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Diffraction from Synchrotron Radiation Sources

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DIFFRACTION FROMSYNCHROTRON RADIATION SOURCES.

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Abstract. The Fraunhofer diffraction properties for a slit are recapitulated. The same technique is then used to deduct the diffraction relation for a gaussian transverse distribution of the light field. .' ynchrotron light source, could be a bending magnet or an undulator, is then treated in the same way as used for the other cases. Finally, the matching between the parameters of the electron beam and those for the light fields is discussed.

1. The single slit. A plane electromagnetic wave is passing a slit as shown in fig. 1. In the Frauenhof diffraction picture, each line segment at the slit is treated as a source point for spherical waves. Since the light intensity is constant over the slit, so is the electric field vector E. The difference in path length between different source points is

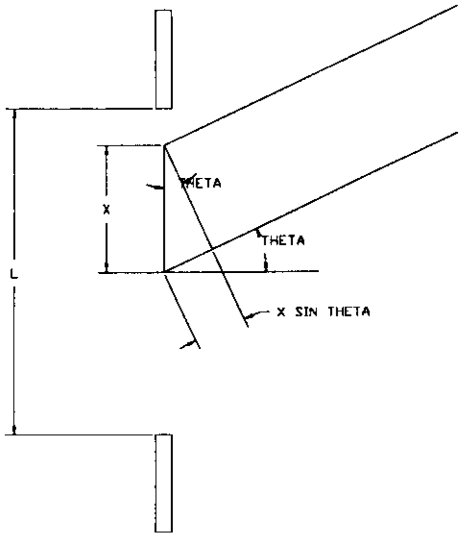

Fig. 1. Slit diffraction.

$\Delta \lambda = x \sin \theta = x\, w$ where $w = \sin \theta$

The difference in phase is given by

$$\Delta \Phi = \frac{\Delta \lambda}{\lambda} 2 \pi = x\, w\, k \text{ where } k = \frac{2 \pi}{\lambda}$$

We can integrate over the slit to get the distant angle distribution of the light field

$$E = const \int_{-L/2}^{L/2} e^{i x k w}\, dx = \frac{2 \sin (k\, w\, L/2)}{k\, w}$$

The light intensity is just the square of the field which gives us the the intensity distribution

$$I = const \frac{\sin^2 (k\, w\, L/2)}{(k\, w)^2}$$

The intensity distribution is seen in fig. 2 and we see that the first minimum is when

$k\, w\, L/2 = \pi$ or $L \sin \theta = \lambda$

which is the diffraction relation for a single slit.

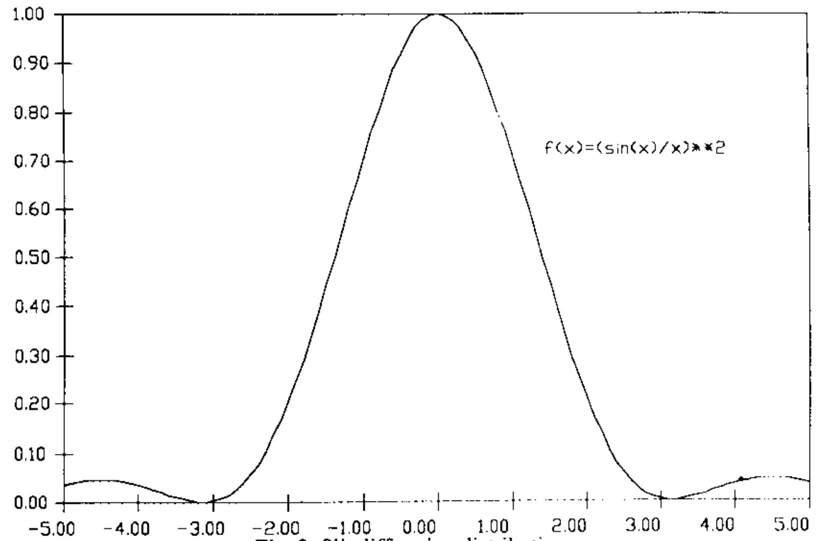

Fig. 2. Slit diffraction distribution.

2. Gaussian distributed beam.

We now proceed in the same way but replace the slit, which gives us a rectangular intensity distribution of the intensity with a light beam of gaussian distribution. The application is evident, the electron beam in a storage has a gaussian distributed intensity.

We treat here the one-dimensional case. The two-dimensional case gives easily the same result.

We assume a gaussian distributed intensity

$$I = const\ e^{-x^2/2\sigma^2}$$

We then get

$$E = const\ e^{-x^2/4\sigma^2}$$

Now, looking for the angular distribution we integrate the contributions from all parts of the field

$$E = const \int e^{\ x^2/4\sigma^2}\ e^{ikxw}\ dx = const \int e^{-1/4\sigma^2 (x - 2i\sigma^2 kw)^2}\ e^{-(\sigma k w)^2}\ dx$$

This integral is easily sovled and we get

$$E = const\ e^{-(\sigma k w)^2}$$

The intensity angular distribution is then given by

$$I = const\ e^{-2(\sigma k w)^2} = const\ e^{-w^2/2\sigma_w^2}$$

This gives us the standard deviation for the angular distribution of the intensity

$$\sigma_w = \frac{1}{2\sigma k}$$

or the well-known dispersion relation for gaussian distributions

$$\sigma_w \sigma = \frac{\lambda}{4\pi}.$$

3. Long light sources.

A relativistic electron passing a magnet structure will emitt synchrotron radiation in the forward direction. Let us start with a very simple model.

The light is now assumed to be emitted at an angle θ towards the particle trajectory. An observer will see an shining disk placed at the middle of the magnet structure as seen in fig 3. For small θ , we can approximat the diameter of this disk, the apparent source size to

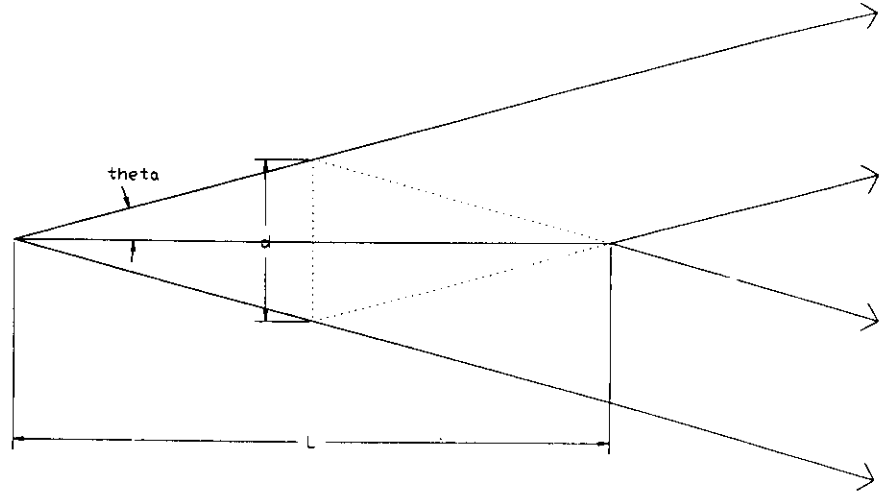

Fig. 3. Light from a lon source.

$$d = L\,\theta$$

The dispersion relation for slits will then give

$$d\,\theta = \lambda$$

or

$$L\,\theta^2 = \lambda \textit{ which gives us } \theta = \sqrt{\lambda/L}$$

Let us now look a bit deeper into the problem (Fig. 4.). At time t=0, the electron is at position x=0 and emitts a spherical light wave. A moment later, when the particle is at position x, it likewise emitts another spherical wave. These waves are out of phase

$$\Delta\Phi = x\,((1 - \cos\theta) + (1 - \beta))\,k$$

where βc is the effective speed in the x-direction of the particle.

The angle-dependent part of the phase shift can then be written for small angles

$$\Delta\Phi = x\,\frac{\theta^2}{2}\,k$$

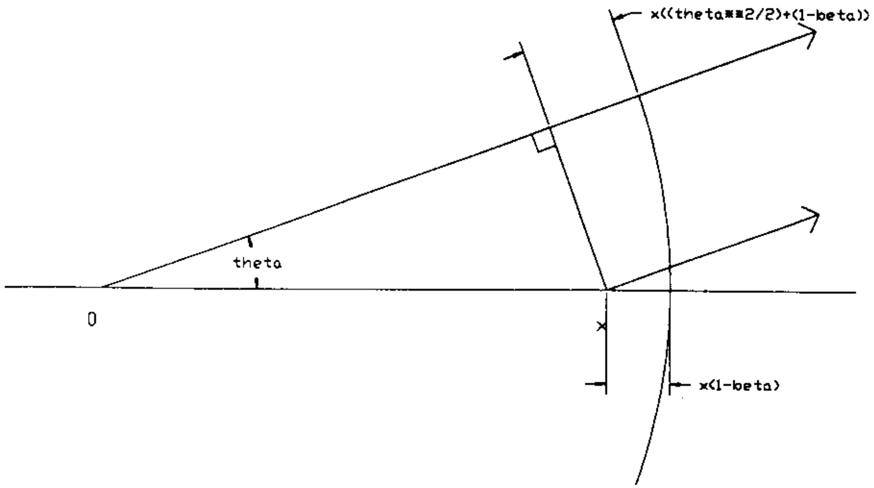

Fig. 4. Wavelength slip at long sources.

Integration over the total length of the magnet structure

$$E = const \int_{-L/2}^{L/2} e^{i k x \frac{\theta^2}{2}} \, dx$$

The light intensity is then given by

$$I = const \frac{\sin^2\left(k \frac{L}{2} \frac{\theta^2}{2}\right)}{\left(k \frac{\theta^2}{2}\right)^2}$$

The angular intensity distribution is seen in fig. 5. The first intensity zero takes place for

$$\theta = \sqrt{2\lambda/L}$$

4. The gaussian approximation.

We are left with one problem: the electron beam is gaussian distributed and we must take this into account when calculating the total effective source size and angular spread of the light. We will now make a gaussian approximation of the angular spread σ'_r induced by diffraction. Then, we will use the diffraction relation for gaussian distributions to calculate the source size induced by diffraction, σ_r. We can then add the standard deviations for the electron beam and the diffraction related ones quadratically to get the peak light intensity in phase space, the brilliance.

The goal to get the peak light intensity in phase space leads us to choose a standard deviation for the gaussian approximation so that the peak of the two distributions coincide.

A rotational symmetric gaussian distribution has the following normalized form

$$f_{gauss}(\theta) = \frac{1}{2\pi\sigma^2} e^{-\theta^2/2\sigma_\theta^2}$$

and the corresponding one for the diffraction distribution

$$f_{diff}(\theta) = \frac{2a}{\pi^2} \frac{\sin^2(a\theta^2)}{(a\theta^2)^2} \quad where\ a = \frac{2\pi}{\lambda} \frac{L}{4}$$

Equalizing the peak values of the two distributions yields

$$\sigma_\theta = \sigma'_r = \sqrt{\frac{\lambda}{2L}}$$

The corresponding (not to equal-looking) distributions are seen in fig. 5.

The diffraction relation for gaussian distributions gives us now

$$\sigma_r = \frac{\sqrt{\lambda L}}{2\sqrt{2}\,\pi}$$

In this approximation we get the total intensity distribution in phase space

$$\frac{d^4 I}{d\theta_x\, d\theta_y\, dx\, dy} = \frac{I_o}{(2\pi)^2} \frac{1}{\Sigma'_x \Sigma'_y \Sigma_x \Sigma_y} e^{-\theta_x^2/2\Sigma'^2_x} e^{-\theta_y^2/2\Sigma'^2_y} e^{-x^2/2\Sigma_x^2} e^{-y^2/2\Sigma_y^2}$$

where Io is the total number of photons per time unit and energy interval.

$$\Sigma_i = \sqrt{\sigma_i^2 + \sigma_r^2}$$

$$\Sigma'_i = \sqrt{\sigma'^2_i + \sigma'^2_r}$$

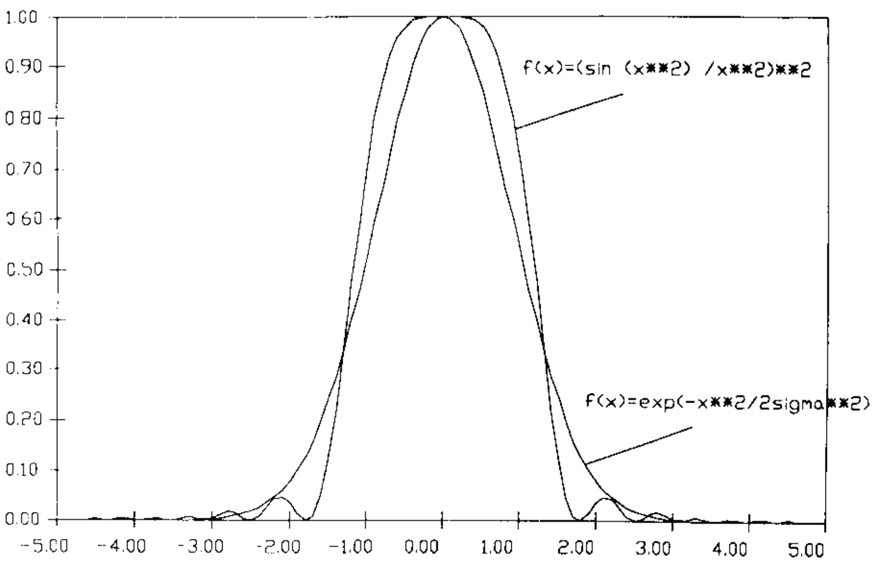

Fig. 5. Gaussian approximation.

It is now natural to define the brilliance as the maximum intensity density

$$B = \frac{I_o}{(2\pi)^2} \frac{1}{\Sigma_x \Sigma_y \Sigma'_x \Sigma'_y}$$

5. Matching.

From the brilliance definition, we see that it pays off to decrease the electron beam emittance

$$\varepsilon_i = \sigma_{i,e}\, \sigma'_{i,e}$$

until it reaches the diffraction-induced emittance $\sigma_r\, \sigma'_r$ or

$$\varepsilon_i = \frac{\lambda}{4\pi}$$

This is, however, not enough. The electron beam must also be matched to the diffraction conditions.The latter can be expressed as

$$\frac{\sigma_r}{\sigma'_r} = \frac{L}{2\pi}$$

The corresponding relation for the electron beam emittance can be written

$$\frac{\sigma_{i,e}}{\sigma'_{i,e}} = \beta_i \quad \textit{where } \beta \textit{ is the Twiss function.}$$

The design aim must then be to get

$$\beta_i = \frac{\sigma_r}{\sigma'_r}$$

Let first concider a bending magnet source. In this case σ'_r approximately equals $1/\gamma$ which through the dispersion relation for gaussian distributions yields $\sigma_r = \frac{\lambda\gamma}{4\pi}$.

The optimum β value is then

$$\beta = \frac{\sigma_r}{\sigma'_r} = \frac{\lambda\gamma^2}{4\pi}$$

which for most typical cases is around 10^{-5} - 10^{-6} m while the minimum β function which can be attained is around 0.1 m. The electron beam is thus heavily mismatched to the light characteristics. The electron beam size is generally three orders of magnitude to large while the electron beam angulars spread is much smaller than the diffraction defined one.

A much better matching is achieved when long undulators can be used.The minimum mean β function value which can be attained in a straight section of length L is around β=L/2. This is only a factor of π from a perfect match.

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