INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

H4.SMR/650-8

Workshop on Three-Dimensional Modelling
of Seismic Waves Generation
Propagation and their Inversion

30 November - 11 December 1992

*Frequency-Time Analysis of Surface Waves*

N. M. Shapiro

Russian Academy of Sciences
International Institute of Earthquake Prediction
Theory & Mathematical Geophysics
Moscow
Russian Federation





# FREQUENCY-TIME ANALYSIS OF SURFACE WAVES

## 1 Introduction.

This lecture is devoted to the principles of surface wave record processing. The general properties of surface wave signals will be discussed, appropriate techniques of data processing and representation will be developed.

If the seismological data is to be used in geophysical interpretation, it should be translated into the language of concepts and quantities that from theoretical model; this is the principal aim of data processing.

In the case of surface waves many physical problems are conveniently stated in terms of amplitude and phase spectrum of Rayleigh or Love wave, and the initial information is presented in form of seismogram: record of ground displacement as function of time, containing both Rayleigh and Love waves as the volume waves and seismic noise. So we must identify a wave of interest and separate it from another part of record, and to measure its spectral parameters. In our discussion the term ´measurement´ implies digital calculation of signal characteristics rather than the physical act of measurement.

It is with the direct processing of a one-dimensional surface wave record that the present lecture is concerned. We shall list the assumptions relating to surface waves when these are regarded as ´signals´, discuss the parameters that describes there general properties, and see how the principal processing techniques follow from this.

We first state explicitly what the ´model language´ is into which we wish to ´translate´ a trace recorded by a seismometer. Suppose the record is given in the form of a real-valued function r(t) (Fig 1).

In theory the phrase ´surface wave´ denotes a certain term of the total wave motion expansion. Corresponding to a ´wave´ is the term signal in processing theory. The signal is understood to be that portion of record which carries information on the wave of interest, all the rest being considered as noise with respect to this signal.

$$r(t) = w(t) + n(t) \qquad (1.1)$$

where $w(t)$ is a signal, and $n(t)$ a noise. We first identify a signal (that is, recognize the typical wave features), separate it from out the background noise and, lastly, identify its characteristics with the corresponding wave characteristics. The ultimate end of data processing is thus to measure signal characteristics.

´Model statements´ corroborated with respect to surface waves are not complete, and nearly every seismogram presents some unexpected features that demands informal (frequently personally based) decision making. So, signal identification is not completely formal problem. In such situation a convenient, corresponding to signal and noise properties, data representation is needed. The surface waves processing techniques called Frequency-Time Analysis (FTAN) being discussed in this lecture, suggests such convenient, graphic method of data representation.

## 2 Signal properties.

In this section we shall define characteristics of a signal and its spectrum, and discuss general properties of surface waves as ´signals´.

Suppose a signal is given in the form of a real-valued function $w(t)$ (1.1). We define the signal spectrum as

$$K(\omega) = |K(\omega)| e^{i\psi(\omega)} = \begin{cases} \sqrt{\frac{2}{\pi}} \int_{-\infty}^{\infty} w(t)\, e^{-i\omega t}\, dt, & \omega > 0 \\ \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} w(t)\, dt, & \omega = 0 \\ 0, & \omega < 0 \end{cases} \qquad (2.1)$$



where $|K(\omega)|$, $\psi(\omega)$ are spectral amplitude and phase, respectively (Fig 2).

The above definition allows a complex-valued signal, $W(t)$, to be constructed that corresponds to $w(t)$ and is related to $K(\omega)$ through the ordinary Fourier transformation on the infinite axis:

$$W(t) = |W(t)|\, e^{i\varphi(t)} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} K(\omega)\, e^{i\omega t}\, d\omega$$

$$(2.2)$$

$$K(\omega) = |K(\omega)|\, e^{i\psi(\omega)} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} W(t)\, e^{-i\omega t}\, dt$$

where $|W(t)|$, $\varphi(t)$ are time-related (or instantaneous) amplitude and phase. In many cases the functions $|W(t)|$ and $\varphi(t)$ have visible meaning of ´envelope amplitude´ and ´carrier phase´.

From (2.1) and (2.2) it follows that $W(t)$ and $w(t)$ are connected by

$$w(t) = \mathrm{Re}\, W(t) \qquad (2.3)$$

that is, the complex-valued signal contains the whole information about the real-valued one, and both are solutions to the same differential equations; $W(t)$ is called the analytic signal (See appendix 1).

Later, the next characteristic, called group time, will be serviceable

$$\tau(w) = -\psi'(w) \qquad (2.4)$$

because it is directly connected with medium properties.

If the medium is assumed to be ´linear´ and ´stationary´ (that is, its properties do not change with time), it affects a signal as a linear filter, whose response equals Green´s function of relevant differential equations. A filter is conveniently fixed by given its frequency response

$$K_s(\omega) = K_i(\omega)\, Z(\omega, \vec{R}) \qquad (2.5)$$

where index $o$ denotes values measured on station, and index $i$ – ones in epicenter; $\bar{R}$ is the station coordinate vector.

In the simplest case of a surface wave in a laterally homogeneous medium this response is the next

$$Z(\omega,\bar{R}) = M(\omega,r)\, e^{-ik(\omega)r} \qquad (2.6)$$

where $k(w)$ is the wavenumber, $r$ the distance the wave has travelled. Real-valued functions $M(w,r)$ and $k(w)$ are controlled by the medium alone and do not depend on wave shape; they contain the information on the medium provided by travelling waves.

The most easily interpretable characteristic is the phase response of the filtering medium, which controls time features of a propagating wave. It usually given by fixing phase velocity

$$C(\omega) = \frac{\omega}{k(\omega)} \qquad (2.7)$$

We shall see below that surface wave processing requires measuring another parameter, namely group velocity

$$U(\omega) = \frac{1}{k'(\omega)} \qquad (2.8)$$

The functions $U(w)$ and $C(w)$ are called dispersion curves.They are related by

$$\frac{1}{U(\omega)} = \frac{1}{C(\omega)} + \omega \frac{d}{d\omega}\left(\frac{1}{C(\omega)}\right) \qquad (2.9)$$

From (2.1), (2.5) and (2.6) spectral phase is given by

$$\psi(\omega) = -k(\omega)r + \psi_o(\omega) \qquad (2.10)$$

where $\psi_o(\omega)$ is the spectral phase at the source. Differentiation of (2.10) yields

$$\tau(\omega) = \frac{r}{U(\omega)} - \psi_o'(\omega) \qquad (2.11)$$

This relation explains the term ´group time´, $\tau(\omega)$ being



that particular characteristic of signal which is used to determine group velocity. For the same reason we shall sometimes employ the phrase ´spectral dispersion curve of the signal´ for the function $\tau(\omega)$.

Seismic sources usually act during a time that is short compared with the typical values of $\tau(\omega)$, enabling $-\psi_o'(\omega)$ in (2.11) to be replaced by a constant (´source time´) to within the required accuracy. We shall often set $\psi_o'(\omega) = 0$, and use

$$\tau(w) = \frac{r}{U(\omega)} \qquad (2.12)$$

Thus, to study the velocity characteristics of the medium one must have measurements of $\psi(\omega)$ and $\tau(\omega)$ for the signal.

The routine aspect of the processing consists in functional transformations. Some of these, such as filtering, are signal-oriented. This kind of transformations works the more effectively, the better we know our signal and, generally speaking, the noise. Bellow we list some well-known general properties of surface waves signals (manly in relation to teleseismic records).

*´Finiteness´ in time and frequency.* A surface waves involves limited interval, both on the seismogram and in the spectrum, signal amplitude being small and hidden in the noise outside that interval. The behavior of a model signal there can be chosen arbitrarily, provided the amplitude does not exceed some threshold. In particular, both functions, $|W(t)|^2$ and $|K(\omega)|^2$, can always be assumed to vanish at $\pm\infty$, and to be integrable. Processing results should not significantly depend on intervals where amplitude is small and cannot be reliably determined.

*Deterministic character.* Apart from some special cases, surface make up the largest, easily identifiable portion of a seismic record. Noise plays a subordinate role. For this reason a surface wave signal can be treated as a deterministic with relatively smooth curves of $|K(\omega)|$ and $\tau(\omega)$.

*Noise.* Stationary noise is small compared with surface waves on most seismograms. Local impulsive nonstationary noise is the main source of trouble. This kind of noise may be largely generated by the surface wave itself. Because it is nonstationary, regular noise models are useless in most of the cases. The common practice is to treat 'everything that is unlike the signal' as noise.

*The dispersion* of $\tau(w)$ associated with group velocity dispersion is the most striking feature of the signal discussed. As a result surface wave shows no definite front, but has a broad spectral band and is long in duration. The typical dispersion curves are shown on Fig 3.

## 3 Signal parameterization.

The general properties of signal and spectrum closely related to processing techniques in use are here described by a parameterization that is widely employed in physics, namely, by using moments and quadratic norms.

We define normalized amplitudes of signal

$$A(t) = \frac{|W(t)|}{\sqrt{\int_{-\infty}^{\infty} |W(t)|^2 \, dt}} \qquad (3.1)$$

and spectrum

$$B(\omega) = \frac{|K(\omega)|}{\sqrt{\int_{-\infty}^{\infty} |K(\omega)|^2 \, d\omega}} \qquad (3.2)$$

They are related by Fourier transformation

$$A(t)\, e^{i\varphi(t)} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} B(\omega)\, e^{i[\psi(\omega)+\omega t]}\, d\omega$$

$$(3.3)$$

$$B(\omega)\, e^{i\psi(\omega)} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A(t)\, e^{i[\varphi(t)-\omega t]}\, dt$$



Than (Parceval's equality)

$$\int_{-\infty}^{\infty} A^2(t)\, dt = \int_{-\infty}^{\infty} B^2(\omega)\, d\omega = 1 \qquad (3.4)$$

Treating $A^2(t)$ and $B^2(\omega)$ as 'energy distribution', we define parameters that characterize 'mean' properties of the signal: the typical time

$$\langle t\rangle = \int_{-\infty}^{\infty} tA^2(t)\, dt \qquad (3.5)$$

the typical frequency

$$\langle\omega\rangle = \int_{-\infty}^{\infty} \omega B^2(\omega)\, d\omega \qquad (3.6)$$

the mean group time

$$\langle\tau\rangle = \int_{-\infty}^{\infty} \tau(\omega) B^2(\omega)\, d\omega \qquad (3.7)$$

the duration $D_t$

$$D_t^2 = 2\int_{-\infty}^{\infty} (t-\langle t\rangle)^2\, A^2(t)\, dt \qquad (3.8)$$

the spectral bandwidth $D_\omega$

$$D_\omega^2 = 2\int_{-\infty}^{\infty} (\omega-\langle\omega\rangle)^2\, B^2(\omega)\, d\omega \qquad (3.9)$$

the 'interval' of signal group time $D_\tau$

$$D_\tau^2 = 2\int_{-\infty}^{\infty} (\tau(\omega)-\langle\tau\rangle)^2\, B^2(\omega)\, d\omega \qquad (3.10)$$

In appendix 2 is shown that

$$\langle t\rangle = \langle\tau\rangle \qquad (3.11)$$

The typical signal time $\langle t \rangle$ equals $\langle \tau \rangle$, the group time averaged over the spectrum.

Equation (3.11) explains how a spectral quantity, group velocity $U(\omega)$, can be the ´velocity of propagation in space and time´. A wave propagating in a dispersive medium varies in shape from point to point, so the concept ´velocity of propagation´ needs a special definition. We take for the ´time at which a wave is at point $r$´ the value of $\langle t \rangle$. And we can obtain from (2.12) and (3.11)

$$\left\langle \frac{1}{U} \right\rangle = \frac{\langle t \rangle}{r} \qquad (3.12)$$

where

$$\left\langle \frac{1}{U} \right\rangle = \int_{-\infty}^{\infty} \frac{B^2(\omega)}{U(\omega)}\, d\omega \qquad (3.13)$$

In the sense indicated, a wave travels in a dispersive medium at a slowness $\langle 1/U \rangle$, that equals the mean ´group slowness´.

## 4 Duration and spectral bandwidth. Uncertainty relation.

In this part we´ll discuss well known for waves ´uncertainty relation´. Using parameters, defined above, it can be written

$$D_\omega D_t \geq 1 \qquad (4.1)$$

We shall discuss this inequality in more detail, and show, that for dispersed signals it can become

$$D_\omega D_t \gg 1 \qquad (4.2)$$

Such signals have a large ´uncertainty´. In next sections we shall show how this uncertainty can be diminished using the frequency- time representation of a signal.

The signal duration and the group time interval are related

$$D_t^2 = D_\tau^2 + 2\int_{-\infty}^{\infty} B'^2(\omega)\, d\omega \qquad (4.3)$$



( see appendix 3 ).

Multiplying (4.3) by $D_\omega^2$ we obtain the accurate expression of the uncertainty

$$D_\omega^2 D_t^2 = D_\omega^2 D_\tau^2 + 2D_\omega^2 \int_{-\infty}^{\infty} B'^2(\omega)\, d\omega \qquad (4.4)$$

It is convenient to introduce the next dimensionless parameter

$$I^2 = 2D_\omega^2 \int_{-\infty}^{\infty} B'^2(\omega)\, d\omega \qquad (4.5)$$

to rewrite the uncertainty relation

$$D_\omega^2 D_t^2 = D_\omega^2 D_\tau^2 + I^2 \qquad (4.6)$$

Thus the signal uncertainty consists of the two different parts. The first is $I^2$, it can be evaluated ( appendix 3 )

$$I^2 \geq 1 \qquad (4.7)$$

It does not depend on phase spectrum and, if we suppose $M(\omega, r) = M(\omega)$ ( see (2.6) ), on distance $r$. And what is more, it is readily verified that the substitution $|K(\omega)| \rightarrow |K(a\omega)|$, where $a$ is a constant, does not affect $I$. Thus, $I$ is independent on the spectral bandwidth, and describes only a ´nonoptimality´ of an amplitude spectrum form.

The accurate equality in (4.7) occures when $B' \sim (\omega - \langle \omega \rangle)B$, that is, for a signal having a Gaussian amplitude spectrum, $B(\omega) = c_1 \exp[c_2(\omega - \langle \omega \rangle)]$. We must require $c_2 < 0$ for $B(\omega)$ to vanish at infinity.

We shall later use the normalized Gaussian function

$$|B(\omega)| = H(\omega) = \frac{1}{\sqrt{2\pi}\,\beta}\, e^{-\frac{(\omega - \omega^H)^2}{2\beta^2}} \qquad (4.8)$$

where $\omega^H = \langle\omega\rangle$ and $\beta = D_\nu$ .

In actual practice, most signals with spectrum close to the Gaussian one have $I$ that is close to 1. It is true for separated spectral peaks.

The second part of uncertainty $D_\nu^2 D_\tau^2$ is directly related to the behavior of phase spectrum and of $\tau(\omega)$ at first, because it is proportional to $D_\tau^2$. The statement $D_\tau^2 = 0$ is equivalent to $\tau(\omega) = const$. We shall occasionally call signals with $D_\tau > 0$ ´dispersed signals´. Thus a separated, nondispersed spectral peak is nearly optimal in the sense of uncertainty.

When

$$D_\tau^2 > \frac{I^2}{D_\nu^2} \qquad (4.9)$$

the nonlinearity in the phase spectrum seriously affects signal shape. We introduce the dimensionless parameter $q$ to characterize the power of signal dispersion

$$q^2 = \frac{D_\tau^2 D_\nu^2}{I^2} \qquad (4.10)$$

When $q \ll 1$ we say that the signal is ´weakly dispersed´, and ´strongly dispersed´ if $q > 1$.

Using (2.12) and (3.8) we obtain

$$D_\tau^2 = r^2 \int_{-\infty}^{\infty} \left[ \frac{1}{U(\omega)} - \left\langle \frac{1}{U} \right\rangle \right]^2 B^2(\omega)\, d\omega \qquad (4.11)$$

In this formula the integral does not depend on the distance from the source, therefore the group time interval is proportional to $r$. So, the influence of the phase spectrum rises with $r$. That is the reason the surface wave signals becomes ´strongly dispersed´ on teleseismic distances.

Frequently the entire signal may be recorded within a spectral interval in which $\tau(\omega)$ is monotonic. In such cases a useful model is $\tau'(\omega) = const$, here called a ´linearly dispersed signal´. Within the framework of this model the group time interval is proportional to the spectral bandwidth



$$D_\tau = |\tau'|\, D_\nu \qquad (4.12)$$

and expression for the signal duration becomes

$$D_t^2 = \frac{I^2}{D_\nu^2} + (\tau' D_\nu)^2 \qquad (4.13)$$

Parameter $q$ for such signal

$$q = \frac{|\tau'|\, D_\nu^2}{I} \qquad (4.14)$$

Parameter $|\tau'|$ is proportional to the distance from the source

$$|\tau'| = r \frac{|U'|}{U^2} \qquad (4.15)$$

## 5 Locally narrow-band signal.

If a signal has $q \ll 1$ and $I$ about 1, than any transformation that diminishes $D_\nu$ will, according to uncertainty relation, leads to an increase in $D_t$ . But surface waves presents a different situation. When $q \gg 1$, both $D_\nu$ and $D_t$ can be diminished simultaneously. We are going to demonstrate this for the simplest case of a linearly dispersed signal ($\tau' = const$) with a sufficiently broad band and $I$ about 1 (this last condition means a single spectral peak is being considered).

Let us pass that signal through a filter with a real-valued frequency response $H(\omega)$ (4.8) . Signal parameters at the filter output are distinguished by placing ~ above a letter.

$$\tilde{K}(\omega) = H(\omega)\, K(\omega) \qquad (5.1)$$

$$\tilde{W}(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tilde{K}(\omega)\, e^{i\omega t}\, d\omega \qquad (5.2)$$

We also suppose that $H(\omega)$ is a Gaussian function (4.8). As $H(\omega)$ is real-valued, we have $\tilde{\tau}' = \tau'$, and the time duration for filtered signal becomes

$$\tilde{D}_t^2 \approx (\tau'\beta)^2 + \frac{1}{\beta^2} \qquad (5.3)$$

the parameter $\tilde{q}$

$$\tilde{q} \approx |\tau'|\,\beta^2 \qquad (5.4)$$

$\tilde{D}_\omega$ and $\tilde{I}$ have been replaced by the filter parameters $D_\omega^H = \beta$ and $I^H = 1$, which would not incur serious errors when $D_\omega^H \ll D_\omega$ i.e., when the filter bandwidth is narrow compared with the bandwidth of the original signal, and $\tilde{B}(\omega) \approx const\cdot H(w)$.

Till the filtered signal remains ´strongly dispersed´ ($\tilde{q} \gg 1$) its time duration remains approximately equal its group time interval (the second term in (5.3) is small compared with the first one)

$$\tilde{D}_t^2 = \tilde{D}_\tau^2 = (\tau'\beta)^2 \qquad (5.5)$$

and its average frequency equals the filter´s one

$$\langle\tilde{\omega}\rangle = \omega^H \qquad (5.6)$$

and its average time

$$\langle\tilde{t}\rangle = \langle\tilde{\tau}\rangle = \tau(\omega^H) \qquad (5.7)$$

but the filtered signal has the smaller duration and spectral bandwidth compared with the original one.

Several filters with different $\omega^H$ can separate out an original signal into a sequence of relatively short, narrow-band



trains that arrive at different times ( $\tau(\omega^H)$ ). Loosely speaking, each narrow spectral band of a strongly dispersed signal is significant only within a limited interval of time that is shorter than the total duration. Similarly, one may say that a strongly dispersed signal has a time-dependent, relatively narrow spectral band around any instant of time $t$. (If the $\tau(\omega)$ curve is not monotonic, there may be several spectral bands of this kind corresponding to a given time moment.) A signal of this kind can appropriately be called a locally narrow-band signal. While being broadband as a whole, it behaves like a narrow-band in the vicinity of a given instant of time.

But the signal duration can´t be diminished infinitely. When a filter becomes too narrow ($\tilde{q} < 1$), $\tilde{D}_t$ is significantly dependent on the second term in (5.3). So, the function $\tilde{D}_t(\beta)$ is not monotonic, it has a minimum when

$$\beta^2 = \frac{1}{|\tau'|} \qquad (5.8)$$

## 6 Frequency-time representation.

We have in fact used the idea of a frequency-time representation when discussing special properties of dispersed signals. If a signal is passed through a system of parallel relatively narrow-band filters $H(\omega-\omega^H)$ with varying central frequencies $\omega^H$, then each resulting filter will, according to (5.7), concentrate around the time $t = \tau(\omega^H)$. (This conclusion is not related to the above requirement $q \gg 1$, and is true for any signal with a smoothly varying $\tau(\omega)$ ). The combination of all the filtered signals $\tilde{W}^H(t)$ will now be treated as a (complex) function of two variables $S(\omega^H, t)$. According to (5.1) and (5.2)

$$S(\omega^H, t) = \tilde{W}^H(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} H(\omega-\omega^H)\, K(\omega)\, e^{i\omega t}\, d\omega \qquad (6.1)$$

which is what we call the frequency-time representation of a signal. The signal analysis based on (6.1) will be abbreviated further on as FTAN (frequency-time analysis). (We shall always use Gaussian function as a filter $H(\omega-\omega^H)$ in (6.1) ).

A contour map of $|S(\omega^H,t)|$ called a FTAN map is used for visual representation. For $\omega^H$ fixed, $|S(\omega^H,t)|$ is the signal envelope at the output of the relevant filter. For this reason, corresponding to each input signal is a 'mountain ridge' (increased values) in the FTAN map extending along the dispersion curve $t(\omega^H) = \tau(\omega^H)$ (Fig 4).

The introduction of $S(\omega^H,t)$ enables us to employ a convenient terminology based, in particular, on the concept of the frequency-time region of a signal. This is understood to be that part of the $(\omega^H,t)$-plane occupied by the relevant ridge. It can be seen from a Figure 4 that the picture of a frequency-time region gives a much clearer notion of a dispersed signal that the parameters $\langle t\rangle$, $D_t$, $\langle\omega\rangle$, $D_\omega$ can. The statement 'the energy of a signal concentrates (on the $(\omega^H,t)$-plane) around its dispersion curve' acquires a defined meaning in terms of $|S(\omega^H,t)|$. From this one can also see that the large uncertainty $D_\omega D_t$ of a dispersed signal is in some sense fictitious; the area of the relevant rectangle on the $(\omega^H,t)$-plane considerably exceeds that of the frequency-time region.

This small area of the frequency-time region provides extra possibilities for separation of signals with different dispersion curves (for example Rayleigh and Love waves) (Fig 5). For the same reason the portion of noise that falls into the region $(\langle\omega\rangle\pm D_\omega, \langle t\rangle\pm D_t)$, but lies far from the signal dispersion curve, can be separated from it (Fig 6). Thus, FTAN gives additional opportunities of filtering of the dispersed signals. This problem will be discussed in the next part.

There is an important feature of the frequency-time representation that makes it essentially different from the more usual spectral and time representations. The function $S(\omega^H,t)$ is not a property of the original signal alone, but also involves the filter characteristic $H(\omega-\omega^H)$ chosen by investigator. Different choices of $H(\omega-\omega^H)$ will transform one and the same signal to different $S(\omega^H,t)$ functions. (The spectrum and the seismogram can be regarded as two extreme cases in the class of such functions:



$K(\omega)$ is for the infinitesimally narrow filters $H(\omega-\omega^H) = \delta(\omega-\omega^H)$, $W(t)$ for infinitely broad filter $H(\omega-\omega^H) = const.$)

A complete frequency-time representation involves two functions: $S(\omega^H,t)$ and $H(\omega-\omega^H)$, we are in fact dealing with a whole class of signal representations. This leads us to the question of the choice of such representation (function $H(\omega-\omega^H)$), which is the most relevant to the processing problem in hand (so called problem of 'optimal FTAN filtering'). In the case of Gaussian filters the form of representation is completely described by the function of filter bandwidth $\beta(\omega^H)$.

We shall use the filters which minimize time cross-section of frequency-time area $\tilde{D}_t$. For a linearly dispersed signal the bandwidth of such filter is given by (5.8)

$$\beta^2(\omega^H) = \frac{1}{|\tau'(\omega^H)|} \qquad (6.2)$$

In terms of the velocities this condition can be rewritten

$$\beta^2(\omega^H) = \frac{U^2(\omega^H)}{r\left|\frac{dU}{d\omega}(\omega^H)\right|} \qquad (6.3)$$

that is, the FTAN optimal filter width decreases with the epicentral distance increasing.

## 7 Frequency-time filtering.

In this part we shall discuss the opportunities the frequency-time representation gives to signal filtering.

The first method is to put a two-dimensional amplitude window $G(\omega^H,t)$ over the frequency-time representation of a signal to cut the noise lying out of the frequency-time area. Thus we obtain filtered frequency-time representation $S^f(\omega^H,t)$ (Fig 6).

$$S^f(\omega^H,t) = G(\omega^H,t)\, S(\omega^H,t) \qquad (7.1)$$

(The index $f$ above a letter is used to distinguish parameters at the frequency-time filter output.) To obtain the filtered spectrum $K^f(\omega)$ we must restore it using $S^f(\omega^H, t)$.

The frequency-time representation is the combination of the narrow-band signals $\tilde{W}^H(t)$ which are the results of the inverse Fourier transformation of corresponding local spectrums $\tilde{K}^H(\omega)$ (see (5.1), (5.2)).

$$\tilde{K}^H(\omega) = \tilde{K}(\omega, \omega^H) = H(\omega - \omega^H)\, K(\omega) \qquad (7.2)$$

$$\tilde{W}^H(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tilde{K}^H(\omega)\, e^{i\omega t}\, d\omega \qquad (7.3)$$

The whole spectrum can be restored as the sum of the local ones

$$K(\omega) = \frac{\int_{-\infty}^{\infty} \tilde{K}(\omega, \omega^H)\, d\omega^H}{\int_{-\infty}^{\infty} H(\omega - \omega^H)\, d\omega^H} \qquad (7.4)$$

The frequency-time representation at the filter output is the combination of the filtered time signals

$$\tilde{W}^{Hf}(t) = G(\omega^H, t)\, \tilde{W}^H(t) \qquad (7.5)$$

Fourier transform gives us the filtered local spectrums

$$\tilde{K}^{Hf}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tilde{W}^{Hf}(t)\, e^{-i\omega t}\, dt \qquad (7.6)$$

Similar to the formula (7.4) we define the filtered spectrum as

$$K^f(\omega) = \frac{\int_{-\infty}^{\infty} \tilde{K}^f(\omega, \omega^H)\, d\omega^H}{\int_{-\infty}^{\infty} H(\omega - \omega^H)\, d\omega^H} \qquad (7.7)$$



or

$$K^f(\omega) = \frac{1}{\sqrt{2\pi}} \int \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S^f(\omega^H, t)\, e^{-i\omega t}\, dt}{\int_{-\infty}^{\infty} H(\omega - \omega^H)\, d\omega^H}\, d\omega^H \qquad (7.8)$$

In the limit of a very large frequency-time window $G(\omega^H, t)$ $K^f(\omega) \to K(\omega)$.

We shall call the formula (7.8) as the inverse FTAN transformation and the formula (6.1) as the direct FTAN transformation.

One can image the operation of frequency-time filtering as a 'time filter whose parameters vary with frequency'. The filter band is 'floating' along the dispersion curve. This is the reason the frequency-time filtering is sometimes called 'floating filtering'.

If the width of such 'floating' filter does not depends on frequency,the frequency-time filtering can be reduced to the ordinary time filtering. In this case $G(\omega^H, t)$ is the region of constant width. For a nondispersed signal it becomes analogous to a time window.

If we know the dispersion curve $\tau(\omega)$ we can transform the signal into the nondispersed one, using operation of the phase equalization. Inverting the group time definition (2.4) we get the phase spectrum

$$\psi(\omega) = -\int_0^{\omega} \tau(x)\, dx + c_1 \omega + c_2 \qquad (7.9)$$

were $c_1$, $c_2$ are constants.

Applying the next transformation

$$\hat{\psi}(\omega) = \psi(\omega) + \int_0^{\omega} \tau(x)\, dx \qquad (7.10)$$

$$\hat{K}(\omega) = |K(\omega)|\, e^{i\hat{\psi}(\omega)} \qquad (7.11)$$

$$\hat{W}(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{K}(\omega)\, e^{i\omega t}\, d\omega \qquad (7.12)$$

we obtain the weakly dispersed signal $\hat{W}(t)$. ( Index $\hat{}$ above a letter denotes a values after the phase equalization.) Its amplitude envelope has a form of a narrow peak. And a large part of the noise, which was not distinguished in a time or frequency domain, lies now out off the signal region and we cut this noise with the amplitude time window $F(t)$ to obtain the filtered signal $\hat{\tilde{W}}(t)$.

$$\hat{\tilde{W}}(t) = F(t)\, \hat{W}(t) \qquad (7.13)$$

Fourier transform gives us the filtered spectrum

$$\hat{\tilde{K}}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{\tilde{W}}(t)\, e^{-i\omega t}\, dt \qquad (7.14)$$

The last step is to restore the phase spectrum

$$\psi(\omega) = \hat{\psi}(\omega) - \int_{0}^{\omega} \tau(x)\, dx \qquad (7.15)$$

The information about $\tau(\omega)$ for such type of filtering can be evaluated using from the axis line of corresponding ridge on the FTAN map (Fig 4). The accuracy of such method of measurement is sufficient for filtering.

The sequence of FTAN maps corresponding to such type of filtering is shown on Fig 7.

The combination of two filters discussed above also can be used. In this case phase equalization (7.9) is made before the direct FTAN transformation.



## REFERENCES


Dziewonski A., Bloch S. and Landisman M., 1969 A technique for the analysis of transient seismic signals. Bull. Seismol. Soc. Amer., vol. 59, p. 427-444.

Keilis-Borok V.I. (ed), 1989. Seismic surface waves in laterally inhomogeneous Earth. Kluwer Publ. House, Dordrecht.

Lander A.V., 1974. On interpretation of frequency-time analysis results. In: Computer Analysis of Digital Seismic Data. Moscow, Nauka, p.279-315 (Computational seismology, 7) (in Russian).

LAnder A.V., 1975. Frequency-time representation of a linearly dispersed signal with a gaussian spectrum. In: Interpretation of Seismological and Neotectonic Evidence. Moscow, Nauka, p. 122-128 (Computational seismology, 8) (in Russian).

Levshin A.L., Pisarenko V.F. and Pogrebinsky G.A., 1972. On a frequency-time analysis of oscillations. Ann. Geophys.,vol. 28, p.211-218.

## *Appendix* 1: Connection between analytic signal and record.

We introduce the additional spectrum $L(\omega)$, which is related to real-valued signal $w(t)$ through the Fourier transform on an infinite axis:

$$w(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} L(\omega)\, e^{i\omega t}\, d\omega \qquad (\text{A1.1})$$

$$L(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} w(t)\, e^{-i\omega t}\, dt \qquad (\text{A1.2})$$

$K(\omega)$ and $L(\omega)$ are related

$$K(\omega) = \begin{cases} 2L(\omega), & \omega > 0 \\ 0, & \omega < 0 \end{cases} \qquad (\text{A1.3})$$

$$L(\omega) = \overline{L(-\omega)} \qquad (\text{A1.4})$$

$$\int_{-\infty}^{0} L(\omega)\, e^{i\omega t} d\omega = \overline{\int_{0}^{\infty} L(\omega)\, e^{i\omega t} d\omega} \qquad (\text{A1.5})$$

$$w(t) = \int_{-\infty}^{\infty} L(\omega)\, e^{i\omega t} d\omega = 2\mathrm{Re}\left( \int_{0}^{\infty} L(\omega)\, e^{i\omega t} d\omega \right) = \qquad (\text{A1.6})$$

$$\mathrm{Re}\left( \int_{-\infty}^{\infty} K(\omega)\, e^{i\omega t} d\omega \right) = \mathrm{Re}\, W(t)$$

## *Appendix* 2: The mean time and group time.

We shall use the next two expressions

$$\int_{-\infty}^{\infty} e^{i\omega(t-s)}\, d\omega = 2\pi\delta(t-s) \qquad (\text{A2.1})$$

and

$$\int_{-\infty}^{\infty} B'B\, d\omega = \frac{1}{2} B^2 \Big|_{-\infty}^{\infty} = 0 \qquad (\text{A2.2})$$



(We used the properties of surface wave spectrum discussed in section 2: $K(\omega) = 0$ (and $B(\omega) = 0$) when $\omega \to \pm\infty$. )

We differentiate both sides of (3.3)-2 with respect to the frequency

$$(B' - iB\tau)\, e^{i\psi} = -\frac{i}{\sqrt{2\pi}} \int_{-\infty}^{\infty} tA\, e^{i[\varphi-\omega t]}\, dt \qquad (\text{A2.3})$$

Now form the product of (A2.3) and the complex conjugate of (3.3)-2, and integrate it over $\omega$

$$\int_{-\infty}^{\infty} [B'B - i\tau B] d\omega = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} tA(t)A(s) e^{i[\varphi(t)-\varphi(s)]} \left[ \int_{\infty}^{\infty} e^{i\omega(t-s)} d\omega \right] dt\, ds$$

Using (A2.1) and (A2.2) we get

$$-i \int_{\infty}^{\infty} \tau\, B^2(\omega)\, d\omega = -i \int_{-\infty}^{\infty} t\, A^2(t)\, dt$$

And using (3.5) and (3.7)

$$\langle\tau\rangle = \langle t\rangle$$

## *Appendix* 3: Uncertainty relation and parameter I.

We add (A2.3) to (3.3)-2. Multiplying the result by $\langle t\rangle$ and using (3.11) we get

$$[B' - iB(\tau - \langle\tau\rangle)]e^{i\psi} = -\frac{i}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (t - \langle t\rangle)\, A\, e^{i(\varphi-\omega t)}\, dt \qquad (\text{A3.1})$$

Regarding this formula as the Fourier transform we apply Parceval's equality. The result is the formulas (4.3)

$$D_t^2 = D_\tau^2 + 2\int_{-\infty}^{\infty} B'^2(\omega)\, d\omega$$

To evaluate $I^2$ we use Parceval's equality (3.4)

$$\int_{-\infty}^{\infty} B^2(\omega)\,d\omega = 1$$

using $B^2\omega\big|_{-\infty}^{\infty} = 0$ and $\langle\omega\rangle \int_{-\infty}^{\infty} B'B\,d\omega = 0$ (see (A2.2)) we rewrite it

$$\int_{-\infty}^{\infty} B^2(\omega)\,d\omega = B^2\omega\big|_{-\infty}^{\infty} - 2\int_{-\infty}^{\infty} B'B\,\omega\,d\omega = -2\int_{-\infty}^{\infty} (\omega-\langle\omega\rangle)B'B\,\omega\,d\omega = 1 \qquad (A3.2)$$

And now we apply the Cauchy's inequality

$$\left(\int_{-\infty}^{\infty} F_1F_2\,d\omega\right)^2 \le \left(\int_{-\infty}^{\infty} |F_1|^2\,d\omega\right)\left(\int_{-\infty}^{\infty} |F_2|^2\,d\omega\right) \qquad (A3.3)$$

substituting $F_1$ by $B'$ and $F_2$ by $(\omega-\langle\omega\rangle)B$ and applying (A3.3) to (A3.2) and using (3.9) we get

$$2D_\omega^2 \int_{-\infty}^{\infty} B'^2(\omega)\,d\omega \ge 1$$



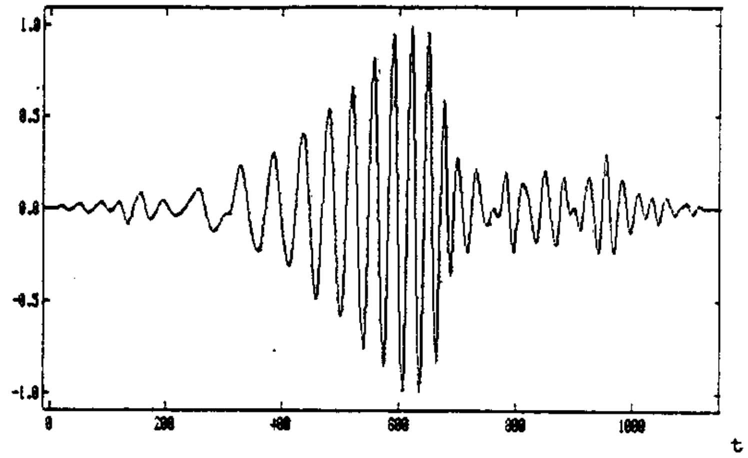



Fig 1. Seismogram.

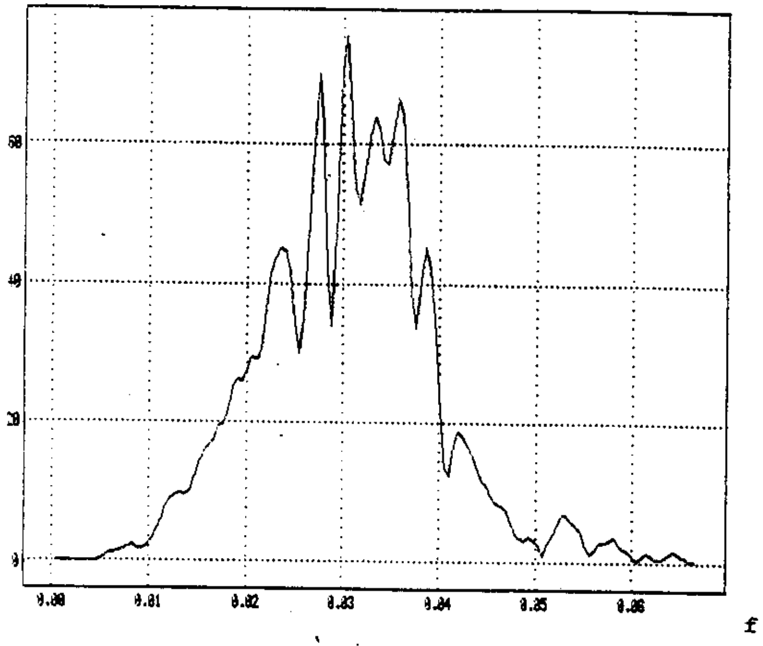



Fig 2. Spectrum of the signal drawn on Fig 1.

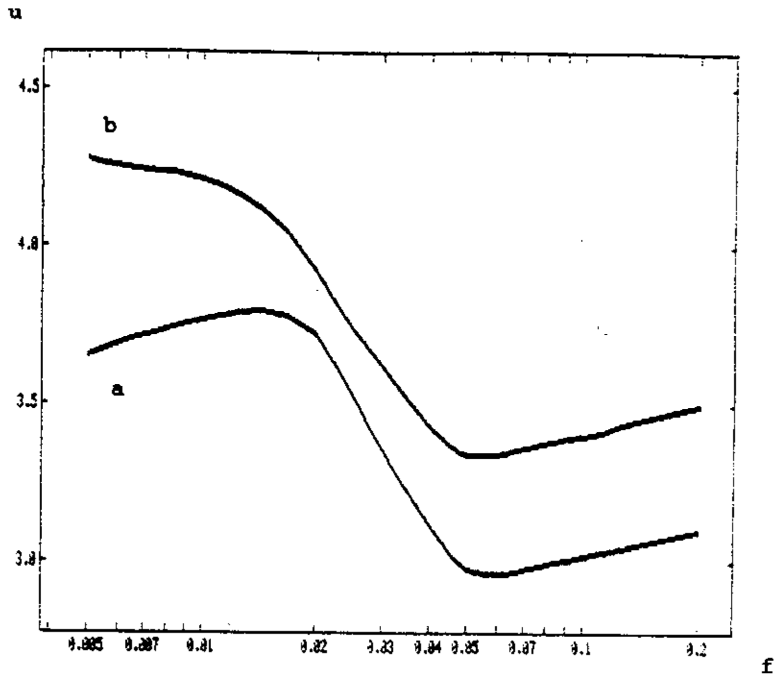



Fig 3. Typical group velocities curves of Rayleigh (a) and Love (b) waves.

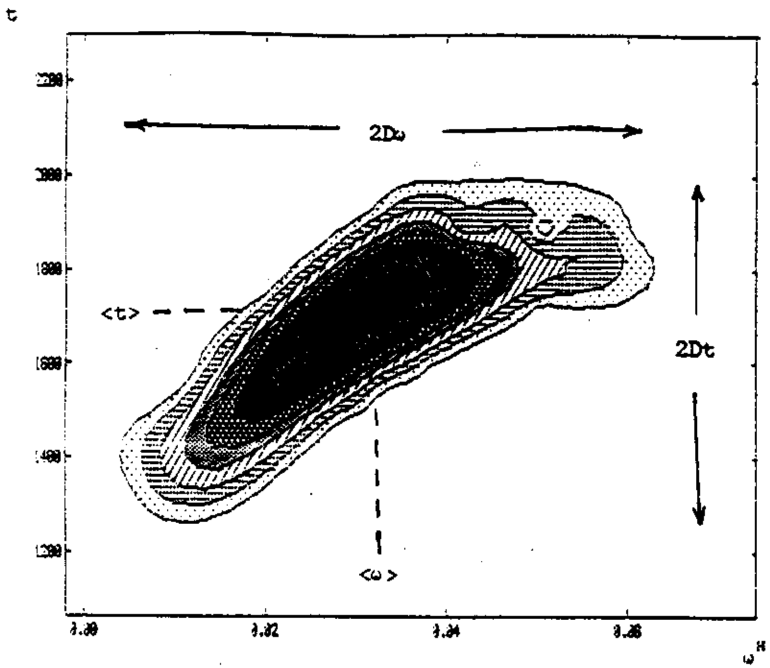



Fig 4. FTAN map.

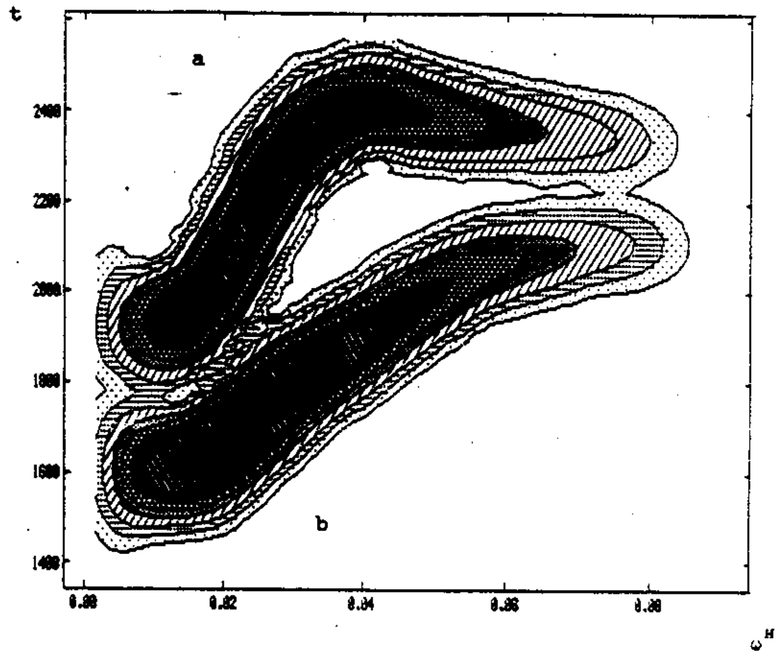
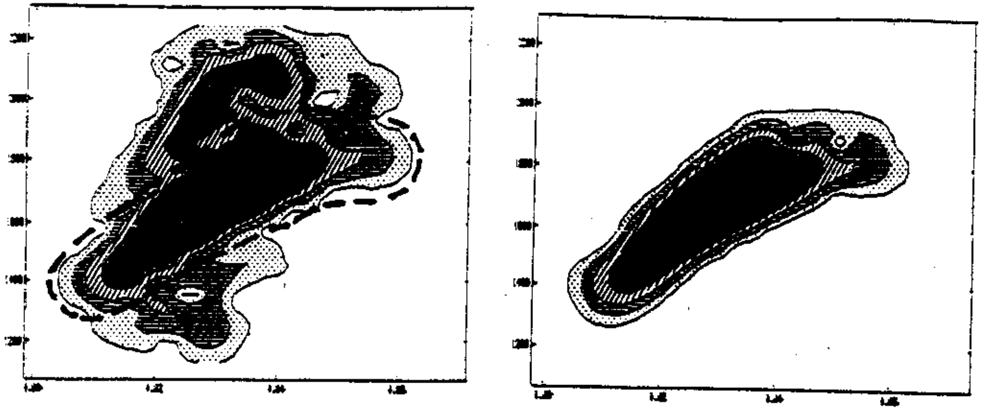
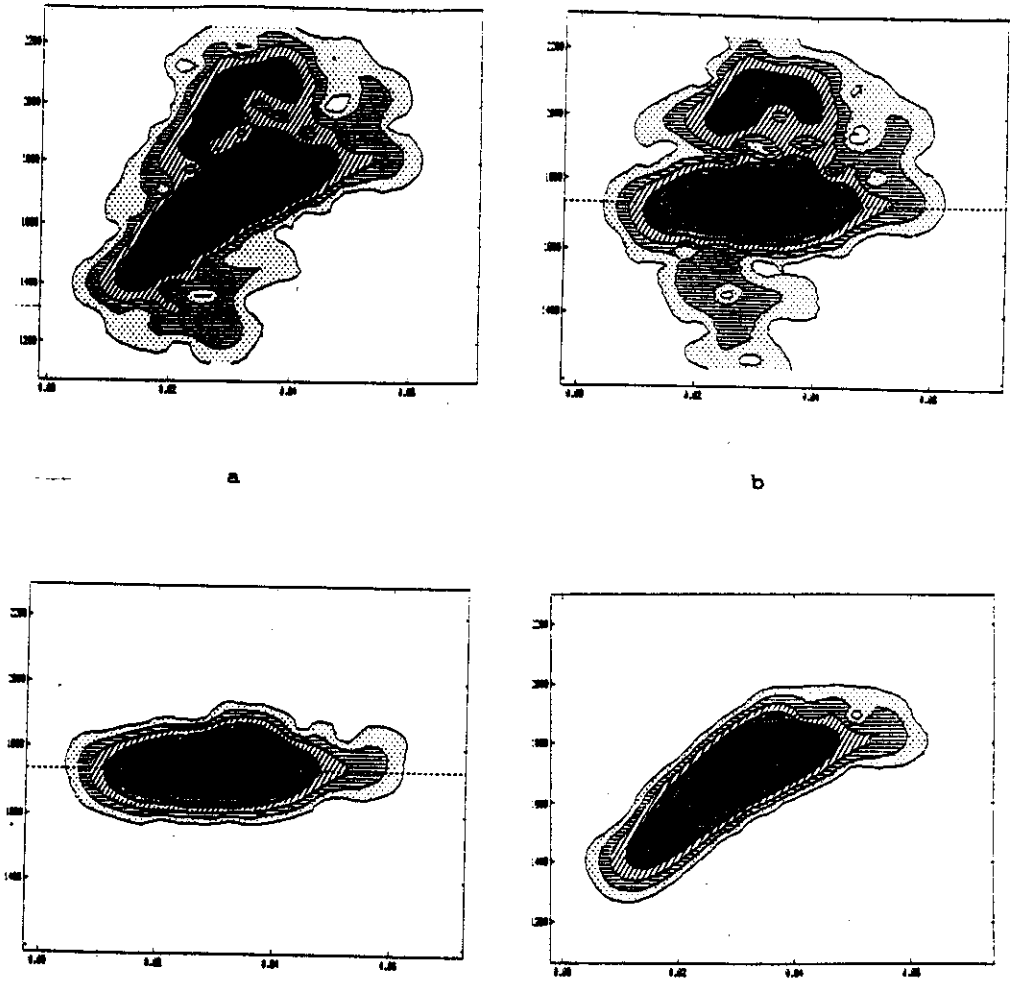



Fig 5. Separation of Rayleigh (a) and Love (b) waves on the FTAN map.



a b

Fig 6. Frequency-time filtering: (a) is the initial FTAN map; (b) is the filtered FTAN map.



a b

c d

Fig 7. (a) FTAN map of the original signal; (b) FTAN map after phase equalization; (c) FTAN map after filtering; (d) resulting FTAN map.