the object matrix. Assuming that the injected substance is concentrated in some parts (substructures) of the object we can display better the shape and the size of these substructure. This is done in particular for the artery, injecting in the blood circle an iodinated medium, this technique is commonly know as angiography. There are several problems with angiography especially if we attempt to perform dynamic studies. In fact, if we want detect the perfusion of the iodinated medium in the circle we must inject it instantly, also in this case, if we use only one energy it is not easy to determine the size of the vassels and of the obstructions. Now there are several attempt to perform angiographic studies using two energies, one above the K- absorption edge of the iodine and one below. In particular the use in this case of a synchrotron source is very attractive because you can produce two intense monochromatic beams very near the K-absorption edge, so we can completely observe advantage of the discontinuity of the iodine linear absorption coefficient.

The K-edge fluoroscopy is the technique with which we can quantitatively determine the concentration of the contrast medium in a region of the observed object or the size of this region. In fact, if μ^T_1 and μ^I_1 are the linear absorption coefficients below the K-edge of the

tissue and of the iodinated medium respectively, and μ^T_2 and μ^I_2 above the K-edge, we can model these coefficients as follow:

$$\mu_1 = \mu^T_1\, w_T + \mu^I_1\, w_I \qquad (46)$$

$$\mu_2 = \mu^T_2\, w_T + \mu^I_2\, w_I$$

with $w_T + w_I = 1$

The two parts of the linear absorption coefficients are referred to the tissue (T) and to the contrast medium (I) respectively. Being the two energies very near we can assume that:

$$\mu^T_1 \approx \mu^T_2 \quad \text{and} \quad \mu^I_2 \approx \mu^I_1 + \Delta\mu_k$$

so $\mu_2 = \mu_1 + \Delta\mu_k\, w_I$

where the $\Delta\mu_k$ is the jump of the linear absorption coefficient at the K-edge.

We can so write the two following relations:

$$\ln(\frac{I_0}{I}) = \mu_1\, dl \qquad (47)$$

$$\ln(\frac{I'_0}{I'}) = (\mu_1 + \Delta\mu_k\, w_I)\, dl$$

solving the system for the w_I we obtain:

$$w_I = \frac{\mu_1}{\Delta\mu_k}\left[\frac{\ln(\frac{I_0}{I})}{\ln(\frac{I'_0}{I'})} - 1\right] \qquad (48)$$

The I_0, as in the case of the dual energy, are the intensity detected passing trough a section contenting only tissue. In this case the first absorptiometric relation, that at an energy below the K-edge, is used in order to correct for the thickness of the substructure with inside the contrast medium, whereas the weight fraction of the iodine is determined through the second relation.

Appendix I

The concept of resolving power of a measuring system. If x is the quantity we want esteem and f(x) the measured quantity, the resolving power x is related to the response function f(x) of the device and to the estimated standard error of the measure $\sigma(f(x))$ by the following relation:

$$\Delta x = \frac{b\left[\sigma^2(f(x_1)) + \sigma^2(f(x_0))\right]^{\frac{1}{2}}}{\frac{df(x)}{dx}} \qquad (1a)$$

Where the $\frac{df(x)}{dx}$ is the sensitivity and $\sigma^2(f(x))$ is the variance of the measured quantity. In the above equation if we assume that measured quantities $f(x_{1,0})$ have the same variance we can simply the relation as follow:

$$\Delta x = \frac{b\left[2\,\sigma^2(f(x))\right]^{\frac{1}{2}}}{\frac{df(x)}{dx}} \qquad (1a')$$

The parameter b can assume values (1-3) that are related to the level of uncertainty of the estimation the we can accept. In the case in which we measure independently n time this variation the σ^2, is the calculated variance. If we measure only one time the value f(x) we should make assumptions on the statistical distribution of this quantity. This can be easily done

in the case of the measurement of total counts in a certain time interval (or of the counting rate), because in this case we can assume a Poisson density distribution. In order to look inside the model we are now proposing for the study of a measuring system, let us speak one moment about the response function of x and/or γ) ray spectrometric system. It consists of a detector with an intrinsic ability to discriminate photons of different energies, an electronic chain and a multichannel analyzer. In fig.21 a typical spectrum as measured with this system is presented. It consist of discrete structure, the peaks, and a continuous background which originate, with different mechanism, either in the spectrometer either outside it. In any case this background cant be suppressed, as can't be suppressed the noise that produces the fluctuation in the total number of counts in a particular peak. Let me consider now the measure of an unknown activity of one particular radionuclide. In this case we can have two different possibilities (i) the activity is very low and in this case we wish know if some activity is present in our sample or (ii) the case in which the presence of the radionuclide is evident and we want measure it or variation of it. In the fist case the variance that we must consider in the above equation is that of the background counts, whereas in the second case the variance to be used is that of the detected counts. In particular we can stress that in the first case we are measuring the minimum detectable activity, whereas in the second case we are estimating the minimum detectable variation. These concepts can be extrapolated usefully to the absorptiometric measurements.