



INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
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UNITED NATIONS INDUSTRIAL DEVELOPMENT ORGANIZATION



INTERNATIONAL CENTRE FOR SCIENCE AND HIGH TECHNOLOGY

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**SECOND SCHOOL ON THE USE OF SYNCHROTRON
RADIATION IN SCIENCE AND TECHNOLOGY:
"JOHN FUGGLE MEMORIAL"**

25 October - 19 November 1993

Miramare - Trieste, Italy

Crystal Monochromators

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**2. School on the Use of Synchrotron Radiation in
Science and Technology**

Miramare (Trieste)

25. Oct. -19. Nov. 1993

Crystal Monochromators

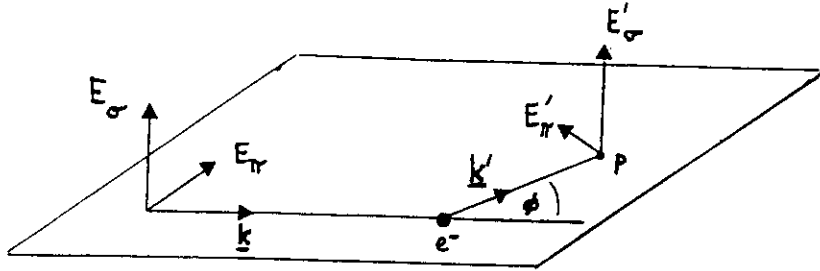
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2. Introduction into the elementary theory of elastic X-ray Scattering

2.1 Scattering at a free electron



Let us assume that an electro-magnetic wave having an electrical field $\underline{E}_0 \exp[i(\omega t - \underline{k} \cdot \underline{x})]$ impinges onto an electron. The electric field induces oscillations of the electron charge and thus an oscillating dipole of size:

$$\underline{M} = -(e^2 / m\omega^2) \underline{E}_0 \exp(i\omega t) \quad (1)$$

This oscillating dipole emits now also a wave. This wave has a wave vector $\underline{k}'$ (with $|\underline{k}'| = |\underline{k}|$), and its electrical field - in a sufficiently large distance r_p from the dipole - is given by (see Jackson):

$$\underline{E}_p = -(e^2 / mc^2)(1/r_p)(\underline{n}' \times \underline{E}_0) \times \underline{n}' \exp[i(\omega t - \underline{k}' \cdot \underline{r}_p)] \quad (2)$$

where $\underline{n}'$ is the unit vector in $\underline{k}'$ direction and $r_0 = e^2 / mc^2 = 2.818 \text{ (exp -15) m}$ is the classical electron radius.

In the case of σ polarisation we have

$$\underline{E}_\sigma' = -\underline{E}_\sigma (e^2 / mc^2)(1/r_p) \exp[i(\omega t - \underline{k}' \cdot \underline{r}_p)] \quad (3)$$

while for the case of π polarisation we get

$$\underline{E}_\pi' = -\underline{E}_\pi (e^2 / mc^2)(1/r_p) \exp[i(\omega t - \underline{k}' \cdot \underline{r}_p)] \cos \phi \quad (4)$$

where ϕ is the scattering angle.

This last case shows the possibility to use scattering in order to linearly polarise a beam which previously was not polarised. Indeed, for $\phi = 90^\circ$ only the σ component will be scattered.

2.2 Scattering by an atom; the Atomic Scattering Factor

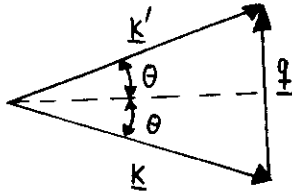
The scattering amplitude for a collection of electrons in an atom is the sum of the scattering amplitudes of all electrons taking into account the phase difference arising from the spatial distribution of the electrons which are assumed to be free.

This assumption is true for photon energies much higher than the binding energy of the innermost shell (for lower photon energies the bound character of the atomic electrons would become noticeable).

Let us now consider a X-ray beam which impinges onto an atom whose n electrons have the position vectors $\mathbf{r}_n$. For simplicity let us use σ polarisation. Then, in a point P in a sufficiently large distance, the scattered wave is given by the sum of all the n terms of type (3) created by n single atoms:

$$\begin{aligned} E_p &= -E_0(e^2/mc^2) \sum_{n=1}^Z 1/(|\mathbf{r}_p - \mathbf{r}_n|) \exp[-i\mathbf{k}' \cdot (\mathbf{r}_p - \mathbf{r}_n)] \exp[-i\mathbf{k} \cdot \mathbf{r}_n] \\ &= -E_0(e^2/mc^2) (1/r_p) \exp[-i\mathbf{k}' \cdot \mathbf{r}_p] \sum_{n=1}^Z \exp[-i\mathbf{r}_n \cdot \mathbf{q}] \quad (5) \end{aligned}$$

with $\mathbf{q} = \mathbf{k}' - \mathbf{k}$ = scattering vector and Z =atomic number.



Now we pass to the more realistic case in which each electronic state "i" has a continuous electron density ($\rho_i(\mathbf{r})$) distribution. Then, summing over all states "i" and integrating over all volume elements dv , equation (5) can be written as:

$$E_p = -E_0(e^2/mc^2)(1/r_p) \exp[-i\mathbf{k}' \cdot \mathbf{r}_p] \sum_i \int_V \rho_i(\mathbf{r}) \exp[-i\mathbf{r} \cdot \mathbf{q}] dv \quad (6)$$

The last part of equation (6) is called Atomic Scattering Factor or Form Factor:

$$f = \sum_i f_i = \sum_i \int_V \rho_i(\mathbf{r}) \exp[-i\mathbf{r} \cdot \mathbf{q}] dv \quad (7)$$

Thus the intensity scattered by an atom is given by:

$$I_p = I_0 (e^2/mc^2)^2 (1/r_p)^2 f^2 \quad (8)$$

f is therefore the relation between the intensities which are elastically scattered by an atom and by a classical electron.

If the $\rho_i(\mathbf{r})$ have spherical symmetries, then f can be written as:

$$f = \sum_i \int_0^\infty 4\pi r^2 \rho_i(r) (\sin kr / kr) dv \quad (9)$$

with $k = |\mathbf{q}| = 4\pi \sin \theta / \lambda$.

We note that in the case of $\mathbf{q} = 0$ (forward scattering)

$$f = Z \quad (10)$$

Up to now we assumed that the photon energy E_{ph} would be much larger than the binding energies E_B of the atom. With this approximation effects of Anomalous Scattering are not taken into account. These effects however can be included if correction terms are added to f :

$$f = f_0 + f' + if'' \quad (11)$$

where f_0 is the structure factor for $E_{ph} > E_B$

f' and f'' are real and imaginary part of the contribution due to the presence of a natural frequency ω' in the atom close to the frequency ω of the incoming photon beam.

It can be shown that f' and f'' are related by the Kramers-Kronig relationship

$$f'(\omega) = 2/\pi \int [(\omega' f''(\omega')) / (\omega^2 - \omega'^2)] d\omega'$$

Furthermore,

$$f''(\omega) = (m \omega / 4\pi e^2) (A/N) \mu(\omega) \quad (12)$$

where N = Avogadro's number
 A = atomic weight
 $\mu(\omega)$ = linear absorption coefficient

2.3 Scattering from a crystal: the Structure Factor

We consider now an assembly of n atoms, each characterised by a Scattering Factor f_n . In the crystal lattice the atoms are defined by the unit vectors $\underline{a}_1, \underline{a}_2$, and $\underline{a}_3$. The dimensions of the crystal are given by N_1, N_2 , and N_3 . Then the position of the n^{th} atom in the m^{th} unit cell can be written as:

$$\underline{L}_m = \underline{R}_m + \underline{r}'_n \quad (13)$$

where

$$\underline{R}_m = m_1 \underline{a}_1 + m_2 \underline{a}_2 + m_3 \underline{a}_3 \quad (14)$$

defines the unit cell and $\underline{r}'_n$ the atom in it.

The electric field in a sufficient far away point P is given by:

$$E_P = -E_0(e^2/mc^2)(1/r_P) \exp[-ik' \cdot r_P] \times \sum_n f_n \exp[-i\underline{L}_n' \cdot \underline{q}] \sum_m \exp[-i\underline{R}_m \cdot \underline{q}] \quad (15)$$

The factor

$$F_q = \sum_n f_n \exp[-i\underline{L}_n' \cdot \underline{q}] \quad (16)$$

is called Structure Factor.

In eq. (15) the terms in the sum over the unit cells can be divided into three separate sums, using eq. (14). Then each sum has the following form:

$$\sum_{m_1=0}^{N_1-1} \exp[-iq m_1 \underline{a}_1] = (\exp[iq N_1 \underline{a}_1] - 1) / (\exp[iq \underline{a}_1] - 1) \quad (17)$$

The intensity is proportional to the product of the square brackets of each of these terms:

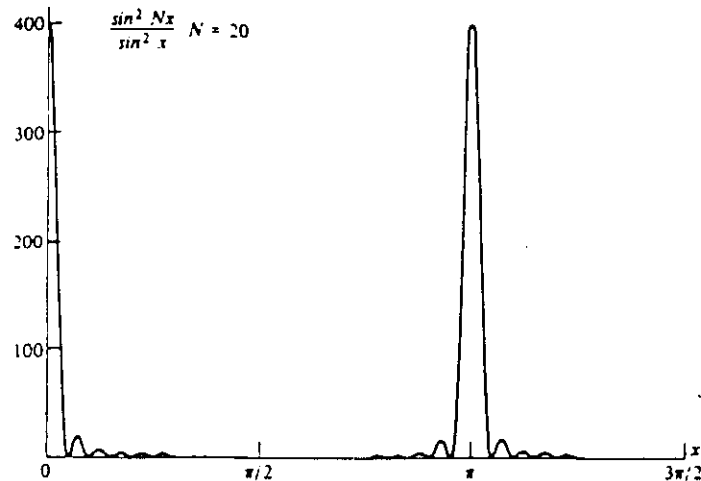
$$\{\sin^2 (1/2) \underline{q} \cdot N_1 \underline{a}_1\} / \{\sin^2 (1/2) \underline{q} \cdot \underline{a}_1\} \quad (18)$$

This function has a maximum only for $\underline{q} \cdot \underline{a}_1 = 2\pi h$, where h is an integer.

Equivalent conditions are valid for the other sums; thus an diffraction peak can be obtained only if:

$$\begin{aligned} \underline{q} \cdot \underline{a}_1 &= 2\pi h \\ \underline{q} \cdot \underline{a}_2 &= 2\pi k \\ \underline{q} \cdot \underline{a}_3 &= 2\pi l \quad \text{with } h, k, l \text{ integers} \end{aligned} \quad (19)$$

Note that the width of each peak is proportional to $1/N_1 N_2 N_3$, i.e. inverse proportional to the number of involved atoms.



The function $(\sin^2 Nx) / \sin^2 x$ for $N = 20$. The function peaks at values of x which are integral multiples of π , and it is essentially zero everywhere else.

The above equations can be rewritten using the vectors of the reciprocal lattice. If we define:

$$\underline{b}_1 = 2\pi (\underline{a}_2 \times \underline{a}_3) / \{\underline{a}_1 (\underline{a}_2 \times \underline{a}_3)\} \quad (20)$$

and the cyclic combinations to obtain $\underline{b}_2$ and $\underline{b}_3$, then

$$\underline{a}_i \cdot \underline{b}_j = 2\pi \delta_{ij} \quad (21)$$

These conditions are equivalent to

$$\underline{q} = \underline{H} \quad (22)$$

with the vector $\underline{H}$ of the reciprocal lattice equal to

$$\underline{K} = h \underline{b}_1 + k \underline{b}_2 + l \underline{b}_3 \quad (23)$$

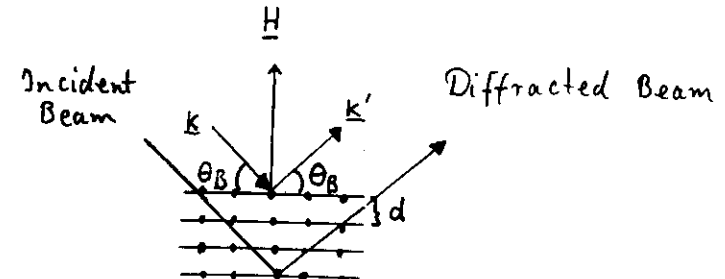
The condition (23) means that the scattering vector is equal to a vector of the reciprocal lattice.

We remember that

$$|\underline{H}| = 2\pi / d_{\{h,k,l\}} \quad (24)$$

Thus the relation (23) is equivalent to the well known Bragg's equation:

$$2 d_{\{h,k,l\}} \sin \theta_B = n \lambda \quad (25)$$



The Structure Factor

$$F_{\mathbf{H}} = \sum f_n \exp[-i\mathbf{L}_n \mathbf{H}]$$

determines if the diffraction on lattice planes, which are defined by $\mathbf{H}$, takes place or not.

For example, for the Zinc Blende structure the following rules for the components of $\mathbf{H}$ are valid:

for: $h + k + l = 4n$	$F_{hkl}^2 = 16 (f_A + f_B)^2$
$h + k + l = 2(2n+1)$	$F_{hkl}^2 = 16 (f_A - f_B)^2$
h, k, l all odd	$F_{hkl}^2 = 16 (f_A^2 + f_B^2)$
h, k, l partly even, odd	$F_{hkl}^2 = 0$

For Si and Ge the same rules are valid with $A = B$. Using these rules it is possible to calculate which reflexes, and which harmonics of them, are allowed.

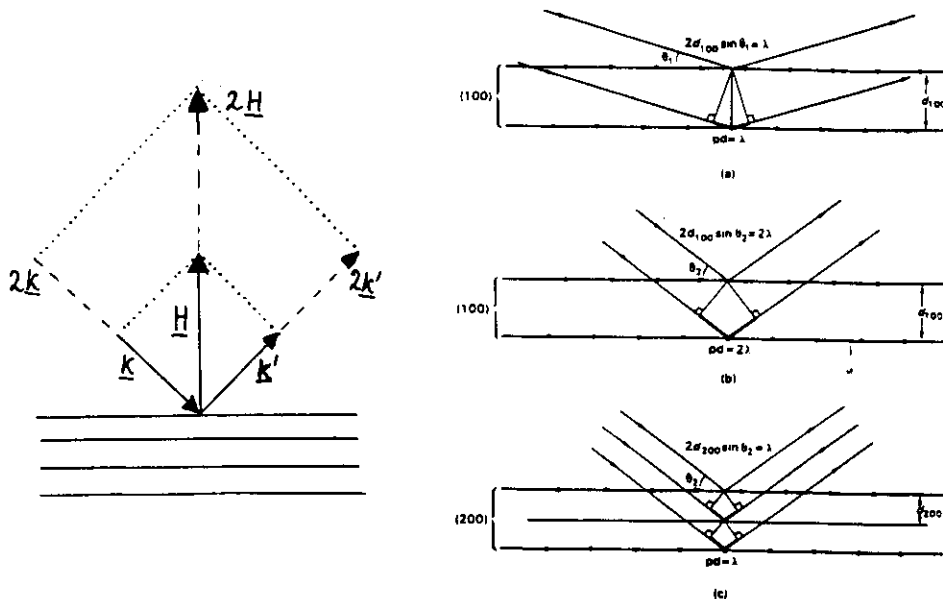


Fig. 3.7 Formation of (a) the first-order reflection from the (100) planes, (b) the second-order reflection from the (100) planes and (c) the first-order reflection from the (200) planes: pd = path difference. Note that (b) and (c) are equivalent.

3. Short Introduction into the Dynamical Theory of X-ray Diffraction

The intensity and form of a diffracted X-ray beam depends on the quality of a single crystal. For a "perfect" crystal one has to take into account that a diffracted beam is again diffracted. The double diffracted beam has again the direction of the incident beam.

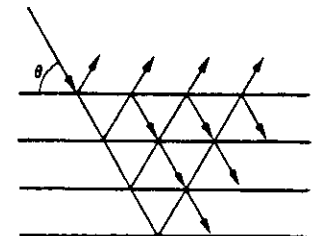
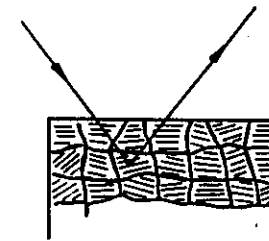
The Kinematical Diffraction Theory neglects this effect, while the Dynamical Diffraction Theory includes it.

Kinematical Theory

- highly imperfect crystals
- mosaic crystals
- small crystals

Dynamical Theory

- highly perfect crystals
- multiple scattering
- extinction



1914 Darwin 1. dynamical theory

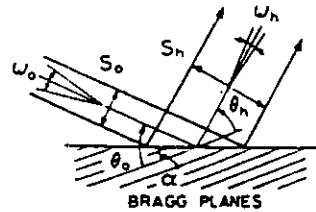
1916/17 Ewald independently developed theory; different, more sophisticated treatment of dynamical diffraction

1931 von Laue Ewald's theory reformulated; problem now based on the solution of the Maxwell equations for a medium with a periodic complex dielectric constant; with period comparable to the wavelength of incident X-rays

Intensity of a diffracted X-ray beam as function of the grazing angle

The Dynamical Theory allows to calculate the amplitude of the wave field in a crystal and the intensity of the diffracted beam.

Here we consider only the (asymmetric) Bragg case:



We define:

$$\eta = [-b \Delta\theta_0 \sin 2\theta_B + (1/2) \Gamma F_0 (1+b)] / \Gamma |P| |b|^{1/2} F_H$$

with

$$b = [\sin(\theta - \alpha)] / [\sin(\theta + \alpha)] \quad \text{asymmetry parameter} \\ (b=1 \text{ for sym. Bragg case})$$

$$\Delta\theta_0 = \theta_0 - \theta_B$$

$$\Gamma = (\lambda^2 / V\pi) (e^2/mc^2) \quad \text{with } V = \text{volume of unit cell}$$

F_H Structure Factor for diffraction at lattice planes defined by the reciprocal lattice vector $\underline{H}$

$$F_0 = (1 - \epsilon) / \Gamma \quad \text{Structure Factor for } \underline{H} = \underline{0}$$

$$P = 1 \quad \text{for } \sigma \text{ polarisation} \\ = \cos 2\theta_B \quad \text{for } \pi \text{ polarisation}$$

η is generally a complex number: $\eta = \eta' + i\eta''$, but gets real if absorption effects are neglected.

Now follows

$$|E_H|^2 / |E_0|^2 = |b| |\eta \pm (\eta^2 - 1)^{1/2}|^2$$

where

E_0 is the incoming electrical field

E_H is the diffracted electrical field

"-" is taken for $\text{Re}\{\eta\} > 0$ and "+" for $\text{Re}\{\eta\} < 0$

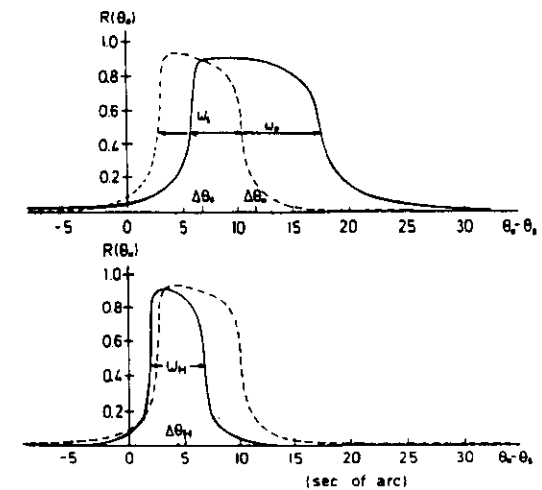


Fig. 4a. Reflection curves for Si(111) at 1.6 Å. $R(\theta_0)$ is the reflectivity as a function of the incidence angle θ , while $R(\theta_h)$ represents the intensity reflected at a reflection angle θ_h for a plane wave incident at θ_0 . (Solid lines for $b = 0.4$; broken lines for $b = 1$ where $R(\theta) = R(\theta_{-1})$). From ref. [20].

We note that:

1. In a small angular range $\Delta\theta$ around θ_B the incident wave gets
 - quasi completely reflected
 - exponentially attenuated in the crystal

This attenuation is described by the Extinction Coefficient, which is much larger than the ordinary absorption coefficient (e.g. for Si, 20 keV: the extinction length is 20 μm , while the ordinary absorption length is 1.1 mm).

2. Outside of $\Delta\theta$ occurs no diffraction
3. the center of the diffraction curve is at $\eta'=0$
and $\Delta\theta_0 = (1/2) (1 + 1/b) \Delta\theta_S$
 $\Delta\theta_H = (1/2) (1 + b) \Delta\theta_S$
with $\Delta\theta_S = \Gamma F_0' / \sin 2\theta_B$

This shift of the diffraction curve is due to the fact that the refraction index $n \neq 1$

4. diffraction occurs in the range $-1 \leq \eta' \leq 1$
with Darwin width $\omega_0 = \omega_S / b^{1/2}$
 $\omega_H = \omega_S b^{1/2}$
where $\omega_S = 2 \Gamma |F| F_H / \sin 2\theta_B$

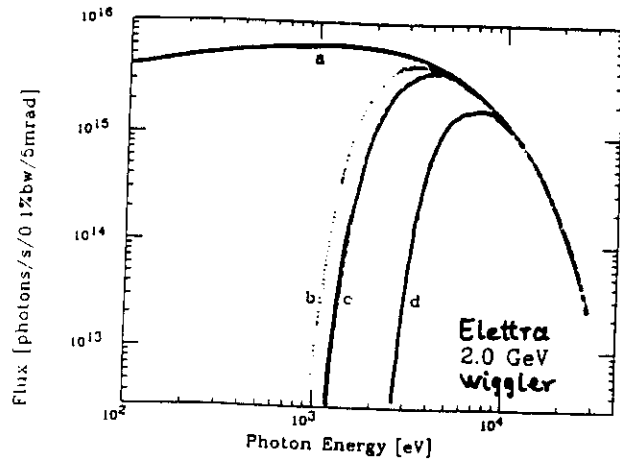
Bragg reflection widths ω , energy resolutions $\Delta E/E$, and integral reflecting powers I of some reflections of Si, Ge and α -quartz perfect crystals at $\lambda = 1.54 \text{ \AA}$. From ref. [20].

Crystal	hkl	ω (second of arc)	$\Delta E/E - \epsilon$ ($\times 10^{-4}$)	I ($\times 10^9$)
Silicon	111	7.395	14.1	39.9
	220	5.459	6.04	29.7
	311	3.192	2.90	16.5
	400	3.603	2.53	19.3
	331	2.336	1.44	11.8
	422	2.925	1.47	15.5
	333			
	(511)	1.989	0.88	9.9
	440	2.675	0.96	14.0
	531	1.907	0.60	9.3
Germanium	111	16.338	32.64	85.9
	220	12.449	14.46	67.4
	311	7.230	6.92	37.1
	400	7.951	5.94	42.3
	331	5.076	3.34	25.4
	422	6.178	3.34	32.4
	333			
	(511)	4.127	2.00	20.2
	440	5.339	2.14	27.5
	531	3.719	1.33	17.7
α -quartz	100	3.798	10.00	18.8
	101	7.453	15.26	40.9
	110	2.512	3.69	12.2
	102	2.488	3.36	12.9
	200	2.252	2.81	11.5
	112	2.927	3.03	15.5
	202	2.072	1.93	10.6
	212	2.042	1.47	10.7
	203	2.430	1.74	12.9
	301	2.368	1.69	12.6

T. Matsushita & H. Hashizume, 1983

Solutions for heat load problem

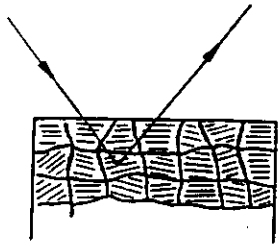
①



use power filters:

8 μm C
20 μm C
250 μm C

②



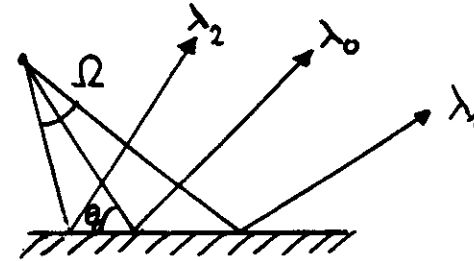
Large band-pass optic:
multilayers
mosaic crystals

③ cool 1. crystal very efficiently (3kW absorbed!),
heat 2. crystal to match temperature:

diffraction beamline, Si(111) : $\Delta T \leq 13^\circ\text{C}$
Si(220) : $\Delta T \leq 7^\circ\text{C}$

3. Description of different monochromators

3.1 Single Crystal monochromator

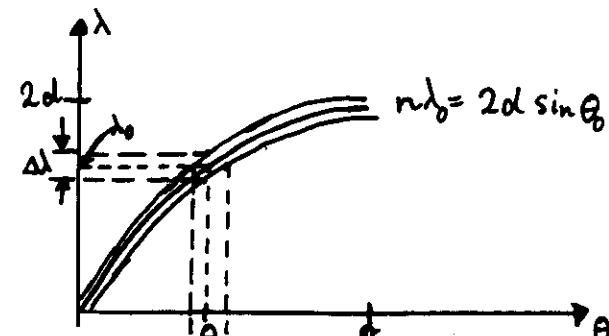


polychromatic point source with divergence Ω

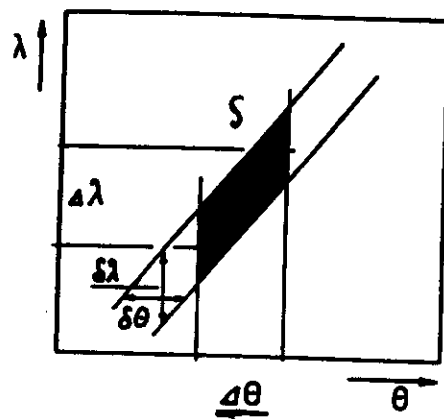
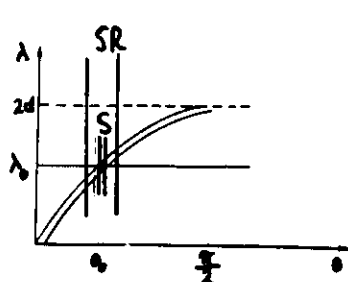
$$\begin{aligned} \rightarrow \lambda_0 &= 2d \sin \theta_0 \\ \lambda_1 &= 2d \sin(\theta_0 - \Omega/2 - \omega/2) \\ \lambda_2 &= 2d \sin(\theta_0 + \Omega/2 + \omega/2) \end{aligned}$$

If Ω and ω small $\rightarrow$

$$\begin{aligned} \frac{\Delta\lambda}{\lambda} &= (\Omega + \omega) \cot \theta_0 \\ &= \text{band pass} \end{aligned}$$



Du Mond diagram



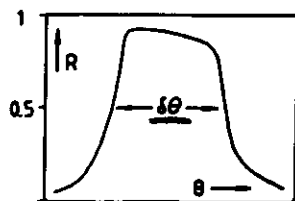
acceptance region composed by a crystal and a collimating slit S

- accepted photon flux $\sim$ red area $\uparrow$

$\delta\theta$ = limited reflex width (Darwin width)
 $\rightarrow$ intrinsic spectral acceptance $\delta\lambda = \lambda \cot\theta \delta\theta$

$\Delta\theta$ = Divergence of incoming light beam

$\rightarrow$ spectral resolution is given through red area $\uparrow$



as

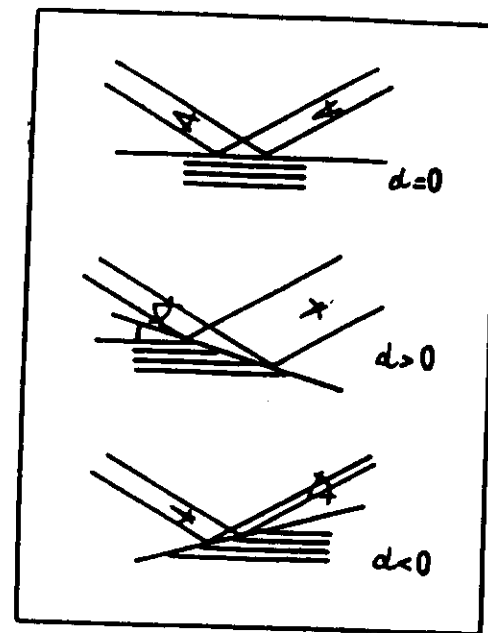
$$\frac{\Delta\lambda}{\lambda} = \cot\theta \Delta\theta + \frac{|\chi_h|}{\sin^2\theta}$$

with $|\chi_h| = \lambda^2 \frac{r_0}{\pi} \frac{|F_h|}{V_c}$

$r_0 = 0.28 \cdot 10^{-12} \text{ cm}$
 V_c = volume of unit cell

$\rightarrow$ accepted flux $\sim \delta\lambda \cdot \Delta\theta$

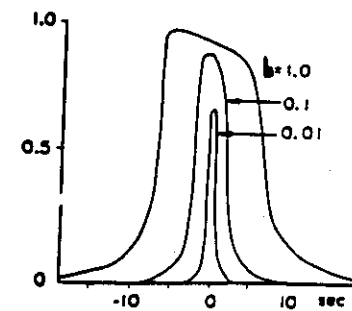
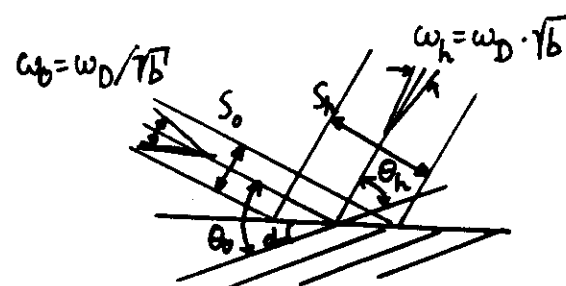
Asymmetric Cut Crystals



$\rightarrow$ possibility to vary reflex width $\delta\theta$:

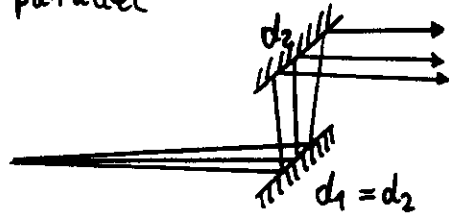
$$S_h \cdot \delta\theta_h = S_0 \cdot \delta\theta_0$$

cross section S_0 of incoming beam
 S_h of diffracted beam $\left\{ : \frac{S_h}{S_0} = \frac{\sin(\theta-d)}{\sin(\theta+d)} =: b \right.$

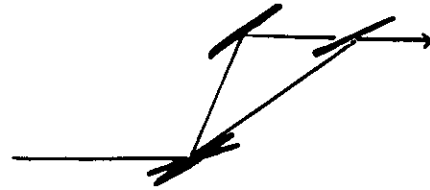


3.2 Two Crystal monochromators

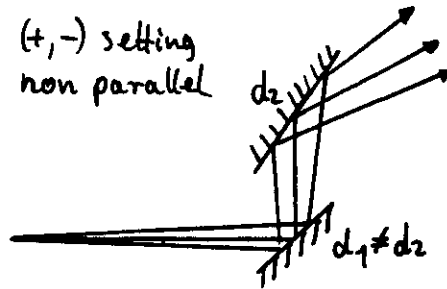
a) (+, -) setting
parallel



- non dispersive
- fixed exit possible

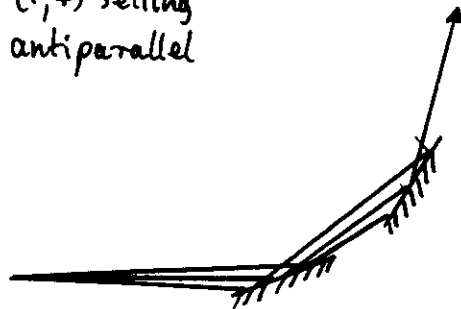


b) (+, -) setting
non parallel



- dispersive (slightly)

c) (+, +) setting
antiparallel



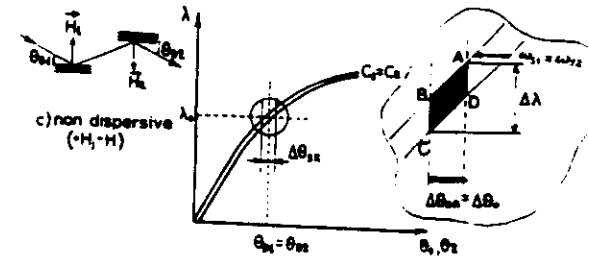
- highly dispersive

$$\Delta\lambda/\lambda = \omega \cot \theta_0$$

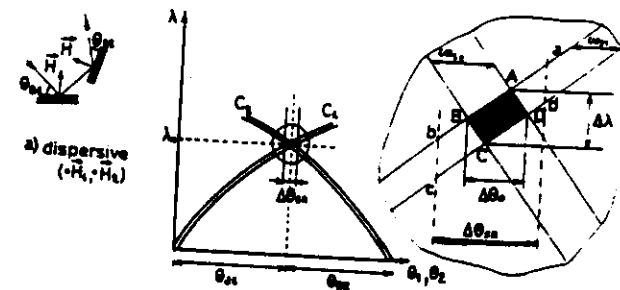
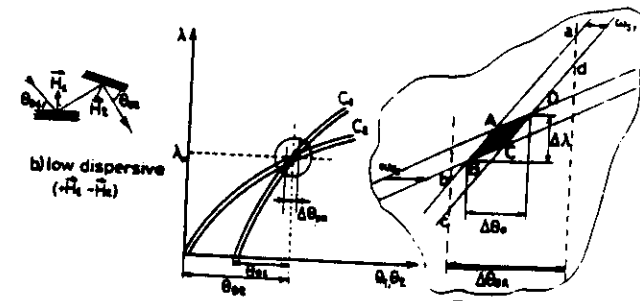
↑ intrinsic
Darwin width



Du Mond diagrams



- highest intensity
- very good alignment required



- low intensity
- highly monochr.
- very stable configuration

Channel-Cut monochromator

one large monocrystal with a "channel"

-
- simplest two crystal monochromator
 - naturally aligned
 - cheap

but: • exit beam shifts vertically by

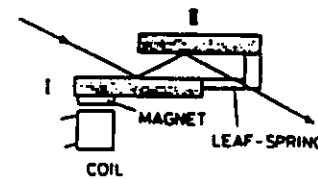
$$\Delta h = 2D \cos \theta_B, \text{ with } D = \text{distance between crystal surfaces}$$

- higher order problem

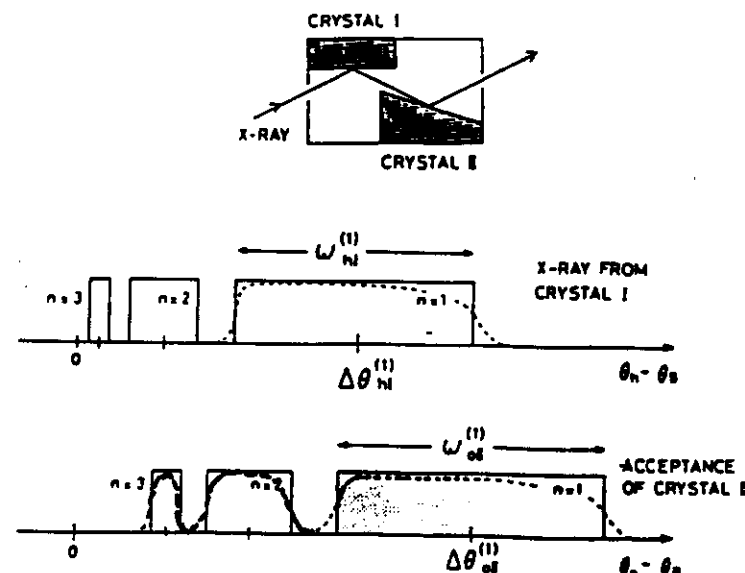
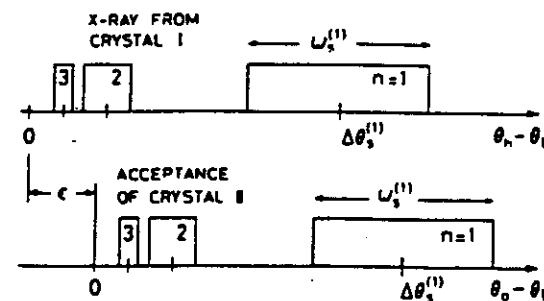
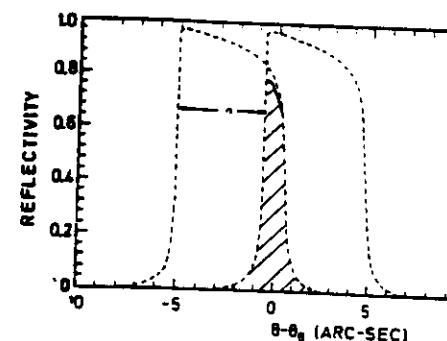
resolution: $\frac{\Delta\lambda}{\lambda} = (\Omega_{\text{rad Source}} + \Omega_{\text{slit}} + \omega_h / \lambda) \cdot \cot \theta_B$

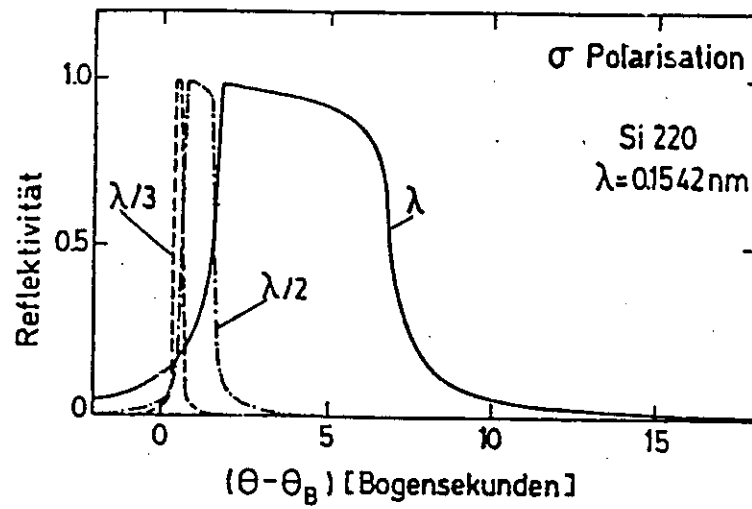
with $\Omega_{\text{rad Source}}$ = vertical divergence on crystal given by the vertical source size = S/L

Ω_{slit} = vertical divergence on crystal given by the entrance slit = F/L



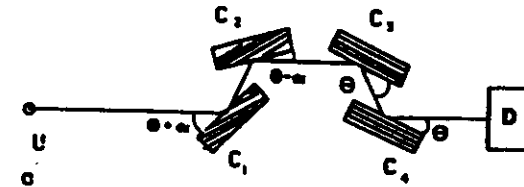
(a)





4 crystal monochromator

- highly dispersive + fixed exit



4 crystals
dispersive

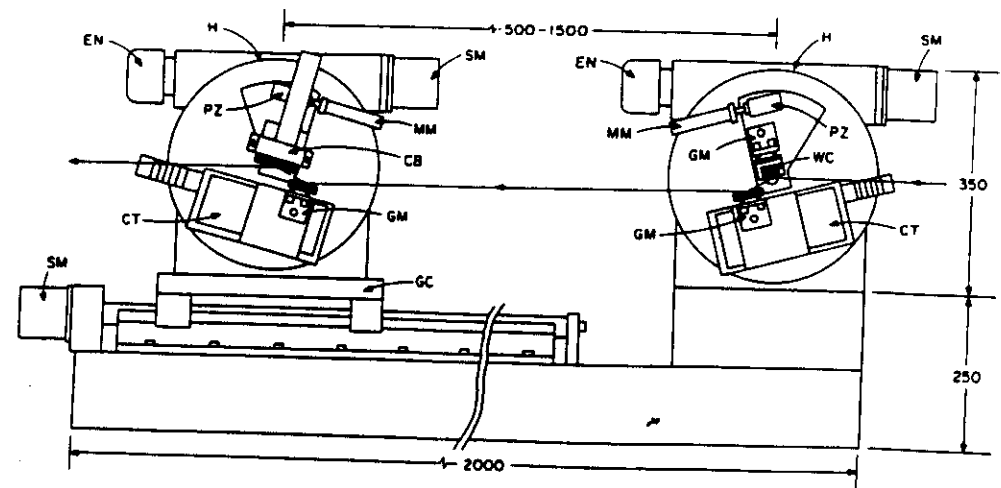


Fig. 1. Schematic of the monochromator. CB: crystal bender; CT: crystal translation stage; EN: encoder; GC: goniometer carriage for 1 m translation; GM: gimbal mount for prealigning crystal; H: Huber model 421 goniometer; MM: motorized micrometer for coarse alignment; PZ: piezoelectric translator for fine crystal alignment; SM: stepping motor; WC: water cooled crystal. Not shown is the piezoelectric adjuster for adjusting the angle of the first goniometer.

S. Heald et. al., NIM 222, 460 (1984)

Focussing X-rays

mirror

- 1 optical element for sagittal + tangential focussing possible
- cut-off frequency $\rightarrow$ very grazing incidence $\rightarrow$ very long mirrors (typ. $> 1\text{m}$!)
- high requirements for
 - slope errors ($< 2\text{arcsec}$)
 - surface roughness ($< 5\text{\AA}$)
 - alignment
- expensive

crystal

- 2 crystals needed
- since radius of curvature $R_{\text{crystal}} \gg R_{\text{mirror}} \rightarrow \sim 10\times$ acceptance (geometrical)
- small crystals $\rightarrow$ cheaper

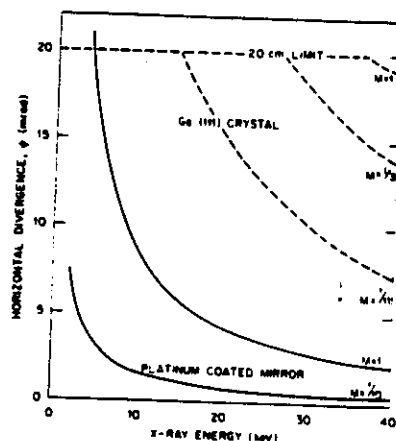
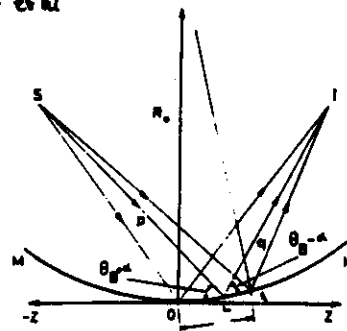
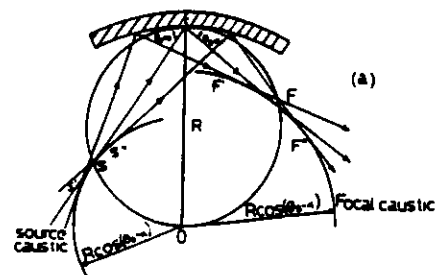


Fig. 1. Comparison of the horizontal divergence intercepted with sagittal focusing by a Ge(111) crystal and platinum-coated mirror located 10 m from the radiation source as a function of photon energy and magnification, M . The crystal dimension is limited to 20 cm with no size restriction placed on the mirror.

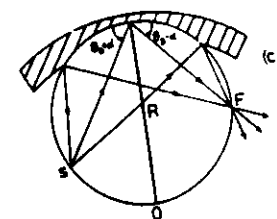
R. Caciuffo et al
1987



$$\frac{2}{R} = \frac{\sin(\theta_B + \alpha)}{\rho} + \frac{\sin(\theta_B - \alpha)}{q}$$



Johann



Johansson

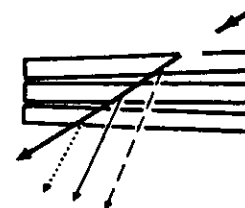
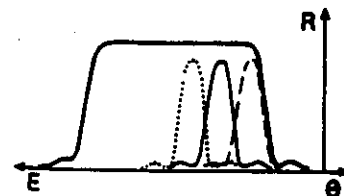
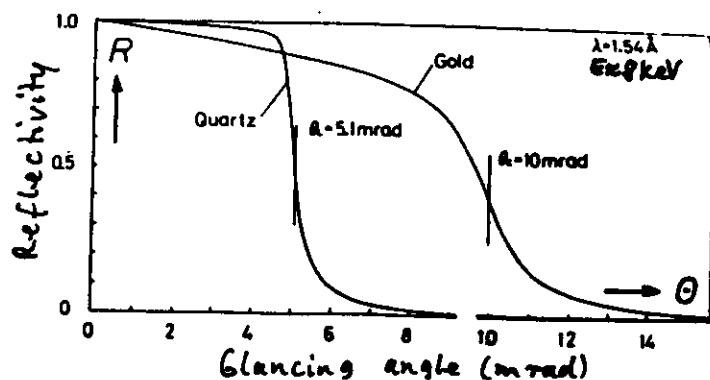


FIG. 3. Calculating of the reflectivity curve from the lamellar model. Reflection of a parallel polychromatic beam is shown above, and below the reflectivity curves of individual lamellae with their sum, the reflectivity curve of the bent crystal R , as a function of energy E , or angle θ .



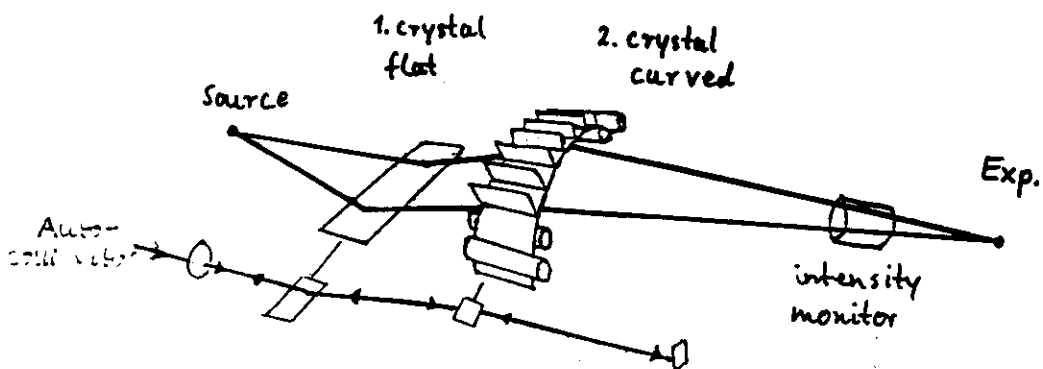
P. Suortti, Rev. Sci. Instrum., Vol. 63, No. 1, p. 942 May 1992

$$\sin \theta_c = \lambda \sqrt{\frac{e^2 N}{m c^2 \pi}}, \quad N = \frac{\rho}{A} \cdot \frac{e}{\text{unit volume}}$$



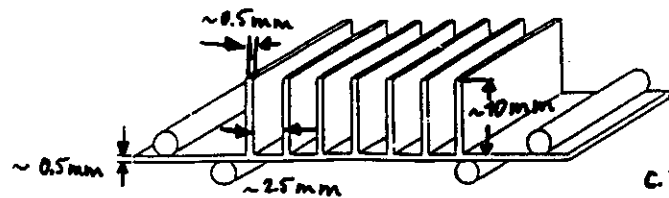
C.J. Sparks et al
NIM 172, 237 (1980)

Heating effects



Two feedback-systems:

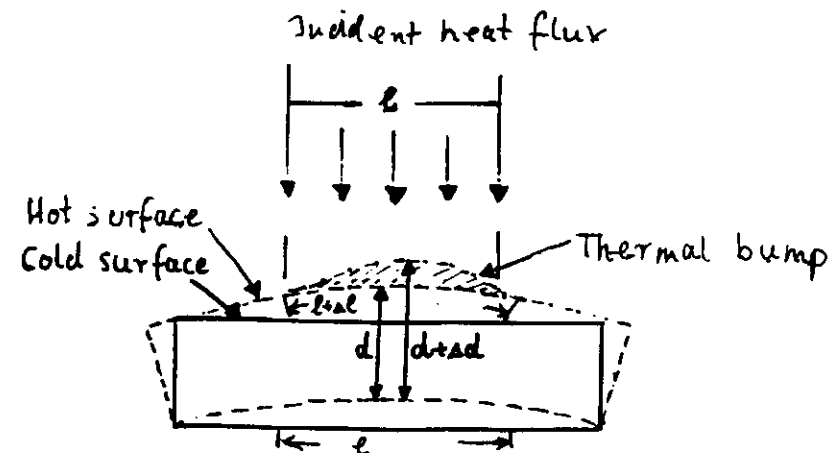
1. keeps crystals $\parallel$ to $1 \mu\text{sec}$
2. ionisation chamber $\rightarrow$ detuned mode for higher order suppression



C. J. Sparks et al.
Nucl. Instr. & Meth.
194, 73 (1982)

4-bar bending device $\rightarrow$ cylindrical and conical shapes

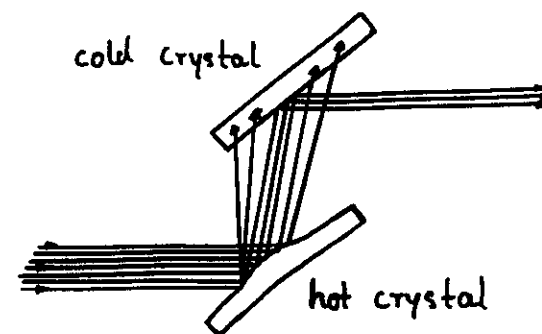
$$R = 1.5 \leftrightarrow 40 \text{ m}$$



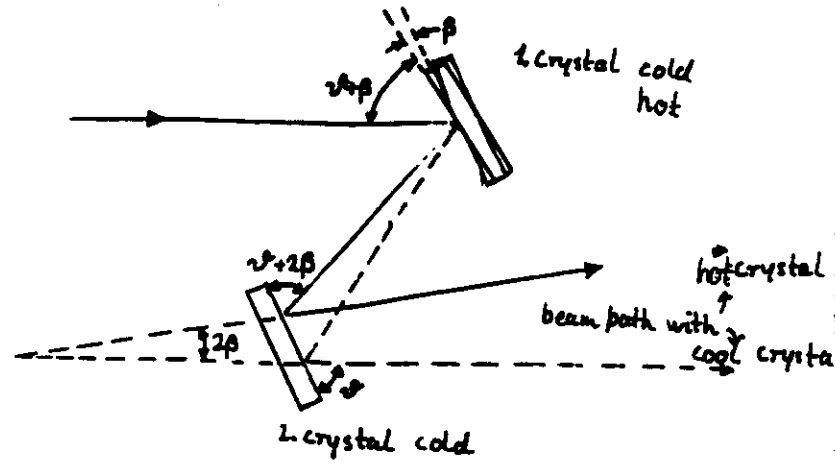
Deformations of a monochromator crystal:

1. overall bending
2. "thermal bump"

① loss of throughput intensity

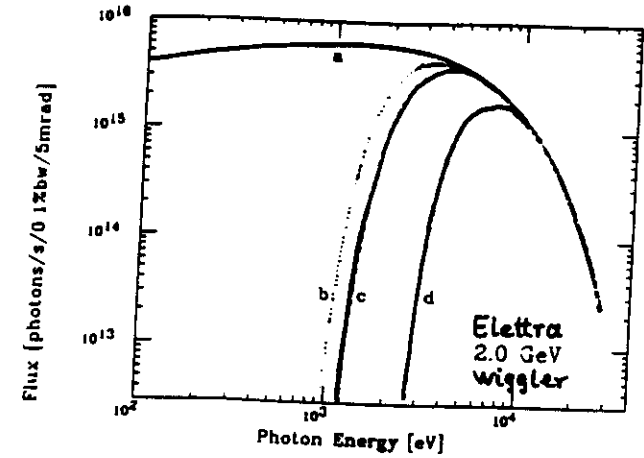


② Loss of intensity requires detuning $\rightarrow$ shift of exit beam



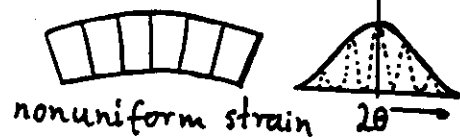
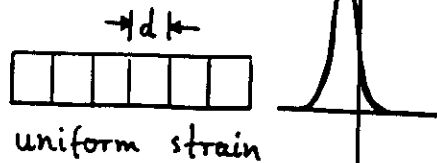
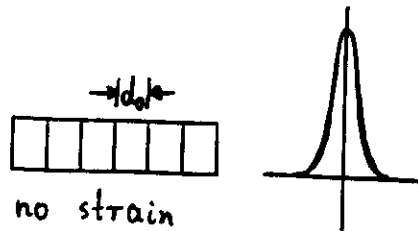
Solutions for heat load problem

①



use po
filters

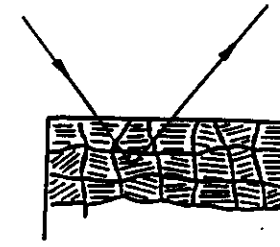
8 μ m
20 μ m
250 μ m



③ loss of energy calibration

④ line broadening
$$b = 2 \frac{\Delta d}{d} \tan \theta$$

②



Large band-pass of
multilayers
mosaic crystals

③ cool 1. crystal very efficiently (3 kW absorb heat)
2. crystal to match temperature:

diffraction beamline, Si(111) : $\Delta T \leq 13^\circ\text{C}$
Si(220) : $\Delta T \leq 7^\circ\text{C}$

The diagram illustrates the jet evaporation process for growing a monocrystal. It consists of several parts:

- Left side:** A 3D perspective view of a rectangular block with a series of vertical ridges on its top surface. A label points to the top surface, stating "top surface thickness $\approx 0.1 \text{ mm}$ ". Below this, a note says "Use organic free water" and " $\approx 1 \text{ liter } 2 \text{ mm } 5 \text{ to } 10 \text{ cm}$ ".
- Right side:** A series of four cross-sectional diagrams labeled a, b, c, and d, showing the progression of a jet of water being applied to the top surface of the block.
 - a:** Shows a jet of water being applied to the top surface of the block.
 - b:** Shows the jet continuing to apply pressure to the top surface.
 - c:** Shows the jet continuing to apply pressure to the top surface.
 - d:** Shows the jet continuing to apply pressure to the top surface.
- Center:** A handwritten note with an arrow pointing from the 3D view to the cross-sections: "Single jet monocrystal, top surface thickness $\approx 0.1 \text{ mm}$, monocrystal melted in a steel can which could be pressurized to form the top surface to 'monocrystal'".

waterjet +
dynamic bending
(Hart, Uni Manchester)

Technical drawing of a mechanical assembly, likely a mold or die. The drawing shows a cross-section of the assembly. Key dimensions and labels are as follows:

- Dimensions:**
 - Top left horizontal dimension: 58 mm
 - Top right horizontal dimension: 65 mm
 - Right vertical dimension: ~30 mm
 - Internal diagonal dimension: 5R
- Labels:**
 - A:** Points to the top surface of the main block.
 - B:** Points to the bottom surface of the main block.
 - C:** Points to the left side of the main block.
 - D:** Points to the right side of the main block.
 - E:** Points to the bottom surface of the main block.

Crystal materials

Requirements:

- energy range + resolution (given by experiment)
- no absorption edges
- heat and radiation resistance
- high reflectivity
- available in sufficient sizes
- "easy" to manufacture
- low cost
- small mosaic spread

Energy resolutions of the monochromator required in various experiments.

Experiments	$\Delta E/E$ or $\Delta \lambda/\lambda$
X-ray microscopy	$10^{-1} - 10^{-2}$
Structure analysis	$10^{-2} - 10^{-3}$
Compton scattering	$10^{-2} - 10^{-4}$
EXAFS	$10^{-3} - 10^{-4}$
Topography (white beam)	$(10^{-3} - 10^{-4})^*$
Anomalous scattering	$\sim 10^{-4}$
Interferometry	$\sim 10^{-4}$

Table 5
Soft X-ray monochromator crystals.

Crystal	$2d$	Energy region/eV	Comments
Si (111)	6.27164	2000-3000	Good stability
Ge (111)	6.532	2000-3000	
In Sb (111)	7.4806	1600-3000	
α -Quartz (10 $\bar{1}0$)	8.512	1400-3700	
Beryl (10 $\bar{1}0$)	15.92	800-2000	0.6 eV resolution at 1000 eV
(Be ₃ Al ₂ (SiO ₃) ₆)			
Mica (002)	19.84	840-1600	Resolution ~ 1 eV radiation damage
Sodium β -alumina (Na ₂ O-11Al ₂ O ₃)	22.49	550-1500	
KAP (Potassium acid phthalate)	26.6 Å	470-1300	Enormous possibilities
Synthetic layered structures, e.g., C-W, B-W, Nb-C	from 10 Å upwards		

Si (220)

YB₆₆

6-30 keV

not used

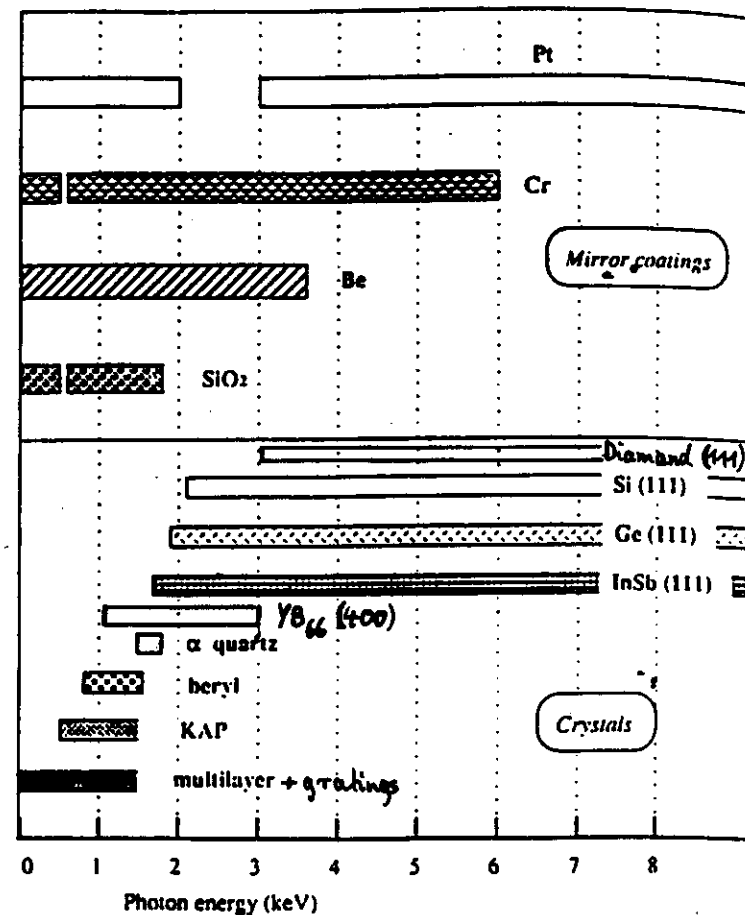


Fig. 3. Energy regions for reflecting coatings ($\alpha = 0.45^\circ$) and monochromator crystals ($10^\circ < \theta < 80^\circ$)

Monochromators

- 5-30 eV : normal incidence, grating
- 10-1000 eV : grazing incidence, grating
- > 800 eV : crystals

Absorption edges in crystals

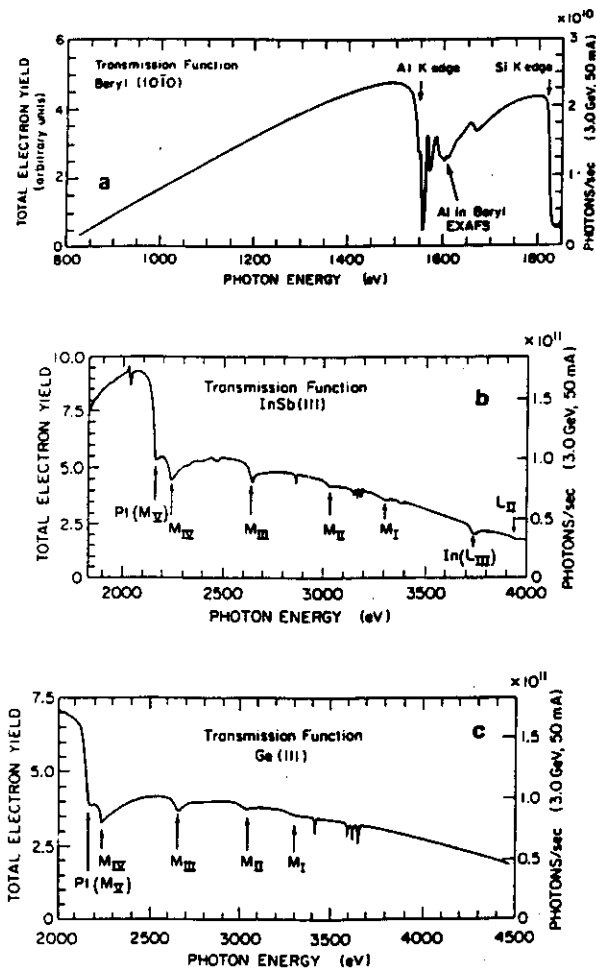


Fig. 39 Transmission functions for the SSRL double crystal monochromator: (a) Beryl (1010), (b) InSb (111), (c) Ge (111) (courtesy Lawrence Berkeley Laboratory, University of California).

Literature

First publications on crystal theories:

1. C. G. Darwin, Phil. Mag. 27, 315, 675 (1914)
2. P. P. Ewald, Ann. Physik 49, 1, 117 (1916); 54, 519 (1917)
3. M. von Laue, Ergeb. Exakt. Naturw., 10, 133 (1931)

Introductions into the different forms of Dynamical Theory:

4. R. W. James, Optical Principles of the Diffraction of X-Rays, G. Bell and Sons, London, 1950
5. R. W. James, Solid State Physics, Vol 15, Academic Press, New York, 1963
6. W. H. Zachariasen, Theory of X-Ray Diffraction in Crystals, John Wiley and Sons, Inc., New York, 1945 (New edition by Dover, New York, 1967)

Important extensions to and summaries of the Dynamical Theory are contained in e.g. the following articles:

7. P. B. Hirsch, Acta Cryst. 5, 176 (1952), and P. B. Hirsch and G. N. Ramachandran, Acta Cryst. 3, 187 (1950)
8. N. Kato, J. Phys. Soc. Japan 7, 397 (1952)
9. G. Borrmann, Trends in Atomic Physics, Interscience Publishers, Inc., New York, 1959
10. K. Kohra, X-Ray Crystallography (Maruzen Co. Ltd, Japan 1961), Vol. II, 9, P. 849
11. A. Authier, Bull. Soc. Franc. Mineral. Crist., 84, 51 (1961)

A complete summary of the Ewald - von Laue formulation of the Dynamical Theory is given in the review article by

12. B. W. Batterman and H. Cole, Rev. Mod. Phys. 36, 681 (1964)

Further Review Articles on Crystal Theories and X-Ray Optics:

- Jackson, Classical Electrodynamics, Wiley
- R. W. James, The Optical Principles of the Diffraction of X-Rays, Bell, London, (1965)
- B. E. Warren, X-Ray Diffraction, Addison-Wesley Publishing Company, Reading, M.A., (1969)
- T. Matsushita & H. Hashizume, Handbook on Synchrotron Radiation, Vol. 1A, Ch. 4, ed. E.E. Koch, North Holland, (1983)
- R. Caciuffo, S. Melone, F. Rustichelli and A. Boeuf, Physics Reports 152, 1, (1987)

Recommended Journals for X-ray Optics:

- Nucl. Instr. & Methods (with special sections on use of Synchrotron Radiation)
- Review of Scientific Instruments (with special sections on use of Synchrotron Radiation)
- Journal of Applied Crystallography