INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

H4.SMR/676-4

SECOND SCHOOL ON THE USE OF SYNCHROTRON RADIATION IN SCIENCE AND TECHNOLOGY:
"JOHN FUGGLE MEMORIAL"

25 October - 19 November 1993

Miramare - Trieste, Italy

INSERTION DEVICES # 1

R.P. Walker
Sincrotrone, Trieste
Italy

MAIN BUILDING STRADA COSTIERA, 11 TEL. 22401 TELEFAX 224163 TELEX 460392 ADRIATICO GUEST HOUSE VIA GRIGNANO, 9 TEL. 224241 TELEFAX 224531 TELEX 460449

Second School on the Use of Synchrotron Radiation
in Science and Technology

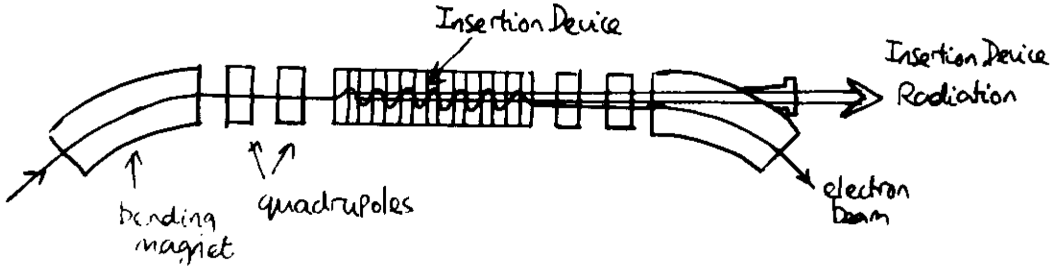
Trieste, October 25th - November 19th, 1993

Insertion Devices # 1

R.P.Walker, Sincrotrone Trieste, Italy

1. Introduction
2. Electron Motion and Basic Features of Emitted Radiation in Standard Insertion Devices
3. Undulator Radiation
4. Multipole Wigglers

1. INTRODUCTION



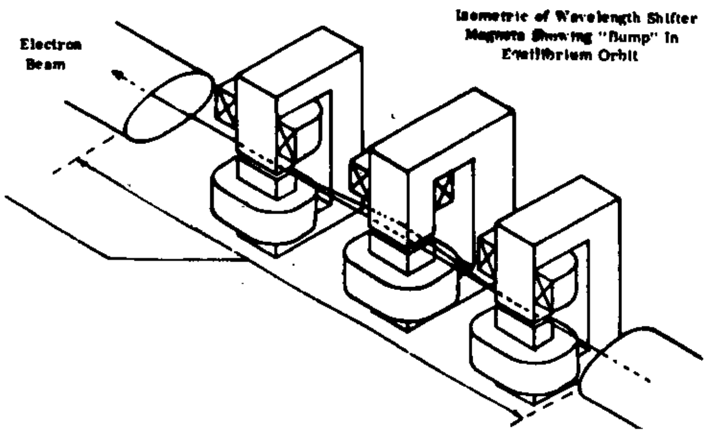
- magnetic elements located in a straight section of a storage ring
⇒ localized deflection of the electron beam
⇒ emission of synchrotron radiation with special properties

Advantages over bending magnet sources :

• <u>higher photon energies</u>
- higher field, and hence critical photon energy, $\epsilon_c(keV) = 0.665\, B E^2$
e.g. 3-pole *"wavelength shifter"*, Wisconsin, (1971)

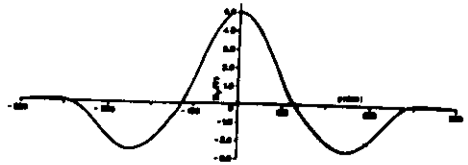
A "WAVELENGTH SHIFTER"
FOR THE UNIVERSITY OF WISCONSIN ELECTRON STORAGE RING *

Walter S. Trzeciak
University of Wisconsin - Physical Sciences Laboratory
Stoughton, Wisconsin



superconducting devices - 3.5 T, VEPP3 (1979),
5.0 T, SRS (1982)
others at DCI, Photon Factory, UVSOR, VEPP-2M etc.

e.g. SRS



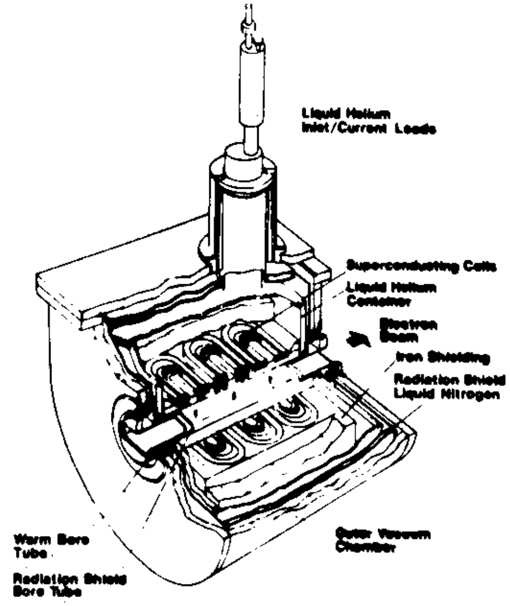
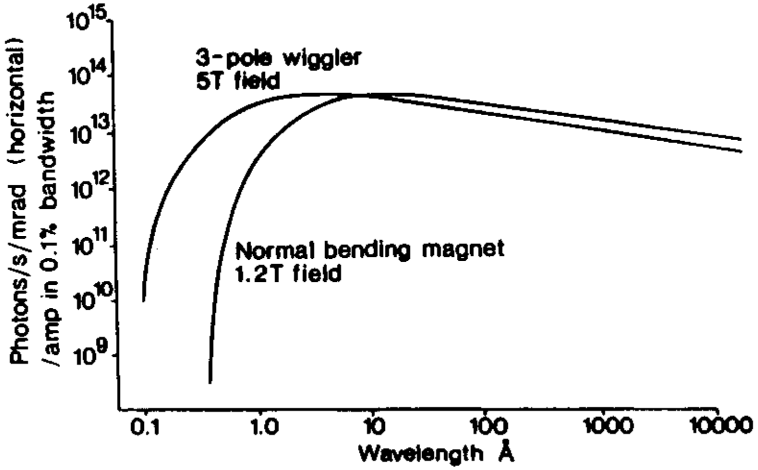
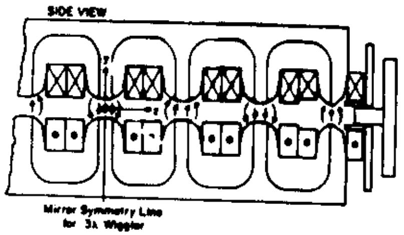


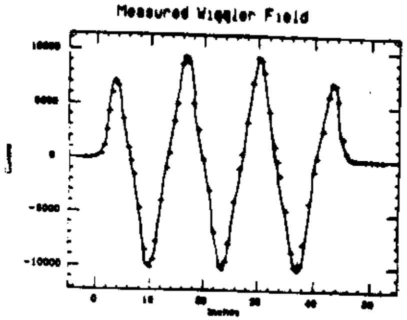
Fig. 2. Cut-away view of the 5.0 T superconducting wiggler magnet.



- **increased photon flux**
  - many emitting poles, *"multipole wiggler"*

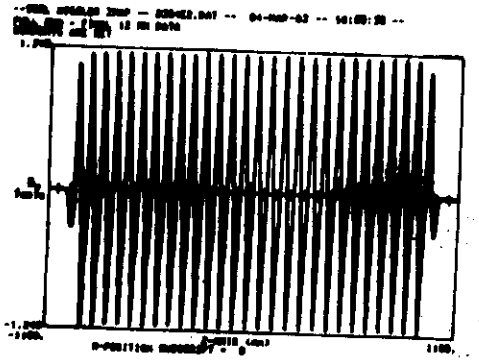
e.g. 7-pole (3λ) wiggler, SSRL, Stanford (1979)
electromagnet, period length = 34.3 cm, peak field 1.8 T





Later multipole wigglers, with many more poles, were constructed with permanent magnets

e.g. 55-pole (27λ) wiggler at SSRL, Stanford (1983)
period length = 7 cm, peak field 1.2 T (1.2 cm gap)



- **increased brightness** (photon flux per unit area and solid angle)
  - smaller field strength and period length gives rise to interference effects, *"undulator"*

used in original experiments of Motz, Stanford, (1951) :

Applications of the Radiation from Fast Electron Beams

H. Motz
Microwave Laboratory, Stanford University, California
(Received July 3, 1950)

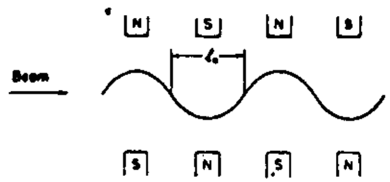
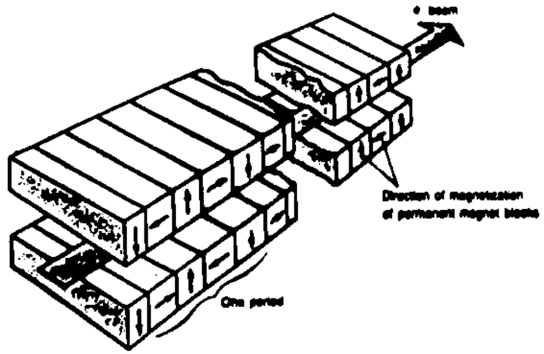


Fig. 1. Schematic arrangement of undulator magnets.

- and in subsequent development of the Free-electron Laser

First use in storage ring, VEPP3, Novosibirsk (1979) and SPEAR, Stanford, (1980) :
30-periods of 6.1 cm; peak field of 0.28 T



- different radiation polarization characteristics
- circular, elliptical or variable

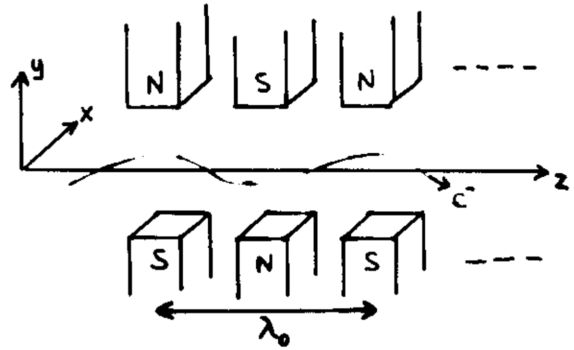
using different magnetic structures and hence electron trajectories (part 3 of lectures)

At present > 50 Insertion Devices are in operation in Synchrotron Radiation Sources worldwide.

Many more are being built for the most recent "third generation" synchrotron radiation sources :
~ 10 IDs in 1-2 GeV rings, ~ 30 IDs in 6-8 GeV rings

2. Basic features of ID radiation.

Electron motion in a plane sinusoidal magnet:



$$B_y = B_0 \cos\left(\frac{2\pi z}{\lambda_0}\right) \qquad \left(\begin{matrix}\gamma = E/mc^2 \\ \beta = v/c\end{matrix}\right)$$

X-motion: $$\frac{dx}{dz} = \beta_x = \left(\frac{K}{\gamma}\right) \sin\left(\frac{2\pi z}{\lambda_0}\right)$$

$$x = -\left(\frac{K}{\gamma}\frac{\lambda_0}{2\pi}\right)\cos\left(\frac{2\pi z}{\lambda_0}\right)$$

"deflection parameter" $$\boxed{K = \frac{eB_0\lambda_0}{2\pi m_e c} = 93.4\, B_0\, \lambda_0}$$

e.g. (1.5 GeV)

	$K \lesssim 3$ "undulator"	$K \gtrsim 3$ "wiggler"	
B_0	0.5	1.5	(T)
λ_0	0.05	0.15	(m)
K	2.3	21.0	
K/γ	0.8	7.2	(mrad)
$K/\gamma\ \lambda_0/2\pi$	6.3	171	(μm)

Z-motion: ($\beta = v/c$)
$$\beta_x^2 + \beta_z^2 = \beta^2 \quad \text{(constant)}$$

$$\therefore \quad \beta_z = \beta\left(1 - \frac{K^2}{4\gamma^2} + \underbrace{\frac{K^2}{4\gamma^2}\cos\frac{4\pi z}{\lambda_0}}_{}\right)$$

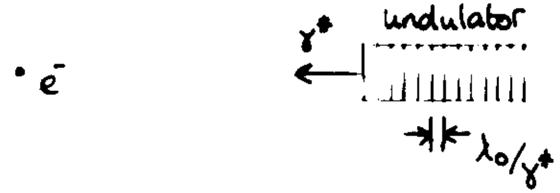
modulation in Z-velocity at twice X-oscillation frequency

average velocity: $$\beta^* = \langle\beta_z\rangle = \beta\left(1 - \frac{K^2}{4\gamma^2}\right)$$

(i) Electron rest frame:

reference frame moving with electrons average velocity along the z-axis, $\beta^* c$

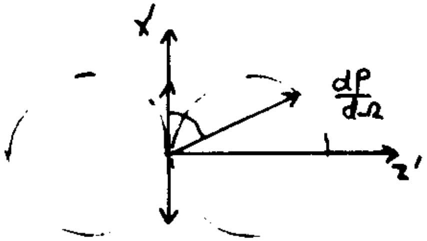
relativistic factor $\gamma^* = \dfrac{1}{(1-\beta^{*2})^{1/2}} \simeq \dfrac{\gamma}{(1+K^2/2)^{1/2}}$



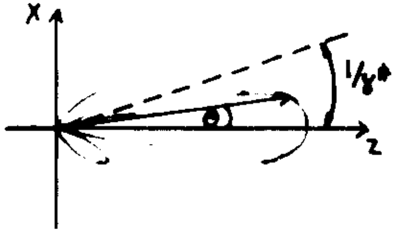
(Lorentz contraction)

dipole emission in moving frame:

wavelength $\lambda' = \dfrac{\lambda_0}{\gamma^*}$



lab frame:



$\lambda = \lambda' \gamma^* (1-\beta^* \cos\theta) = \dfrac{\lambda'}{2\gamma^*}(1+\gamma^{*2}\theta^2)$ (relativistic Doppler shift)

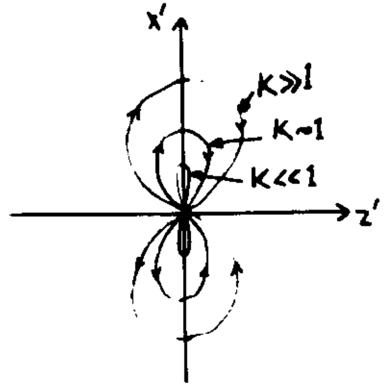
ie

$$\boxed{\lambda = \frac{\lambda_0}{2\gamma^2}\left(1+\frac{K^2}{2}+\gamma^2\theta^2\right)}$$

($\gamma = 1957\, E[\mathrm{GeV}]$)

In moving frame z oscillation amplitude multiplied by γ^*

ie) $\dfrac{z' \text{ amplitude}}{x' \text{ amplitude}} = \dfrac{K}{8(1+K^2/2)^{1/2}}$



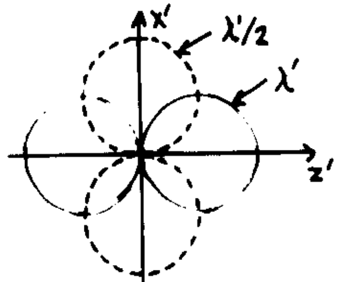
$K < 1$ z' motion small, simple harmonic motion
→ single frequency emission

$K > 1$ "figure-of-eight" motion
increased emission in harmonics

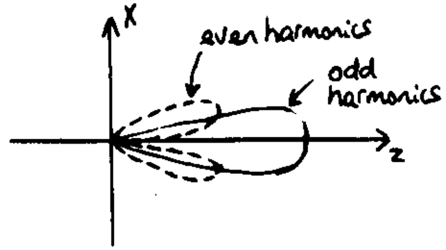
$\dfrac{\text{Power in fundamental}}{\text{Total power}} = \dfrac{1}{(1+K^2/2)^2}$

K	ratio
0.1	0.99
0.5	0.79
1.0	0.44
5.0	0.005

decompose into x' and z' oscillations:



electron rest frame

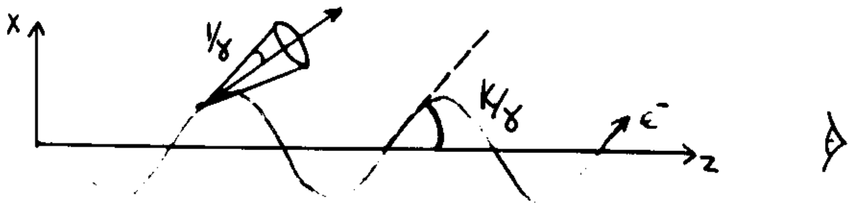


lab. frame

) Electric field (in time & frequency domains)

$$K = \frac{\text{maximum deflection angle of trajectory } (K/\gamma)}{\text{natural opening angle of synchrotron radn. } (1/\gamma)}$$

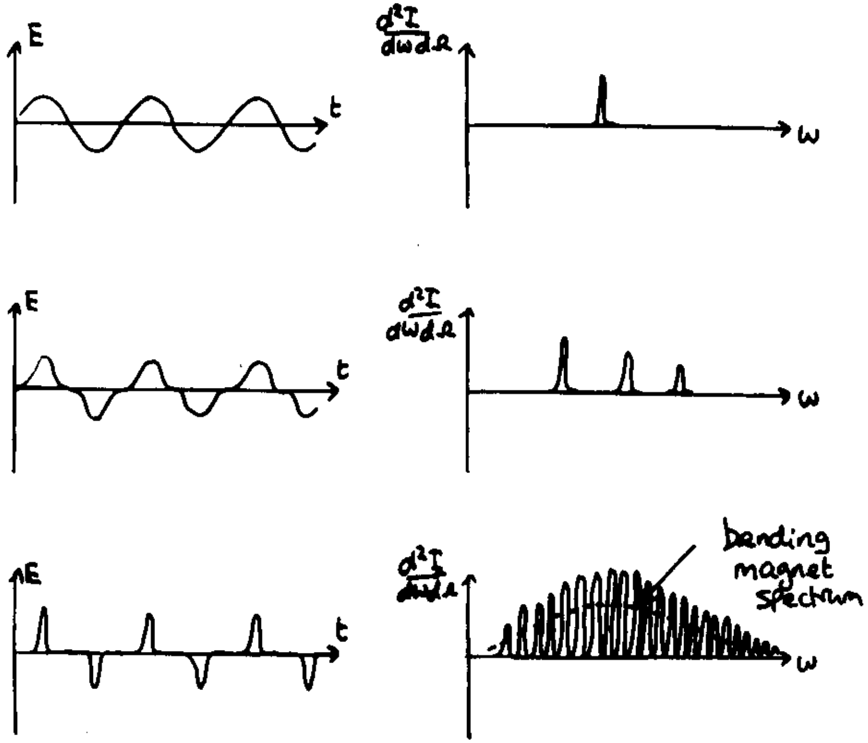
"sweeping searchlight":



K ≪ 1

K ~ 1

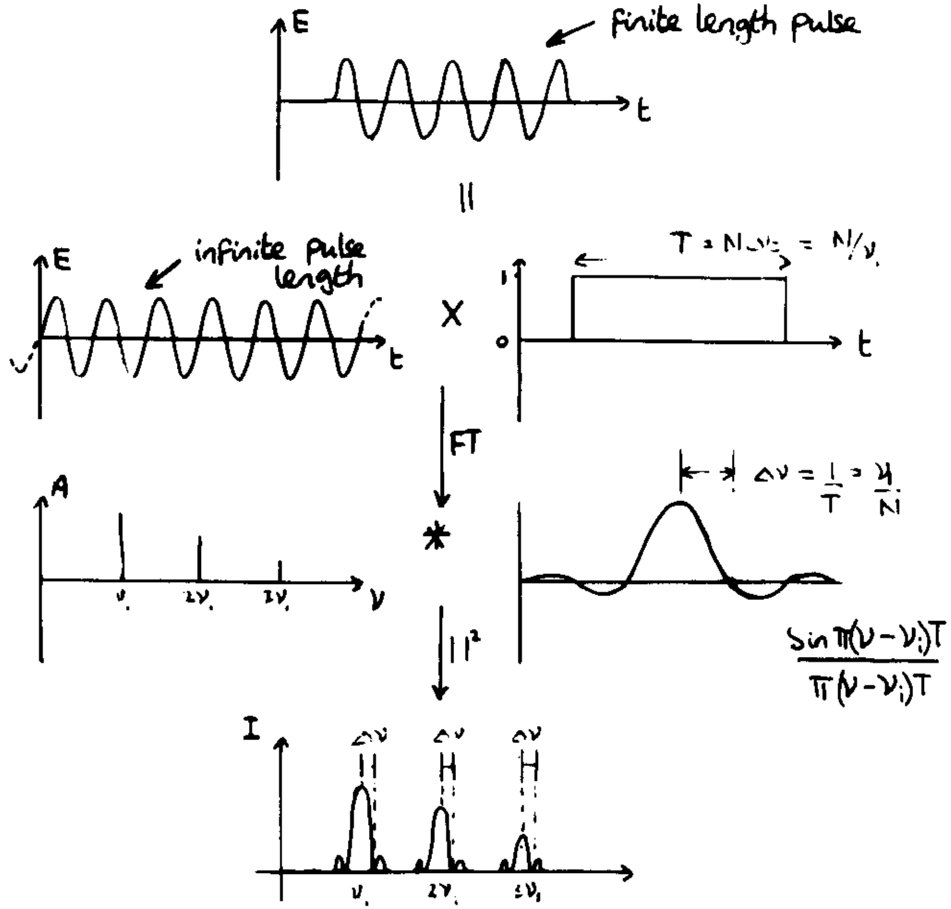
K ≫ 1



time domain $\xrightarrow{FT}$ frequency domain

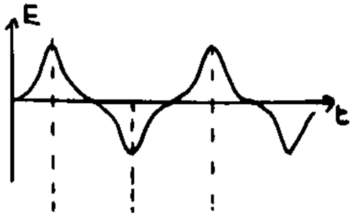
$$A(\omega) = \int E(t)\, e^{i\omega t}\, dt$$

$$I(\omega) = |A(\omega)|^2$$



$$\Delta\nu = \frac{\nu_1}{N} \qquad \frac{\Delta\nu}{\nu} = \frac{1}{iN}$$

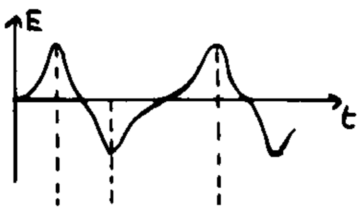
on-axis:



equispaced

→ odd harmonics

off-axis horizontally:

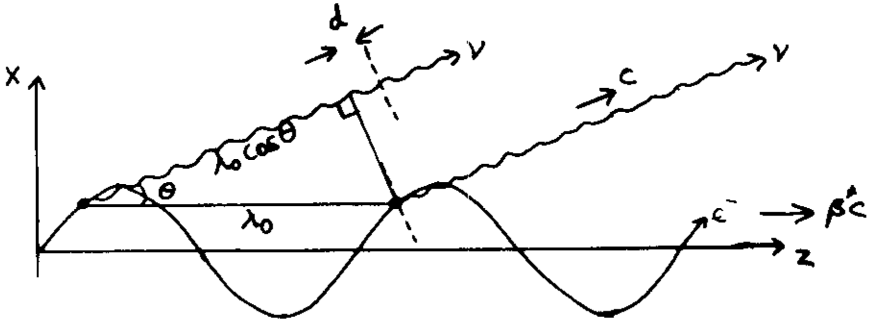


not equispaced

→ even + odd harmonics

(iii) Interference

- of radiation emitted by the same electron at different points in the magnet.



$$d = \frac{\lambda_0}{\beta^*} - \lambda_0 \cos\theta \quad \text{path length difference}$$

$$= i\lambda \quad \text{constructive interference}$$

since $\dfrac{1}{\beta^*} = 1 + \dfrac{1}{2\gamma^2} + \dfrac{K^2}{4\gamma^2}$

$$\boxed{\lambda = \frac{1}{i}\frac{\lambda_0}{2\gamma^2}\left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right)} \qquad (\theta \text{ small})$$

$i = 1, 2, 3 \ldots$ harmonic number

For N periods in a length L:

$$\frac{L}{\beta^*} - L\cos\theta = iN\lambda \qquad \text{constructive interference}$$

change in λ, at fixed θ:

$$\frac{L}{\beta^*} - L\cos\theta = iN\lambda' + \lambda' \qquad \text{destructive interference}$$

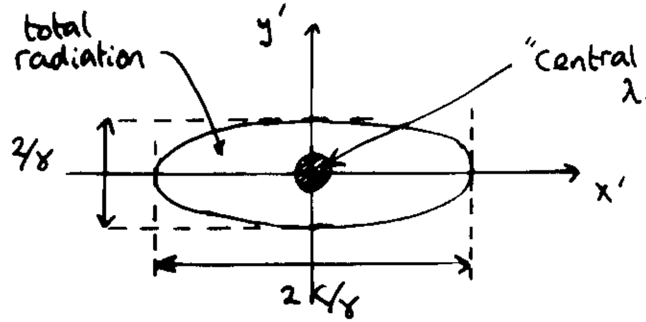
$$\therefore \boxed{\frac{\Delta\lambda}{\lambda} = \frac{1}{iN}}$$

change in θ, at fixed λ:

$$\frac{L}{\beta^*} - L\cos\theta' = iN\lambda + \lambda \qquad \text{destructive interference}$$

$$\Delta(\cos\theta) = \frac{\lambda}{L} = \frac{\Delta(\theta^2)}{2}$$

at $\theta = 0$ $\quad \boxed{\Delta\theta = \sqrt{\frac{2\lambda}{L}} = \frac{1}{\gamma}\sqrt{\frac{1 + K^2/2}{iN}}}$



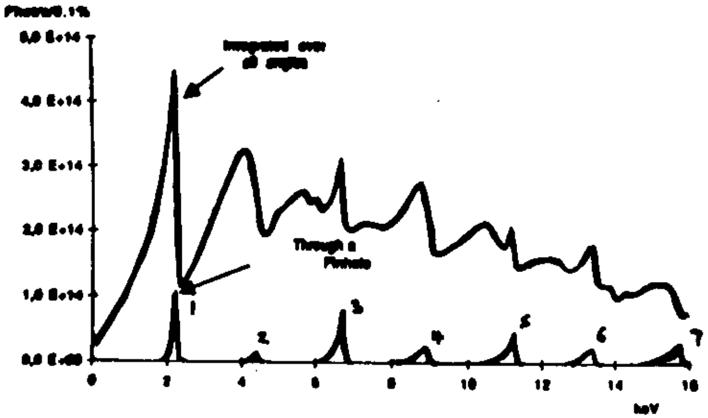
Summary :

- radiation spectrum consists of a series of harmonics

- number of harmonics increases rapidly with K

- radiation wavelength varies with angle, θ
→ spectrum integrated over a wide range of angles is broad

- at a given θ, the wavelength variation is small
→ selecting a narrow range of angles, with a "pinhole" aperture, gives a quasi-monochromatic spectrum :



3. Undulator radiation

Angular flux density

$$\frac{d^2 I}{d\omega/\omega \, d\Omega} = 1.74 \; 10^{14} \; F_i(K,\theta,\phi) \; L\left(\Delta\omega/\omega_1\right) E_{GeV}^2 \, I_b$$

unit : photons / sec / mrad² / 0.1% bandwidth

$$L = \text{lineshape function} = \frac{\sin^2 N\pi \Delta\omega/\omega_1}{(\pi \Delta\omega/\omega_1)^2}$$

- includes interference between periods

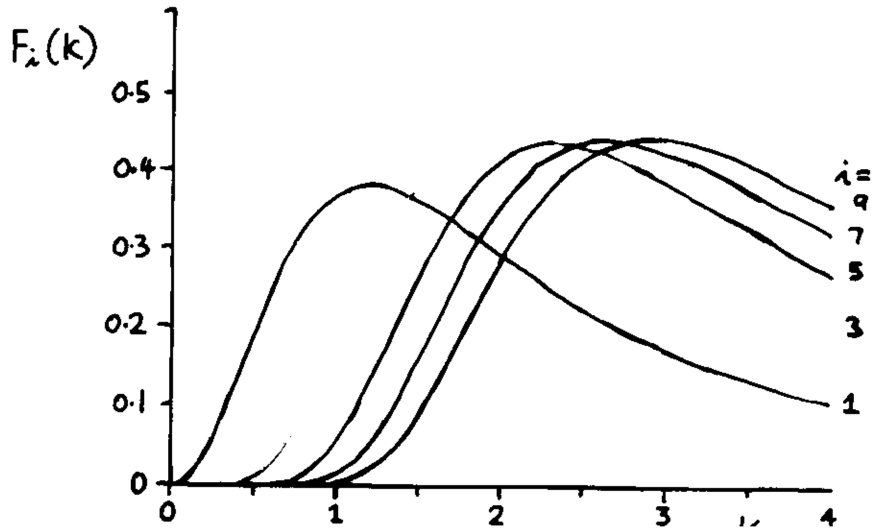
F_i = angular distribution

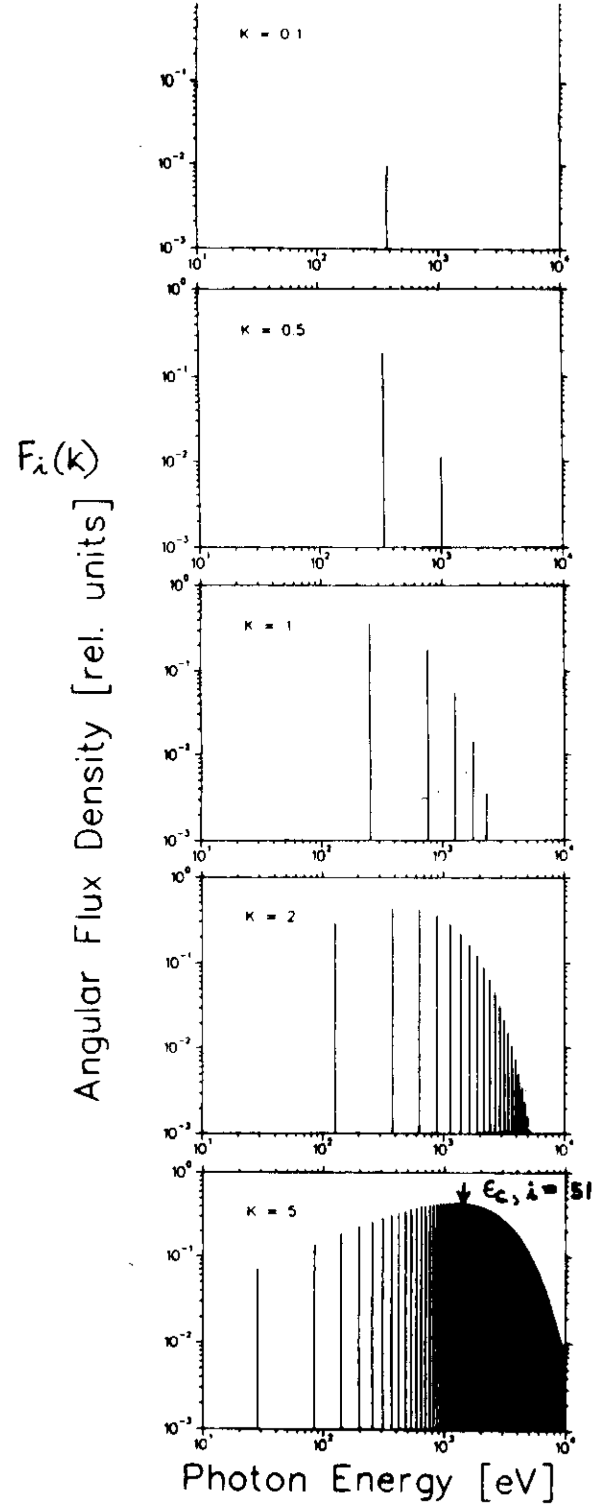
- includes interference within one period, between the positive and negative poles

on-axis, $\Delta\omega = 0$: $$\frac{d^2 I}{d\omega/\omega \, d\Omega} = 1.74 \; 10^{14} \; N^2 \; F_i(K) \; E_{GeV}^2 \, I_b$$

where,

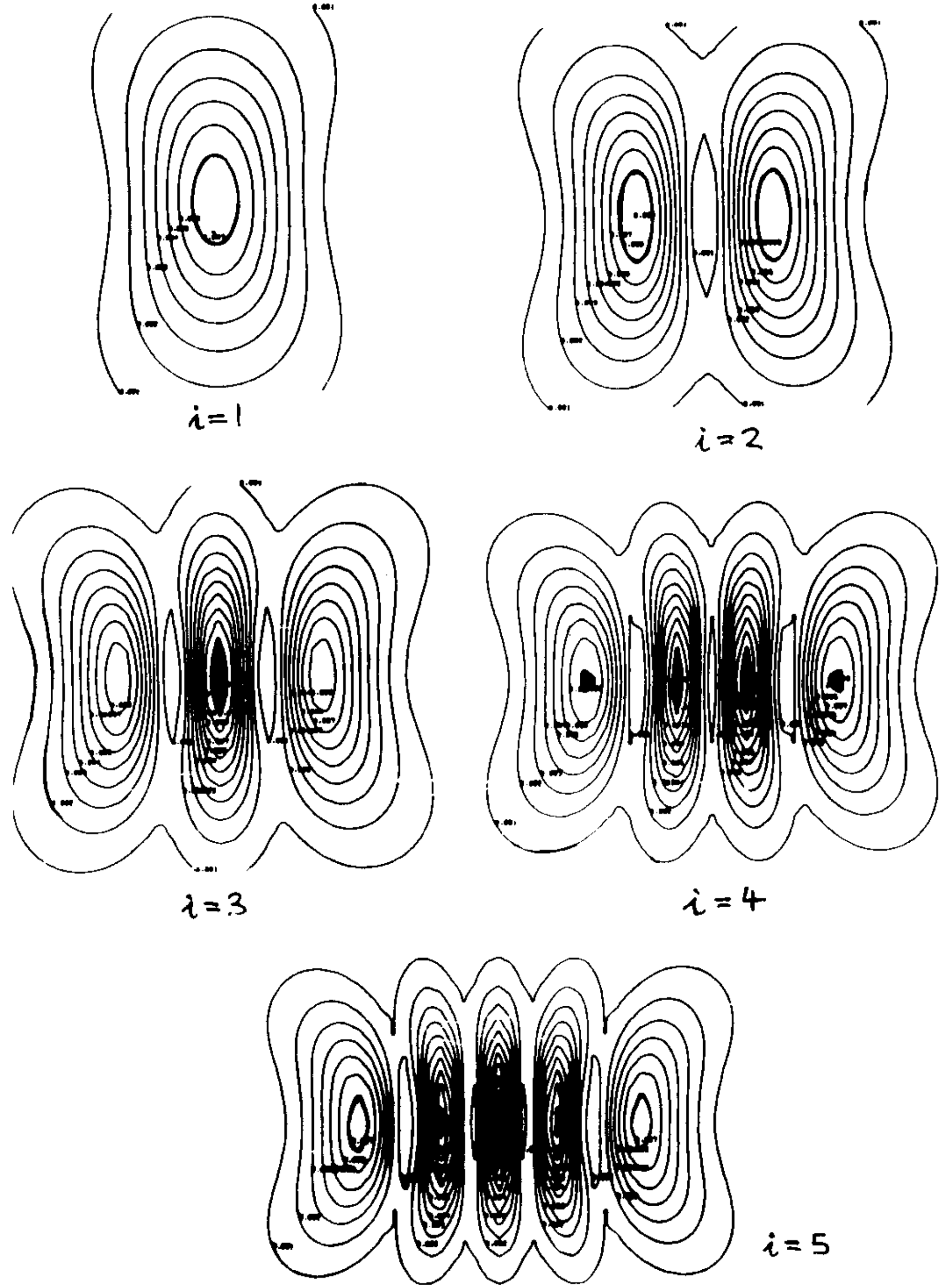
$$F_i(K) = \frac{i^2 K^2}{(1+K^2/2)^2}\left[J_{\frac{i-1}{2}}\left(\frac{i^2K^2}{1+K^2/2}\right) - J_{\frac{i+1}{2}}\left(\frac{i^2K^2}{1+K^2/2}\right)\right]^2$$





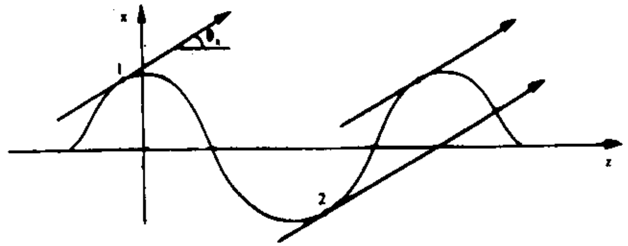
$\lambda_0 = 0.1\,m$
$E = 2\,GeV$
$N = \infty$
$\epsilon_{x,y} = 0$

Off-axis angular flux density (K=2)



Why is the angular distribution so complicated ?

- because of the interference that occurs within each period, between the positive and negative poles :



The complex angular distribution can be understood using a simple model, consisting of a series of dipole magnets.
For a single dipole the energy radiated per unit frequency and solid angle is :

$$\frac{d^2 I}{d\omega d\Omega} = \frac{3e^2}{16\pi^3 \epsilon_0 c}\gamma^2 y^2 (1+X^2)^2 \left[|A_x|^2, |A_y|^2\right] \qquad y = \omega/\omega_c,\quad X = \gamma\psi,\quad \xi = \frac{y}{2}(1+X^2)^{3/2}$$

where $$\left[A_x, A_y\right] = \left[K_{2/3}(\xi),\ i\frac{X}{(1+X^2)^{1/2}} K_{1/3}(\xi)\right] \equiv \left[A_{x0}, A_{y0}\right]$$

For two dipoles we have :

$$\left[A_x, A_y\right] = \left[(1-e^{i\Delta})A_{x0},\ (1+e^{i\Delta})A_{y0}\right]$$

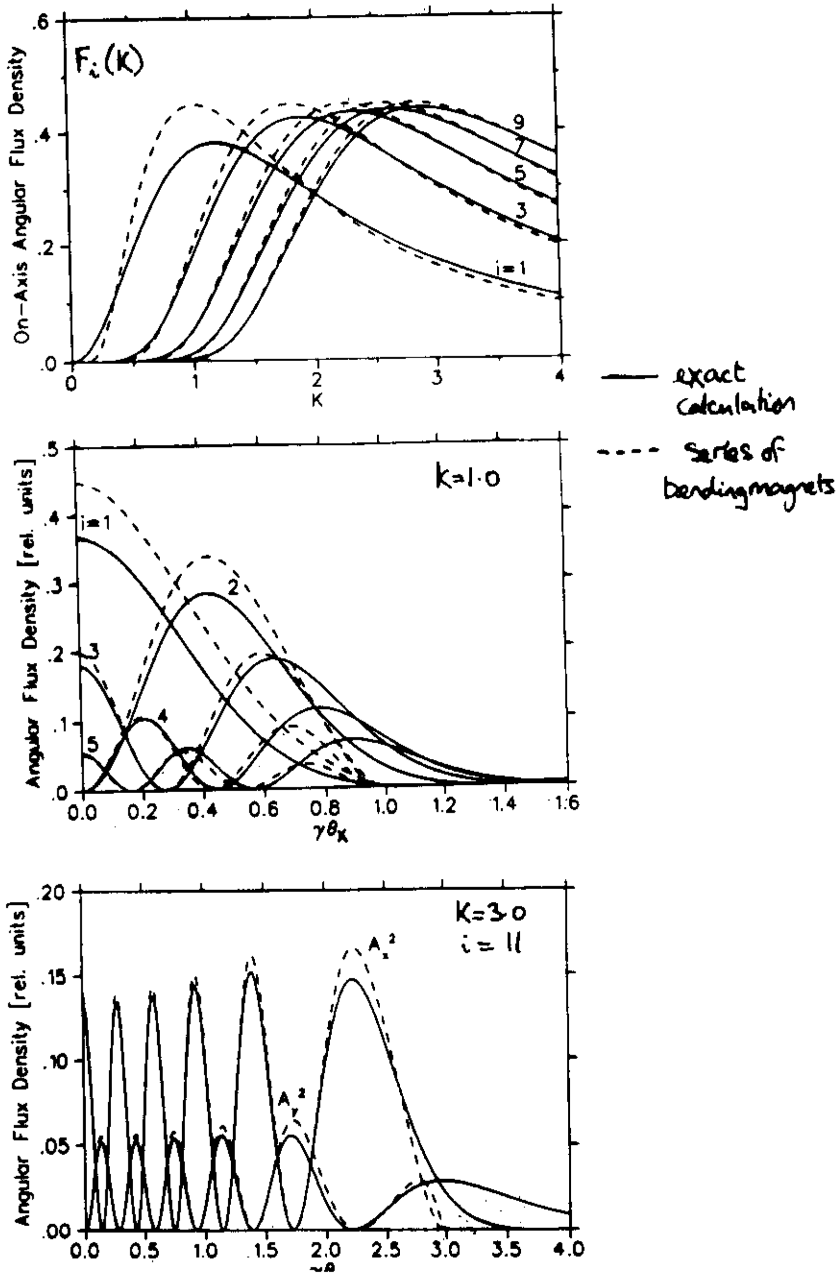
where Δ is the phase difference between the poles.

Taking the modulus-square, and including N periods, the intensity becomes :

$$\left[|A_x|^2, |A_y|^2\right] = 4\left[\sin^2\frac{\Delta}{2}|A_{x0}|^2,\ \cos^2\frac{\Delta}{2}|A_{y0}|^2\right]\frac{\sin^2 N\pi\omega/\omega_1}{\sin^2 \pi\omega/\omega_1}$$

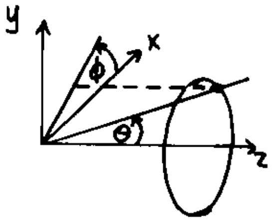
As the horizontal angle varies from zero to maximum (K/γ) the phase (Δ) varies from iπ to i2π.
There is therefore a sinusoidal variation of intensity with Δ and hence angle.

In fact the model of an undulator as a series of dipole magnets, with an appropriate interference function, is quite accurate even for small K value and small i :



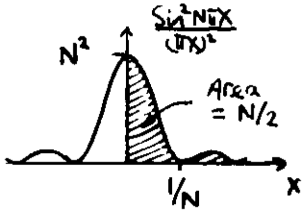
Flux in the "central cone"

$$\frac{dI}{d\omega/\omega} = \int_0^{\infty}\!\!\int_0^{2\pi} \underbrace{\frac{d^2I}{d\omega/\omega\; d\Omega}}_{\text{angular flux density}} \; \underbrace{\theta\, d\theta\, d\phi}_{\text{solid angle}}$$



close to the axis; odd harmonics:

$$\frac{dI}{d\omega/\omega} = 1.74\;10^{14}\; E^2_{GeV}\, I_b\, F_i(K) \int_0^{\infty} \frac{\sin^2 N\pi x}{(\pi x)^2}\, \pi\, d\theta^2$$



$$x = \frac{\omega - \omega_1}{\omega_1} = \frac{i\gamma^2\theta^2}{1+K^2/2}$$

$$\int_0^{\infty} \frac{\sin^2 N\pi x}{(\pi x)^2}\, \pi\, d\theta^2 = \frac{N}{2}\,\pi\, \frac{1+K^2/2}{i\gamma^2}$$

$$\boxed{\frac{dI}{d\omega/\omega} = 0.71\;10^{14}\; N\, \frac{F_i(K)(1+K^2/2)}{i}\; I_b}$$

photons/sec/
0.1% bandwidth

Gaussian approximation:

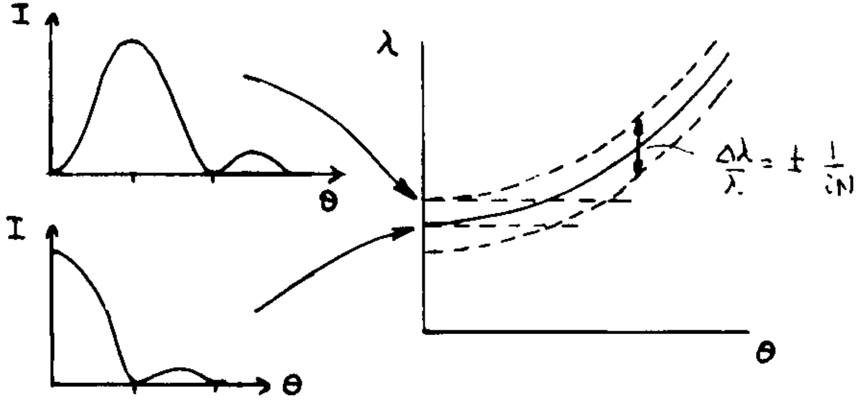
$$\frac{\text{integrated flux}}{\text{peak angular flux density}} = 2\pi\sigma_R'^2 = \frac{\pi}{2N}\,\frac{1+K^2/2}{i\gamma^2}$$

$$\therefore\; \sigma_R' = \sqrt{\frac{\lambda}{2L}}$$

Effect of "detuning"

- Selecting with the monochromator a wavelength slightly different to the on-axis value, λ_i

$$\lambda_i = \frac{\lambda_0}{2\gamma^2}\left(1+K^2/2\right)$$



ie) 2x integrated flux at $\lambda = \lambda_i(1+iN)$
but angular distribution not peaked on-axis

Comparison with bending magnet source

angular flux density:

peak bending magnet angular flux density

$= 1.95\ 10^{13}\ E_{GeV}^2\ I_b$ photons/sec/mrad²/0.1% bandwidth

$$\frac{\text{undulator}}{\text{bending magnet}} = 8.9\ N^2\ F_i(K)$$

eg $N=50$, $K=1$, $i=1$
$F_1(1) =$ [illegible]

$\boxed{= 8206}$

flux:

peak bending magnet flux, integrated over vertical angle $= 2.23\ 10^{13}\ E_{GeV}\ I_b$ photons/sec/mrad/0.1% bandwidth

$$\frac{\text{total undulator flux}}{\text{bend. mag. flux / mrad horiz.}} = \frac{6.4\ N}{E_{GeV}}\ \frac{F_i(K)\ 1+K^2/2}{i}$$

eg $N=50$, $K=1$, $i=1$
$E = 1.5$ GeV

$\boxed{= 119}$

Electron beam emittance

To a good approximation the effect of a finite electron angle is only to shift the radiation pattern

ie) the effective angular distribution is given by a convolution of the undulator radiation angular distribution $F_i(K,\theta,\phi)$ and the Gaussian distribution of electron angles.

$\therefore$ total flux remains same, but:

- radiation opening angle increases and so peak angular flux density decreases
  combining radiation opening angle σ_R' and electron beam divergence σ_x', σ_y' gives:

$$\Sigma_x' = (\sigma_x'^2 + \sigma_R'^2)^{1/2} \qquad \Sigma_y' = (\sigma_y'^2 + \sigma_R'^2)^{1/2}$$

  and the peak angular flux density is reduced approximately by: $(\Sigma_x' \Sigma_y') / \sigma_R'^2$

ie
$$\frac{d^2 I}{d\psi_x\, d\psi_y} = \frac{dI/d\omega/\omega}{2\pi\ \Sigma_x'\ \Sigma_y'}$$

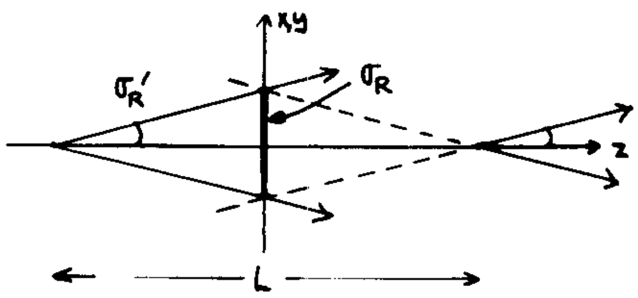
- pinhole size must increase to obtain same flux
- spectrum broader for given acceptance angle

Brightness

- includes not only the angular extent of the source, but also its size

units: photons / sec / mrad² / mm² / 0.1% bandwidth

finite source length ("depth of field"):



equivalent effects

$$\sigma_R \sim \frac{\sigma_R' L}{2}$$

diffraction: $\sigma_R \sigma_R' \sim \frac{\lambda}{4\pi}$ diffraction limit

$$\therefore \sigma_R \sim \frac{\sigma_R' L}{2\pi}$$

convolution with electron beam dimensions:

$$\Sigma_x = (\sigma_x^2 + \sigma_R^2)^{1/2} \qquad \Sigma_y = (\sigma_y^2 + \sigma_R^2)^{1/2}$$

source brightness:

$$\mathcal{B} = \frac{dI/d\omega/\omega}{4\pi^2 \Sigma_x \Sigma_x' \Sigma_y \Sigma_y'}$$

Peak or central density in 4-D phase space, equivalent to definition of electron beam brightness

diffraction limit $\mathcal{B} = \frac{Flux}{(\lambda/2)^2}$

electron beam limit $\mathcal{B} = \underline{Flux}$

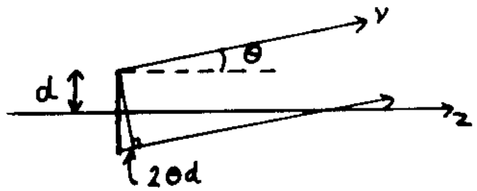
Coherence

temporal coherence - defined by a coherence length over which the waves remain in phase, related to the spectral bandwidth:

$$l_c = \frac{\lambda}{\Delta\lambda/\lambda}$$

in practise, determined by monochromator bandpass since usually $(\Delta\lambda/\lambda)_{mono} < (\Delta\lambda/\lambda)_{\text{undulator radiation}}$

spatial coherence - defined by the phase-space area of the source, related to the source brightness



$$\therefore \theta d \leq \lambda/4$$

thus, a "diffraction limited" source is fully spatially coherent $(\sigma_R \sigma_R' \sim \lambda/4\pi)$

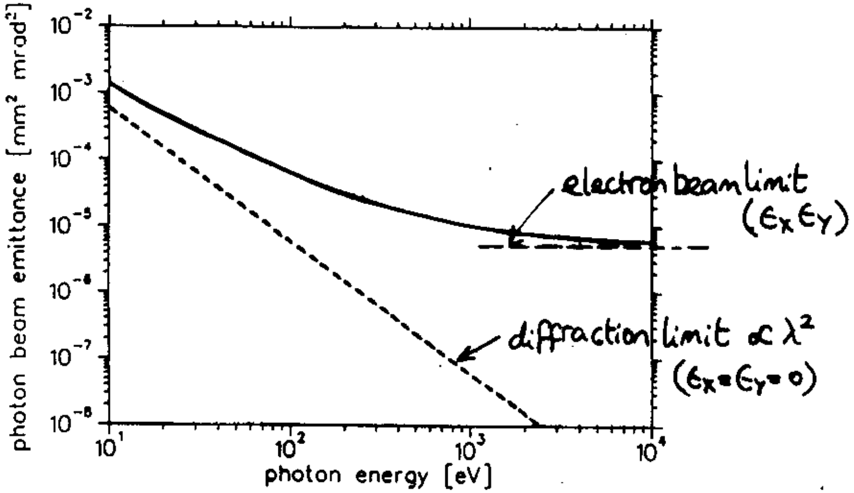
given the definition of brightness, define

$$\text{spatially coherent flux} = \text{brightness} \times \left(\frac{\lambda}{2}\right)^2$$

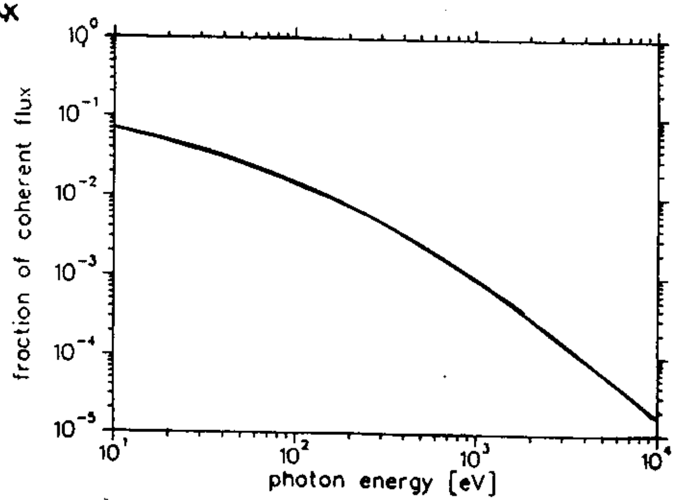
hence also,

$$\frac{\text{coherent flux}}{\text{total flux}} = \frac{(\lambda/2)^2}{4\pi^2 \Sigma_x \Sigma_x' \Sigma_y \Sigma_y'}$$

$\Sigma_x \Sigma_x' \Sigma_y \Sigma_y'$



$\dfrac{\text{coherent flux}}{\text{total flux}}$



ELETTRA $\quad \epsilon_x = 7.2\ 10^{-9}$

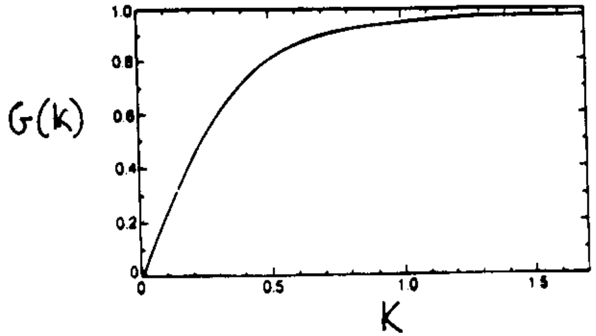
$\epsilon_y = 7.2\ 10^{-10}$

Power, and power density

$$P_{TOT} = 0.633\ E_{GeV}^2\ B_0^2\ L\ I_b \qquad \text{kW}$$

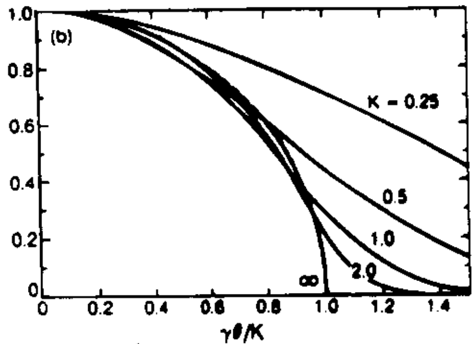
$$\frac{d^2P}{d\Omega} = 10.84\ \underline{E_{GeV}^4}\ B_0\ N\ G(k)\ I_b \qquad \text{W/mrad}^2$$

note strong energy dependence

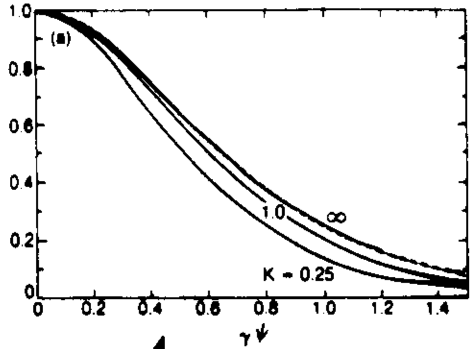


angular distribution of power density:

horizontal plane



vertical plane



as K increases, rapidly approaches situation where $dP/d\Omega$ determined by B at appropriate tangent point

similar to bending magnet

$$\left(B = B_0 \cos kz \quad \theta = K/\gamma \sin kz, \quad \therefore (B/B_0)^2 + (\theta/(K/\gamma))^2 = \text{Const} \right)$$

4. Multipole wigglers

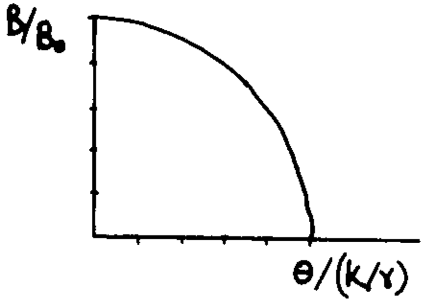
angular flux density / vertically integrated flux

in the absence of interference effects both quantities are the same as for a bending magnet except:

- flux multiplied by the number of poles
- critical wavelength (λ_c) and energy (ϵ_c) depend on viewing angle in the horizontal plane:

$$B = B_0 \cos\left(\frac{2\pi z}{\lambda_0}\right)$$

$$\theta = K/\gamma \, \sin\left(\frac{2\pi z}{\lambda_0}\right)$$



$$\epsilon_c\,(\text{keV}) = 0.665\, E_{GeV}^2\, B = 0.665\, E_{GeV}^2\, B_0 \left\{1 - \left(\theta/(K/\gamma)\right)^2\right\}^{1/2}$$

source size, divergence, brightness

size increases with acceptance angles $\Delta\theta, \Delta\psi$ due to finite length:

$$\Sigma_x \simeq \left(\sigma_x^2 + a^2 + \frac{\sigma_x'^2 L^2}{12} + \frac{\Delta\theta^2 L^2}{36}\right)^{1/2}$$

$a =$ trajectory amplitude $= \dfrac{K}{\gamma}\dfrac{\lambda_0}{2\pi}$

$$\Sigma_y \simeq \left(\sigma_y^2 + \frac{\sigma_y'^2 L^2}{12} + \frac{\Delta\psi^2 L^2}{36}\right)^{1/2}$$

$$\Sigma_y' = \left(\sigma_y'^2 + \sigma_R'^2\right)^{1/2} \quad \text{as for bending magnet}$$

on-axis brightness:

$$\mathcal{B} = \frac{\text{Flux/mrad}}{(2\pi)^{3/2}\left[\left(\sigma_x^2 + a^2 + L^2\sigma_x'^2\right)\left(\sigma_y^2 + L^2\sigma_y'^2\right)\left(\sigma_y'^2 + \sigma_R'^2\right)\right]^{1/2}}$$

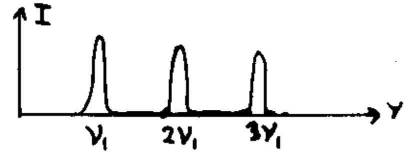
when does an undulator become a wiggler?

- sufficient number of harmonics ($K \gtrsim 3$)
- λ not too close to fundamental ($i \gtrsim 10$)
- interference effects "smoothed out"

  intensities from each pole add, not amplitudes giving smooth bending magnet type spectrum

"smoothing" effects:

- large wavelength acceptance

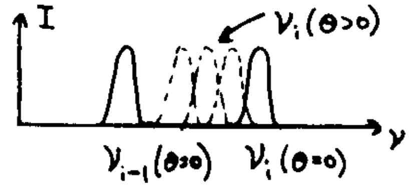


$\Delta\nu = \nu_1$ spacing

$$\frac{\Delta\nu}{\nu_i} = \frac{\nu_1}{\nu_i} = \frac{1}{i}$$

$\therefore$ smoothed if $\left(\dfrac{\Delta\nu}{\nu}\right)_{\text{acceptance}} \gtrsim \dfrac{1}{i}$

- large angular acceptance



$\therefore$ smoothed if

$$\frac{1}{i}\,\frac{\lambda_0}{2\gamma^2}\left(1+\frac{K^2}{2}+\gamma^2\theta^2\right) = \frac{1}{i-1}\,\frac{\lambda_0}{2\gamma^2}\left(1+\frac{K^2}{2}\right)$$

$$\left(\gamma^2\theta^2\right)_{acceptance} \gtrsim \frac{1+K^2/2}{i}$$

- large electron beam emittance

  equivalent effect to acceptance angle

$\therefore$ smoothed if $\left(\gamma^2\sigma'^2\right)_{divergence} \gtrsim \frac{1+K^2/2}{i}$

- large magnet field or periodicity errors

$$\lambda_1 = \frac{\lambda_0}{2\gamma^2}\left(1+K^2/2\right) \qquad \lambda_c = \frac{4\pi}{3}\frac{\rho}{\gamma^3} = \frac{2}{3}\,\frac{\lambda_0}{\gamma^2}\,\frac{1}{K}$$

$$\therefore \frac{\lambda_1}{\lambda_c} = \frac{3}{4}\left(1+K^2/2\right)K$$

eg.

K	i at λ_c	$\gamma\theta = \sqrt{\frac{1+K^2/2}{i}}$
3	12	0.67
5	51	0.52
10	383	0.37
20	3015	0.26

in most cases $(\Delta\gamma/\gamma)_{acceptance}$ and $(\gamma^2\sigma'^2)$ not sufficient to smooth out the spectrum

dominant factor is $(\gamma^2\theta^2)_{acceptance}$

The spectrum of a mulipole wiggler is therefore complex, and in general consists of three parts :

- the fundamental and first few harmonics where there are strong interference effects
- the high photon energy part, at high harmonic number, where the spectrum is essentialy smooth i.e. no interference effects
- an intermediate zone, with moderate spectral modulation

The limits of the various parts depend on the various "smoothing" mechanisms discussed previously.

An example to illustrate the complexity of the spectrum, (and the difficulties of calculating it !) :