INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

H4.SMR/650-7

Workshop on Three-Dimensional Modelling of Seismic Waves Generation Propagation and their Inversion

30 November - 11 December 1992

Regional Stress Parameters Reconstruction from Seismological Observations

A.V. Kiryushin

Russian Academy of Sciences
International Institute of Earthquake Prediction
Theory & Mathematical Geophysics
Moscow
Russian Federation

A.V.Kiryushin

Regional stress parameters reconstruction from seismological observations

Introduction.

Field observations of the orientation of striation on fault surfaces and/or earthquake focal mechanisms can be used to determine regional principal stress directions together with a scalar which characterizes relative stress magnitudes. The basic inverse technique solving this problem was published by Carey & Brunier (1974) and then it was widely applied by number of workers who has also made some improvements and modifications (Gerhart and Forsyth 1984, Etchecopar et al. 1981).

It should be noticed, however, that result of the inversion can be strongly affected by uncertainties of earthquake fault plane solutions. This fact is especially important when regional information is used and leads recently to new algorithms, by which principal stress directions and a scalar coefficient are reconstructed jointly with the population of the focal mechanisms from first motion data or using some a priory information (Angelier et al 1982, Rivera 1989).

While having a noticeable difference between various inversion techniques and using different kinds of data all of this methods are, nevertheless, based on the same underlying hypothesis first time explicitly formulated by Bott (1959). According to his assumption, failure in rocks can be represented as a slip within some preferred pre-existing plane of quite arbitrary orientation which depends on reological properties of the region and/or some mechanical reasons such as stress field evaluation in the past. While strength is exceeded by shearing stress within previously "unnoticed" preferred plane and fracture occurs, the initial slip must be in the direction of the maximum shearing stress within that plane.

In addition to this basic hypothesis it is usually supposed that interaction between different fault motions is negligible, and regional tectonic stress field is thought to be homogeneous within some subregions under consideration.

A formalism of the regional stress parameters and focal mechanisms joint reconstruction governed by the above assumptions will be described below.

Description of a seismic source model.

As it was mentioned above, Bott's hypothesis of fracture generation provides a constructive relation of stress tensor reconstruction techniques. The initial slip information which is necessary for the reconstruction may, in turn, be obtained from seismological first motion data using dislocation model of a seismic source, which is well known for more than thirty years after Vvedenskaya 1960 and Knopoff & Gilbert 1960 introduced it to seismology. Now it will be briefly described.

Dislocation model of seismic source.

Symmetric stress tensor σ_{ij} and displacements u_i of perfectly elastic body Ω bounded by free surface $\partial\Omega$ and subjected to an external loading satisfies equation of motion

$$\sigma_{ij,j} + f_i = \rho \ddot{u}_i \quad , \quad i,j = 1,2,3 \quad , \tag{1a}$$

boundary condition

$$\sigma_{ij} \nu_j \big|_{\partial\Omega} = 0 \tag{1b}$$

and initial conditions which are assumed to be homogeneous:

$$u_i \equiv \dot{u}_i \equiv 0 \ , \ t < t_0 \tag{1c}$$

Here $\sigma_{ij,j}$ - spatial derivative of the stress tensor, $\dot{u}_i$, $\ddot{u}_i$ - first and second temporal derivatives of displacement, ρ - density, f_i - i-component of an external force, ν_j - components of unit vector normal to the free surface, and usual summation convention is assumed.

If displacements are defined uniquely at any point of Ω then an elastic strain connected with stress by Hook's law

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl} \tag{1d}$$

can be introduced as follows:

$$\varepsilon_{kl} = 0.5(u_{k,l} + u_{l,k}) \ . \tag{1e}$$

Suppose now that a shear dislocation occurs within some imbedded surface Σ. It means that a relative motion of two blocks divided by this surface leads to the displacement discontinuity on it.

It is convenient now to consider two different surfaces Σ^+ and Σ^- (instead of Σ) moving separately but occupying the same points of Ω. Thus, the discontinuity can be formulated in terms of additional boundary condition, i.e.

$$u_i \big|_{\Sigma^+} - u_i \big|_{\Sigma^-} = [u_i] \ , \tag{1f}$$

where $[u_i]$ is a known function in space and time.

Note that (1*e*) and thus (1*d*) are no longer valid within Σ.

Solution of the problem (1*a*-*f*) is well known and may be expressed by the following formula

$$u_i(x,t) = \int_{t_0}^{t} d\tau \int_{\Omega} G_{ij}(x;y;t-\tau) f_j(y,\tau) d\Omega_y$$
$$+ \int_{t_0}^{t} d\tau \int_{\Sigma} G_{ip,q}(x;y;t-\tau) c_{jkpq} n_k [u_j(y,\tau)] d\Sigma \quad , \tag{2}$$

where G_{ij} is Green's function and n_k - normal to the Σ.

Thus as it is seen from (2) dislocation itself generates seismic oscillation field even at the absence of any external force, when the above expression becomes

$$u_i(x,t) = \int_{t_0}^{t} d\tau \int_{\Sigma} G_{ip,q}(x;y;t-\tau) m_{pq}(y,\tau) d\Sigma \ . \tag{3}$$

Here $m_{pq} = c_{ijpq}(n_i [u_j] + n_j [u_i])$ is called seismic moment density tensor and the symmetry of c_{ijpq} over the first two indexes is taken into account. In the case of structure isotropy and ideally shear dislocation seismic moment density tensor is given by the formula

$$m_{pq} = \mu(n_i [u_j] + n_j [u_i]) \ , \tag{4}$$

where μ is the shear modulus.

For teleseismic distances a point source approximation is usually valid and so seismic moment density tensor may be expressed as

$$m_{pq}(x,t) = M_{pq}(t)\delta(x-x_0) \ , \tag{5}$$

while (3) becomes

$$u_i = \int_{t_0}^{t} M_{jk}(t) G_{ij,k}(x;x_0;t-\tau)\,dt. \qquad (6)$$

It is evident that the symmetric sum in (4) have a sense of non-elastic deformation produced by the seismic dislocation and, hence, while acting as a seismic source, moment density tensor generates tension and compression body waves according to the directions of its eigenvectors and its eigenvalue magnitudes. It is easy to show that moment density tensor have one eigenvalue equals identically zero and two others which differ only by sign. Moreover, vector n bisects the vertical angles made by the eigenvectors of m or M corresponding to nonzero eigenvalues, while vector [u] bisects the other pair.

While having been exited, tension and compression body waves are then "transmitted" to the points of registration by the solution (3), (6). For the sake of convenience let's assume that registration is made on teleseismic distances and thus the first motion corresponds to the P-wave. If first motion on seismogram is positive the P-wave should be identified as compression wave and vice versa.

It is well known that body waves are transmitted trough the media with a negligible change of the waveform. Hence, if the structure (and so P-wave Green's function) is assumed to be known, then first motion data can be recalculated onto the unit focal sphere. In ideal case when there are no registration errors and slip dislocation model of fracture is true the signs of the first motions, i.e. polarities, should be divided on a focal sphere by two nodal planes, defined by slip vector [u] and normal n. On the other hand, it is thus possible to determine an earthquake fault plane solution, i.e. an orthogonal pair (n,a), where a stands for normalized [u], from the polarities observed on some seismic stations. This can be done by use of radiation function defined on a focal sphere.

In addition I wish to note that the fault plane solution determined from first motion data corresponds to the beginning of the fracture process. This is of importance when focal mechanisms are used for the stress parameters reconstruction.

Radiation function of a point dislocation seismic source.

The radiation function A(q) of a point dislocation source representing a displacement field on a focal sphere can be defined by the following simple formula

$$S(q,t) = M(t)q \ ,$$

where q is a unit vector and $M(t)$ satisfies (4) and (5).

The above expression can be decomposed to the sum of P- and S- wave radiation functions

$$S(q,t) = S_p + S_s = (q^t M\, q)q + (Mq - (q^t M\, q)q) \ .$$

Hereafter the upper script t denotes transposition.

Thus, $\mathrm{sign}(q^t M(t_0)q)$,with M satisfying (4) and(5), provides a synthetic first motion polarity for the whole range of focal mechanisms, i.e. for any possible orientation of orthogonal pair (n,a) at some starting moment of fracture.

Let s_i denotes a polarity (+ or −), registered on i-th station and then "back projected" to the focal sphere. Let q_i be a unit vector corresponding to s_i . Then, if shear dislocation model of the source is valid the synthetic polarities must fit the data, i.e.

$$s_i = \mathrm{sign}(q_i^t M(t_0)q_i) \quad , \ i=1,\dots,N \ . \qquad (7)$$

Here N - number of seismic stations.

Thus, varying orientation of the pair (n,a) and comparing synthetic and registered polarities one can find an earthquake fault plane solution. However, as it is seen from (4),(5) and (7), it remains impossible to distinguish between vectors n and a.

Bott's hypothesis.

Let T be regional stress tensor, then stress vector t defined on a fault plane with normal n is

$$t = T\cdot n \ .$$

The components of t on slip vector a and n are, respectively

$$n^t\cdot t = n^t\cdot T\cdot n \ ,$$
$$a^t\cdot t = a^t\cdot T\cdot n \ .$$

The Bott's hypothesis in this terms means that t has no components on vector orthogonal to both a and n, i.e.

$$T\cdot n - (n^t\cdot T\cdot n)n = (a^t\cdot T\cdot n)a \qquad (8)$$

and moreover

$$a^t \cdot T \cdot n \geq 0 \qquad (9)$$

Thus, Bott's hypothesis governs the relationship between unknown regional stress tensor T and available focal mechanisms which can be determined as it was previously described.

Let's note, however, that both (8) and (9) are verified by any other tensor of the form $\tilde{T} = \alpha T + \beta I$, where α is a positive constant and βI is any isotropic tensor. It means that only four of six degrees of freedom of the symmetric stress tensor can be determine from focal mechanisms data, while two others depends on an arbitrary choice of the constants α and β.

Let

$$\alpha = 1/\,[SpT^2 - \frac{1}{3}\cdot(SpT)^2]^{1/2},$$

$$\beta = \frac{1}{3}\cdot SpT/\,[SpT^2 - \frac{1}{3}\cdot(SpT)^2]^{1/2},$$

where $SpT \equiv T_{ii}$ and $SpT^2 \equiv T_{ij}T_{ji}$ are first and second invariants of T. Then $\tilde{T}$ have sense of normalized deviatoric stress tensor which components obey the following constrains:

$$Sp\tilde{T} \equiv \tilde{T}_{ii} \equiv \Sigma\lambda_i = 0 \quad ,$$

$$Sp\tilde{T}^2 \equiv \tilde{T}_{ij}\tilde{T}_{ji} \equiv \Sigma\lambda_i^2 = 1 \; . \qquad (10)$$

Here λ_i denotes eigenvalue of $\tilde{T}$ corresponding to its i-th eigenvector.

Whenever eigenvalues of $\tilde{T}$ differs from that of the regional stress tensor, its eigenvectors coincide with eigenvectors of T and hence define the principal stress directions.

Moreover, as it follows from (10) and definition of $\tilde{T}$, both relative magnitudes of the principal stresses and eigenvalues of $\tilde{T}$ may be characterized by the same scalar. It can be, for example, the Lode-Nadai coefficient μ_σ ($-1 \leq \mu_\sigma \leq 1$), which is defined as follows:

$$\mu_\sigma = [2\cdot(\sigma_2 - \sigma_3)/(\sigma_1 - \sigma_3)] - 1 \; , \qquad \sigma_1 \geq \sigma_2 \geq \sigma_3 \; , \qquad (11)$$

where σ_i denote principal stress, i.e. eigenvalue of the regional stress tensor. It is obvious that any of the λ_i is uniquely determined by μ_σ and vise versa.

Thus, in general case relations (8) and (9) can be used to determine the regional stress parameters, namely principal stress directions and Lode-Nadai coefficient without any ambiguity described above. Also, it becomes possible by means of this relations to discriminate the fault planes of earthquake mechanisms, i.e. to make difference between vectors a and n.

Nevertheless, there are some cases when Bott's hypothesis provides no information about the unknown regional stress. It is so when n coincides with one of the eigenvectors of $\tilde{T}$. Then $T \cdot n = \lambda n$, and both (8) and (9) are verified for any $\tilde{T}$.

It is also important to take into account another example of an ambiguity, which is not connected with formalism. It has mechanical character of uniaxial loading and arises when two eigenvalues of the normalized deviatoric stress tensor are equal. Let $\lambda_1 = \lambda_2$ and v denotes eigenvector appropriate to λ_3. Then any vector within the plane for which v is normal can be referred to as eigenvector, and so is when principal stress directions are considered.

Reconstruction of stress tensor parameters and earthquake mechanisms from first motion data.

As it was shown above, Bott's hypothesis and dislocation source model together with the assumptions of non-interacting fault motions and homogeneous tectonic stress field provide a necessary formalism for the reconstruction of regional stress tensor parameters and earthquake mechanisms.

It is obvious, however, that the above assumptions should be considered just as a more or less advantageous model governing the relationship between stress parameters, the range of possible initial fault motions within preferred planes and data. The usefulness of this model for joint reconstruction may be influenced by a number of mechanical and reological factors (for example, by presence of internal friction, Calerier,1988), which it doesn't take into account.

Also, the data used, i.e. polarities on a focal sphere, may contain some errors due to improper registration or unsufficient knowledge of the regional structure.

Thus, all the governing relations should be considered to be valid unperfectly and we shall use a statistical Bayes approach to solve the problem of joint reconstruction.

Now, let $\mathbb{T}$ and $\mathbb{M}_i$ denote any sets of parameters which uniquely determine the normalized deviatoric stress tensor $\tilde{T}$ and fault plane solution of the i-th earthquake from considered population. In the following the joint set of $\mathbb{T}$ and all $\mathbb{M}_i$, $i=1,\dots,N$, will be referred to as *model* and will be denoted by $\mathbb{H}$. Also, let S_i be a set of polarities corresponding to i-th earthquake, and each polarity is considered to be a realization of random variable continuously distributed on a focal sphere.

Let's denote a conditional probability density of the *data* given $\mathbb{H}$ by $p(S_1,\dots,S_N|\mathbb{H})$ and an a prior probability density of $\mathbb{H}$ by $p(\mathbb{H})$. Then, according to Bayes theorem, posterior conditional probability density of the model given *data* is defined as follows:

$$p(\mathbb{H}|S_1,\dots,S_N) = \frac{p(S_1,\dots,S_N|\mathbb{H})\cdot p(\mathbb{H})}{p(S_1,\dots,S_N)} . \qquad (12)$$

Here $p(S_1,\dots,S_N)$ is defined by

$$p(S_1,\dots,S_N) = \int_{\Omega_H} p(S_1,\dots,S_N|\mathbb{H})\cdot p(\mathbb{H})d\mathbb{H} , \qquad (13)$$

where Ω_H is space of model parameters.

As it follows from (12), varying $\mathbb{H}$ over Ω_H one can obtain posterior probability density distribution of the model parameters given a fixed set of observed data. Thus, to solve the problem of joint reconstruction one can take as an estimate of the model parameters that set of $\mathbb{H}$, for which $p(\mathbb{H}|S_1,\dots,S_N)$ is maximal.

Now we can use the formalism governing by the basic assumptions to give concrete expressions to the introduced probability density functions.

First let's assume that all polarities are independent. Then, as it is follows from the slip dislocation model of the seismic source, any set S_i given $\mathbb{M}_i$ is independent of $\mathbb{T}$ and any other $\mathbb{M}_j$, $j\neq i$. Thus it is possible to rewrite the conditional probability density of the data given $\mathbb{H}$ in the form

$$p(S_1,\dots,S_N|\mathbb{H}) = \prod_i p(S_i|\mathbb{M}_i). \qquad (14)$$

Note that multipliers in the product (14) have a sense of

likelihood function in standard focal mechanism reconstruction techniques. Its concrete expression are well known (see, for example, Brillinger et al,1980) and can be introduced as follows

$$p(S_i|\mathbb{M}_i) = 0.5\cdot\prod_j (1+(2\varkappa-1)\cdot f(A_j)\cdot \mathrm{sign}(A_j)\cdot s_j) \qquad (15)$$

where j indicates seismic station, s_j denotes registered polarity (+ or −), $A_j = q_j^t\cdot M\cdot q_j$ (see (7)), $\varkappa$ is probability of polarity true registration, $\varkappa > 0.5$, and $f(A_j)$ is any function of the first motion amplitude, which provides less weights to the polarities registered near the nodal planes. For example it can be defined by

$$\mathrm{erf}(\alpha A_j) = 1/(2\pi)^{1/2}\int_0^{\alpha A_j}\exp(-x^2)dx,$$

where α – a parameter.

While concrete expression of $p(S_1,\dots,S_N|\mathbb{H})$ is now described, let's consider another multiplier: $p(\mathbb{H})$.

As Bott's hypothesis provides a relationship between regional stress tensor and a population of the fault plane solution, the unknown parameters of the model are not independent and this is the intrinsic feature of the problem.

To use the formalism of the Bott's hypothesis let us first introduce the symbolic form of $p(\mathbb{H})$ as follows

$$p(\mathbb{H}) = p(\mathbb{M},\mathbb{T}) = p(a_1,n_1,\dots,a_N,n_N,\mathbb{T})$$

$$a_i = a(\mathbb{M}_i),\quad n_i = n(\mathbb{M}_i),\qquad i=1,\dots,N \qquad (16)$$

According to the Bott's assumptions preferred pre-existing planes seems to be independent of the present regional stresses. However, the direction of the slip vector should be in the direction of the maximum shear stress within the fault plane. Thus, we can rewrite (16) in the following form:

$$p(\mathbb{H}) = p(a_1,\dots,a_N|n_1,\dots,n_N,\mathbb{T})$$

Finally, by one of the assumptions, different fault motions are considered to be independent and so

$$p(\mathbb{H}) = \prod_i p(a(\mathbb{M}_i)|n(\mathbb{M}_i),\mathbb{T})\cdot p(n(\mathbb{M}_i))\cdot p(\mathbb{T}) \qquad (17)$$

Now, let's return to formulae (8) and (9) to write a concrete expression of $p(a(M_i)|n(M_i),T)$. We denote maximum shear stress within fault plane with normal n by P_s, i.e. $P_s = T \cdot n - (n^t \cdot T \cdot n)n$. Suppose that some additional small force F (for example due to pre-existed slikenslides) acts within the fault plane during the fault motion. We also suppose that action of F within the plane is proportional to the P_s magnitude, i.e. we shall substitute combination

$$P_s + |P_s| F_s$$

for P_s in (8) and then rewrite to equivalent form:

$$\Delta \equiv \frac{P_s + |P_s| F_s}{|P_s + |P_s| F_s|} - a = 0 , \qquad (18)$$

where F_s denotes tangential component of F.

It is obvious that F may be considered as a "noise", which we add to the model of fracture generation. Moreover, if we assume that F_s is distributed isotropicaly within fault plane, it can be shown that the first order approximation of (18) implies the following expression for $p(a(M_i)|n(M_i),T)$:

$$p(a(M_i)|n(M_i),T) = 1/(\sigma(2\pi)^{1/2})\cdot(\exp(-\Delta^2/(2\sigma^2)) , \qquad (19)$$

where small σ guarantees that inequality (9) holds for reasonable values of $p(a(M_i)|n(M_i),T)$.

Finally, all expressions described above can be substituted to (12) and (13) together with the concrete expressions for $p(n(M_i))$ and $p(T)$ which can be only defined by use of a prior regional information. Ideally, if some model parameters are chosen, one can find then a solution of the problem, i.e. the set of parameters for which $p(M|S_1,...,S_N)$ is maximal. Both gradient and direct trial methods can be used for this. However it is a question of computational facilities to work with such multi parametric case. Thus it is useful to limit the space of possible solutions. We do this maximizing (14) and determining sets of best focal mechanisms for each earthquakes, which then are used in the second step of joint reconstruction. Some examples will be demonstrated.

References.

Angelier, J., Tarantola, A., Valette, B.and Manoussis,S.,1982. Inversion of field data in fault tectonics to obtain the regional stress. I.Single phase fault populations: a new method of computing the stress populations. Geophis.J.R.Astron.Soc., 69:607-621

Bott, M.H.P., 1959. The mechanics of oblique slip faulting. Geol.Mag.,96:109-117.

Brillinger D.R., Udias A.S.J. and Bolt B.A.,1980, A probability model for regional focal mechanisms solutions., Bull.Seis.Soc.Am., 70:149-170.

Carey E. and Brunier B.,1974 Analyse theorique et numerique d'un modele mechanique elementaire applique a l'etude d'une population de failles.C.R.Acad.Sci.Paris.Ser D.,287:891-894.

Celerier, B., 1988. How much does slip on a reactivated fault plane constrain the stress tensor? Tectonics,7:1257-1278.

Etchecopar, A., Vasseur, G. and Daignnieres, M. 1981. An inverse problem in microtectonics for the determination of stress tensors from fault striation analysis. J.Struct.Geol.,3:51-65.

Gerphart, J.W. and Forsyth, D.W., 1984. An improved method for determining the regional stress tensor using earthquakes focal mechanism data: application to San-Fernando earthquake sequence.J.Geophys.Res.,89:9305-9320.

Knopoff L. and Gilbert F.,1960, First motions from from seismic sources., Bull.Seis.Soc.Am.,50:117-134.

Rivera L-A.,1989, Inversion du tenseur des contraintes et des mechanismesau foyer a partir des donees de polarite pour une population de seismes.,Ph.D. thesis., l'Universite Louis Pasteur de Strasbourg.

Vvedenskaya A.V.,1960., Determination of the stress active in the Foci of earthquakes, based on observations at seismological staitions., Bull. (Izvestya) Ac.of Sciences USSR. Geophysics Series. 1960,4:513-519.