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Synthetic Seismograms from Multimode Summation: Theory and Computational Aspects

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SYNTHETIC SEISMOGRAMS FROM MULTIMODE SUMMATION: THEORY AND COMPUTATIONAL ASPECTS

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1. INTRODUCTION

In spite of the very considerable efforts made by seismologists and theoreticians, it is still missing a satisfactory theory which describes accurately wave propagation in three-dimensional models of the Earth. If the extremely time consuming numerical procedures, based on finite differences or finite element methods, are excluded, all the existing analytical methods involve significant approximations. The modal summation method (Panza, 1985; Florsh et al., 1991) is practically free from approximation in the one dimensional case and can be efficiently extended, introducing approximations of variable and to some extent quantifiable size, to two- and three-dimensional cases (Vaccari et al., 1989). The method allows to construct very realistic signals, also in the relatively simple one-dimensional case, and can be very easily applied for a quantitative and realistic earthquake hazard assessement.

2. WAVES IN MULTILAYERED MEDIA

The medium is assumed to consist of homogeneous layers, separated by first-order discontinuities. If a medium is continuously inhomogeneous, it is replaced by a number of homogeneous layers. The advantage of the homogeneous-layer approximation is that inside each layer the equation of motion takes a relatively simple form and can be solved exacly. Its disadvantage is that boundary conditions have to be fulfilled at many interfaces. Analytical methods for inhomogeneous layers - in contrast to numerical, e.g. finite-difference methods - are not yet developed to a point where they really can compete with the methods for homogeneous layers. At present, within the framework of the activities of the Istituto di Geodesia e Geofisica dell'Universita' di Trieste, it is under development a large project for the formulation of the theory and related computer code for the construction of complete synthetic seismograms for three-dimensional anelastic media, based on modal summation.

The equation of motion for a homogeneous, isotropic elastic medium is

$$\rho \mathbf{u}_{tt} = (\lambda+2\mu)\ \text{grad div}\ \mathbf{u} - \mu\ \text{rot rot}\ \mathbf{u} \qquad (1)$$

where **u** is the displacement vector, $\mathbf{u}_{tt}$ its time second order derivative, ρ is the density and λ and μ are the Lamé parameters. Body forces due to gravity and seismic sources are not included in equation (1): it is assumed that gravity has no other effect than to determine, via self compression, the constant values of ρ, λ and μ, and sources of seismic waves are included through their known contribution to **u** (Harkrider, 1964). In order to simplify the discussion as far as possible, we shall consider solutions of the elastic equations of motion in the form of plane waves rather than attempt to treat the more complex case of waves diverging from a point-source. This does not involve loss of generality in the computation of the dispersion function since the point-source solution may be developed by integration of plane-wave solutions (Harkrider, 1964), with a preassigned precision depending upon source-receiver distance (Panza et al., 1973).

The x axis is taken parallel to the layers with the positive sense in the direction of propagation. The positive z axis is taken as directed into the medium.

3. P-SV WAVES

For the m-th layer let r_m= density, d_m= thickness, λ_m and $\mu_m = \rho_m\beta_m^2$= Lamé elastic constants, α_m= velocity of propagation of dilatational waves, β_m=velocity of propagation of rotational waves, $k=\omega/c$=horizontal wave number, ω angular frequency, c phase velocity, $\gamma_m=2(\beta_m/c)^2$, u_m=displacement component in the x direction, w_m=displacement component in the z direction, σ_m=normal stress, τ_m=tangential stress.

For $m<n$ $r_{\alpha_m}=[(c/\alpha_m)^2-1]^{1/2}$ if $c>\alpha_m$ and $r_{\alpha_m}=-i[1-(c/\alpha_m)^2]^{1/2}$, if $c<\alpha_m$; furthermore $r_{\beta_m}=[(c/\beta_m)^2-1]^{1/2}$, if $c>\beta_m$ and $r_{\beta_m}=-i[1-(c/\beta_m)^2]^{1/2}$ if $c<\beta_m$. Finally, if $m=n$, $r_{\alpha_m}=-i[1-(c/\alpha_m)^2]^{1/2}$ and $r_{\beta_m}=-i[1-(c/\beta_m)^2]^{1/2}$.

Then periodic solutions of the elastic equation of motion for the m-th layer may be found by combining dilatational wave solutions,

$$\Delta_m=(\partial u_m/\partial x)+(\partial w_m/\partial z)= \exp[i(pt-kx)][\Delta'_m\exp(-ikr_{\alpha_m}z)+\Delta''_m\exp(ikr_{\alpha_m}z)] \qquad (2)$$

with rotational wave solutions

$$\delta_m=(1/2)[(\partial u_m/\partial z)-(\partial w_m/\partial x)]= \exp[i(pt-kx)][\delta'_m\exp(-ikr_{\beta_m}z)+\delta''_m\exp(ikr_{\beta_m}z)] \qquad (3)$$

where Δ'_m, Δ''_m, δ'_m and δ''_m are constants.

With the sign conventions defined above, the term in Δ'_m represents a plane wave whose direction of propagation makes an angle $\cot^{-1}r_{\alpha_m}$ with the +z direction when r_{α_m} is real, and a wave propagated in the +x direction with amplitude diminishing exponentially in the +z direction when r_{α_m} is imaginary. Similarly, the term in Δ''_m represents a plane wave making the same angle with the -z direction when r_{α_m} is real and a wave propagated in the +x direction with amplitude increasing exponentially in the +z direction when r_{α_m} is imaginary. The same applies to the terms in δ'_m and δ''_m with r_{β_m} substituted for r_{α_m}.

Droping the term $\exp[i(\omega t-kx)]$ the displacements and the pertinent stress components corresponding to the dilatation and rotation, given by equations (2) and (3), can be written:

$$u_m=-(\alpha_m/\omega)^2(\partial\Delta_m/\partial x)-2(\beta_m/\omega)^2(\partial\delta_m/\partial z) \qquad (4)$$

$$w_m=-(\alpha_m/\omega)^2(\partial\Delta_m/\partial z)+2(\beta_m/\omega)^2(\partial\delta_m/\partial x) \qquad (5)$$

$$\sigma_m=\rho_m\{\alpha^2_m\Delta_m+2\beta^2_m[(\alpha_m/\omega)^2(\partial^2\Delta_m/\partial x^2)+2(\beta_m/\omega)^2(\partial^2\delta_m/\partial x\partial z)]\} \qquad (6)$$

$$\tau_m=2\rho_m\beta^2_m\{-(\alpha_m/\omega)^2(\partial^2\Delta_m/\partial x\partial z)+(\beta_m/\omega)^2[(\partial^2\delta_m/\partial x^2)-(\partial^2\delta_m/\partial z^2)]\} \qquad (7)$$

The boundary conditions at an interface between two layers require that these four quantities should be continuous. Continuity of the displacements is assured if the corresponding velocity components $\dot{u}_m$ and $\dot{w}_m$ are made continuous and, since c is the same in all layers, we may take the dimensionless quantities $\dot{u}_m/c$ and $\dot{w}_m/c$ to be continuous. Substituting the expressions (2) and (3) in equations (4) to (7) and expressing the exponential functions of ikrz in trigonometric form, we find

$$c\dot{u}_m=A_m\cos p_m-iB_m\sin p_m+r_{\beta_m}C_m\cos q_m-ir_{\beta_m}D_m\sin q_m \qquad (8)$$

$$c\dot{w}_m=-ir_{\alpha_m}A_m\sin p_m+r_{\alpha_m}B_m\cos p_m+iC_m\sin q_m-D_m\cos q_m \qquad (9)$$

$$\sigma_m=\rho_m(\gamma_m-1)A_m\cos p_m-i\rho_m(\gamma_m-1)B_m\sin p_m+\rho_m\gamma_m r_{\beta_m}C_m\cos q_m- i\rho_m\gamma_m r_{\beta_m}D_m\sin q_m \qquad (10)$$

$$\tau_m=i\rho_m\gamma_m r_{\alpha_m}A_m\sin p_m-\rho_m\gamma_m r_{\alpha_m}B_m\cos p_m-i\rho_m(\gamma_m-1)C_m\sin q_m+ \rho_m(\gamma_m-1)D_m\cos q_m \qquad (11)$$

where

$A_m=-\alpha^2_m(\Delta'_m+\Delta''_m)$, $B_m=-\alpha^2_m(\Delta'_m-\Delta''_m)$, $C_m=-2\beta^2_m(\delta'_m-\delta''_m)$, $D_m=-2\beta^2_m(\delta'_m+\delta''_m)$, $p_m=kr_{\alpha_m}[z-z^{(m-1)}]$, $q_m=kr_{\beta_m}[z-z^{(m-1)}]$, $z^{(m-1)}$ is the depth of the upper interface of the m-th layer and Δ'_m, Δ''_m, δ'_m, δ''_m are the constants defined in (6) appearing in the depth-dependent part of the dilatational and rotational wave solutions:

$$\Delta'_m\exp(-ikr_{\alpha_m}z)+\Delta''_m\exp(ikr_{\alpha_m}z) \qquad (12)$$
$$\delta'_m\exp(-ikr_{\beta_m}z)+\delta''_m\exp(ikr_{\beta_m}z) \qquad (13)$$

3.1. EVALUATION OF EIGENVALUES AND EIGENFUNCTIONS

For a continental model, the vanishing of the two components of stress at the free surface yields:

$$-\rho_1(\gamma_1-1)A_1-\rho_1\gamma_1 r_{\beta_1}C_1=0 \qquad (14)$$
$$\rho_1\gamma_1 r_{\alpha_1}B_1-\rho_1(\gamma_1-1)A_1=0 \qquad (15)$$

Thus the submatrix $\Lambda^{(0)}$ defined in (7,8) can be written in the form

$$\Lambda^{(0)}=\begin{vmatrix} -\rho_1(\gamma_1-1) & 0 & -\rho_1\gamma_1 & 0 \\ 0 & \rho_1\gamma_1 & 0 & -\rho_1(\gamma_1-1) \end{vmatrix} \qquad (16)$$

At the m-th interface, the continuity of displacement and stress yields

$$A_m\cos P_m-iB_m\sin P_m+r_{\beta_m}C_m\cos Q_m-ir_{\beta_m}D_m\sin Q_m=A_{m+1}+r_{\beta_{m+1}}C_{m+1}, \qquad (17)$$
$$-ir_{\alpha_m}A_m\sin P_m+r_{\alpha_m}B_m\cos P_m+iC_m\sin Q_m-D_m\cos Q_m=r_{\alpha_{m+1}}B_{m+1}-D_{m+1} \qquad (18)$$
$$\rho_m(\gamma_m-1)A_m\cos P_m-i\rho_m(\gamma_m-1)B_m\sin P_m+\rho_m\gamma_m r_{\beta_m}C_m\cos Q_m-i\rho_m\gamma_m r_{\beta_m}D_m\sin Q_m$$
$$=\rho_{m+1}(\gamma_{m+1}-1)A_{m+1}+\rho_{m+1}\gamma_{m+1}r_{\beta_{m+1}}C_{m+1}, \qquad (19)$$
$$i\rho_m\gamma_m r_{\alpha_m}A_m\sin P_m-\rho_m\gamma_m r_{\alpha_m}B_m\cos P_m-i\rho_m(\gamma_m-1)C_m\sin Q_m+\rho_m(\gamma_m-1)D_m\cos Q_m$$
$$=\rho_{m+1}\gamma_{m+1}r_{\alpha_{m+1}}B_{m+1}+\rho_{m+1}(\gamma_{m+1}-1)D_{m+1} \qquad (20)$$

where $P_m=kr_{\alpha_m}d_m$, $Q_m=kr_{\beta_m}d_m$ and d_m is the layer thickness. Thus the interface submatrices defined in (6) have the form

$$\Lambda^{(m)}=\begin{vmatrix} \cos P_m & -i\sin P_m/r_{\alpha_m} & \cos Q_m & -ir_{\beta_m}\sin Q_m & -1 & 0 & -1 & 0 \\ -ir_{\alpha_m}\sin P_m & \cos P_m & i\sin Q_m/r_{\beta_m} & -\cos Q_m & 0 & -1 & 0 & 1 \\ \rho_m(\gamma_m-1)\cos P_m & -i\rho_m(\gamma_m-1)\sin P_m/r_{\alpha_m} & \rho_m\gamma_m\cos Q_m & -i\rho_m\gamma_m r_{\beta_m}\sin Q_m & -\rho_{m+1}(\gamma_{m+1}-1) & 0 & -\rho_{m+1}\gamma_{m+1} & 0 \\ i\rho_m\gamma_m r_{\alpha_m}\sin P_m & -\rho_m\gamma_m\cos P_m & -i\rho_m(\gamma_m-1)\sin Q_m/r_{\beta_m} & \rho_m(\gamma_m-1)\cos Q_m & 0 & \rho_{m+1}\gamma_{m+1} & 0 & -\rho_{m+1}(\gamma_{m+1}-1) \end{vmatrix} \qquad (21)$$

and, noting that, when imposing surface waves conditions, in the half space $\Delta''_n=\delta''_n=0$, $A_n=B_n=-\alpha^2_n\Delta'_n$ and $C_n=D_n=-2\beta_n\omega'_n$, the submatrix representing the (n-1)th interface has the form

$$\Lambda^{(n-1)}=\begin{vmatrix} \cdots & -1 & -r_{\beta_n} \\ \cdots & -r_{\alpha_n} & 1 \\ \cdots & -\rho_n(\gamma_n-1) & -\rho_n\gamma_n r_{\beta_n} \\ \cdots & \rho_n\gamma_n r_{\alpha_n} & -\rho_n(\gamma_n-1) \end{vmatrix} \qquad (22)$$

where the first four columns are the same as those of $\Lambda^{(m)}$ with $m=n-1$. For each layer, $\Lambda^{(i)}$ $(i=1,n)$ submatrices represent the denominators of Cramer's system solutions when the boundary conditions are applied. In more compact notation it can be written

$$\Delta_R=\begin{vmatrix} |\Lambda^{(0)}| & & & & \\ & |\Lambda^{(1)}| & & & \\ & & |\,.\,| & & \\ & & & |\Lambda^{(n-2)}| & \\ & & & & |\Lambda^{(n-1)}| \end{vmatrix} \qquad (23)$$

where the non zero elements only are pictured. A condition for surface waves to exist is $\Delta_R=0$, which defines the dispersion function for Rayleigh waves:

$$F_R(\omega,c)=\Delta_R=0. \tag{24}$$

The discrete solutions (ω,c) of the equation (24) describe, in each of the layers, body waves or surface waves depending upon the real or imaginary nature of r_{α_m} and r_{β_m}. More precisely real values of r_{α_m} and r_{β_m} correspond to P- and S-waves while imaginary values of r_{α_m} and r_{β_m} correspond to surface waves. Therefore the modal summation method allows to solve in an exact and complete way the full wave equation in a preassigned (ω,c) interval. In other words it is possible to describe all the rays propagating with phase velocity less than a preassigned maximum value. It is easy to prove that using the modal representation the upper limit for the phase velocity is represented by the S-wave velocity value assigned to the half space used to terminate the structure at depth.

Once the eigenvalue problem is solved it is possible to determine eigenfunctions, i.e. displacements and stresses.

The algorithmic details of eigenfunction evaluation by Knopoff's method are rather involved (Schwab et al., 1984). The problem consists in the determination of the constants A_m, B_m, C_m, D_m for the layers above the homogeneous half-space and the constants A_n and D_n for the deepest structural unit. The starting point is therefore the linear, homogeneous system of 4n-2 equations in 4n-2 unknowns

$$\begin{vmatrix} |\Lambda^{(0)}| & & & & \\ & |\Lambda^{(1)}| & & & \\ & & |\;\cdot\;| & & \\ & & & |\Lambda^{(n-2)}| & \\ & & & & |\Lambda^{(n-1)}| \end{vmatrix} \begin{vmatrix} A_1 \\ r_{\alpha_1}B_1 \\ r_{\beta_1}C_1 \\ D_1 \\ A_2 \\ r_{\alpha_2}B_2 \\ r_{\beta_2}C_2 \\ D_2 \\ \\ A_{n-1} \\ r_{\alpha_{n-1}}B_{n-1} \\ r_{\beta_{n-1}}C_{n-1} \\ D_{n-1} \\ A_n \\ D_n \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ 0 \\ 0 \end{vmatrix} \tag{25}$$

where the submatrices $\Lambda^{(i)}$ (i=1,n) are given by equations (16), (21) and (22). Once the dispersion or eigenvalue problem is solved we are ready to determine the layer constants. This is done by deleting the last equation of the system and transposing the terms containing D_n to the right-hand side of the equations, thus forming a vector of inhomogeneous terms. If we arbitralily set D_n to unity, this will force all $r_{\alpha_m}B_m$ and D_m to be real, and all A_m and $r_{\beta_m}C_m$ to be imaginary. At this stage Cramer's rule can be applied to obtain A_n. The remaining layer constants can be determined by iteration. For more details about the computations of eigenfunctions see Schwab et al. (1984).

4. SH WAVES

With the same notations and geometry of Section 3 we may write, for the m-th layer, the following expressions for the displacement and the stress

$$u_m=w_m=\sigma_m=\tau_m=0 \tag{26}$$

$$v_m=\exp i(\omega t-kx)[v'_m\exp(-ikr_{\beta_m}z)+v''_m\exp(ikr_{\beta_m}z)] \tag{27}$$

$$\upsilon_m=\mu_m(\partial v_m/\partial z)=ik\mu_m r_{\beta_m}\exp i(\omega t-kx)[v''_m\exp(ikr_{\beta_m}z)-v'_m\exp(-ikr_{\beta_m}z)] \tag{28}$$

Neglecting, here too, the term $\exp i(\omega t-kx)$, at the m-th interface the continuity of displacement and stress yields

$$\dot v_m/c=(\dot v_{m-1}/c)\cos Q_m+i\upsilon_{m-1}(\mu_m r_{\beta_m})^{-1}\sin Q_m \tag{29}$$

$$\upsilon_m=i(\dot v_{m-1}/c)\mu_m r_{\beta_m}\sin Q_m+\upsilon_{m-1}\cos Q_m \tag{30}$$

4.1. EVALUATION OF EIGENVALUES AND EIGENFUNCTIONS

From equations (29) and (30) the layer matrix can be defined:

$$a_m=\begin{bmatrix} \cos Q_m & \dfrac{i\sin Q_m}{\mu_m\cdot r_{\beta_m}} \\ i\mu_m\cdot r_{\beta_m}\cdot\sin Q_m & \cos Q_m \end{bmatrix}$$

For the multimode surface-wave eigenvalue computations, using notation of (Schwab and Knopoff, 1972), the dispersion function can be written as the modified product for layer-matrices :

$$F_L(\omega, c) = b_n \cdot b_{n-1} \cdot b_{n-2} \cdot \ldots \cdot b_1 \tag{31}$$

where n is the number of layers, including the lower halfspace. In equation (31) b_n is given by:

$$\begin{aligned} b_n &= (s,-1) \text{ if the halfspace is solid} \\ b_n &= (0,-1) \text{ if the halfspace is liquid} \\ b_n &= (1,0) \text{ if the halfspace is rigid} \end{aligned} \tag{32}$$

where

$$s = -\mu_n \cdot \left(1 - \left(\frac{c}{\beta_n}\right)^2\right)^{\frac{1}{2}} \tag{33}$$

b_m (0<m<n) is given by:

$$b_m = \begin{bmatrix} \cos Q_m & \dfrac{\sin Q_m}{\mu_m \cdot r_{\beta_m}} \\ \mu_m \cdot r_{\beta_m} \cdot \sin Q_m & \cos Q_m \end{bmatrix} \quad \text{if } c > \beta_m \tag{34}$$

$$b_m = \begin{bmatrix} \cosh Q^*_m & \dfrac{\sinh Q^*_m}{\mu_m \cdot r^*_{\beta_m}} \\ -\mu_m \cdot r^*_{\beta_m} \cdot \sinh Q^*_m & \cosh Q^*_m \end{bmatrix} \quad \text{if } c < \beta_m \tag{35}$$

$$b_m = \begin{bmatrix} 1 & \dfrac{p \cdot d_m}{\mu_m \cdot c} \\ 0 & 1 \end{bmatrix} \quad \text{if } c = \beta_m \tag{36}$$

where we have introduced the real part of imaginary quantities

$$\left.\begin{aligned} r^*_{\beta_m} &= -\left(1 - \left(\frac{c}{\beta_m}\right)^2\right)^{\frac{1}{2}} \\ Q^*_m &= \frac{p \cdot r^*_{\beta_m} \cdot d_m}{c} = k \cdot r^*_{\beta_m} \cdot d_m \end{aligned}\right\} \quad \text{if } c < \beta_m \tag{37}$$

The modified matrix product of b_m and b_{m-1} is defined as follows:

$$\left[b_m \cdot b_{m-1}\right]_{jk} = \begin{cases} (b_m)_{jl} \cdot (b_{m-1})_{lk} & \text{if } (j+k) \text{ is even} \\ (-1)^{j+i} \cdot (b_m)_{jl} \cdot (b_{m-1})_{lk} & \text{if } (j+k) \text{ is odd} \end{cases} \tag{38}$$

The mathematical solution of the surface wave propagation allows two types of waves in the solid halfspace, exponentially increasing and decreasing with depth. To avoid infinite values of the solution, the coefficient of exponentially increasing wave in the halfspace must vanish (surface waves condition). If the halfspace is supposed to be liquid, the deepest interface is at the analogy of the mantle-core boundary. In analogy with the case of P-SV waves, imaginary values of r_{β_m} correspond to surface waves, while real values of r_{β_m} correspond to S-waves. More precisely real values of r_{β_m} correspond to S-waves while imaginary values of r_{β_m} correspond to surface waves. Therefore also for SH-waves, the modal summation method allows to solve in an exact and complete way the full wave equation in a preassigned (ω,c) interval. In other words it is possible to describe all the rays propagating with phase velocity less than a preassigned maximum value. It is easy to prove that using the modal representation the upper limit for the phase velocity is represented by the S-wave velocity value assigned to the half space used to terminate the structure at depth.

The computation of the eigenfunctions at the layer interfaces can be performed as follows (10):

$$\begin{bmatrix} v_m \\ \upsilon_m \end{bmatrix} = \begin{bmatrix} \cos Q_m & \dfrac{\sin Q_m}{k \cdot \mu_m \cdot r_{\beta_m}} \\ -k \cdot \mu_m \cdot r_{\beta_m} \cdot \sin Q_m & \cos Q_m \end{bmatrix} \cdot \begin{bmatrix} v_{m-1} \\ \upsilon_{m-1} \end{bmatrix} \quad \text{if } c > \beta_m \tag{39}$$

$$\begin{bmatrix} v_m \\ \upsilon_m \end{bmatrix} = \begin{bmatrix} \cosh Q_m^* & \dfrac{\sinh Q_m^*}{k \cdot \mu_m \cdot r_{\beta_m}^*} \\ k \cdot \mu_m \cdot r_{\beta_m}^* \cdot \sinh Q_m^* & \cosh Q_m^* \end{bmatrix} \cdot \begin{bmatrix} v_{m-1} \\ \upsilon_{m-1} \end{bmatrix} \quad \text{if } c < \beta_m \tag{40}$$

$$\begin{bmatrix} v_m \\ \upsilon_m \end{bmatrix} = \begin{bmatrix} 1 & \dfrac{d_m}{\mu_m} \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} v_{m-1} \\ \upsilon_{m-1} \end{bmatrix} \quad \text{if } c = \beta_m \tag{41}$$

where v_m is the displacement and υ_m the stress at the interface m. Notice that :

$$\dot{v}_m = i\omega v_m \tag{42}$$

For the last interface, supposing a solid terminating halfspace, we shall use:

$$\begin{cases} v_{n-2} \cdot \cos Q_{n-1} + \dfrac{\upsilon_{n-2} \cdot \sin Q_{n-1}}{k \cdot r_{\beta_{n-1}} \cdot \mu_{n-1}} & \text{if } c > \beta_{n-1} \\ v_{n-2} \cdot \cos Q_{n-1}^* + \dfrac{\upsilon_{n-2} \cdot \sin Q_{n-1}^*}{k \cdot r_{\beta_{n-1}}^* \cdot \mu_{n-1}} & \text{if } c < \beta_{n-1} \\ v_{n-2} + \upsilon_{n-2} \cdot \dfrac{d_{n-1}}{\mu_{n-1}} & \text{if } c = \beta_{n-1} \end{cases} \tag{43}$$

These computations are performed using the initial values $(v_0, \upsilon_0)=(1,0)$ at the free surface.

5. COMPUTATION OF GROUP VELOCITIES

Following (Schwab and Knopoff, 1972) the group velocity, u, is obtained from

$$u = \frac{c}{1-(\omega/c)(\partial c/\partial \omega)} \tag{44}$$

where standard implicit function theory is applied to the dispersion function F to obtain

$$dc/d\omega = -(\partial F/\partial \omega)_c / (\partial F/\partial c)_\omega \tag{45}$$

Equation (45) is obviously valid both when F indicates Love as well as Rayleigh dispersion function.

6. ENERGY INTEGRAL

Along with eigenvalues and eigenfunctions, the integrals:

$$I_{1_R} = \int_0^\infty \rho(z)\{y_1^2(z) + y_3^2(z)\}dz \tag{46}$$

where $y_1=w(z)/w(0)$ and $iy_3=u(z)/w(0)$, are required in multimode synthesis of theoretical seismograms. For a sequence of homogeneous solid layers, these integrals can be written as

$$I_{1_R} = c^2 \, [\, r_{\alpha_1} B_1 - D_1]^{-2} \sum I_{(m)} \qquad \text{with } m=1, 2, \ldots\ldots n.$$

The integrals $I_{(m)}$ are given by equations (51) and (53) of (9).

For SH-waves we have:

$$I_{1_L} = \begin{cases} \left(\dfrac{c}{\dot{v}_0}\right)^2 \cdot \displaystyle\sum_{m=1}^{n} I_{(m)} & \text{for the } (S-L) \text{ case} \\ \left(\dfrac{c}{\dot{v}_0}\right)^2 \cdot \left(\left(\displaystyle\sum_{m=1}^{n} I_{(m)}\right) + I_{(S-S)}\right) & \text{for the } (S-S) \text{ case} \end{cases} \tag{48}$$

with:

$$I_{(m)} = \int_{z_{m-1}}^{z_m} \rho_m \cdot \left(\frac{\dot{v}(z)}{c}\right)^2 dz$$

$$I_{(S-S)} = \int_{z_{n-1}}^{\infty} \rho_n \cdot \left(\frac{\dot{v}(z)}{c}\right)^2 dz \qquad (49)$$

The integrals $I_{(m)}$ can be computed analytically, both for Rayleigh and Love waves (Schwab et al., 1984; Florsh et al., 1991).

7. MODE FOLLOWER AND STRUCTURE MINIMIZATION

Since all the problems connected with the loss of precision at high frequencies have been solved (Schwab et al., 1984) the summation of higher modes of surface waves allows the generation of complete strong motion synthetics even at high frequencies. The key point in the use of multimode summation, both for Love and Rayleigh modes, is an efficient computation of the phase velocity for the different modes at sufficiently small frequency intervals Δf with sufficient precision. To be efficient it is not advisable to determine at each frequency and for each mode the zeros of the dispersion function using the standard root-bracketing and root-refining procedure (Schwab and Knopoff, 1972). This must be used only when strictly necessary, as for instance at the beginning of each mode. For all other points i of each mode, the phase velocity can be estimated by cubic extrapolation, using the values of the phase slowness $s=1/c$ and df/ds already determined at frequencies f_{i-2} and f_{i-1}. However, the precision that can be reached in this way is not satisfactory, thus the phase velocity value must be refined. This can be done by an iterative cubic fit in the F-c plane.

Once the problem of an efficient determination of phase velocities is overcome, two other main problems must be solved at each frequency: (a) to correctly follow a mode and (b) to determine the minimum number of layers to be used. The problem of correctly following a mode arises in the high-frequency domain (f>0.1Hz), where several higher modes are very close to each other. The determination of the minimum number of layers to be used - structure minimization - is critical in order to reach a high precision in phase velocity determination spending the minimum possible computer time. In order to ensure high efficiency in the computation of synthetic seismograms, it is necessary to compute the phase velocity, phase attenuation, group velocity, ellipticity, energy integral and eigenfunctions and their maximum depth of penetration at constant frequency intervals. To reach a maximum frequency of 10 Hz, a satisfactory step is 0.05 Hz. To determine the total number of modes present in the frequency interval considered, we fix $c=c_0$ a value close to β_n, where β_n is the S-wave velocity in the half-space, and we increment f to find its values corresponding to zeros of the dispersion function $F(f,c_0)$ (8). Obviously, starting from f=0, the first zero in $F(f,c_0)$ corresponds to the fundamental mode, the second to the first higher mode, and so on. The values of f for which $F(f,c_0) = 0$ are used as starting frequencies (the lowest frequencies) for the computation of the different modes. Once the starting frequency for each mode is defined, it is possible to compute, beginning from the fundamental mode, all dispersion relations. This is accomplished by keeping f fixed and varying c, the procedure being applied at all of the equally spaced frequency points of the chosen frequency interval.

More details about the mode follower and the complete description of the procedure for structure minimization are given in (Panza and Suhadolc, 1989).

8. ATTENUATION DUE TO ANELASTICITY

The treatment of anelasticity requires, for causality reasons, the introduction of body wave dispersion (BWD) (Futterman, 1962). In a medium with constant Q, the P- and S-wave phase-velocity can be expressed:

$$A_1(\omega) = \frac{A_1(\omega_0)}{1 + \frac{2}{\pi} \cdot A_1(\omega_0) \cdot A_2(\omega_0) \cdot \ln\left(\frac{\omega_0}{\omega}\right)} \qquad (50)$$

$$B_1(\omega) = \frac{B_1(\omega_0)}{1 + \frac{2}{\pi} \cdot B_1(\omega_0) \cdot B_2(\omega_0) \cdot \ln\left(\frac{\omega_0}{\omega}\right)} \qquad (51)$$

The layer index m is omitted in equations (50) and (51). $A_1(\omega_0)$ and $A_2(\omega_0)$ are the P-wave velocity and the P-wave phase attenuation, while $B_1(\omega_0)$ and $B_2(\omega_0)$ are the S-wave velocity and the S-wave phase attenuation at the reference angular frequency ω_0 (see also Panza and Suhadolc, 1989). The quantities A_1 and A_2 and B_1 and B_2 are related to the complex body-wave velocity α and β (Schwab and Knopoff, 1972):

$$\frac{1}{\alpha} = \frac{1}{A_1} - i \cdot A_2 \qquad (52)$$

$$\frac{1}{\beta} = \frac{1}{B_1} - i \cdot B_2 \qquad (53)$$

In the computation we have chosen the reference angular frequency $\omega_0=2\pi$ radians. In anelastic media the surface wave phase velocity c must be expressed as a complex quantity:

$$\frac{1}{c} = \frac{1}{C_1} - i \cdot C_2 \qquad (54)$$

C_1 is the attenuated phase velocity and C_2 is the phase attenuation, which is necessary for the computation of seismograms. C_2 can be estimated by using the variational technique (Takeuchi and Saito, 1972; Aki and Richards, 1980). The phase attenuation C_2 is given by (Panza, 1985; Florsh et al., 1991; Panza and Suhadolc, 1989). For Rayleigh waves:

$$C_2 = (2\omega I_3 \bar{k})^{-1} / \, \mathrm{Im}(I_4)$$

where $\bar{k}$ is the wave number in the perfectly elastic case and:

$$I_3 = \int_0^\infty \left\{ \left[(\lambda + 2\mu) - \frac{\lambda^2}{(\lambda + 2\mu)} \right] y_3^2 + \frac{1}{k} \cdot \left(y_1 y_4 - \frac{\lambda}{(\lambda + 2\mu)} y_2 y_3 \right) \right\} dz$$

$$I_4 = \int_0^\infty \left\{ \delta(\lambda + 2\mu) \left[\frac{1}{(\lambda + 2\mu)^2} (y_2^2 + 2k\lambda y_2 y_3) + k^2 \left(1 + \frac{\lambda^2}{(\lambda + 2\mu)^2} \right) y_3^2 \right] \right.$$
$$\left. + \delta\mu \frac{1}{\mu^2} y_4^2 - \delta\lambda \left[\frac{2k}{(\lambda + 2\mu)} (y_2 y_3 + k\lambda y_3^2) \right] \right\} dz$$

and $y_2=\sigma(z)/w(0)$ and $iy_4=\tau(z)/w(0)$,

$$\delta\mu = \rho\left(\beta_1^2 - \beta_2^2 - \bar{\beta}^2\right) + 2i\rho\beta_1\beta_2$$
$$\delta\lambda = \rho\left[\left(\alpha_1^2 - \alpha_2^2 - \bar{\alpha}^2\right) - 2\left(\beta_1^2 - \beta_2^2 - \bar{\beta}^2\right)\right] + i\rho 2(\alpha_1\alpha_2 - 2\beta_1\beta_2)$$
$$\delta(\lambda + 2\mu) = \rho\left(\alpha_1^2 - \alpha_2^2 - \bar{\alpha}^2\right) + i2\rho\alpha_1\alpha_2$$

In these expressions α and β are the compressional and shear-wave velocities in the perfectly elastic case.

All these integrals can be calculated analytically, since simple analytic expressions are known for the eigenfunctions.

The most important effect of the attenuation is the modification of the wave velocities and the decay of amplitude in the final computations of seismograms. As the variational technique is only an approximated method, the C_2 values can be in error by as much as 20 per cent in comparison with the exact method. This error arise mainly from the use of the elastic and therefore real eigenfunctions to compute the phase attenuation.

Recently (Day et al., 1989) showed the limits of the variational technique in the locked mode approximation, which can be obtained by limiting the model with a rigid or liquid halfspace. He showed, that an error in amplitudes up to 100 per cent can occur, when dealing with low Q-values. The error increases when the Q-values undergo large variations with depth. Introducing a solid halfspace in the model and using the structure minimization procedure prevents this kind of error.

9. RESPONSE TO BURIED SOURCES

To include the seismic source in the computations, the formulation due to (Harkrider, 1970) is used. A detailed description of the fault model of an earthquake used in the following computations is given in (Panza et al., 1973). For the double couple point source, the asymptotic expression of the Fourier time transform of the j-th Love(U_L)- or Rayleigh(U_R^R, U_R^V)-mode displacement at the free surface at a distance r from the source can be written as:

$$U_L = R(\omega) \cdot e^{i\Phi_0} \cdot e^{-\frac{i3\pi}{4}} \cdot k_L^{\frac{1}{2}} \cdot \chi_L(\theta, h) \cdot A_L \cdot \frac{e^{-ik_L r}}{\sqrt{2\pi r}} \cdot e^{-\omega r C_{2L}} \qquad (57)$$

$$U^R_R = R(\omega) \cdot e^{i\Phi_0} \cdot |n| \cdot e^{-\frac{i3\pi}{4}} \cdot k_R^{\frac{1}{2}} \cdot \varepsilon_0 \cdot \chi_R(\theta, h) \cdot A_R \cdot \frac{e^{-ik_R r}}{\sqrt{2\pi r}} \cdot e^{-\omega r C_{2R}} \qquad (58)$$

$$U^V_R = e^{-\frac{i\pi}{2}} \cdot \varepsilon_0^{-1} \cdot U_R \qquad (59)$$

where $R(\omega)$ is the Fourier transform of the equivalent point-force time function, n is the unit vector perpendicular to the fault and has units of length, $\Phi_0 = \arg R(\omega)$ is the initial phase and $\varepsilon_0 = -u^*(0)/w(0)$ is the ellipticity. The factors A_R A_L are given by:

$$A_L = \frac{1}{2 \cdot c \cdot u \cdot I_{1_L}} \qquad (60)$$

and

$$A_R = \frac{1}{2 \cdot c \cdot u \cdot I_{1_R}} \qquad (61)$$

where c and u are the phase and group velocities for Love and Rayleigh waves respectively.

The effect of anelasticity is expressed by the term:

$$e^{-\omega r C_2} \qquad (62)$$

where C_2, which indicates the phase attenuation either for Love or Rayleigh waves, can be determined as shown in Section 8.

$\chi(\theta,h)$ is the azimuthal dependence given by:

$$\chi_R(\theta, h) = d_0 + i \cdot (d_{1_R} \sin\theta + d_{2_R} \cos\theta) + d_{3_R} \sin 2\theta + d_{4_R} \cos 2\theta$$

for P-SV waves, and by

$$\chi_L(\theta, h) = i \cdot (d_{1_L} \sin\theta + d_{2_L} \cos\theta) + d_{3_L} \sin 2\theta + d_{4_L} \cos 2\theta$$

for SH waves.

$$d_0 = \tfrac{1}{2} \cdot B(h) \cdot \sin\lambda \cdot \sin 2\delta$$
$$d_{1_R} = -C(h) \cdot \sin\lambda \cdot \cos 2\delta$$
$$d_{2_R} = -C(h) \cdot \cos\lambda \cdot \cos\delta$$
$$d_{3_R} = A(h) \cdot \cos\lambda \cdot \sin\delta$$
$$d_{4_R} = -\tfrac{1}{2} \cdot A(h) \cdot \sin\lambda \cdot \sin 2\delta$$

$$d_{1_L} = G(h) \cdot \cos\lambda \cdot \cos\delta$$
$$d_{2_L} = -G(h) \cdot \sin\lambda \cdot \cos 2\delta$$
$$d_{3_L} = \tfrac{1}{2} \cdot V(h) \cdot \sin\lambda \cdot \sin 2\delta$$
$$d_{4_L} = V(h) \cdot \cos\lambda \cdot \sin\delta$$

θ is the angle between the strike of the fault and the epicenter-station direction, λ is the rake angle, δ is the dip angle and h is the source depth. A(h), B(h), C(h), G(h) and V(h) depend on the values of the eigenfunctions at the hypocenter:

$$A(h) = -\frac{u^*(h)}{w_0}$$

$$B(h) = -\left(3 - 4\frac{\beta^2(h)}{\alpha^2(h)}\right)\frac{u^*(h)}{w_0} - \frac{2}{\rho(h)\alpha^2(h)}\frac{\sigma^*(h)}{\dot{w}_0 / c}$$

$$C(h) = -\frac{1}{\mu(h)}\frac{\tau(h)}{\dot{w}_0 / c}$$

$$G(h) = -\frac{1}{\mu(h)} \cdot \left(\frac{\upsilon^*(h)}{\frac{\dot{v}_0}{c}}\right) = \frac{1}{k \cdot \mu(h)} \cdot \frac{\upsilon(h)}{v_0}$$

$$V(h) = \frac{\dot{v}_s(h)}{\dot{v}_0} = \frac{v_s(h)}{v_0}$$

The asymptotic expressions (57), (58) and (59) allow the computation of synthetic seismograms with at least 3 significant figures as long as kr>10 (Panza et al., 1973) and is equivalent to the expression in terms of the seismic moment [e.g. see equations (7.148), (7.149) and (7.150) in Aki and Richards (1980)].The seismogram related to a given mode is obtained by the inverse Fourier transform of (57), (58) and (59).

The extension of these results to the available formalism for sources with finite dimensions and durations is quite straightforward; the necessary details can be found in (Panza and Suhadolc, 1989).

10. TWO DIMENSIONAL MODELS

The expressions (57), (58) and (59), describing the displacement due to surface-wave modes, have been generalized to the case of two stratified quarterspaces in welded contact (Levshin, 1985; Vaccari et al., 1989). For example the radial component of displacement spectrum can be written:

$$U = R(\omega) \cdot e^{-i(3\pi/4)} \cdot e^{-i\omega(l/c + l'/c')} \cdot e^{-(lC_2 + l'C'_2)}$$
$$\frac{1}{\sqrt{J}} \cdot \sqrt{\frac{\omega}{2\pi}} \cdot \frac{\varepsilon'}{\sqrt{2c'u'I'_{1R}}} \cdot \Gamma_{jj'} \cdot \sqrt{\frac{\cos\varphi}{\cos\varphi'}} \cdot \frac{\chi_R(\theta, h)}{\sqrt{2c^2uI_{1R}}} \qquad (63)$$

It is assumed that the source and the receiver are situated far from the sharp vertical discontinuity in comparison to the biggest wavelength of interest. Equation (63) represents the radial displacement carried by the n-th Rayleigh mode generated by a point source at M(r, l) of a medium j, then transmitted through the boundary between j and j' and recorded at point M'(r', l') on the surface of medium j' as mode n'. r and r' are respectively the distances of source and receiver from the vertical interface, l and l' are the paths travelled by incident and generated waves, ϕ and ϕ' are the angles of incidence and refraction. In Eq.(63) the primed quantities refer to the medium where the receiver is placed while the unprimed quantities refer to the medium containing the source. R(ω) is the Fourier transform of the time function relative to the source, c and c' are the phase velocities, u and u' the group velocities, I and I' are energy integrals e' is the Rayleigh mode ellipticity of the receiver's medium, $\chi_R(\theta,h)$ is the radiation pattern evaluated for the medium containing the source, $J=(\cos\phi \cos\phi'/c)[(rc/\cos^3\phi)+(r'c'/\cos^3\phi')]$ describes the geometrical spreading of the surface wave energy. The effect of anelasticity is expressed by $e^{-\omega(lC_2+l'C'_2)}$ where C_2 and C'_2 are the phase attenuations in the two media (for more details see (1)).

The coupling coefficients $\Gamma_{jj'}(\omega, \phi, \phi', n, n')$, necessary to describe reflection and transmission phenomena at the boundary, will be discussed in Section 10.1.

10.1. COUPLING COEFFICIENTS

The problem of reflection and transmission of surface waves through lateral discontinuities existing inside the earth can not be solved in an analytical way. Several methods based on different approximations have been proposed to define and estimate surface waves reflection and transmission coefficients. Here only the transmission problem will be discussed, but the same procedure can be adopted to describe the reflection problem.

The approximations suggested by (Gregersen and Alsop, 1974) were chosen to evaluate a set of coupling coefficients that gives a picture of how the energy carried by the normal modes, characteristic of the medium with the source, and transmitted through the discontinuity, is redistributed among the normal modes, of the medium with the receiver.

The starting point is the stress-displacement system of the incoming surface wave mode. Decomposing the incident wave into the P-component and the SV-component, we turn to a solvable problem of reflection and transmission of P-SV waves. The problem will be solved for every single section on the vertical interface as if it would be infinite, using well known formulae based on Snell's law and continuity conditions of displacement and traction at the boundary. Since the sections are in reality limited, in this way we neglect the effects arising at the corners between the horizontal interfaces and the vertical one. We can think the corner effects as giving rise to a system of diffracted waves. These arise because it is impossible to satisfy exactly the continuity conditions also on the horizontal interfaces. With this approximation a reflected and a transmitted stress-displacement system can be determined. They contain a P-component and an SV-component but their combination does not give a Rayleigh wave any longer because the continuity conditions at the horizontal boundaries are not matched. The medium with the receiver is characterized by a set of normal modes corresponding to solutions of the wave equation verifying the continuity conditions on the horizontal interfaces. Our aim is to determine how each of these modes is excited by the modes contained in the incident wave, or in other words, how the transmitted system redistributes among the normal modes existing in the medium with the receiver. To obtain this, the transmitted system is projected on the system of normal modes characteristic of the medium with the receiver, using an appropriate definition of scalar product.

A stress-displacement vector for an incident wave identified by the subscript I can be defined

$$\mathbf{A}_I = (u_I, v_I, w_I, p_{xxI}, p_{xyI}, p_{xzI}) \tag{64}$$

where p_{ij} is the j-th component of the stress acting across the plane normal to the i-th axis. The p_{xj} (with j = x, y, z) stress components are considered because for the geometry of the problem only stresses acting on the vertical plane x=0 are involved. Defining similarly a stress-displacement vector relative to the normal modes system of the medium with the receiver, the projection of vector $\mathbf{A}_I$ on the vector $\mathbf{A}_{II}$ is performed via a scalar product, suggested by Herrera's orthogonality relation (Herrera, 1964):

$$\langle \mathbf{A}_I, \mathbf{A}_{II} \rangle = \frac{1}{2i} \int_0^\infty [u_I \bar{p}_{xxII} + v_I \bar{p}_{xyII} + w_I \bar{p}_{xzII} - p_{xxI} \bar{u}_{II} - p_{xyI} \bar{v}_{II} - p_{xzI} \bar{w}_{II}] dz \tag{65}$$

the bar denotes complex conjugate. Indicating the transmitted system by the vector

$$\mathbf{A}_T = (u_T, v_T, w_T, p_{xxT}, p_{xyT}, p_{xzT}) \tag{66}$$

the quantity:

$$\gamma_{jj'}^{(m,m')} = \frac{\langle \mathbf{A}_T^{(m)}, \mathbf{A}_{II}^{(m')} \rangle}{\langle \mathbf{A}_I^{(m)}, \mathbf{A}_I^{(m)} \rangle^{\frac{1}{2}} \langle \mathbf{A}_{II}^{(m')}, \mathbf{A}_{II}^{(m')} \rangle^{\frac{1}{2}}} \tag{67}$$

provides the amplitude of mode n' of $\mathbf{A}_{II}$ due to mode n of $\mathbf{A}_I$, contained in the transmitted vector $\mathbf{A}_T$. If the amplitude of a wave generated by an incident wave of unit amplitude is preferred:

$$\Gamma_{jj'} = \gamma_{jj'}^{(m,m')} \frac{\langle \mathbf{A}_I^{(m)}, \mathbf{A}_I^{(m)} \rangle^{\frac{1}{2}}}{\langle \mathbf{A}_{II}^{(m')}, \mathbf{A}_{II}^{(m')} \rangle^{\frac{1}{2}}} = \frac{\langle \mathbf{A}_T^{(m)}, \mathbf{A}_{II}^{(m')} \rangle}{\langle \mathbf{A}_{II}^{(m')}, \mathbf{A}_{II}^{(m')} \rangle} \tag{68}$$

$\Gamma_{jj'}$ is the coupling coefficient appearing in equation (63).

The approximations used to obtain $\Gamma_{jj'}$ are:

a) the Rayleigh modes for the two media are evaluated assuming each medium as a halfspace instead of a quarterspace; this is a reasonable assumption a few wavelengths from the interface;

b) a system of diffracted waves arising at the corners of the sections is neglected; this is a good approximation for a small contrast in the elastic parameters characterizing the two quarterspaces (Gregersen and Alsop, 1974).

The crucial approximation is contained in this last point. Although it is not easy to estimate quantitatively the accuracy of it, a criterion is given by a reversibility theorem (Vaccari et al., 1989).

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