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SMR.648 - 27

SECOND AUTUMN WORKSHOP ON MATHEMATICAL ECOLOGY

(2 - 20 November 1992)

"The Role of the Biosphere in Climate Change"

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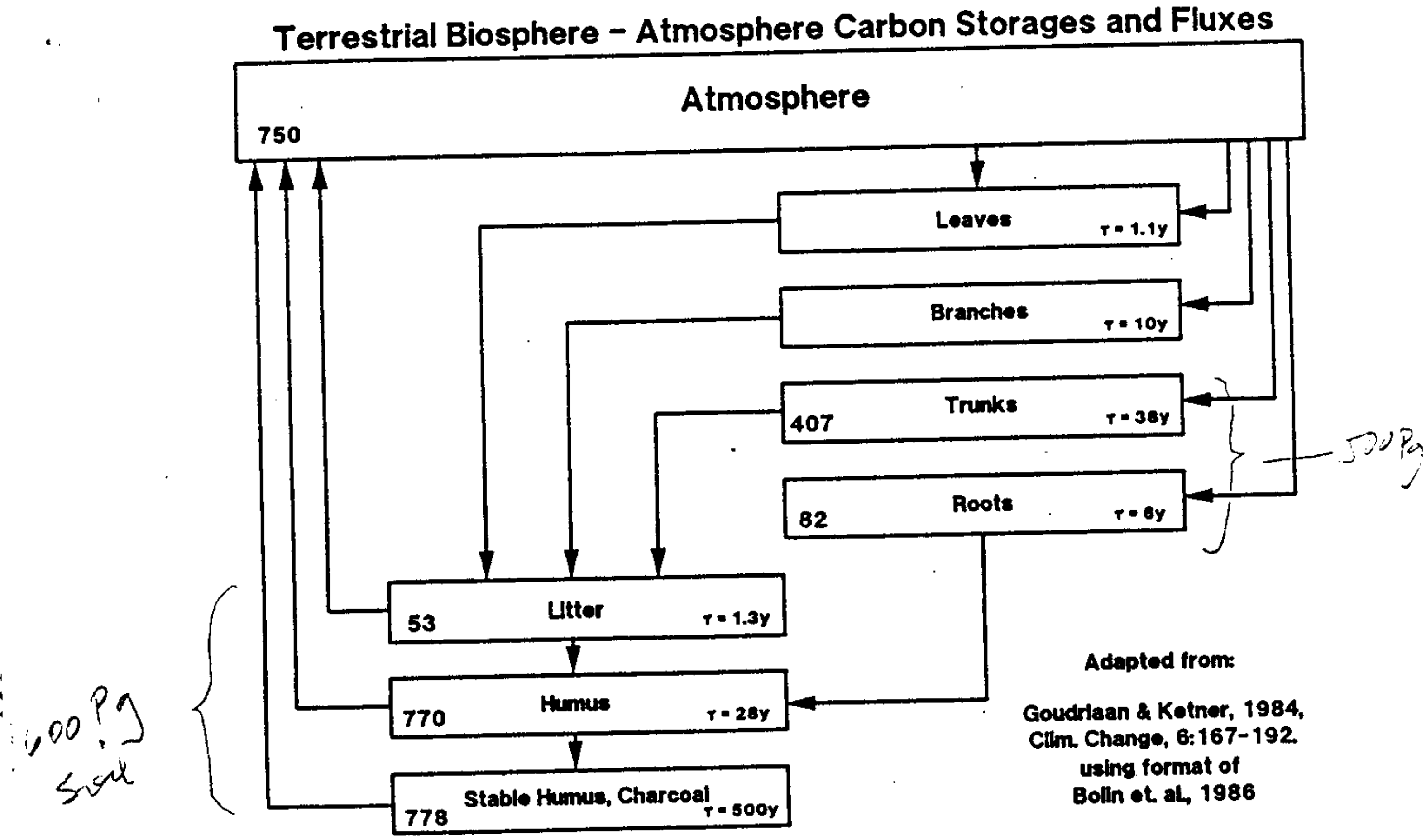
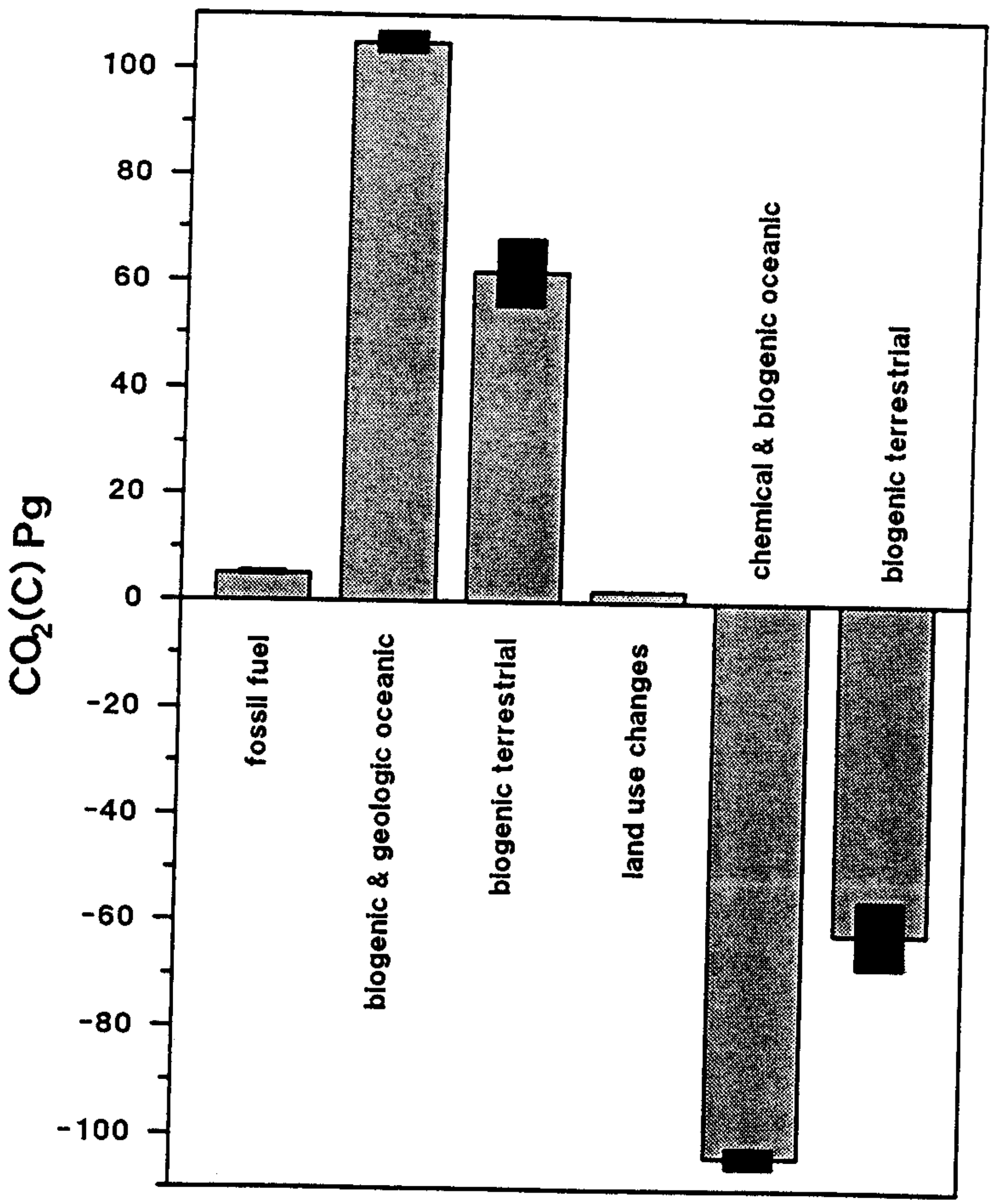
These are preliminary lecture notes, intended only for distribution to participants.

Example of C budget.



Global Sources and Sinks of CO_2(C)



- Biospheric flux > 30x anthropogenic
- Changing conditions could imbalance biospheric flux even a small amount & net biospheric fluxes could dominate anthropogenic

Temporal Patterns

Annual cycle

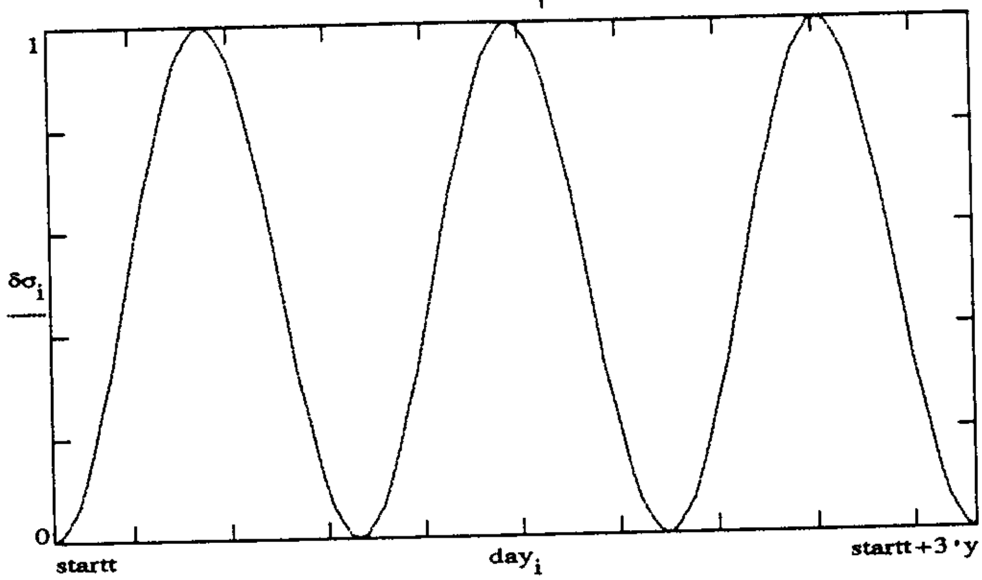
daily phase angle

$$\omega \equiv \frac{2 \cdot \pi}{y}$$

seasons

$$\delta\sigma_i := \frac{1 - \cos\left[\omega \cdot day_i\right]}{2}$$

Seasonality Driver



Litter fall relative intensity

$begin\_fall\_date := 240 \cdot d$ $\quad end\_fall\_date := 285 \cdot d$

$fall\_period := end\_fall\_date - begin\_fall\_date$

$$\omega_{\phi\alpha\lambda\lambda} := 2 \cdot \frac{\pi}{fall\_period}$$

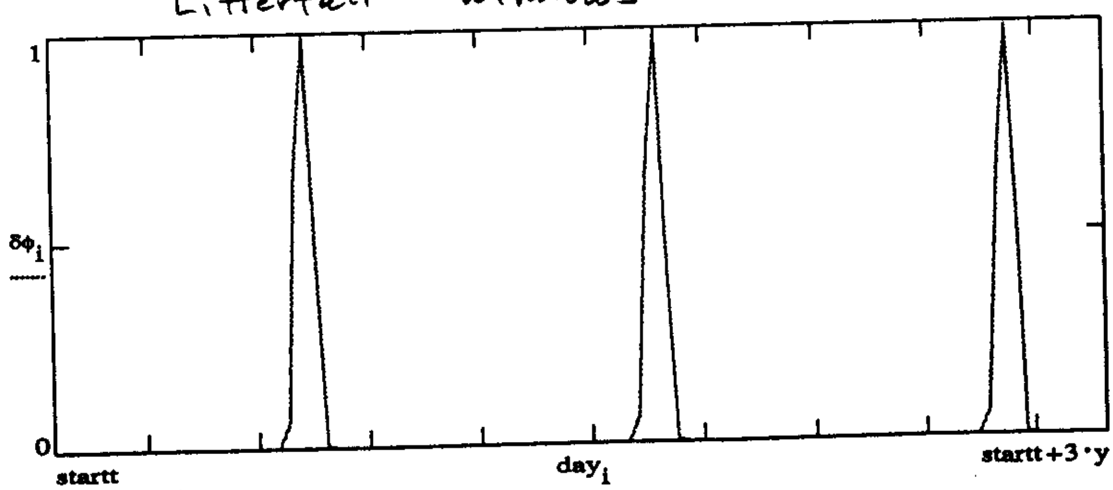
$beginfall(z) := if\left[mod\left[day_z, y\right] < begin\_fall\_date, 0, 1\right]$

$endfall(z) := if\left[mod\left[day_z, y\right] > end\_fall\_date, 0, 1\right]$

$fall(z) := beginfall(z) \cdot endfall(z)$

$$\delta\phi_i := \frac{1 - \cos\left[\omega_{\phi\alpha\lambda\lambda} \cdot \left[mod\left[day_i, y\right] \ldots + (-begin\_fall\_date)\right] \cdot fall(i)\right]}{2}$$

Litterfall "Windows"



Constants

$\mu := 0.0009 \cdot d^{-1}$ maximum photosynthesis rate constant referenced to perennial standing stock

$K := 275 \cdot Pg$ half saturation constant for photosynthesis referenced to total atmospheric C

$lf := 0.2 \cdot d^{-1}$ maximum litterfall rate constant

$wf := 0.02 \cdot y^{-1}$ woody litter annual turnover rate constant

$ploss := 0.02 \cdot y^{-1}$ annual natural loss of perennial vegetation

$ap := \frac{ploss}{\left[0.114 \cdot y^{-1}\right]}$ annual contribution to perennial vegetation, 0.114/y is ratio to achieve steady state at 1800 AD conditions

$f1 := 0.80$ fraction of annual litter production into organic pool 1

$f2 := 0.161$ fraction of annual litter production into organic pool 2

$f3 := 0.029$ fraction of annual litter production into organic pool 3

$f4 := 0.009$ fraction of annual litter production into organic pool 4

$f5 := 0.001$ fraction of annual litter production into organic pool 5

$k_{d1} := 1.4 \cdot y^{-1}$ decomposition rate constant for soil organic carbon pool 1

$$k_{d1}^{-1} = 0.714286 \cdot y$$

$k_{d2} := 0.2 \cdot y^{-1}$ decomposition rate constant for soil organic carbon pool 2

$$k_{d2}^{-1} = 5 \cdot y$$

$k_{d3} := 0.04 \cdot y^{-1}$ decomposition rate constant for soil organic carbon pool 3

$$k_{d3}^{-1} = 25 \cdot y$$

$k_{d4} := 0.004 \cdot y^{-1}$ decomposition rate constant for soil organic carbon pool 4

$$k_{d4}^{-1} = 2.5 \cdot 10^{2} \cdot y$$

$k_{d5} := 0.001 \cdot y^{-1}$ decomposition rate constant for soil organic carbon pool 5

$$k_{d5}^{-1} = 1 \cdot 10^{3} \cdot y$$

$Vp := 1700 \cdot m \cdot y^{-1}$ piston velocity for surface oceanic gas exchange (1700 m/y — Broecker (1974), 1000 m/y S&M, 1981)

$S := .5 \cdot m \cdot d^{-1}$ sinking velocity for particulate C

$fwl := 0.66667$ fraction of woody litter going into soil organic pool 2

Constant Quantities and Initial Values

$KH := 10^{-1.53} \cdot molC \cdot l^{-1} \cdot atm^{-1}$

Henry's constant for CO2, 298 K. Relates pCO2 (atm) to H2CO3*; Given as molC per liter per atm, but note that there is one mol of C per mol of any species of inorganic C

$K1 := 10^{-6}$

$K2 := 10^{-9.11}$

equilibrium constants for inorganic carbon species

$pH := 8.2$

$$\alpha_0 := \left[1 + \frac{K1}{10^{-pH}} + \frac{K1 \cdot K2}{\left[10^{-pH}\right]^2}\right]^{-1}$$

fraction of inorganic carbon diffusing into seawater that remains as H2CO3*

$\alpha_0 = 5.586974 \cdot 10^{-3}$

$$f_{pf} := \frac{(29 \cdot g)}{\left[12 \cdot g \cdot 52 \cdot 10^{17} \cdot kg\right]}$$

conversion factor for mass to mole fraction of C in the atmosphere

$PAconv := f_{pf} \cdot (760 \cdot torr - 23.77 \cdot torr)$

conversion factor for mass of C in the atmosphere to partial pressure of CO2 (atm) at sea level. 29 g/mol atmosphere assumed. 52×10^{17} kg is total mass of atmosphere (S&M, 1981), 23.77 torr is partial pressure of water vapor in saturated atmosphere at 298 K.

$SO := 361 \cdot 10^6 \cdot km^2$

surface area of the ocean

$DO := 75 \cdot m$

depth of the mixed ocean

$VO := SO \cdot DO$

volume of the mixed ocean

← This mass corresponds to projected atm. mass at 2000 AD

$Ma_0 := 880.6456 \cdot Pg$

initial mass of atmospheric carbon (600 Pg corresponds to about the year 1800)

$PAconv \cdot Ma_0 = 3.964738 \cdot 10^2 \cdot ppmv$

initial atmospheric C concentration

$KH \cdot PAconv \cdot Ma_0 = 0.011701 \cdot molC \cdot m^{-3}$

ocean H2CO3* concentration calculated to be in equilibium with the atmospheric concentration this initialization

$Mva_0 := 8.925243 \cdot Pg$

initial mass of annual vegetation (including leaves of woody plants)

$Mo1_0 := 116.9627 \cdot Pg$

initial mass of quickest turnover soil organic carbon

$Mo2_0 := 242.9495 \cdot Pg$

initial mass of second quickest turnover soil organic carbon

$Mo3_0 := 394.8443 \cdot Pg$

initial mass of third slowest turnover soil organic carbon

$Mo4_0 := 356.5545 \cdot Pg$

initial mass of second slowest turnover soil organic carbon

$Mo5_0 := 151.3712 \cdot Pg$

initial mass of slowest turnover soil organic carbon

$MO_0 := \left[KH \cdot Ma_0 \cdot PAconv \cdot VO\right] \cdot \left[\alpha_0\right]^{-1}$

computation of initial mass of CO2 in the mixed upper layer of the ocean, equil with atmosphere

$MO_0 = 6.804364 \cdot 10^2 \cdot Pg$

mass this initialization

$$\frac{MO_0}{VO} \cdot \alpha_0 = 1.170077 \cdot 10^{-5} \cdot molC \cdot l^{-1}$$

concentration this initialization calculated directly; should be identical to initialization from Ma0

$MO_0 := 656.2087 \cdot Pg$

$$L(MVA,f) := lf \cdot MVA \cdot f$$

L(v) is litterfall flux

$$D(w1,w2,w3,w4,w5,s) := s \cdot \left[k_{d1} \cdot w1 + k_{d2} \cdot w2 + k_{d3} \cdot w3 + k_{d4} \cdot w4 + k_{d5} \cdot w5 \right]$$

D( ) is decomposition flux

$$P(MA,MWV,s) := \frac{\mu \cdot MWV \cdot MA \cdot s}{K + MA}$$

P( ) is photosynthesis flux

atmospheric carbon

$$DMa(MA,MWV,MO,w1,w2,w3,w4,w5,s,u) := \begin{bmatrix} F(u) + B(u) + D(w1,w2,w3,w4,w5,s) \ldots \\ + -SO \cdot \left[Vp \cdot \left[KH \cdot MA \cdot PAconv - \frac{\alpha_0 \cdot MO}{VO} \right] \right] \ldots \\ + -P(MA,MWV,s) \end{bmatrix}$$

ODE's defining the system

carbon in woody vegetation

$$DMwv(MA,MWV,s,u) := ap \cdot P(MA,MWV,s) - ploss \cdot MWV - B(u)$$

carbon in annual vegetation

$$DMva(MA,MVA,MWV,f,s) := (1 - ap) \cdot P(MA,MWV,s) - L(MVA,f)$$

pool 1 soil organic carbon

$$DMo1(MVA,w1,f,s) := f1 \cdot L(MVA,f) - k_{d1} \cdot s \cdot w1$$

pool 2 soil organic carbon (assumed to receive the fraction fwl of the woody litter)

$$DMo2(MVA,MWV,w2,f,s) := f2 \cdot L(MVA,f) + fwl \cdot ploss \cdot MWV - k_{d2} \cdot s \cdot w2$$

pool 3 soil organic carbon (assumed to receive the remainder of woody litter

$$DMo3(MVA,MWV,w3,f,s) := f3 \cdot L(MVA,f) + (1 - fwl) \cdot ploss \cdot MWV - k_{d3} \cdot s \cdot w3$$

pool 4 soil organic carbon

$$DMo4(MVA,w4,f,s) := f4 \cdot L(MVA,f) - k_{d4} \cdot s \cdot w4$$

pool 5 soil organic carbon

$$DMo5(MVA,w5,f,s) := f5 \cdot L(MVA,f) - k_{d5} \cdot s \cdot w5$$

oceanic carbon

$$DMO(MA,MO) := SO \cdot \left[Vp \cdot \left[KH \cdot MA \cdot PAconv - \frac{\alpha_0 \cdot MO}{VO} \right] - S \cdot rio \cdot \frac{\alpha_0 \cdot MO}{VO} \right]$$

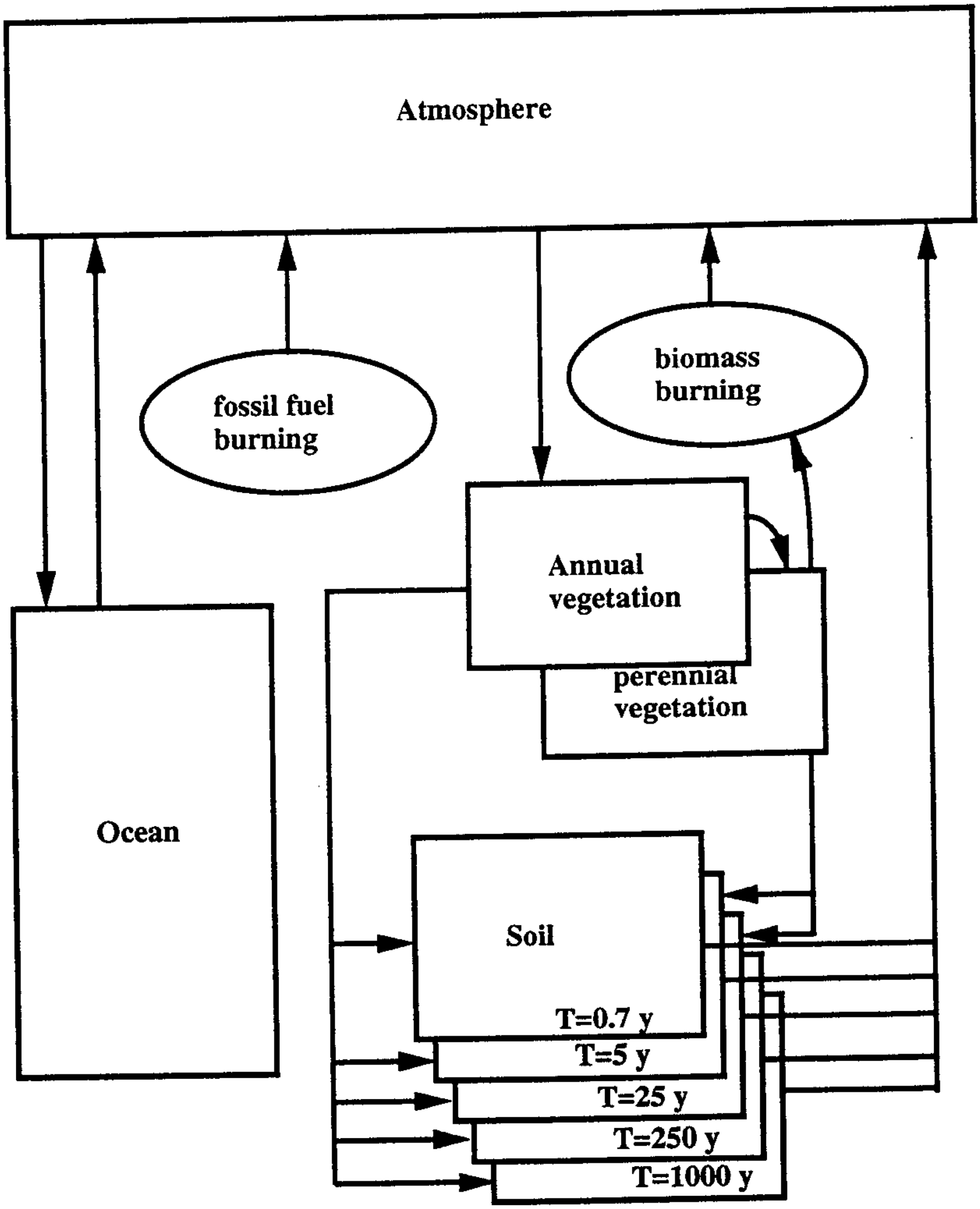
Integration

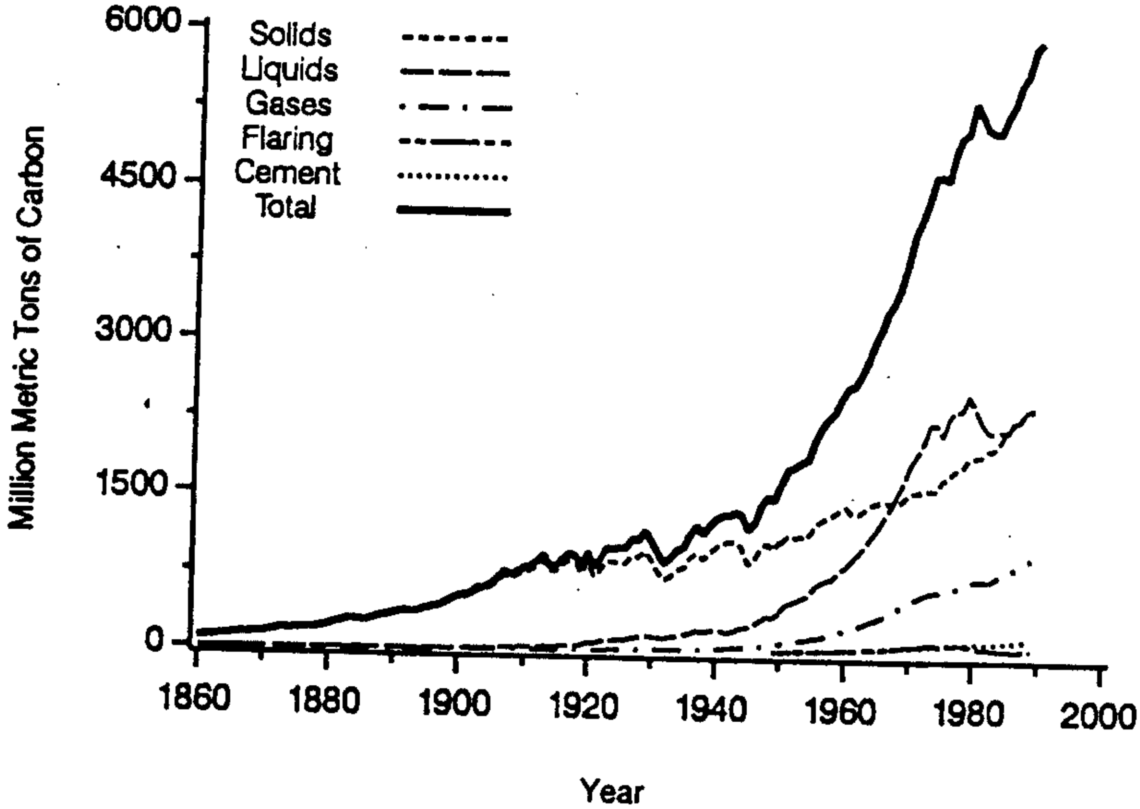
$$\begin{bmatrix} Ma_{[i+1]} \\ Mva_{[i+1]} \\ Mwv_{[i+1]} \\ Mo1_{[i+1]} \\ Mo2_{[i+1]} \\ Mo3_{[i+1]} \\ Mo4_{[i+1]} \\ Mo5_{[i+1]} \\ MO_{[i+1]} \end{bmatrix} := \begin{bmatrix} Ma_i + dh \cdot DMa\left[Ma_i, Mwv_i, MO_i, Mo1_i, Mo2_i, Mo3_i, Mo4_i, Mo5_i, \delta\sigma_i, day_i\right] \\ Mva_i + dh \cdot DMva\left[Ma_i, Mva_i, Mwv_i, \delta\phi_i, \delta\sigma_i\right] \\ Mwv_i + dh \cdot DMwv\left[Ma_i, Mwv_i, \delta\sigma_i, day_i\right] \\ Mo1_i + dh \cdot DMo1\left[Mva_i, Mo1_i, \delta\phi_i, \delta\sigma_i\right] \\ Mo2_i + dh \cdot DMo2\left[Mva_i, Mwv_i, Mo2_i, \delta\phi_i, \delta\sigma_i\right] \\ Mo3_i + dh \cdot DMo3\left[Mva_i, Mwv_i, Mo3_i, \delta\phi_i, \delta\sigma_i\right] \\ Mo4_i + dh \cdot DMo4\left[Mva_i, Mo4_i, \delta\phi_i, \delta\sigma_i\right] \\ Mo5_i + dh \cdot DMo5\left[Mva_i, Mo5_i, \delta\phi_i, \delta\sigma_i\right] \\ MO_i + dh \cdot DMO\left[Ma_i, MO_i\right] \end{bmatrix}$$

computations; dh ≈ 9d

esm for Global Carbon Storages and Flows

extremely simple model





Global CO_2 emissions from fossil fuel burning, cement production, and gas flaring, 1860–1989.

$F_{max} := 40 \cdot Pg \cdot y^{-1}$ maximum annual CO2 input to the atmosphere from future fossil fuel oxidation

$F_0 := 40 \cdot Tg \cdot y^{-1}$ assumed 1800 rate of fossil fuel oxidation

$$A := \frac{[F_{max} - F_0]}{F_0}$$

$a := 0.009 \cdot Pg \cdot y^{-2}$

$b := 0.02782 \cdot y^{-1}$

assumed linear (a) and logistic (b) rates of fossil fuel oxidation

$$F(z) := a \cdot (z) + \frac{F_{max}}{[1 + A \cdot [e^{-b \cdot z}]]} 0$$

$F1_{max} := 33 \cdot Pg \cdot y^{-1}$

$F1_0 := 0.1 \cdot Tg \cdot y^{-1}$

$$A1 := \frac{[F1_{max} - F1_0]}{F1_0}$$

$c := 0.0485 \cdot y^{-1}$

$cc := .0007 \cdot y^{-1}$

$Pre := 2 \cdot Pg \cdot y^{-1}$

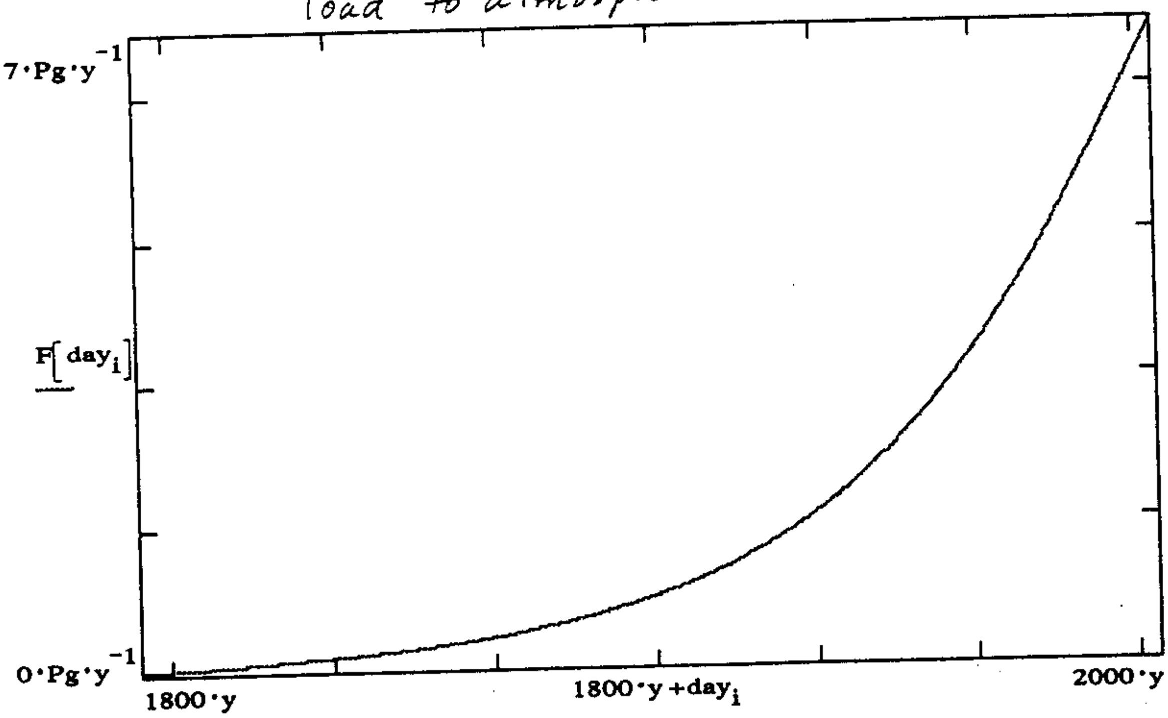
$$F(z) := \frac{F_{max}}{[1 + A \cdot [e^{-b \cdot z}]]} - \frac{F1_{max}}{[1 + A1 \cdot [e^{-c \cdot z}]]} + [Pre \cdot [1 - e^{-[cc \cdot [z]]}]]$$

other functions used for other scenarios

$F(60 \cdot y) = 0.291831 \cdot Pg \cdot y^{-1}$ $F(180 \cdot y) = 4.837744 \cdot Pg \cdot y^{-1}$

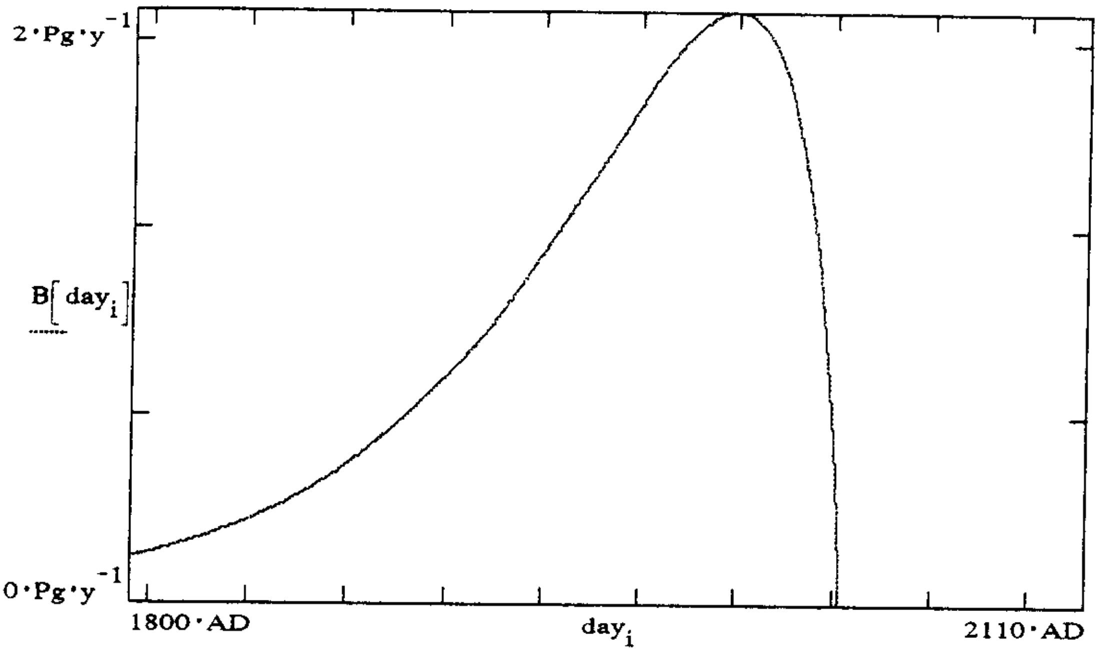
$F(150 \cdot y) = 2.496032 \cdot Pg \cdot y^{-1}$ $F(192 \cdot y) = 6.095853 \cdot Pg \cdot y^{-1}$

Approximation to fossil fuel etc. annual load to atmosphere.

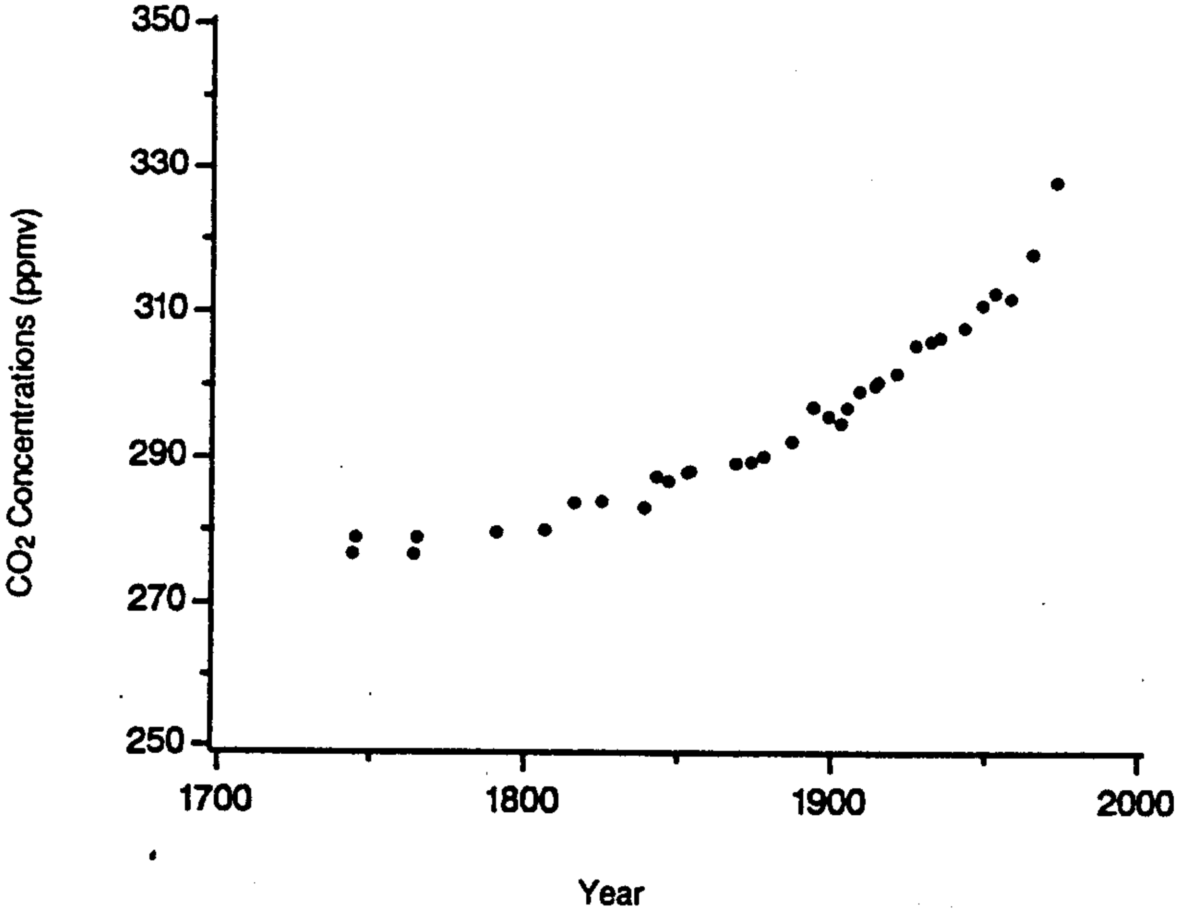


$B(z) := \mathrm{if}\left[z > y_{end}, 0 \cdot Pg \cdot y^{-1}, \left[\left[B_{max} \cdot e^{\left[a \cdot \left[z - y_{max}\right]\right]}\right]\right] \cdot \left[\frac{\left[y_{end} - z\right]}{\left[y_{end} - y_{max}\right]}\right]^{c}\right]$

$B\left[day_0\right] = 0.151232 \cdot Pg \cdot y^{-1}$



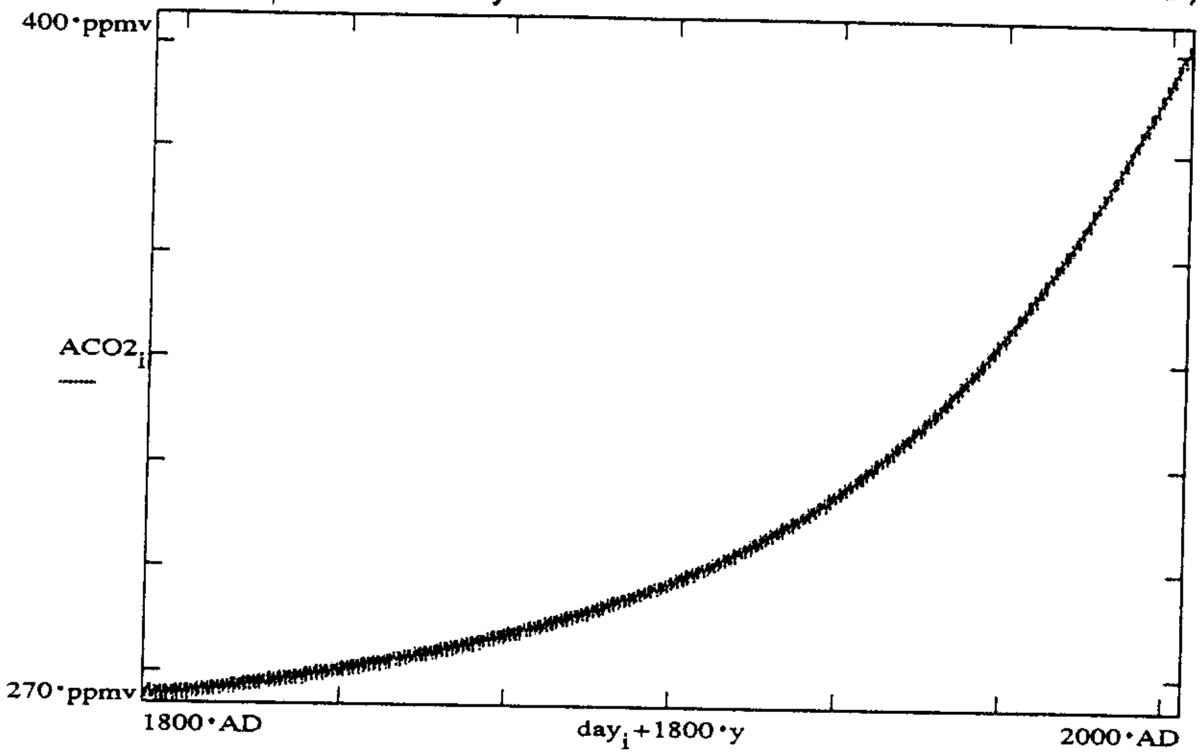
- Function approximating biomass burning annual flux of CO_2 into atmosphere.
- This function also serves as loss rate to perennial vegetation.



Atmospheric CO_2 derived from the Siple ice core.

1. model "calibrated" to steady state with conditions of 1800 A.D. all constituents stable for > 20y simulation

2. Run with fossil fuel & biomass burning inputs & losses (to perennial vegetation) for 1800 AD to 2000 AD.



now := 192·36
y82 := 182·36

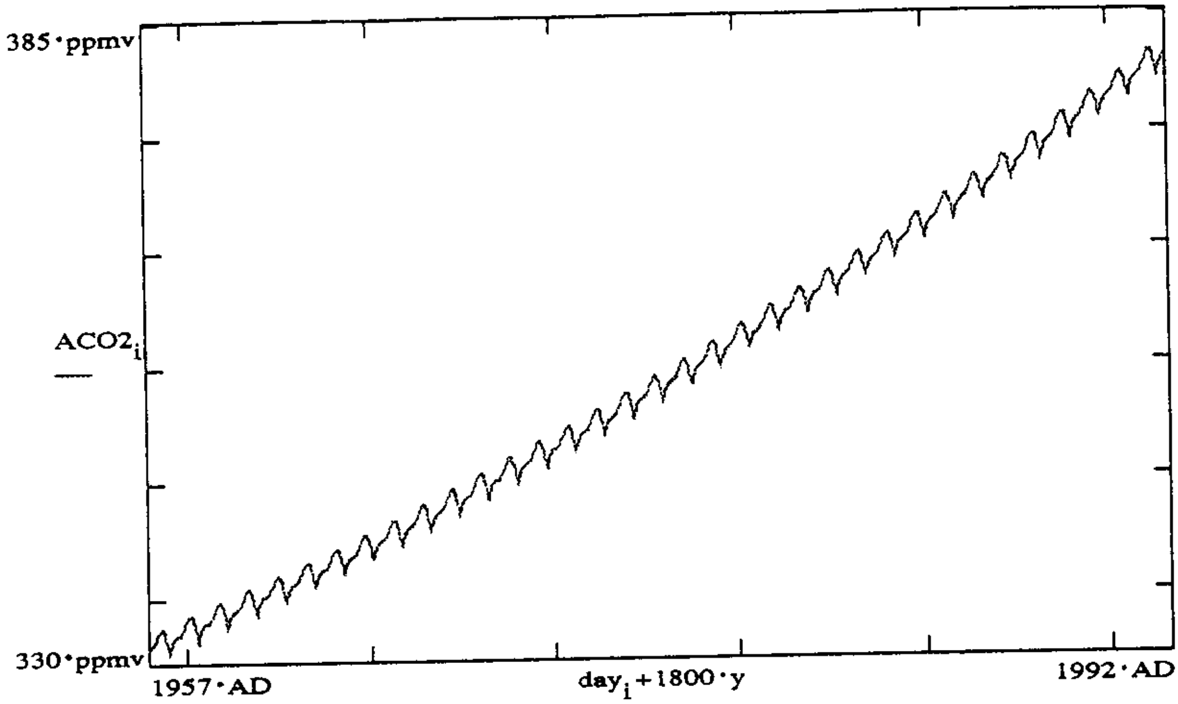
now = 6.912·10^3
y82 = 6.552·10^3

last_decade := y82 .. now

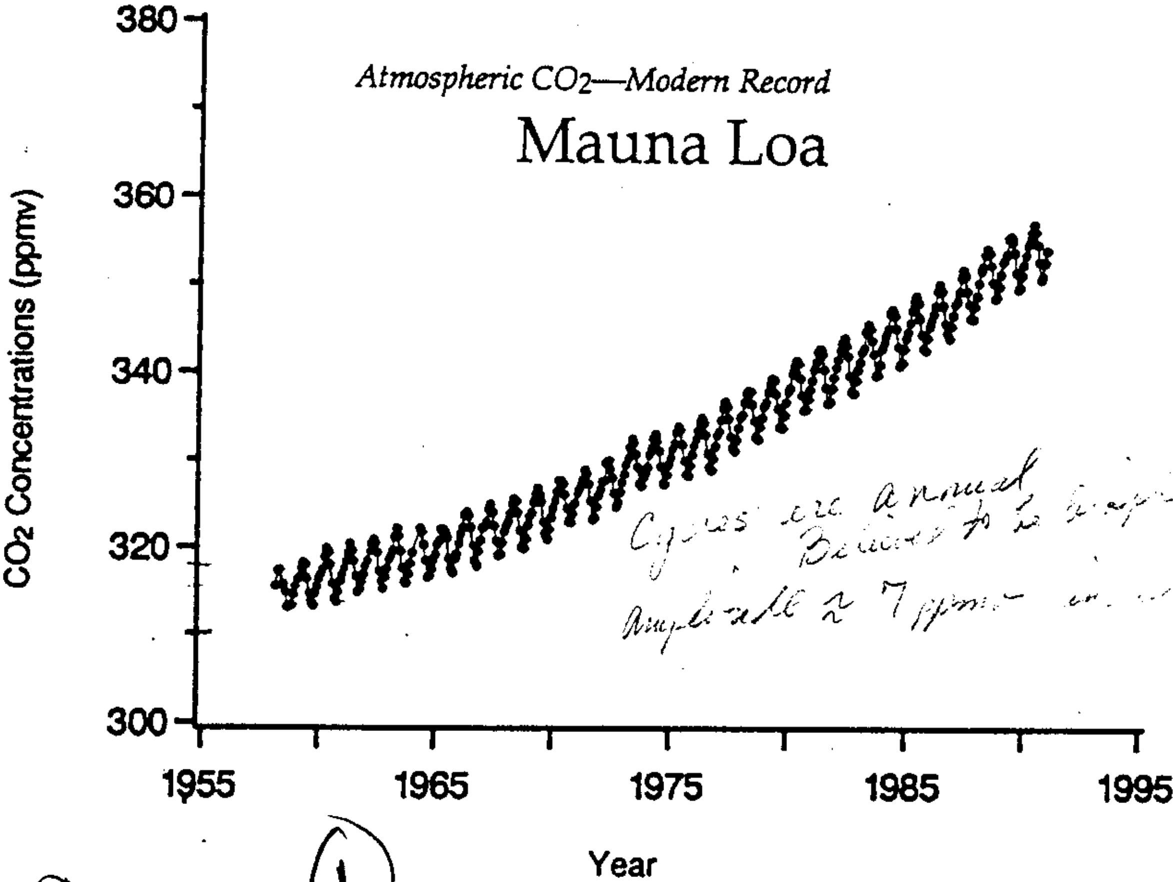
$$\alpha := \frac{Ma_{now} - Ma_{y82}}{dh \cdot \left[\left[\sum_{last\_decade} F\left[day_{last\_decade}\right]\right]\right]}$$

$\alpha = 0.679786$ Airborne fraction

Note $\alpha \approx$ .08 higher than actual.



Details of output for 1957 - 1992 AD, interval of Keeling Mauna Loa atm CO_2 record.

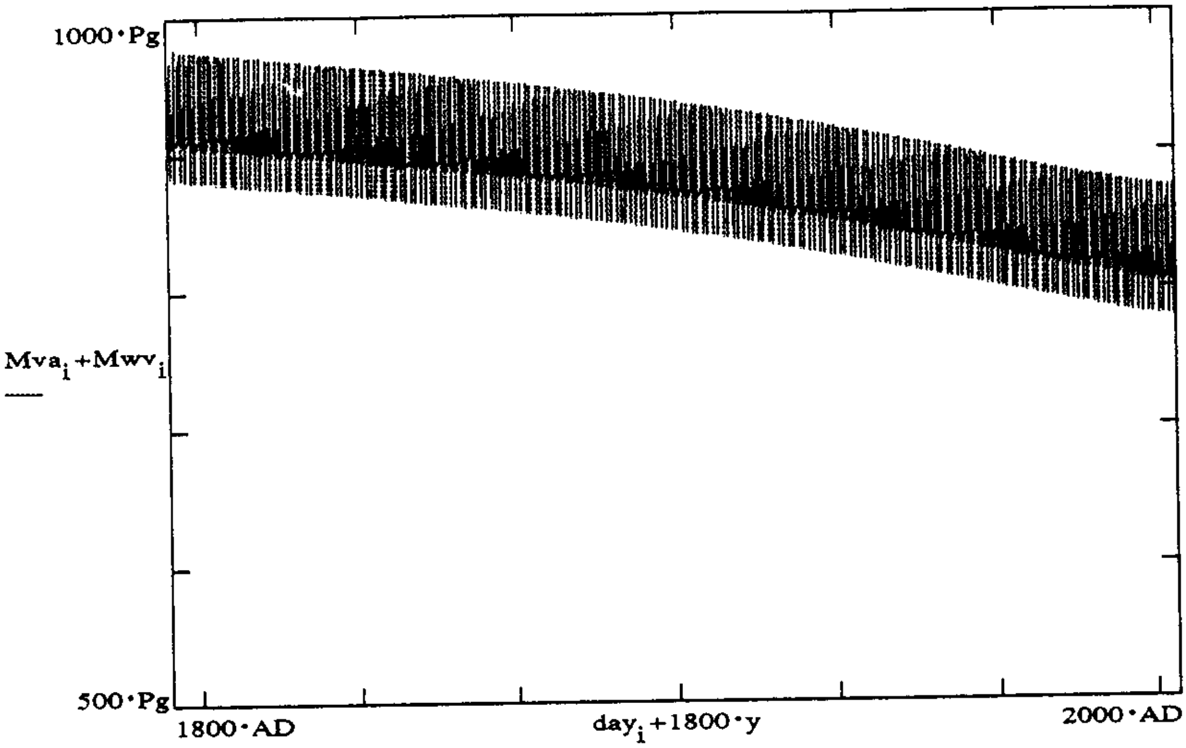


Monthly atmospheric CO_2 concentrations.

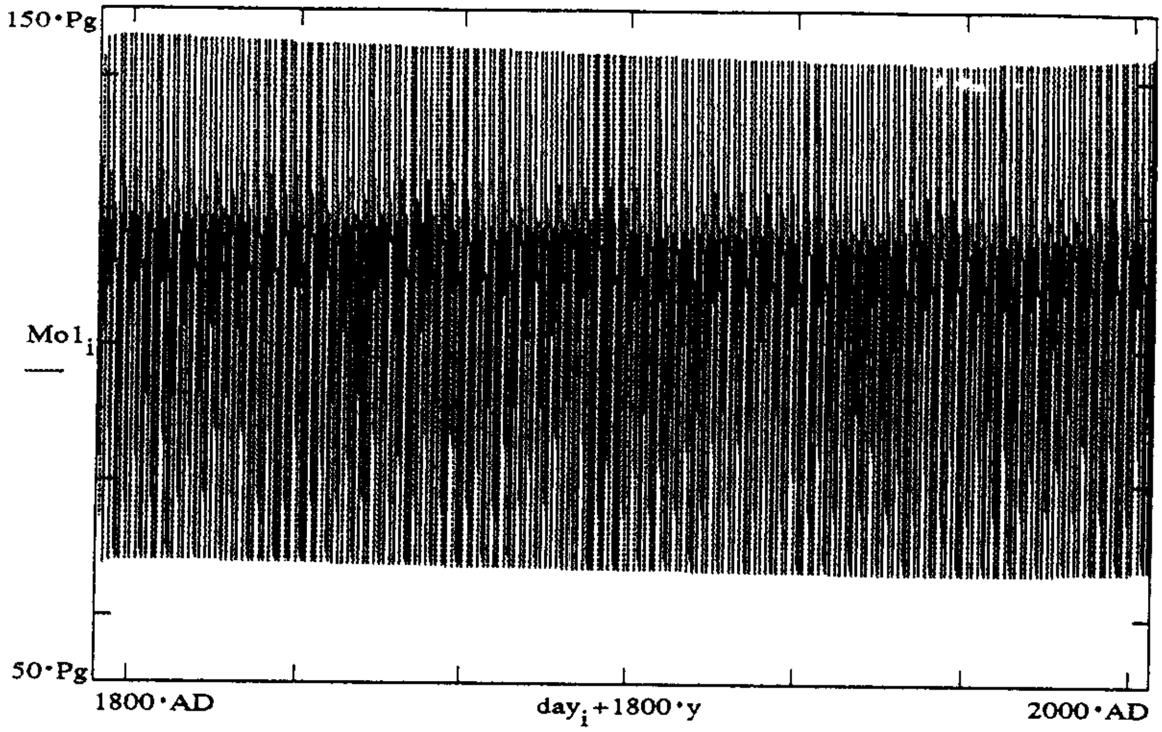
Cycles are annual. Believed to be biosphere. Amplitude ≈ 7 ppm increasing at 0.8%/y

(1) Overall Increase is around 0.4% - 0.5% per year (3 - 4 Pg)

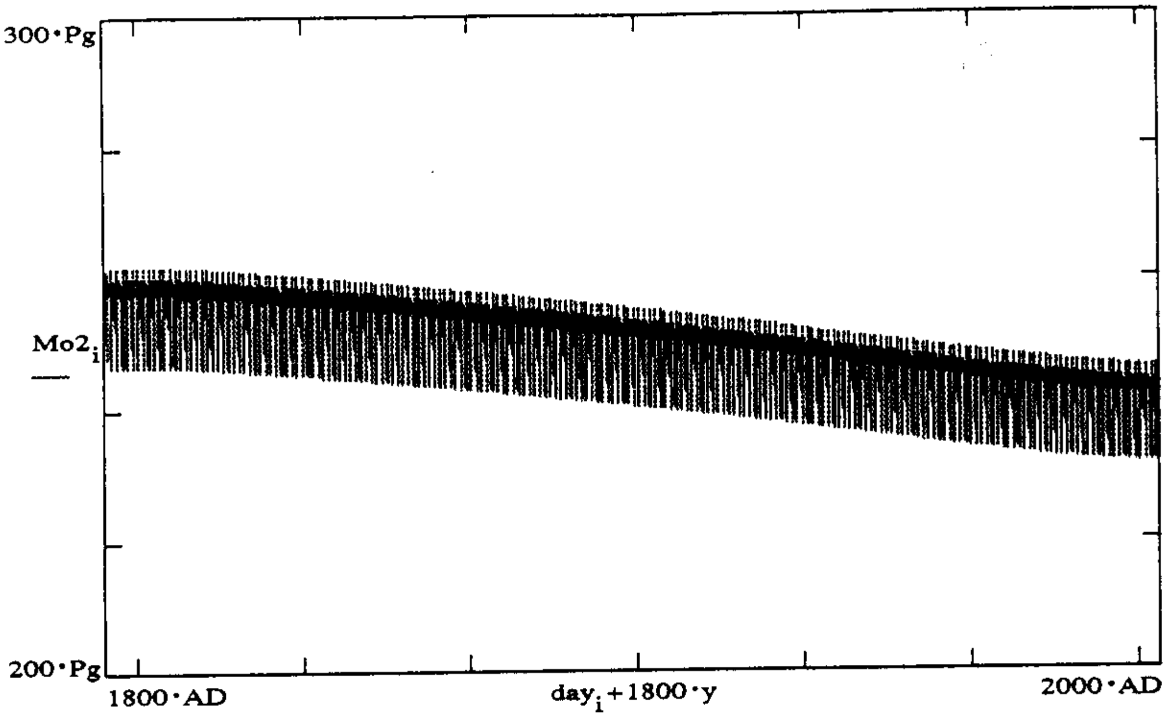
(2) But compare to FF input & note that increase is < input, never mind the BB —



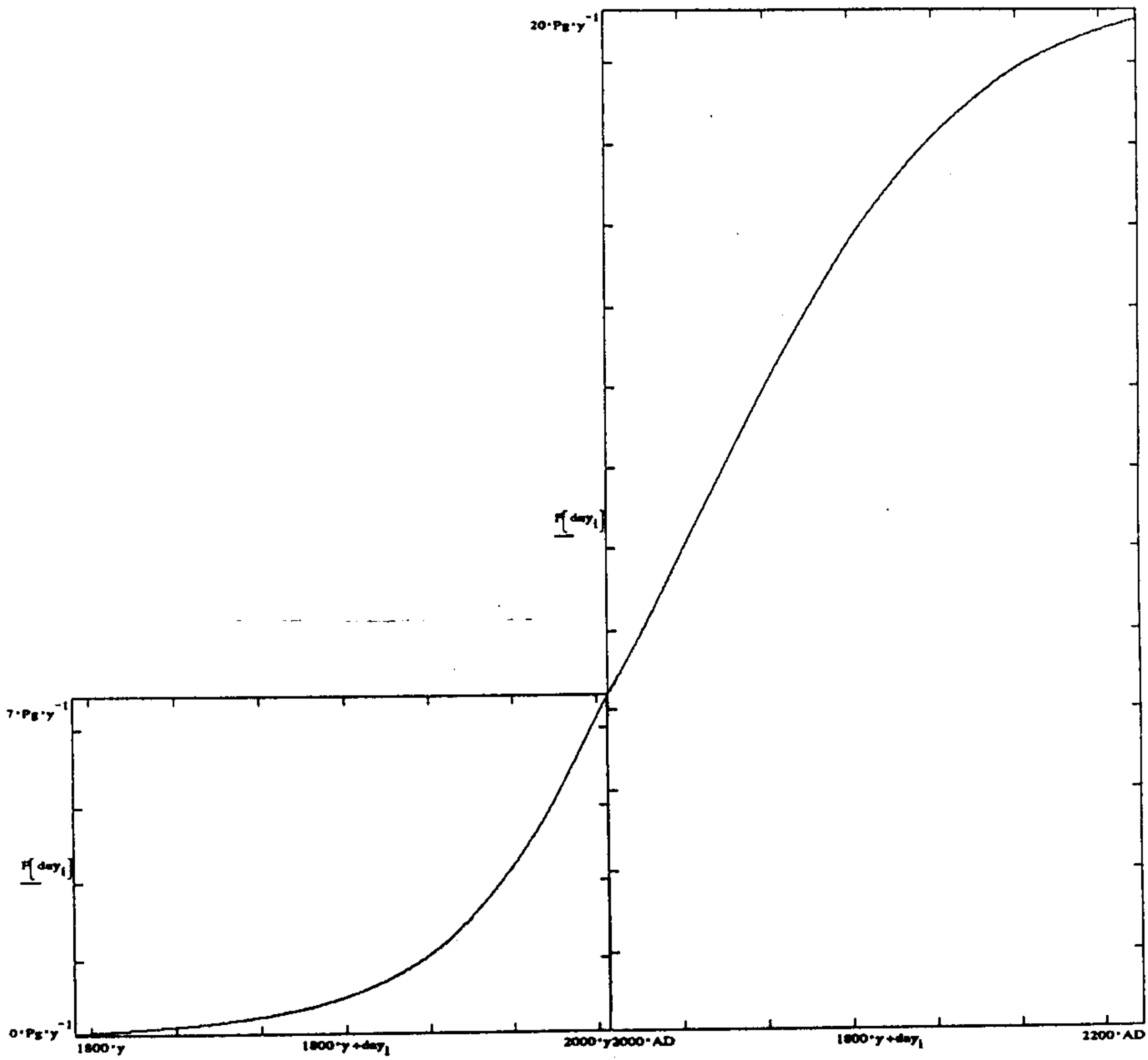
Calculated vegetation. Width of band is magnitude of annual fluctuations. Reduction in overall magnitude due to deforestation (equiv to biomass burning flux in this esm).



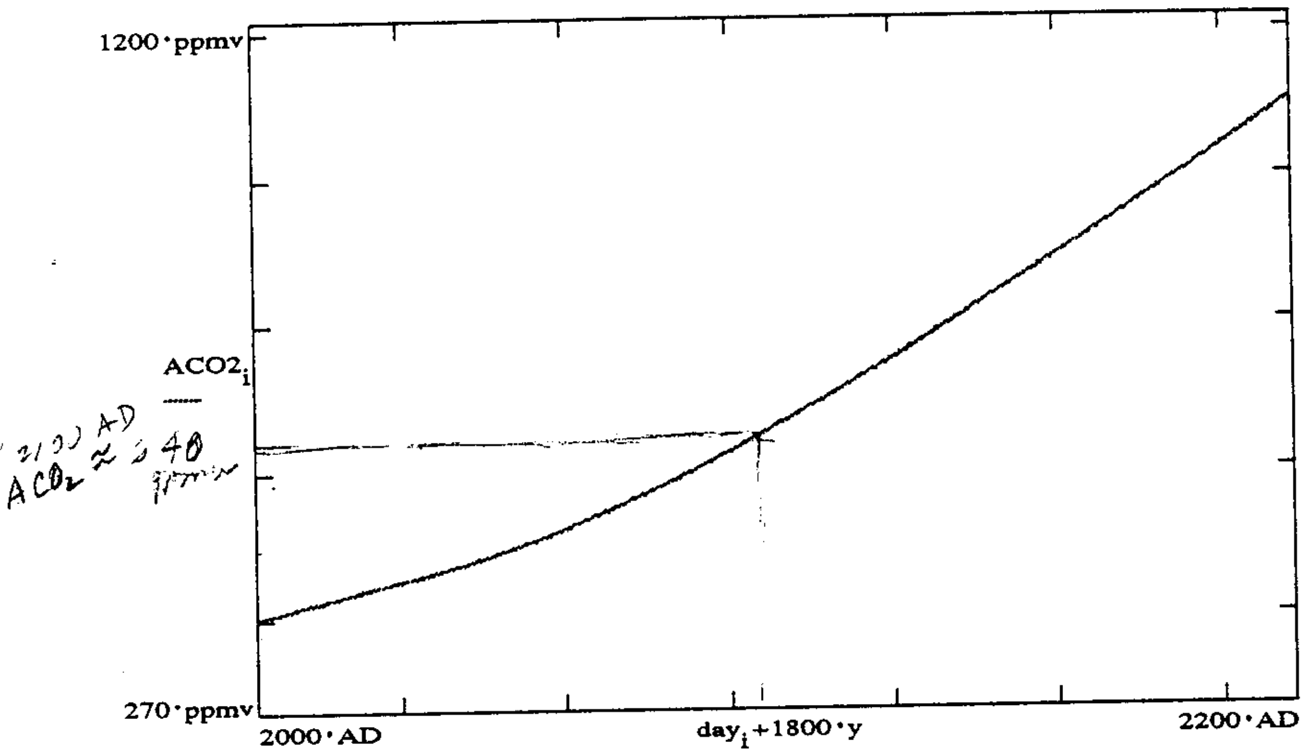
Fastest turnover time soil organic C pool.
Slight increase with t → 2000 AD due to
C fertilization of annual vegetation,



5 yr. turnover time soil organic C. Note decrease due to decrease in inputs from annual vegetation & slowing of decrease with $t \rightarrow$ 2000 AD from C-fertilized inputs from annual vegetation. Higher turnover-time components respond similarly, but with lower magnitudes.



Whole spectrum of fossil fuel inputs for scenario with carrying-capacity-limited vegetation & following scenario without carrying-capacity limitation



$y2100 := 100 \cdot 36$
$y2110 := 110 \cdot 36$

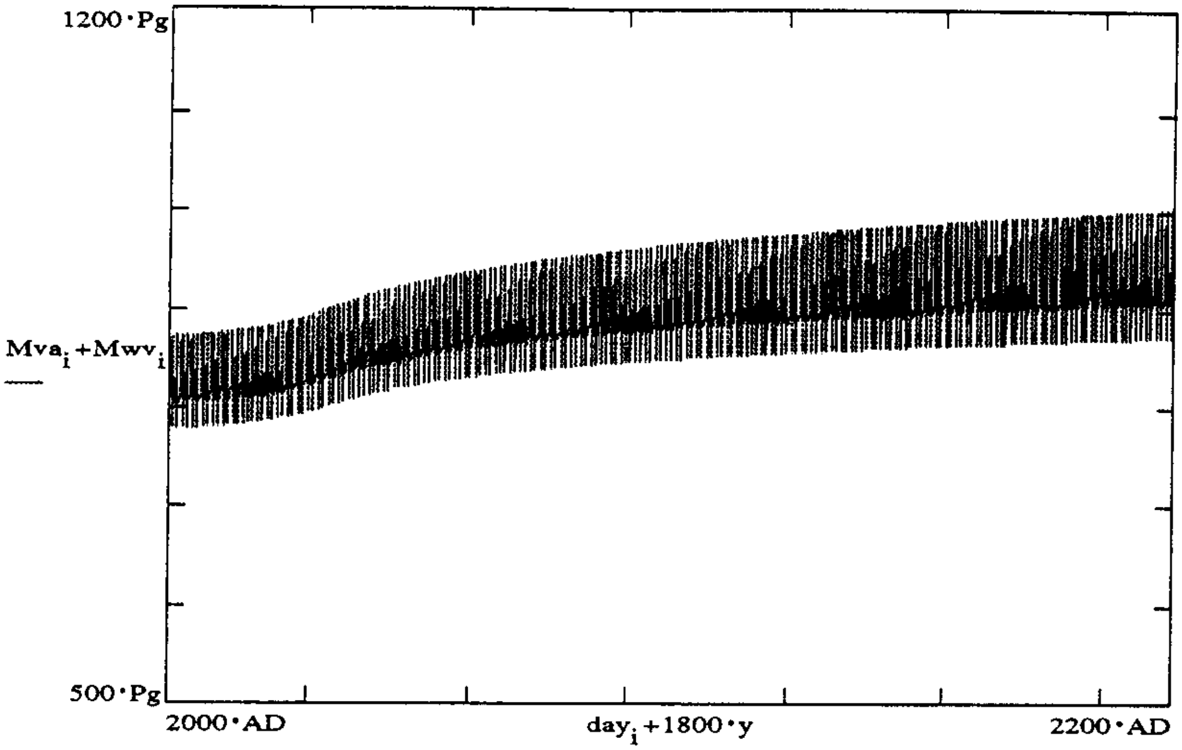
$y2100 = 3.6 \cdot 10^3$
$y2110 = 3.96 \cdot 10^3$

$decade := y2100 .. y2110$

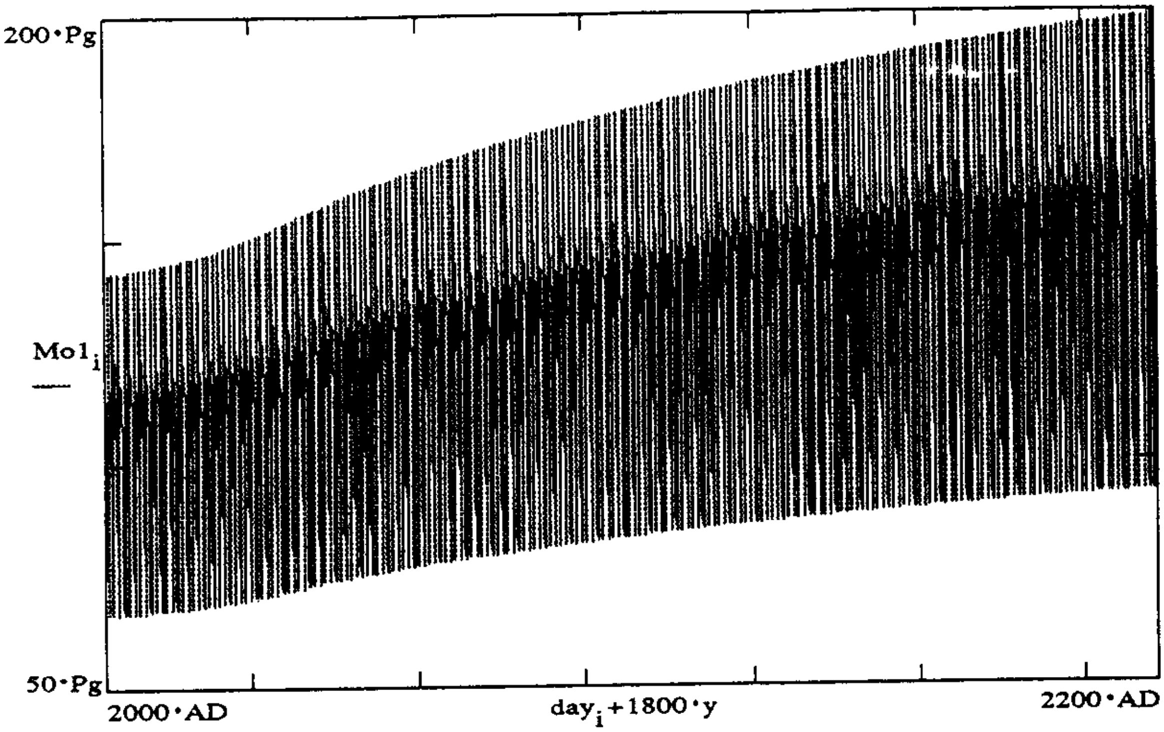
$$\alpha := \frac{Ma_{y2110} - Ma_{y2100}}{dh \cdot \left[\left[\sum_{decade} F\left[day_{decade}\right]\right]\right]}$$

$\alpha = 0.518296$ Airborne fraction

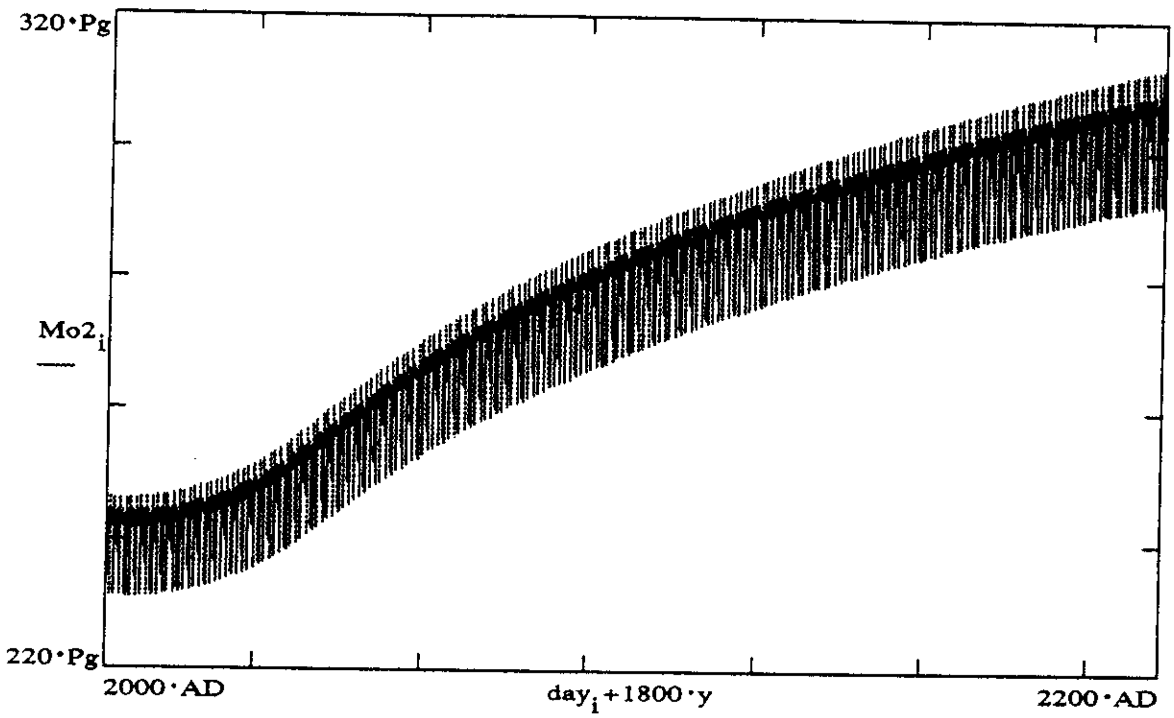
note reduction in α below 1992 level.



With carrying-capacity limitation regrowth to about level of 1800 AD occurs by 2200 AD.

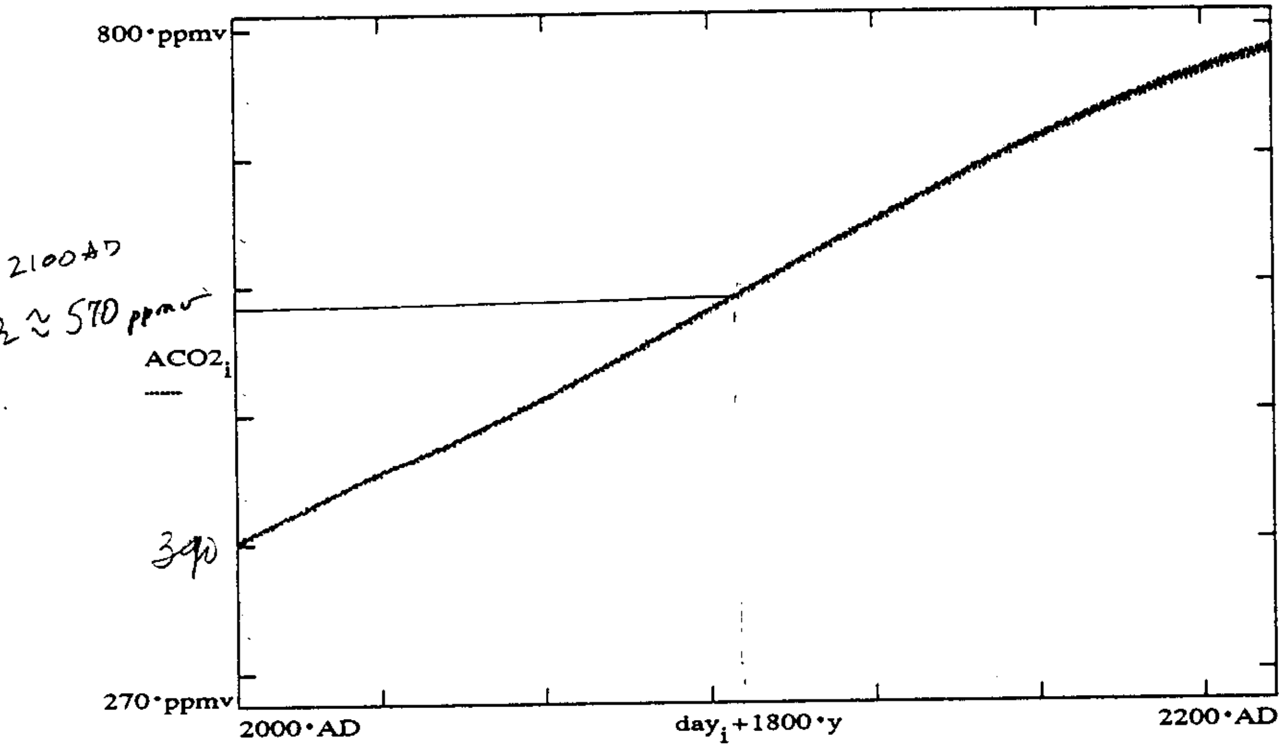


Short-turnover-time soil organic C increases by greater proportion than vegetation & amplitude of annual fluctuations increases.



5-year turnover-time organic C-

Calculated ACO2 2000 - 2200 AD without carrying capacity limitation on vegetation.



y2080 := 80·36
y2070 := 70·36

$y2080 = 2.88 \cdot 10^3$

$y2070 = 2.52 \cdot 10^3$

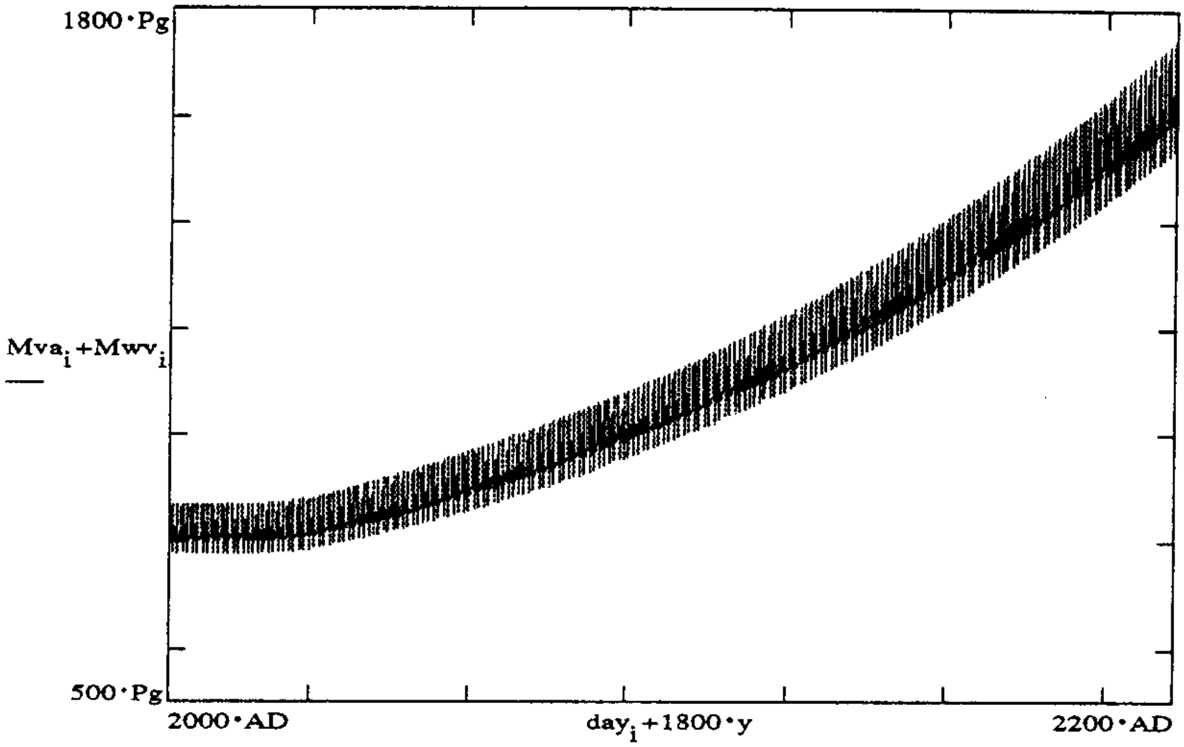
decade := y2070 .. y2080

$$\alpha := \frac{Ma_{y2080} - Ma_{y2070}}{dh \cdot \left[\left[\sum_{decade} F\left[day_{decade}\right]\right]\right]}$$

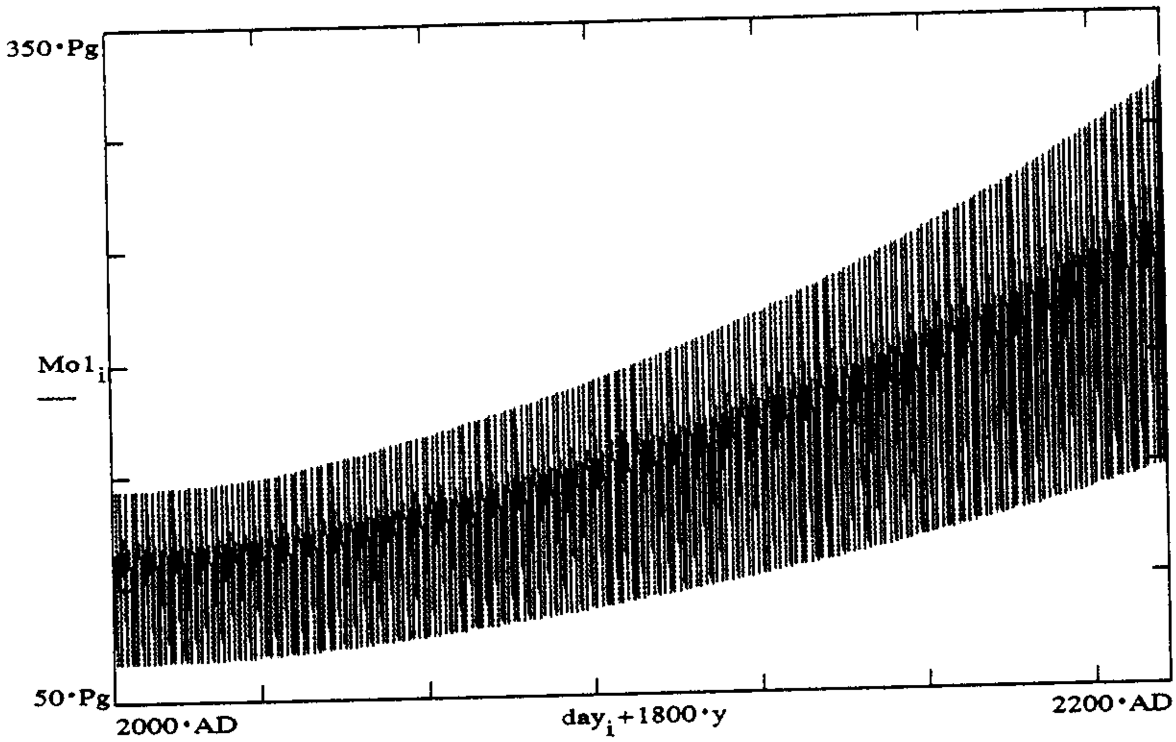
$Ma_{y2080} - Ma_{y2070}$

$\alpha = 0.328689$ Airborne fraction

Reduction in α below level with carrying-capacity limitation on vegetation presumably due to increased plant uptake & storage by soils.



Vegetation reaches nearly double the standing crop of 1800 AD



Large increase in overall magnitude & amplitude of annual fluctuations of rapid turnover soil organic C. Other fractions follow similar pattern.

$F_{max} := 40 \cdot Pg \cdot y^{-1}$ maximum annual CO2 input to the atmosphere from future fossil fuel oxidation $F1_{max} := 33 \cdot Pg \cdot y^{-1}$

$F_0 := 40 \cdot Tg \cdot y^{-1}$ assumed 1800 rate of fossil fuel oxidation

$Pre := 2 \cdot Pg \cdot y^{-1}$ $F1_0 := 0.1 \cdot Tg \cdot y^{-1}$

$cc := .0007 \cdot y^{-1}$

$A := \frac{[F_{max} - F_0]}{F_0}$ $A1 := \frac{[F1_{max} - F1_0]}{F1_0}$ $A = 9.99 \cdot 10^2$ $A1 = 3.29999 \cdot 10^5$

$a := 0.01 \cdot Pg \cdot y^{-2}$ assumed linear (a) and logistic (b) rates of fossil fuel oxidation $a \cdot day_0 = 18 \cdot Pg \cdot y^{-1}$

$b := 0.02782 \cdot y^{-1}$

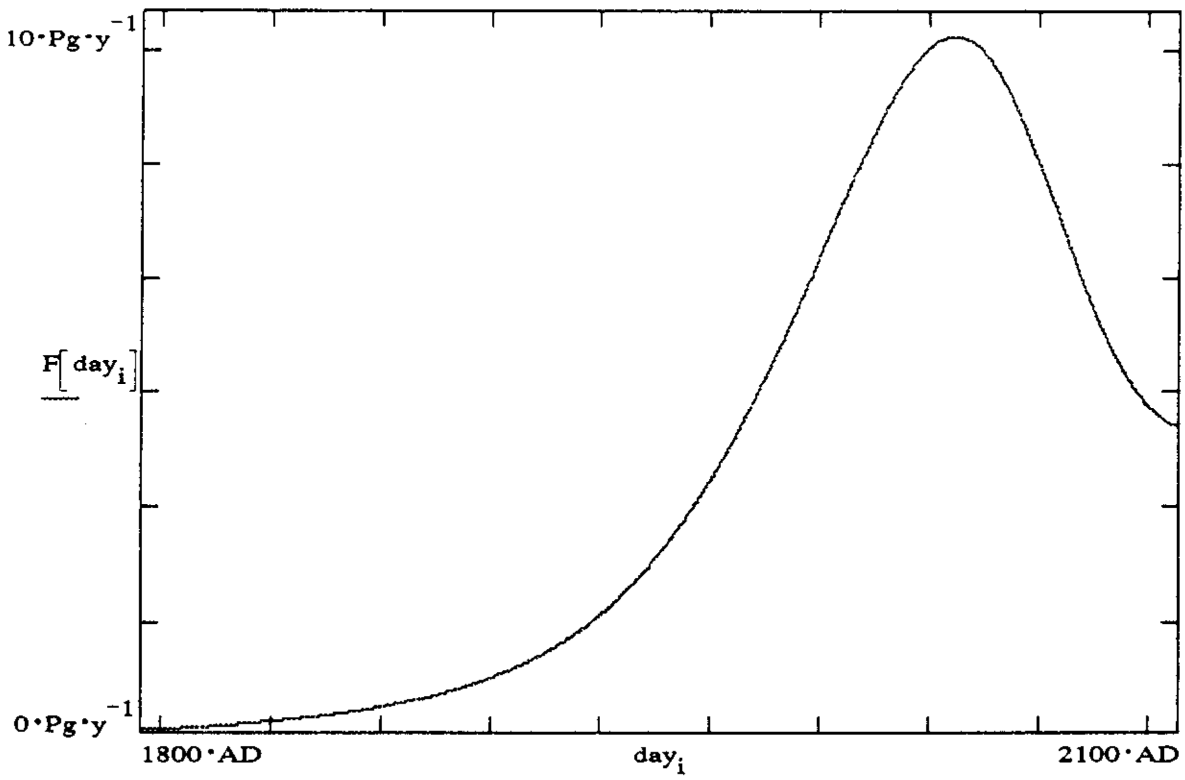
$c := 0.0485 \cdot y^{-1}$ $a \cdot (z - 1800 \cdot y)$ $day_n = 2.11 \cdot 10^3 \cdot y$

$$F(z) := \frac{F_{max}}{[1 + A \cdot [e^{-b \cdot [z - 1800 \cdot y]}]]} - \frac{F1_{max}}{[1 + A1 \cdot [e^{-c \cdot [z - 1800 \cdot y]}]]} + [Pre \cdot [1 - e^{-[cc \cdot [z - 1800 \cdot y]]}]]$$

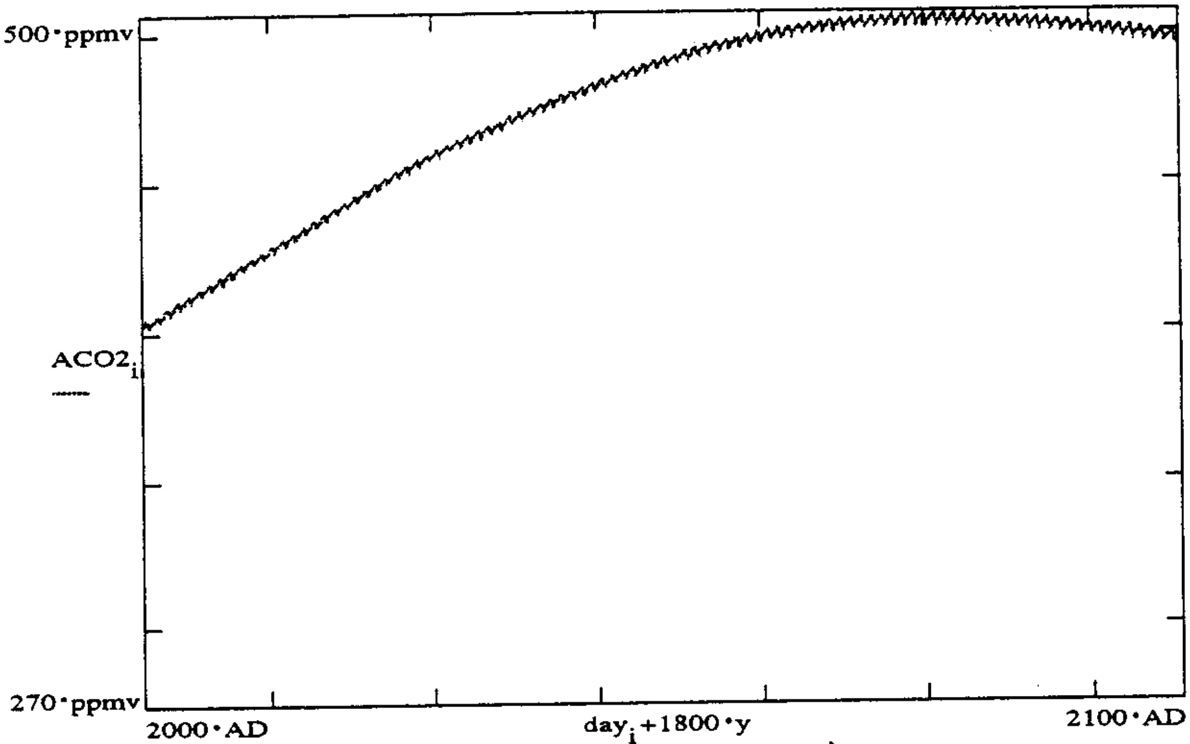
$F(2050 \cdot y) = 8.972674 \cdot Pg \cdot y^{-1}$

$F(1860 \cdot y) = 0.291831 \cdot Pg \cdot y^{-1}$ $F(1980 \cdot y) = 4.837744 \cdot Pg \cdot y^{-1}$ $F(2110 \cdot y) = 4.233108 \cdot Pg \cdot y^{-1}$

$F(1950 \cdot y) = 2.496032 \cdot Pg \cdot y^{-1}$ $F(1992 \cdot y) = 6.095853 \cdot Pg \cdot y^{-1}$



"Optimistic" fossil fuel flux scenario.



Optimistic scenario with maximal ACO2 < 500 ppmv.

$y2080 := 80 \cdot 36$ $\quad y2080 = 2.88 \cdot 10^3$

$y2070 := 70 \cdot 36$ $\quad y2070 = 2.52 \cdot 10^3$

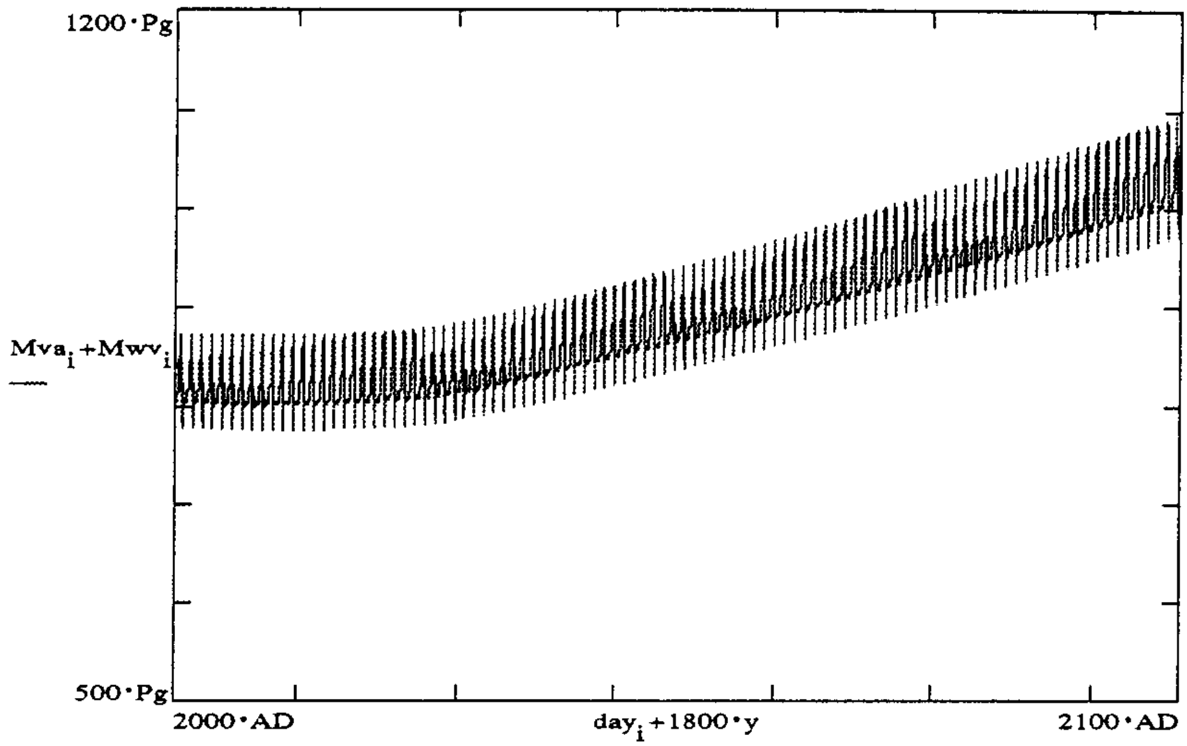
$decade := y2070 .. y2080$

$$\alpha := \frac{Ma_{y2080} - Ma_{y2070}}{dh \cdot \left[\left[\sum_{decade} F\left[day_{decade}\right]\right]\right]}$$

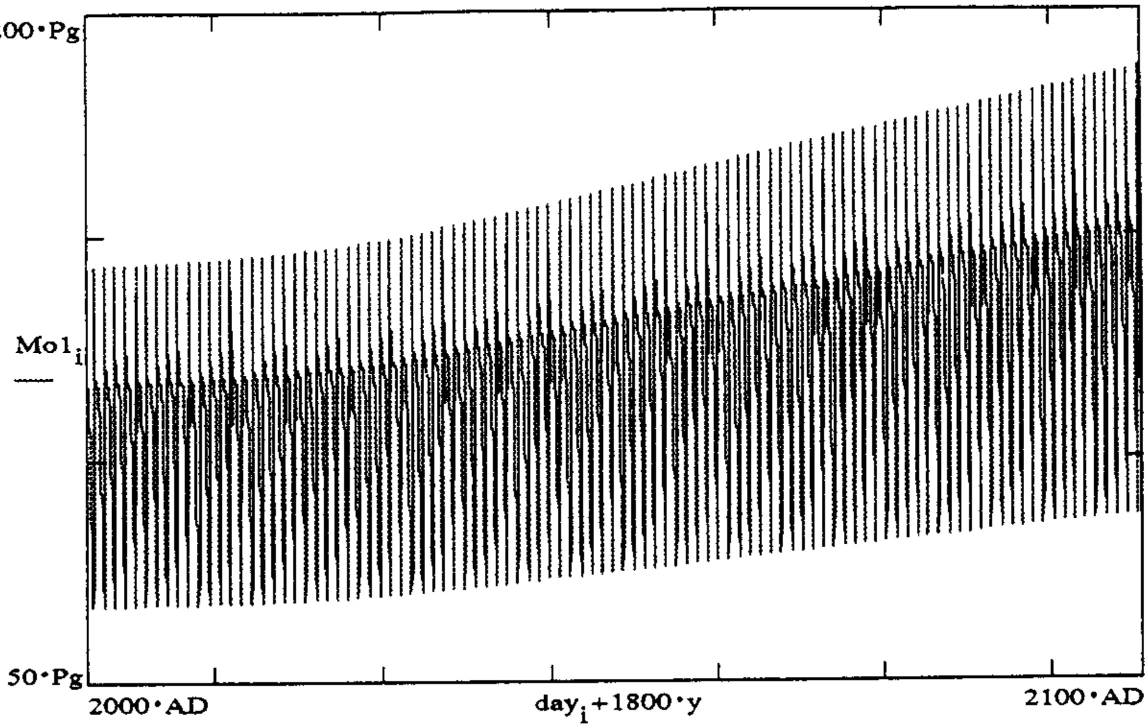
$Ma_{y2080} - Ma_{y2070}$

$\alpha = 0.019631$ Airborne fraction

α can be wide variety of values depending on when it is calculated.



Vegetation exceeds level of 1800 AD, but is considerably lower than levels reached with high fossil fuel input.



Rapid turnover soil organic C. Continuing to accumulate C. Other fractions follow similar pattern.