INTERNATIONAL ATOMIC ENERGY AGENCY
UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION
INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
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H4.SMR/676-8

SECOND SCHOOL ON THE USE OF SYNCHROTRON RADIATION IN SCIENCE AND TECHNOLOGY:
"JOHN FUGGLE MEMORIAL"

25 October - 19 November 1993

Miramare - Trieste, Italy

COHERENCE

M. Eriksson
MAX - LAB
Lund, Sweden

COHERENCE

$P \sim q^2$ FOR POINT CHARGES, BUT HOW BIG CAN THE POINT BE?

1. TEMPORAL OR LONGITUDINAL COHERENCE.

$$P \sim \sum_{k,j} E_k E_j^* \quad , \; E_k \sim e^{i(\omega t + \varphi_k)} \quad , \; \varphi_k = \frac{2\pi}{\lambda} z_k$$

$$P \sim \sum_{k,j} e^{i(\omega t + \varphi_k)} e^{-i(\omega t + \varphi_j)} = \sum_{k,j} e^{i(\varphi_k - \varphi_j)}$$

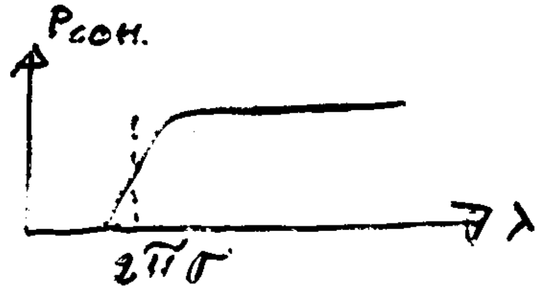
$$= N_e + \sum_{j \neq k} e^{i(\varphi_k - \varphi_j)}$$

INCOHERENT COHERENT

EL. DISTR. $\psi = \frac{N_e}{\sqrt{2\pi}\,\sigma} e^{-z^2/2\sigma^2}$

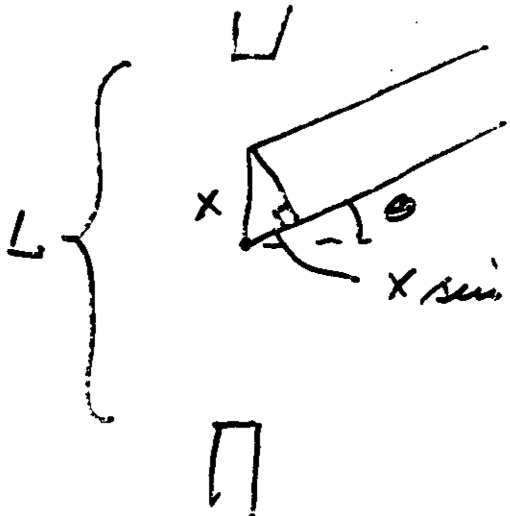
$$P \sim N_e + \frac{N_e(N_e - 1)}{2\pi\sigma^2} \left(\int e^{-\frac{z^2}{2\sigma^2}} e^{i\frac{2\pi z}{\lambda}} dz \right)^2$$

$$P \sim N_e + N_e(N_e - 1)\, e^{-\left(\frac{2\pi\sigma}{\lambda}\right)^2}$$



2. TRANSVERSE COHERENCE (OR DIFFRACTION)

a) SLIT (RECTANGULAR DISTRIBUTION)



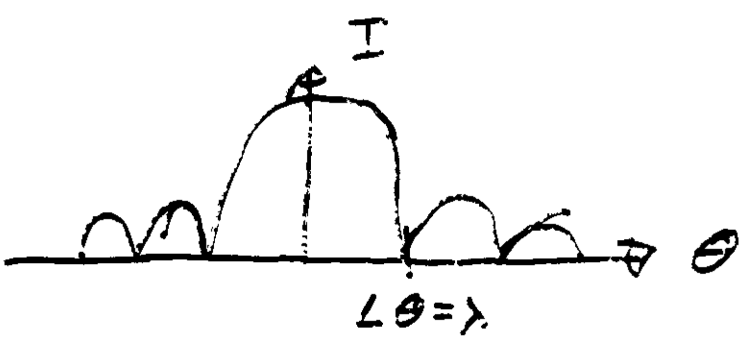
$$x \sin\theta \Rightarrow \Delta\Phi = \frac{\Delta\lambda}{\lambda}\cdot 2\pi = x \sin\theta \cdot k$$

$$k = \frac{2\pi}{\lambda}$$

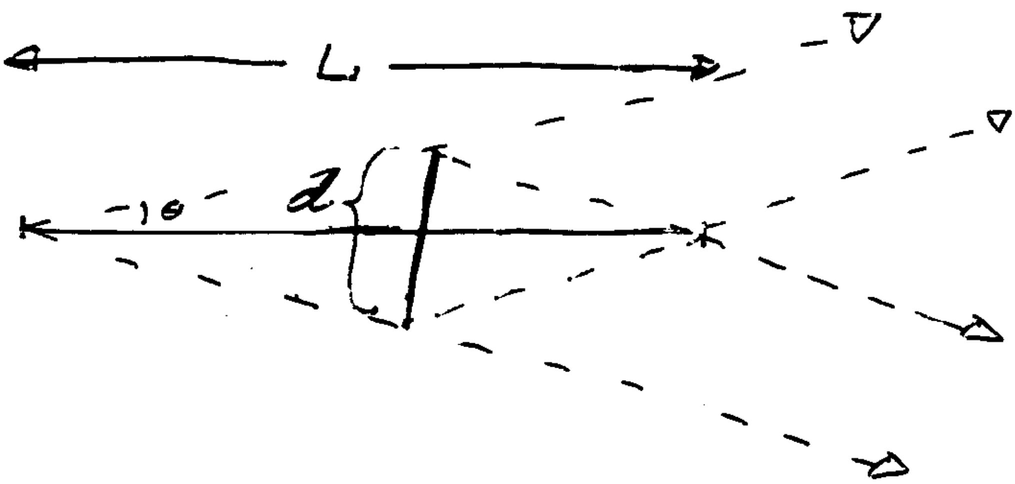
$$E \sim \int_{-L/2}^{L/2} e^{ikwx}\,dx \sim \frac{\sin\left(k\frac{L}{2}w\right)}{k\,w}$$

WHERE $w = \sin\theta$

$$I \sim E^2 \sim \frac{\sin^2\left(k\frac{L}{2}w\right)}{(k\,w)^2}$$



C. LONG LIGHT SOURCE.



DISK DIAMETER $d = L \cdot \theta$

DISP. REL. $d \cdot \theta = \lambda \Rightarrow L\theta^2 = \lambda$ OR $\theta = \sqrt{\frac{\lambda}{L}}$

$$d = \sqrt{\lambda L}$$

EXAMPLE: $E = 500\ MeV \Rightarrow \gamma = 10^3$

$B = 1\ T \Rightarrow \rho = 1.67\ m,\ \lambda_c = 7.4\ \AA$

BENDING MAGNET: $L = \frac{2\rho}{\gamma} = 3.3\ mm$

$\Rightarrow \theta = \sqrt{\frac{\lambda}{L}} = 1.5\ mrad \quad \left(\frac{1}{\gamma} = 1\ mrad\right)$

$d = \sqrt{\lambda \cdot L} = 5\mu m$

UND $L = 1\ m \Rightarrow \theta = 86\ \mu rad,\ d = 86\mu m$

EMISSION ANGLE AND SOURCE SIZE
VARIATION WITH WAVELENGTH

$$\left.\begin{array}{l} \theta = \sqrt{\dfrac{\lambda}{L}} \\ L = 2\theta\rho \end{array}\right\} \Rightarrow \theta^3 = \frac{\lambda}{2\rho} \quad \text{OR} \quad \theta \sim \lambda^{1/3}$$

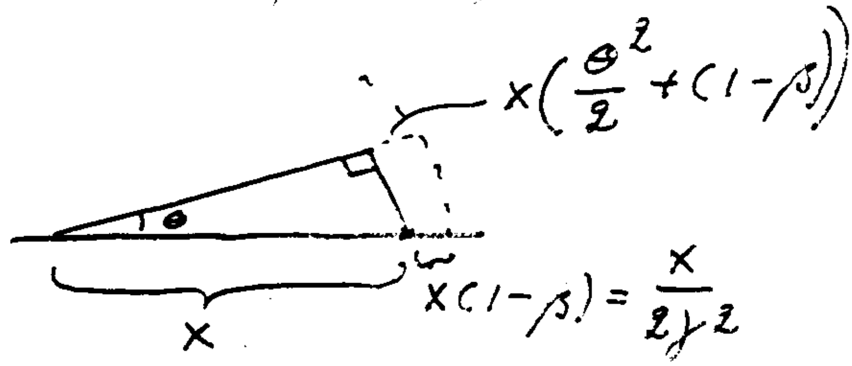
$$\theta = \frac{1}{\gamma}\left(\frac{\lambda}{\lambda_c}\right)^{1/3}$$

AND $d = C\left(\dfrac{\lambda}{\lambda_c}\right)^{2/3}$

THE "DEPTH OF FIELD" EFFECT
AND
DIFFRACTION ARE IDENTICAL

WHAT HAPPEN IF $L_{MAG} < L$?

MORE EXACTLY.



ANGLE DEPENDENT PART

$$E \sim \int_{-L/2}^{L/2} e^{ikx\frac{\theta^2}{2}} dx \sim \frac{\sin\left(k \frac{L}{2} \frac{\theta^2}{2}\right)}{k \frac{\theta^2}{2}}$$

$$I \sim \frac{\sin^2\left(k \frac{L}{2} \frac{\theta^2}{2}\right)}{\left(k \frac{\theta^2}{2}\right)^2}$$

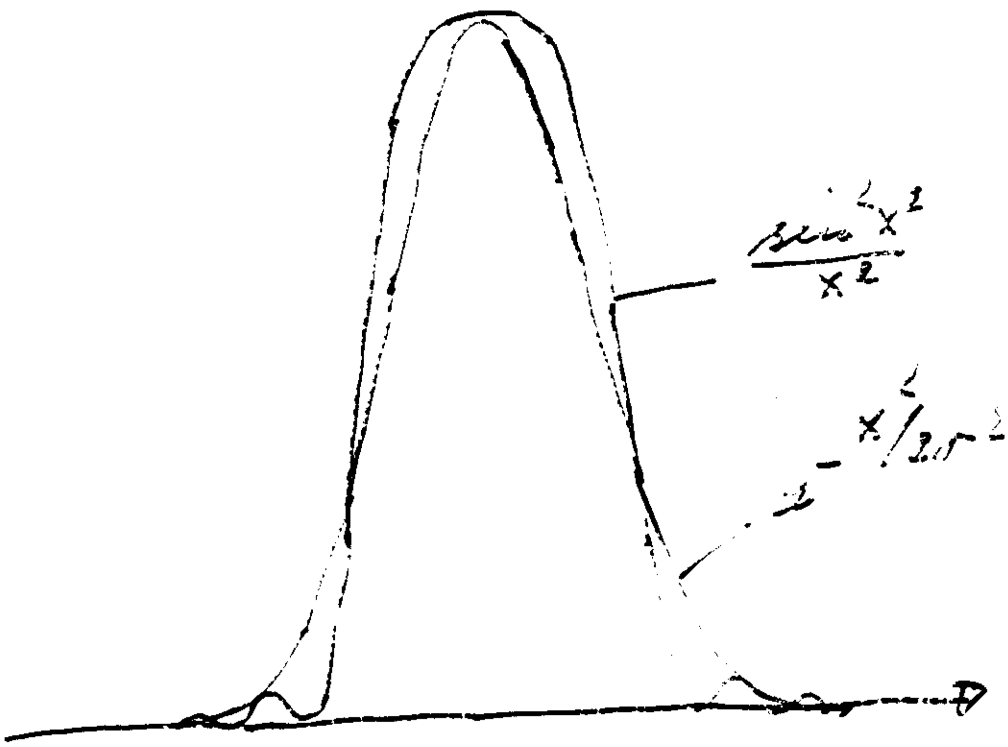
RESEMBLING THE SLIT!

FIRS MIN. FOR $\theta = \sqrt{\frac{2\lambda}{L}}$

TO CALCULATE THE OVER-ALL EFFECT FROM DIFFRACTION AND EL. BEAM EMITTANCE

$\left(\frac{\sin^2(L\theta)^2}{(L\theta)^2} \, , \, e^{-\theta^2/2\sigma_\theta^2} \right)$ WE ARE IN TROUBLE.

GAUSSIAN APPROXIMATION.



EQUALISING THE PEAKS GIVE US $\sigma_\theta = \sigma_{r'} = \frac{1}{\,}\sqrt{\frac{\lambda}{2L}}$

$$\sigma_r \, \sigma_{r'} = \frac{\lambda}{4\pi} \Rightarrow \sigma_r = \frac{\sqrt{\lambda L}}{2\sqrt{2}\,\pi}$$

THE INTENSITY IN PHASE SPACE

CAN NOW BE WRITTEN:

$$\frac{I_0}{(2\pi)^2} \frac{1}{\Sigma_{x'} \Sigma_x \Sigma_{y'} \Sigma_y} e^{-\left(\theta_x^2 / 2\Sigma_{x'}^2 + x^2 / 2\Sigma_x^2 + \theta_y^2 / 2\Sigma_{y'}^2 + y^2 / 2\Sigma_y^2 \right)}$$

WHERE $\Sigma_i = \sqrt{\sigma_i^2 + \sigma_r^2}$

$\Sigma_{i'} = \sqrt{\sigma_{i'}^2 + \sigma_{r'}^2}$

BRILLIANCE IS DEFINED AS $x = y = \theta_x = \theta_y = 0$

$$B = \frac{I_0}{(2\pi)^2} \frac{1}{\Sigma_{x'} \Sigma_x \Sigma_{y'} \Sigma_y} \quad \left(\frac{f}{s,\ mA,\ mm^2,\ mrad^2} \right)$$