INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

H4.SMR/676-7

SECOND SCHOOL ON THE USE OF SYNCHROTRON RADIATION IN SCIENCE AND TECHNOLOGY: *"JOHN FUGGLE MEMORIAL"*

25 October - 19 November 1993

Miramare - Trieste, Italy

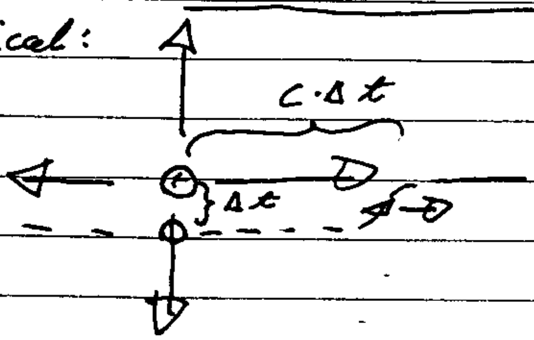
Synchrotron Radiation

M. Eriksson
MAX - LAB
Lund, Sweden

MAIN BUILDING Strada Costiera, 11 Tel. 22401 Telefax 224163 / 224559 Telex 460392 ADRIATICO GUEST HOUSE Via Grignano, 9 Tel. 224241 Telefax 224531 Telex 460449
MICROPROCESSOR LAB. Via Beirut, 31 Tel. 224471 Telefax 224600 GALILEO GUEST HOUSE Via Beirut, 7 Tel. 22401

SYNCHROTRON RADIATION.

Classical:



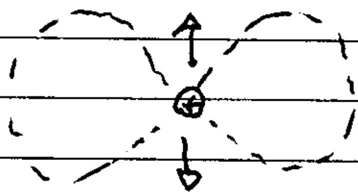
$E \sim q \cdot \dot{v} \cos\theta$

$P \sim E^2$

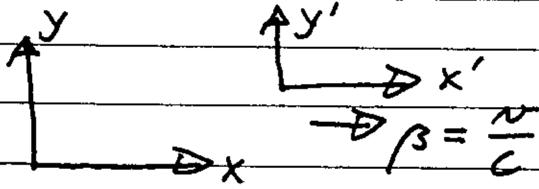
$$P = \frac{2q^2}{3c}\left(\frac{\dot{v}}{c}\right)^2 \cdot \quad (1)$$

E-vector in the same direction as $\dot{v}$

INTENSITY DISTRIBUTION



RELATIVITY AND 4-VECTORS

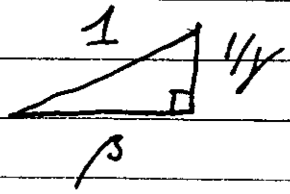


THEN WE HAVE THE LORENTZ TRANSFORMATION

$$\begin{pmatrix} x' \\ y' \\ z' \\ ict' \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & i\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\beta\gamma & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ ict \end{pmatrix}$$

WHERE $\beta = \frac{v}{c}$, $\gamma = \frac{1}{\sqrt{1-\beta^2}}$

NOTE:



A 4-VECTOR $\underline{r} = (x, y, z, ict)$ IS INVARIANT UNDER LORENTZ TRANSFORMATIONS. THAT IS

$$|r| = \sqrt{x^2+y^2+z^2-c^2t^2}$$ TAKES THE SAME VALUE IN ALL SYSTEMS.

$\boxed{\text{EX. 1. : PROVE THIS.}}$

A SPECIAL 4-VECTOR IS THE PROPER TIME

$$\tau = \sqrt{t^2 - \frac{x^2+y^2+z^2}{c^2}} = \cancel{t}\, t\sqrt{1-\frac{v^2}{c^2}} = \frac{t}{\gamma}$$

NOW, THE 4-VELOCITY $\underline{u} \equiv \frac{d\underline{r}}{d\tau} = \frac{d\underline{r}}{dt}\frac{dt}{d\tau} = \underline{\dot{r}}\cdot\gamma$

MUST ALSO BE A 4-VECTOR.

AND SO MUST THE 4-MOMENTUM

$$\underline{P} \equiv m_0 \underline{u} = (\gamma m_0 v_x, \gamma m_0 v_y, \gamma m_0 v_z, ic\gamma m_0)$$

THUS

$$\sqrt{(\gamma m_0 c^2)^2 - p^2c^2} = \sqrt{E^2 - p^2c^2} = E_0$$

WHERE $E = \gamma m_0 c^2$

THE 4 - FORCE

$$\underline{F} \equiv \frac{d\underline{P}}{d\tau} = \frac{d\underline{P}}{dt}\frac{dt}{d\tau} = \gamma\frac{d\bar{P}}{dt} + \gamma\frac{i}{c}\frac{dE}{dt}$$ is also invariant.

LET US REWRITE (1) TO INVARIANT FORM

$$P = \frac{2q^2}{3c^3}\frac{1}{m_0^2}\left(\frac{d\underline{P}}{dt}\right)^2 = \frac{2q^2}{3c^3}\frac{1}{m_0^2}\left(\gamma^2\left(\frac{d\bar{P}}{dt}\right)^2\right) \quad \text{IF } \frac{dE}{dt} = 0$$

NOW

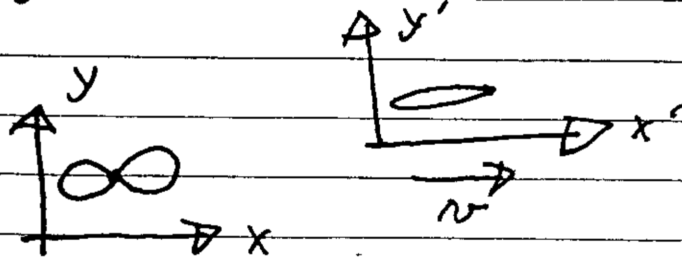
$$\frac{dp}{dt} = \frac{\gamma m_0 v^2}{\rho} = \frac{\gamma m_0 \beta^2 \cdot c^2}{\rho}$$

SO: $\cancel{P = \frac{2}{3}\frac{q^2}{c}}$ $P = \frac{2q^2}{3} c \beta^4 \frac{\gamma^4}{\rho^2}$

$$\text{OR} \quad \Delta E = 88.5 \frac{E^4 (\text{GeV})}{\rho\,(\text{m})} \quad (\text{keV/turn})$$

EX. 2. SHOW THIS

ANGLE DISTRIBUTION.



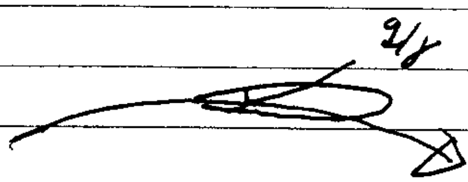
TAKE THE 4-MOMENTUM

$$\begin{pmatrix} p_x' \\ p_y' \\ p_z' \\ \frac{iE'}{c} \end{pmatrix} = \begin{pmatrix} h\nu' \frac{\cos\theta'}{c} \\ h\nu' \frac{\sin\theta'}{c} \\ 0 \\ \frac{ih\nu'}{c} \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & i\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\beta\gamma & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} h\nu \frac{\cos\theta}{c} \\ h\nu \frac{\sin\theta}{c} \\ 0 \\ ih\frac{\nu}{c} \end{pmatrix}$$

$$\Rightarrow \begin{cases} \nu' \cos\theta' = \gamma\, \nu (\cos\theta - \beta) \\ \nu' \sin\theta' = \nu \sin\theta \end{cases}$$

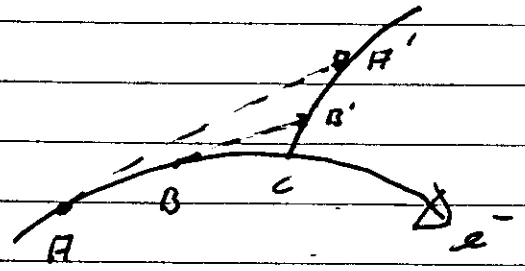
NOTE! A: DOPPLER SHIFT $\nu' = \gamma \cdot 2\nu$ for $\theta = 0$

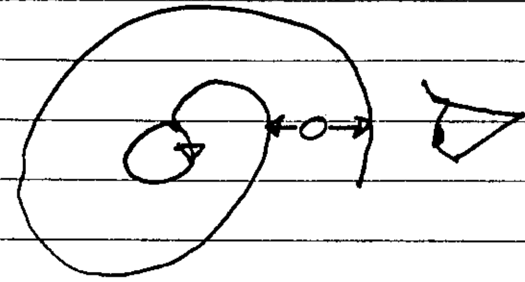
$$B: \tan\theta' = \frac{\sin\theta}{\gamma(\cos\theta - \beta)}$$

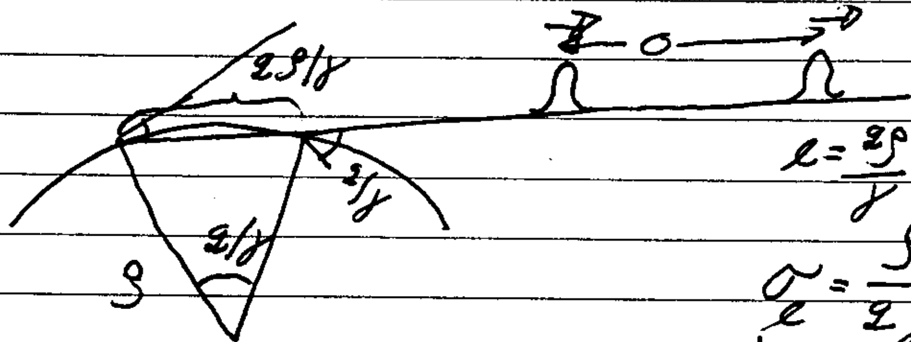


POLARISATION PLANE

ENERGY SPECTRUM

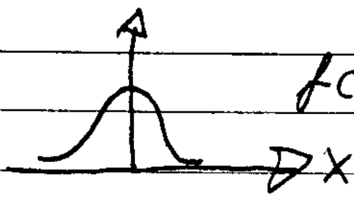






$$\ell = \frac{2\rho}{\gamma}(1-\beta) \simeq \frac{\rho}{\gamma^3}$$

$$\sigma_\ell = \frac{\rho}{2\gamma^3}$$



$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma_\ell}\, e^{-x^2/2\sigma_\ell^2}$$

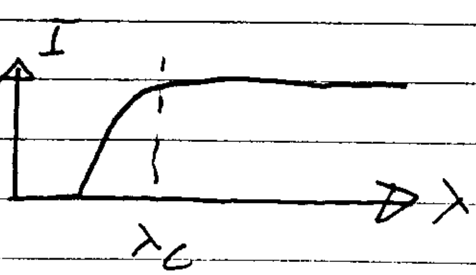
$$f(x) = \sum a_n e^{ikxn}, \quad k = \frac{2\pi}{\ell}$$

$$a_n = C\int e^{ikxn} e^{-x^2/2\sigma^2}\,dx = C\int e^{-\frac{1}{2\sigma^2}(x - i\sigma^2 kn)^2} e^{-\frac{1}{2}\sigma^2 k^2 n^2}\,dx$$

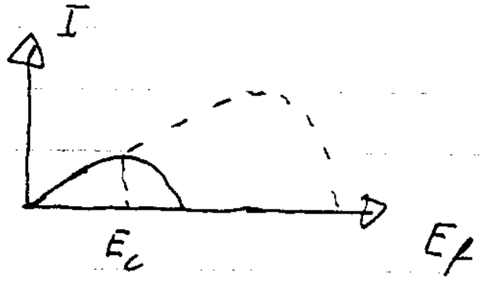
$$= C' e^{-\frac{1}{2}\left(\frac{2\pi\sigma n}{\ell}\right)^2}$$

$$\lambda_c = \frac{\ell}{n} = 2\pi\sigma \Rightarrow \lambda_c = \frac{\pi\rho}{\gamma^3}$$

(EXACT: $\frac{4\pi}{3}\frac{\rho}{\gamma^3}$)



$$\lambda_c(\text{Å}) = \frac{18.6}{E^2(\text{GeV})\,B(\text{T})}$$



$$E_c = \frac{12.000}{\lambda_c(\text{Å})} \ (eV)$$

HALF OF THE POWER ABOVE E_c, HALF BELOW.

$$E_c \sim E_e^3 , \ N_f \sim E \Rightarrow P \sim E^4$$

IF POSSIBLE:
CHOOSE A RING FOR YOUR EXPERIMENT SO YOUR ENERGY MATCHES E_c

POWER: MAX II: 1.5 GeV, $\rho = 3.33$ m, $B = 1.5$ T

$$\Rightarrow \lambda_c = \frac{18.6}{1.5^2 \cdot 1.5} = 5.5 \text{ Å} \Rightarrow E_c = 2.2 \text{ keV}$$

$$\Delta E = 88.5 \cdot \frac{E^4}{\rho} = 130 \text{ keV} \quad (60 \text{ fotons/turn}) \ (3/\text{magnet})$$

$$I = 0.2 \text{ A} \Rightarrow P_F = 26 \text{ kW or } 1.3 \text{ kW/magnet}$$

$$\text{POWER DENSITY} \left.\begin{array}{l} = 4 \text{ W/mrad} \\ \theta/\gamma = 0.6 \text{ mrad} \end{array}\right\}$$

POWER DENSITY ON ABSORBERS 1 m FROM SOURCE POINT 7 W/mm²