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Determination of Spatio-Temporal Characteristics of a Seismic Source from Surface Waves Amplitude Spectra

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DETERMINATION OF SPATIO-TEMPORAL CHARACTERISTICS OF A SEISMIC SOURCE FROM SURFACE WAVES AMPLITUDE SPECTRA.

Stress glut moments of total degree 2 according to Backus and Mulcahy, 1977 determine the geometry, duration of a seismic source and the propagation of rupture. Following Backus and Mulcahy, 1976 we will define source region as a region occupied by nonelastic motion, or region where partial derivative of stress glut tensor with respect to the time $\dot{\Gamma}$ is not identically zero. The definition of stress glut tensor is described in appendix 1.

We will consider a partial case of a seismic source when the stress glut tensor can be expressed by equation

$$\dot{\Gamma}(x,t) = F(x,t)M \ , \qquad (1)$$

where $F(x,t)$ is a nonnegative scalar function and M is a normalized seismic moment tensor. In this case source region can be defined by the condition that $F(x,t)$ is not identically zero and source duration is the time during which unelastic motion occurs at various points within the source region, i.e., $F(x,t)$ is different from zero. Spatio-temporal characteristics of the source can be expressed by correspondent moments of function $F(x,t)$. In general case stress glut moments of spatial degree 2 and higher are not unequally determined by displacement field. But in the case when equation (1) is valid such a uneasiness takes place.

Following equations express the integral estimates of the source characteristics in terms of spatio-temporal moments of $F(x,t)$ of total degree (both in space and time) 0, 1, and 2.

The moment $F^{(m,n)}(q,\tau)$ of spatial degree m and temporal degree n with respect to point q and instant of time τ is a tensor of order m and is given by formula ($k_1,\dots,k_m = 1,2,3$)

$$F^{(m,n)}_{k_1\dots k_m}(q,\tau) = \int_\Omega dV_x \int_0^\infty F(x,t)(x_{k_1} - q_{k_1})\dots(x_{k_m} - q_{k_m})(t-\tau)^n dt. \qquad (2)$$

Source location is estimated by the spatial centroid q_c of the field $F(x,t)$ defined as

$$q_c = F^{(1,0)}(0)/M_0, \qquad (3)$$

where $M_0 = F^{(0,0)}$ is the scalar seismic moment.
In a similar fashion, the temporal centroid τ_c is estimated by formula

$$\tau_c = F^{(0,1)}(0)/M_0 . \qquad (4)$$

The source duration is estimated by $2\Delta\tau$, where

$$(\Delta\tau)^2 = F^{(0,2)}(\tau_c)/M_0 . \qquad (5)$$

Let r be a unit vector. The mean source size along r is estimated by $2l_r$, where

$$l_r^2 = \mathbf{r}^T \mathbf{W} \mathbf{r} \qquad (6)$$

and

$$\mathbf{W} = F^{(2,0)}(q_c)/M_0 . \qquad (7)$$

From (7) it follows that a source region has the least mean size along that eigenvector of W corresponding to the least eigenvalue and the greatest mean size along that eigenvector of the same matrix corresponding to the greatest eigenvalue.
Let v be the mean velocity of the instant spatial centroid (see Bukchin B.G., 1989). Then

$$\mathbf{v} = \mathbf{w}/(\Delta\tau)^2 , \qquad (8)$$

where

$$\mathbf{w} = F^{(1,1)}(q_c,\tau_c)/M_0 .$$

The relation between spectrum of displacement field $u_i(x,\omega)$ and spatio-temporal moments of function $F(x,t)$ can be expressed by formula (see appendix 1)

$$u_i(x,\omega) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^n}{m!\,n!}F^{(m,n)}_{k_1\dots k_m}(0,0)M_{jl}\times$$
$$\times (i\omega)^{n-1}\frac{\partial}{\partial y_{k_1}}\cdots\frac{\partial}{\partial y_{k_m}}\frac{\partial}{\partial y_l}G_{ij}(x,y,\omega)|_{y=0} \qquad (9)$$

(the summation convention for repeated subscripts is used).
Here we assume that point $y=0$ and instant $t=0$ belong to the source region and time of the source activity respectively. $G_{ij}(x,y,\omega)$ is the spectrum of Green function for chosen model of medium and wave type. Correspondent formulae (see Levshin, 1985) are given in appendix 2. Since (9) involve infinite series, these relations

cannot be used to compute the moments $F^{(m,n)}$. However, when th displacement function $u_i(x,\omega)$ and the Green function $G_{ij}(x,y,\omega)$ have been low pass filtered, the terms in (9) start to decreas with m and n increasing at least as rapidly as $(\omega\Delta t)^{m+n}$ $(\omega\Delta t<1;$ Δt - the source duration) and one might then restrict onesel to considering finite sums only. Representing in this for spectrum of displacements in surface waves, we can derive a set o equations for the moments of F of total degree $m+n \le N$.

Let us consider a low frequency part of spectrum of i-t component of displacements carried by some Love or Rayleigh mod $u_i(x,\omega)$. If frequency ω is small (time duration of the source i much smaller than period, and size of the source region is muc smaller than wave length), then we can take into account i formula (9) only the first terms for $m + n \le 2$.

Let $y = 0$ - position of spatial centroid of the source an $t = 0$ - temporal centroid. Then we have $F^{(1,0)} = F^{(0,1)} = 0$. In this case (9) can be written as follows

$$u_i(x,\omega) = \frac{1}{i\omega}M_0 M_{jl}\frac{\partial}{\partial y_l}G_{ij}(x,y,\omega)|_{y=0} +$$
$$+\frac{1}{2i\omega}F^{(2,0)}_{mn}(0,0)M_{jl}\frac{\partial}{\partial y_m}\frac{\partial}{\partial y_n}\frac{\partial}{\partial y_l}G_{ij}(x,y,\omega)|_{y=0} - \qquad (10$$
$$-F^{(1,1)}_{m}(0,0)M_{jl}\frac{\partial}{\partial y_m}\frac{\partial}{\partial y_l}G_{ij}(x,y,\omega)|_{y=0} +$$
$$+\frac{i\omega}{2}F^{(0,2)}(0,0)M_{jl}\frac{\partial}{\partial y_l}G_{ij}(x,y,\omega)|_{y=0} .$$

If all characteristics of medium, depth of the best point sourc and seismic moment tensor are known (determined, for example, usi spectral domain of longer periods) the representation (10) give us a system of linear equations for moments of function F of tota degree 2. Let us consider a plane source. All moments of F o total degree 2 can be expressed in this case by formulas (3)-(8 in terms of 6 parameters: Δt - estimate of source duration, l_{max} - estimate of maximal mean size of the source, φ_l - estimate of th angle between the direction of maximal size and strike axis, l_{min}- estimate of minimal mean size of the source, v - estimate of the absolute value of instant centroid velocity v and φ_v- the angle between v and strike axis.

Using the Bessel inequality for the moments under discussion we can obtain following constrain for parameters considered above (see appendix 3).

$$v^2 \Delta t^2 (\cos^2\varphi \,/\, l_{max}^2 + \sin^2\varphi \,/\, l_{min}^2) \le 1 \qquad (11)$$

Here φ - the angle between direction of maximal size and direction of v. Assuming that the source is a plane fault and representation (1) is valid let us consider a rough grid in space of 6 parameters defined above. This parameters have to follow (11). Let models of media be given and moment tensor be fixed as well as the depth of best point source. Let fault plane (one of two nodal planes) be identified. Using formula (10) we can calculate the amplitude spectra of surface waves at the points of observation for every possible combination of values of varying parameters. Comparison of calculated and observed amplitude spectra give us residual $\varepsilon^{(i)}$ for every point of observation, every wave and every frequency ω. Let $u^{(i)}(r,\omega)$ be any observed value of spectrum, i=1,...,N; $\varepsilon^{(i)}$ - corresponding residual of $|u^{(i)}(r,\omega)|$. We define normalized amplitude residual by formula

$$\varepsilon(\Delta t, l_{max}, l_{min}, \varphi_l, v, \varphi_v) = \left[\left(\sum_{i=1}^{N} \varepsilon^{(i)2}\right) / \left(\sum_{i=1}^{N} |u^{(i)}(r,\omega)|^2\right)\right]^{1/2} . \qquad (12)$$

Fixing the value of one of varying parameters we will put in correspondence to it a minimal value of residual ε on the set of all possible values of other parameters. In this way we will define 6 functions of residual correspondent to the 6 varying parameters: $\varepsilon_{\Delta t}(\Delta t)$, $\varepsilon_{l_{max}}(l_{max})$, $\varepsilon_{l_{min}}(l_{min})$, $\varepsilon_{\varphi_l}(\varphi_l)$, $\varepsilon_v(v)$ and $\varepsilon_{\varphi_v}(\varphi_v)$. The value of parameter for which the correspondent function of residual attains its minimum we will define as estimate of this parameter. In this same time these functions characterize the resolution of correspondent parameters.

The technique described above was used for estimation the characteristics of Georgian earthquake, 29.04.91. The estimation

of source parameters was done by using spectra of Love and Rayleigh fundamental modes in the spectral domain 30 to 80 s. The distribution of stations is shown in Figure 1. The star indicates the position of epicenter, the squares - positions of stations: 1 - №38 (MARS), 2 - OBN (IRIS) and 3 - ARU (IRIS).

Love and Rayleigh fundamental modes were extracted by using a frequency-time analysis program. Analyzing the long period part of the spectra (periods from 50 to 80 seconds) we determined the following focal mechanism of the source: strike 285°, dip 15° and rake 90°. The stereographic projection of nodal planes on the lower hemisphere is shown in Figure 2 The estimate of seismic moment is $4.9 \cdot 10^{19}$ n·m. The best point source depth was found to be about 6km.

To estimate duration and geometry of the source we have used amplitude spectra of fundamental modes of Love and Rayleigh waves in spectral domain from 30 to 50 seconds. The plane dipping to the North was identified as a fault plane (by virtue of aftershocks distribution - see Figure 2). Results of direct trial of possible values of unknown parameters are shown in the Figure 3. Figure 3a shows the residual $\varepsilon_{\Delta t}$ as function of duration Δt. Sampling interval of this function is 2 s. $\varepsilon_{\Delta t}$ attains its minimum at the value of duration equal 16 s. Dependence of residual ε_v on the absolute value of the instant centroid velocity v is given in Figure 3b. Sampling interval here is 0.2 km/s. Value v = 2 km/s corresponds to the minimum of ε_v . Figures 2c and 2d show residuals $\varepsilon_{l_{max}}$ and $\varepsilon_{l_{min}}$ as functions of maximal mean size of the source l_{max} and minimal mean size l_{min} respectively. Sizes sampling is 5 km. These functions give us the following estimates l_{max}= 35 km and l_{min}= 20 km. Functions ε_{φ_l} and ε_{φ_v} defining direction of maximal mean size of the source and direction of the instant centroid velocity are given in Figures 3e and 3f respectively. The residuals were calculated for all possible values of angles φ_l and φ_v while other parameters were fixed equal to their estimates obtained before. Angles are measured in the foot wall of the fault plane clockwise round from the strike axis.

φ_l varies from 0° to 180°, φ_v - from 0° to 360°. The sampling interval is 20° and near the minimum 5°. As one can see in Figures 3e and 3f both of these functions attain there minimum at this same angle 40°. Note that the coincidence of these directions was obtained as a result of there independent variation. Comparison of the values of obtained estimates indicate that a model of unilateral rupture propagating along the direction of maximal size of the source is acceptable. A scheme of such a model is given in the Figure 4.

Appendix 1. Definition of stress glut tensor.

We will start from *motion equation*

$$\sigma_{ij,j} + f_i = \rho \ddot{u}_i \;, \quad i,j = 1,2,3 \tag{13}$$

Here u_i - i-component of displacements; $\ddot{u}_i$ - 2-nd derivative of with respect to the time; σ_{ij} - elements of symmetric stress tens

$\sigma_{ij,j} = \sum_{j=1}^{3} \partial\sigma_{ij}/\partial x_j$ (the summation convention for repeat subscripts is used); ρ - density; f_i - components of external for

The stresses and displacements are connected by *Hooke's law*

$$\sigma_{ij} = \lambda\delta_{ij} e_{kk} + 2\mu e_{ij} \quad \text{(in isotropic case)}, \tag{14}$$

where $e_{ij} = 0.5(u_{i,j} + u_{j,i})$ - elements of strain tensor.

We will assume that before $t=0$ there was not any motion, so *initial conditions* are following

$$\dot{u} \equiv u \equiv 0 \quad \text{for } t < 0 \,. \tag{15}$$

Elastic body under consideration is bounded by free surface S_0. It means that homogeneouse *boundary conditions* have to be satisfied:

$$\sigma_{ij} n_j \Big|_{S_0} = 0 \;, \tag{16}$$

where n_j - components of the normal to the S_0.

The solution of the problem (13)-(16) can be expressed by formula

$$u_i(x,t) = \int_0^t d\tau \int_\Omega G_{ij}(x;y;t-\tau) f_j(y,\tau)\, dV_y \tag{17}$$

or

$$u_i(x,t) = \int_0^t d\tau \int_\Omega H_{ij}(x;y;t-\tau) \dot{f}_j(y,\tau)\, dV_y \,. \tag{18}$$

Here G_{ij} - Green function, $H_{ij}(x;y;t) = \int_0^t G_{ij}(x;y;\tau)\, d\tau$,

and $0 < t < t_*$ - time interval when $\dot{f}$ is not identically zero.

Seismic disturbances most frequently arise from the action of internal sources (earthquakes or explosions) in absence of any external body forces. One must then set $f_j \equiv 0$ in (13), so that the only solution that satisfies the homogeneous initial (15) and boundary (16) conditions, as well as Hooke's law (14), will be $u_i \equiv 0$. Non-zero displacements cannot arise in the medium, unless at least one of the above conditions is not true. Following Backus and Mulcahy (1976), we assume seismic motion to be caused by a departure from Hooke's law within some volume of the medium at some time interval $t_e > t > 0$.

Let $u_i(x,t)$ describe the displacements and $\sigma_{ij}(x,t)$ the stresses that would have existed in the medium had Hooke's law (14) been true everywhere in it. Let $s_{ij}(x,t)$ be the actual stresses. The difference

$$\Gamma_{ij}(x,t) = \sigma_{ij}(x,t) - s_{ij}(x,t), \tag{19}$$

called the ***stress glut tensor***, is not identically zero within the three-dimensional region Ω. That region we define as source region. Within, and only within, that region, the tensor $\dot{\Gamma}_{ij}(x,t)$ too is not identically zero. We shall assume that Ω lies wholly within the medium (does not come out to the surface) and that, since some instant of time $t_e > 0$, $\dot{\Gamma}_{ij}(x,t) = 0$ everywhere in the medium. The integral of Γ_{ij} over Ω is called the seismic moment tensor (Kostrov, 1970; Aki and Richards, 1980). As the true motion obeys the equation $s_{ij,j} = \rho \ddot{u}_i$, in accordance with (13) (f≡0), one derives from (19)

$$\sigma_{ij,j} + g_i = \rho \ddot{u}_i , \tag{20}$$

$$g_i = -\Gamma_{ij,j} , \tag{21}$$

where $g_i(x,t)$ we will define as equivalent force. Then the resulting displacements are given by the same formulas, (17) and (18), with f_i replaced by g_i. Using relation (21) for g_i and the Gauss-Ostrogradsky theorem, we finally get

$$u_i(x,t) = \int_0^{t_e} dt \int_\Omega G_{ij,k}(x;y;t-\tau)\Gamma_{jk}(y,\tau)\,dV_y \tag{22}$$

or

$$u_i(x,t) = \int_0^{t_e} dt \int_\Omega H_{ij,k}(x;y;t-\tau)\dot{\Gamma}_{jk}(y,\tau)\,dV_y . \tag{23}$$

The G_{ij}, H_{ij} are here differentiated with respect to y_k.

If the departure from perfect elasticity is confined to some arbitrary finite area at the inner surface Σ, the stress glut tensor becomes $\Gamma_{jk}(x,t) = m_{jk}(x,t)\delta_\Sigma(x)$, where $\delta_\Sigma(x)$ is a distribution that satisfies

$$\int_V \delta_\Sigma(x)\phi(x)\,dV_x = \int_\Sigma \phi(x)\,d\Sigma_x$$

for any function $\phi(x)$. Integration over the volume V_y in (22),(23) will then reduce to that over the surface Σ:

$$u_i(x,t) = \int_0^t dt \int_\Sigma G_{ij,k}(x;y;t-\tau)m_{jk}(y,\tau)\,d\Sigma_y ,$$

where the points y belong to Σ. If the departure from perfect elasticity is defined as a discontinuity in displacement u at Σ without a stress discontinuity, then we have

$$m_{jk}(x,t) = n_q(x)\left[u_p(x,t)\right]c_{jkpq}(x),$$

where n is the normal to Σ, $[u_p]$ - components of the vector of discontinuity. For an isotropic medium we shall have

$$m_{jk} = \lambda\left[u_p\right]n_p\delta_{jk} + \mu(n_j\left[u_k\right] + n_k\left[u_j\right]) ;$$

in the case of tangential (shear) dislocation we have

$$n_p\left[u_p\right] \equiv 0 \quad \text{and}$$

$$m_{jk} = \mu(n_j\left[u_k\right] + n_k\left[u_j\right]) . \tag{24}$$

If the departure from perfect elasticity is confined to a small vicinity of x_0 (the region Ω shrinks to a point), then

$$\Gamma_{jk}(x,t) = m_{jk}(t)\delta(x-x_0)$$

and the equivalent forces g_i take the dipole form

$$g_j = -m_{jk}(t)\frac{\partial\delta(x-x_0)}{\partial x_k} \tag{25}$$

Such a source excites a field of the form

$$u_i = \int_0^t m_{jk}(t)G_{ij,k}(x;x_0;t-\tau)\,dt, \tag{26}$$

or

$$u_i = \int_0^{t_e} \dot{m}_{jk}(t)H_{ij,k}(x;x_0;t-\tau)\,dt, \tag{27}$$

where the G_{ij}, H_{ij} are differentiated with respect to y_k at the point $y = x_0$.

A point center of expansion (an ideally concentrated explosion) in an isotropic medium will produce (Aki and Richards, 1980)

$$m_{jk} = m(t)\delta_{jk} \quad , \qquad (28)$$

while for a point source of slip we shall have

$$m_{jk} = m(t)(\varkappa_j n_k + \varkappa_k n_j), \qquad (29)$$

where the $\varkappa_j$ are unit vector components in the direction of the discontinuity $[u]$ (slip vector) and $m(t) = \mu|[u]|$. The quantity $m_0 = \lim_{t\to\infty} m(t)$ is called the seismic moment.

Relations between displacement field and the stress glut moments.

We are going to discuss relations that connect observed displacements with the moments of stress glut tensor and can be used to estimate the moments.

The moment $\dot\Gamma^{(m,n)}(q,\tau)$ of spatial degree m and temporal degree n with respect to point q and instant of time τ is a tensor of order $m + 2$ and is given by formula

$$\dot\Gamma^{(m,n)}_{ij;k_1\ldots k_m}(q,\tau) = \int_\Omega dV_x \int_0^\infty \dot\Gamma_{ij}(x,t)(x_{k_1} - q_{k_1})\ldots(x_{k_m} - q_{k_m})(t-\tau)^n dt \ , \qquad (30)$$

$$i, j, k_1, \ldots, k_m = 1,2,3.$$

where Ω is a volume outside of which we have $\dot\Gamma(x,t) \equiv 0$.

Replacing in the expression (23) the function $H_{ij}(x,y,t-\tau)$ by its Taylor series in powers of y and in powers of τ, we get

$$u_i(x,t) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^n}{m!n!}\dot\Gamma^{(m,n)}_{jk_0;k_1\ldots k_m}(0,0) \times \frac{\partial^n}{\partial t^n}\frac{\partial}{\partial y_{k_0}}\frac{\partial}{\partial y_{k_1}}\cdots\frac{\partial}{\partial y_{k_m}}H_{ij}(x,y,t)|_{y=0} \ . \qquad (31)$$

Expanding H_{ij} in powers of y, we assume the elastic parameters to be sufficiently smooth. We have for the Fourier transforms $u_i(x,\omega)$ and $H_{ij}(x,y,\omega)$ from (31):

$$u_i(x,\omega) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^n}{m!n!}\dot\Gamma^{(m,n)}_{jk_0;k_1\ldots k_m}(0,0) \times (i\omega)^n \frac{\partial}{\partial y_{k_0}}\frac{\partial}{\partial y_{k_1}}\cdots\frac{\partial}{\partial y_{k_m}}H_{ij}(x,y,\omega)|_{y=0} \ . \qquad (32)$$

Appendix 2. Green function for media with smooth horizontal inhomogeneity.

We'll consider only media with smooth horizontal inhomogeneity. It means that variation of properties is small along any horizontal direction. For this assumption surface waves spectral parameters are locally determined (they depend on horizontal coordinates x and y) and are the same as in horizontally homogeneous medium with the same structure as under the surface point (x,y).

In this case function $G_{km}(r,s,\omega)$, Fourier transform of surface wave part of Green function, which corresponds to a given Love or Rayleigh mode can be described by formula

$$G_{km}(r,s,\omega) = (-1)^{m(m-1)/2} A W^{(k)}(\omega,0,\varphi)|_{M_r} W^{(m)}(\omega,h,\varphi)|_{M_s} \times \exp(-i\psi) \ , \qquad (33)$$

where r - point of registration; s - radius-vector of the source; $k, m = 1,2,3$; 1 corresponds to vertical coordinate z, 2 and 3 - to horizontal coordinates x and y;

$A = 1/\sqrt{8\pi\omega}\,\exp(-i\pi/4)/\sqrt{(vcI)|_{M_s}(cI)|_{M_r}J(\omega,r)}$; $\psi = \omega L/\hat v$;

v - phase velocity; c - group velocity; M_s marks the medium at the source region and M_r - the medium near the station;

$I = \int_0^\infty \rho[V^{(3)}(\omega,z)]^2 dz$ for Love wave (ρ - density),

$I = \int_0^\infty \rho\{[V^{(1)}(\omega,z)]^2 + [V^{(2)}(\omega,z)]^2\} dz$ for Rayleigh wave;

$V^{(1)}$, $V^{(2)}$ - vertical and radial components of vector eigenfunction of Rayleigh differential operator; $V^{(3)}$ - eigenfunction of Love differential operator; $\hat v = L/\int_L \frac{dL}{v}$ - average phase velocity along the L; L - the ray from s to r; $s = (h,0,0)$; r - a point on the free surface; L - the length of the ray L; J - geometrical spreading;

for Love wave

$$W^{(1)}(\omega,z,\varphi) = 0, \quad W^{(2)}(\omega,z,\varphi) = -i \sin\varphi \; V^{(3)}(\omega,z),$$
$$W^{(3)}(\omega,z,\varphi) = i \cos\varphi \; V^{(3)}(\omega,z);$$

for Rayleigh wave

$$W^{(1)}(\omega,z,\varphi) = V^{(1)}(\omega,z), \quad W^{(2)}(\omega,z,\varphi) = -i \cos\varphi \; V^{(2)}(\omega,z),$$
$$W^{(3)}(\omega,z,\varphi) = -i \sin\varphi \; V^{(2)}(\omega,z);$$

φ - initial azimuth of the ray L.

Derivatives $G_{km,n}$ are determined by equations

$$G_{km,1} = (-1)^{m(m-1)/2} A W^{(k)}(\omega,0,\varphi)|_{M_r} \partial W^{(m)}(\omega,h,\varphi)/\partial z|_{M_s} \times$$
$$\times \exp(-i\psi) \, , \tag{34}$$

$$G_{km,2} = i\xi \cos\varphi \; G_{km} \, , \quad G_{km,3} = i\xi \sin\varphi \; G_{km} \, ,$$

where $\xi = \omega / v$.

Appendix 3. Proof of the inequality (11).

Let us consider a space of such functions $\psi(\mathbf{x},t)$ that integral

$$\int_\Omega dV_x \int_0^\infty F(\mathbf{x},t)\psi^2(\mathbf{x},t)\,dt$$

exists. Here $F(\mathbf{x},t)$ - function from (1), $\mathbf{x} = (x_1, x_2, x_3)$ - spatial vector and t - time. Let us define for this space a scalar product of functions $\varphi_i(\mathbf{x},t)$ and $\varphi_j(\mathbf{x},t)$ by formula

$$(\varphi_i, \varphi_j) = \frac{1}{M_0} \int_\Omega dV_x \int_0^\infty F(\mathbf{x},t)\varphi_i(\mathbf{x},t)\varphi_j(\mathbf{x},t)\,dt, \tag{35}$$

where M_0 - seismic moment.

Let us consider a linear span of independent functions φ_k, where $\varphi_k(\mathbf{x},t) = x_k$ $(k = 1,2,3)$. Let $P(\mathbf{x},t)$ - projection of function $\psi(\mathbf{x},t) = t$ on this sub space. Then for $P(\mathbf{x},t)$ we have

$$P(\mathbf{x},t) = c_i \varphi_i = c_i x_i = \mathbf{c}^T \mathbf{x} = \mathbf{x}^T \mathbf{c} \, , \tag{36}$$

where c_i is minimizing a product (q,q) defined by (35) and

$$q = t - c_i x_i = t - \mathbf{c}^T \mathbf{x} \, . \tag{37}$$

Taking into account that x=0 and t=0 are spatial and temporal centroids of the source we will obtain following formulae for c and $P(\mathbf{x},t)$:

$$\mathbf{c} = W^{-1}\mathbf{w}, \tag{38}$$

and
$$P(\mathbf{x},t) = \mathbf{x}^T W^{-1}\mathbf{w}, \tag{39}$$

where W and w are defined by formulae (7) and (8).

The Bessel inequality gives us

$$(P,P) \le (t,t) \, .$$

But $(t,t) = (\Delta\tau)^2$, and for (P,P) we have

$$(P,P) = \frac{1}{M_0} \int_\Omega dV_x \int_0^\infty F(\mathbf{x},t)\mathbf{w}^T W^{-1}\mathbf{x}\mathbf{x}^T W^{-1}\mathbf{w}\,dt =$$
$$= \mathbf{w}^T W^{-1} W W^{-1}\mathbf{w} = \mathbf{w}^T W^{-1}\mathbf{w}.$$

And finally
$$\mathbf{w}^T W^{-1}\mathbf{w} \le (\Delta\tau)^2 . \tag{40}$$

Inequality (40) is valid in the case of 2-dimensional source region (a plane source) as well. Rewriting (40) in such a case in the coordinates of main axes of matrix W, using formulae (4)-(8) we will obtain inequality (11).

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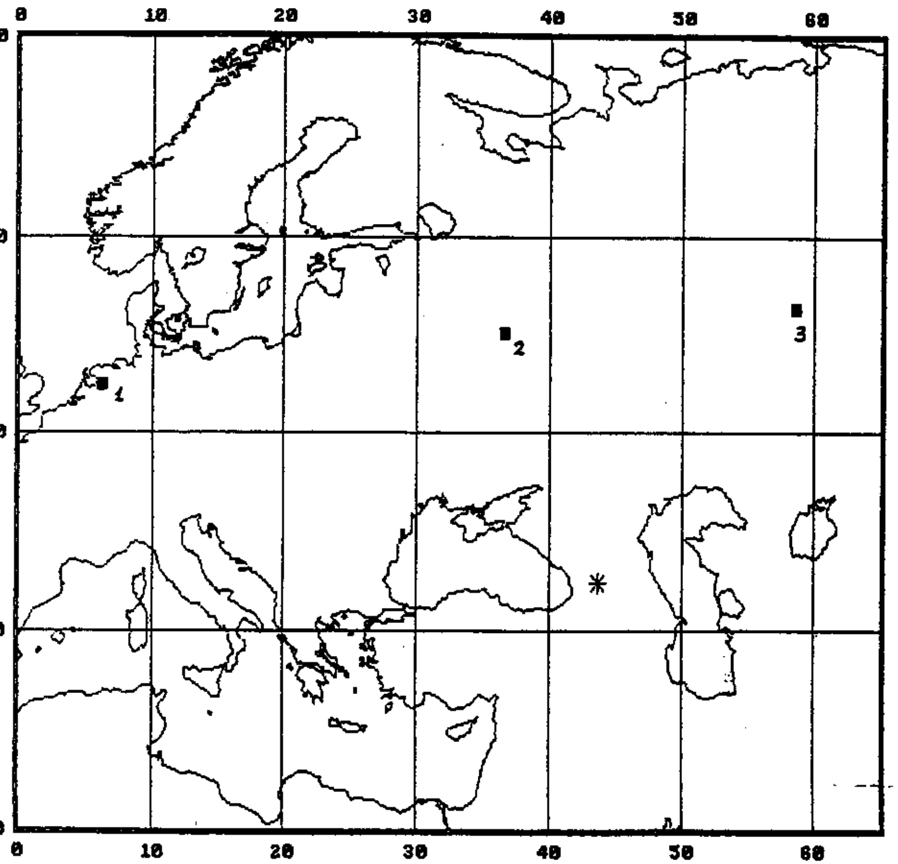
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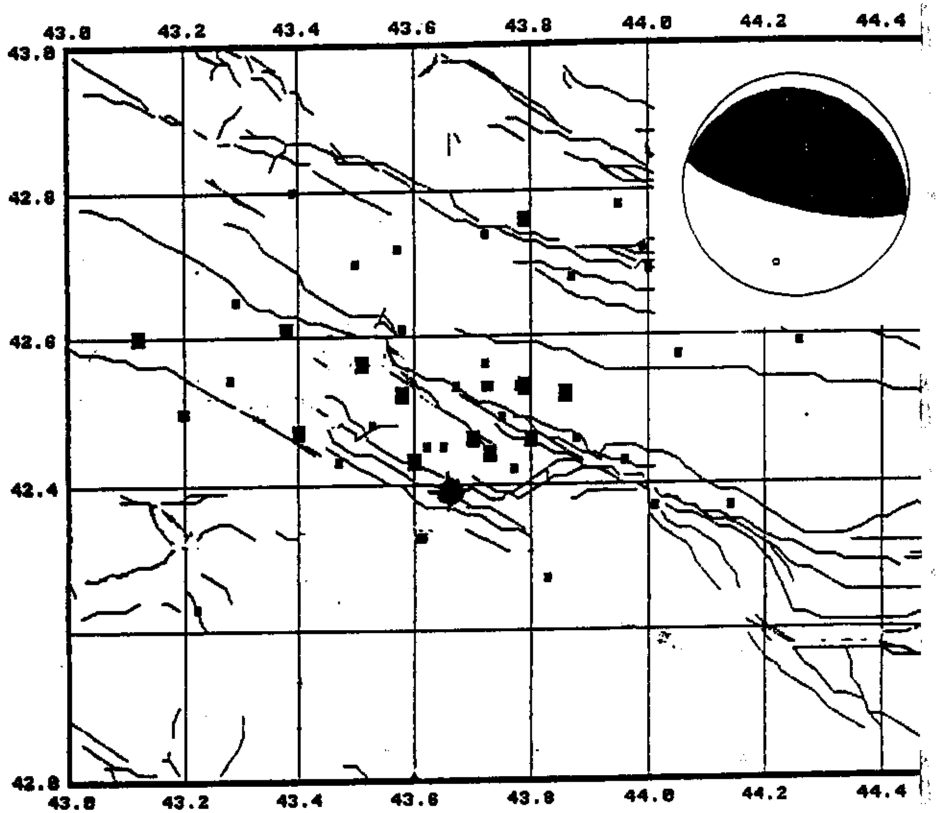
Fig. 1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42.4
42.0

Fig. 2

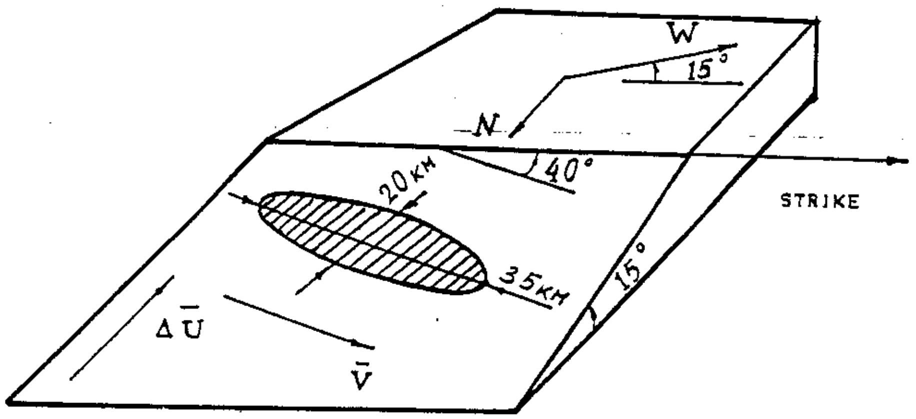
W
15°
N
40°
STRIKE
20 KM
35 KM
15°
ΔU
V

Fig. 4

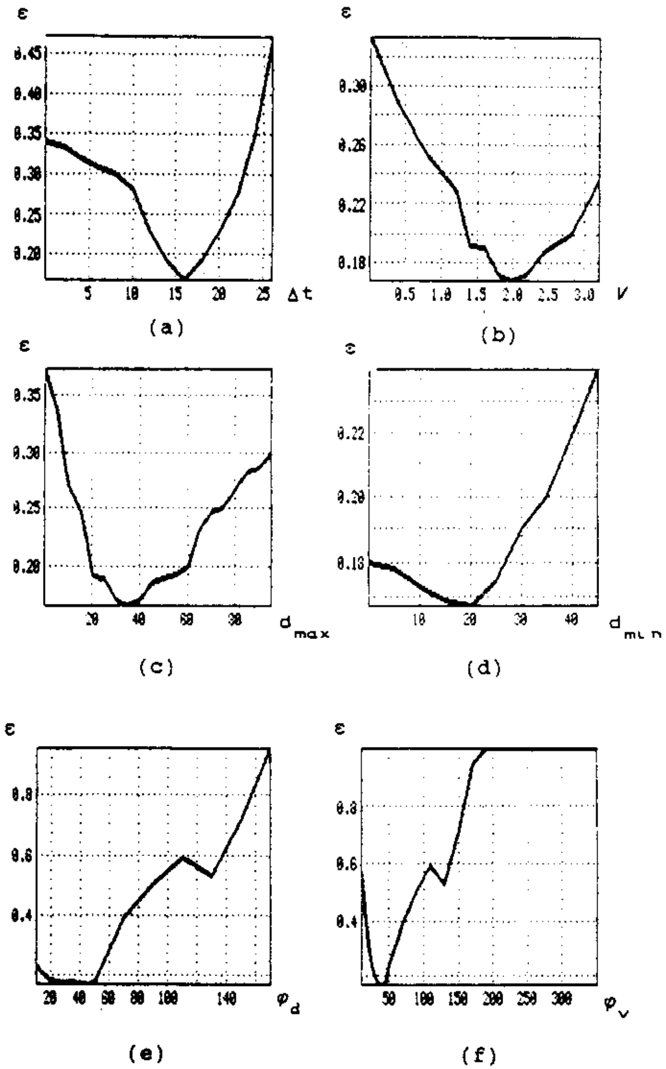
ε
0.45
0.40
0.35
0.30
0.25
0.20
5 10 15 20 25 Δt
(a)
ε
0.30
0.26
0.22
0.18
0.5 1.0 1.5 2.0 2.5 3.0 V
(b)
ε
0.35
0.30
0.25
0.20
20 40 60 80 d max
(c)
ε
0.22
0.20
0.18
10 20 30 40 d min
(d)
ε
0.8
0.6
0.4
20 40 60 80 100 140 φd
(e)
ε
0.8
0.6
0.4
50 100 150 200 250 300 φv
(f)

Fig. 3