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SECOND SCHOOL ON THE USE OF SYNCHROTRON RADIATION IN SCIENCE AND TECHNOLOGY: *"JOHN FUGGLE MEMORIAL"*

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Miramare - Trieste, Italy

Insertion Devices # 3

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Second School on the Use of Synchrotron Radiation
in Science and Technology

Trieste, October 25th - November 19th, 1993

Insertion Devices # 3

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1. Introduction

2. Polarization Properties of Bending Magnets and Conventional Insertion Devices

3. Insertion Device Sources of Circularly Polarized Radiation

1. INTRODUCTION

Applications of CP radiation:

(1) CD - Circular dichroism, difference in <u>absorbtion</u> for Left and Right CP light. Visible & VUV.
→ absolute configuration, electronic structure of organic and biological molecules

(2) CIDS - Circular Intensity Differential Scattering, difference in <u>scattering</u> of Left and Right CP light. Visible & U.V., extending to VUV & soft X-ray
→ sensitive to the helical organization, "super-coiling" of biological macromolecules. (DNA etc.)

(3) MCD - Magnetic Circular Dichroism, difference in absorbtion depending on relative alignment of magnetic field direction and polarization, in magnetic materials. Soft & hard X-rays. → electronic structure of magnetic materials

(4) Spin and angular resolved photoemission, CD & MCD. VUV, soft & hard X-rays

(5) Magnetic Compton Scattering
Hard X-ray.

Polarization Components

$$\underline{E}(t) = \left(E_x(t), E_y(t), 0\right)$$ Electric field seen by observer

$$\underline{A}(\omega) = \int E(t)\, e^{i\omega t}\, dt$$ Amplitude at a given frequency

$$= \left(A_x, A_y, 0\right)$$

In general A_x, A_y are complex quantities

$$I(\omega) = |A(\omega)|^2 = |A_x|^2 + |A_y^2|$$ Radiation intensity

Amplitude polarized in a given direction $\underline{U} = \int \underline{E} \cdot \underline{U}\, e^{i\omega t}\, dt$

direction	$\underline{U}$		
x	$(1,0,0)$		
y	$(0\,1,0)$		
45°	$(1,1,0)/\sqrt{2}$		$A_{45°} = (A_x + A_y)/\sqrt{2}$
−45°	$(1,-1,0)/\sqrt{2}$		
R	$(1,i,0)/\sqrt{2}$	eg	$A_R = (A_x + iA_y)/\sqrt{2}$
L	$(1,-i,0)/\sqrt{2}$		$A_L = (A_x - iA_y)/\sqrt{2}$

Intensity polarized in a given direction $= |A_U|^2$

eg] $$I_R = |A_R|^2 = \left[|A_x|^2 + |A_y|^2 + i\left(A_x^* A_y - A_x A_y^*\right)\right]/2$$

this corresponds to the intensity transmitted by a right circularly polarized filter

ie) it contains the contribution transmitted of linearly polarized light also

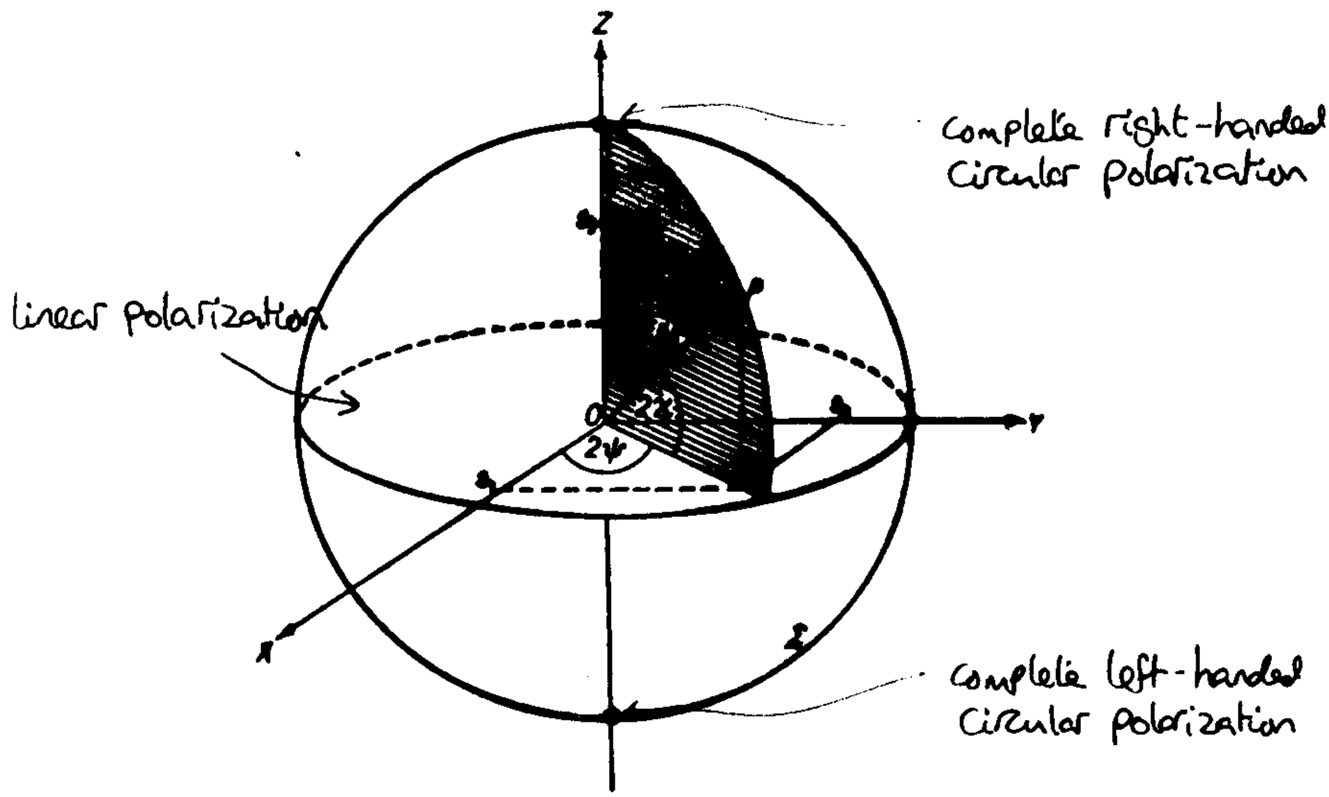
$$I_L = |A_L|^2 = \left[|A_x|^2 + |A_y|^2 - i\left(A_x^* A_y - A_x A_y^*\right)\right]/2$$

$$\therefore\ I_R + I_L = |A_x|^2 + |A_y|^2 = I_{total}$$

To consider radiation exclusively polarized in a given direction, we can use differences in intensity:

$$\left.\begin{aligned} S_1 &= I_x - I_y \\ S_2 &= I_{45^\circ} - I_{-45^\circ} \\ S_3 &= I_R - I_L \\ S_0 &= I_x + I_y \\ &= I_{45^\circ} + I_{-45^\circ} \\ &= I_R + I_L \end{aligned}\right\} \text{Stokes Parameters (1852)}$$

It can be shown that : $\boxed{S_1^2 + S_2^2 + S_3^2 = S_0^2}$



Geometrical representation of the different states of polarization : <u>Poincaré sphere</u> (1892)

We can also define "<u>polarization rates</u>":

$$l_1 = \frac{S_1}{S_0} = \frac{I_x - I_y}{I_x + I_y}$$

$$l_2 = \frac{S_2}{S_0} = \frac{I_{45^\circ} - I_{-45^\circ}}{I_{45^\circ} + I_{45^\circ}}$$

$$l_3 = \frac{S_3}{S_0} = \frac{I_R - I_L}{I_R + I_L}$$

It follows that:

$$l_1^2 + l_2^2 + l_3^2 = 1$$

The above relation holds for a single <u>monochromatic</u> wave.

<u>In general</u> an <u>unpolarized</u> component is introduced:

$$S_0 = \sqrt{S_1^2 + S_2^2 + S_3^2} + S_4$$

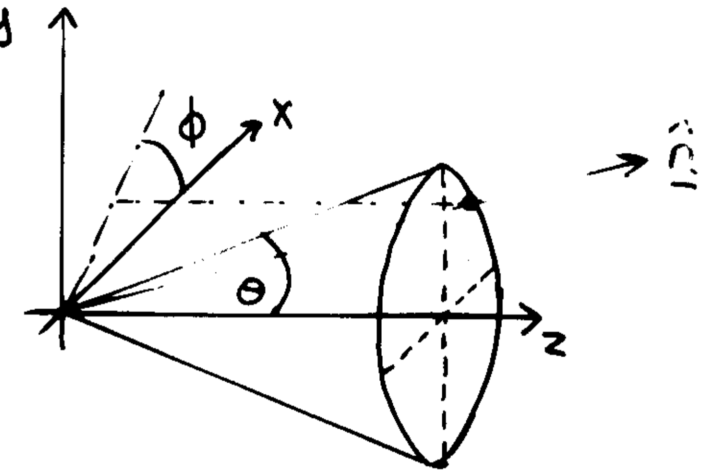
$$1 = \underbrace{\sqrt{l_1^2 + l_2^2 + l_3^2}}_{\text{Polarized fraction}} + \underbrace{l_4}_{\text{un-polarized fraction}}$$

This occurs due to:

- range of wavelengths
- angular acceptance
- electron beam emittance and energy spread
- summation of radiation emitted by different poles

2. Polarization Properties of Bending Magnets and Conventional Insertion Devices.

Co-ordinate system:



$$\underset{\sim}{n} = (\sin\theta\cos\phi,\ \sin\theta\sin\phi,\ \cos\theta)$$
$$\simeq (\theta\cos\phi,\ \theta\sin\phi,\ 1-\theta^2/2)$$

It can be shown that the E-field is related to $(\underset{\sim}{n} \wedge \underset{\sim}{n} \wedge \dot{\underset{\sim}{\beta}})$, from which we obtain that:

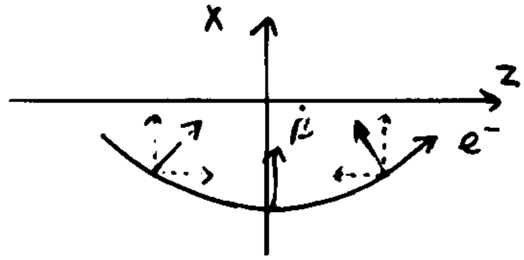
$E_x \sim \dot{\beta}_x$ acceleration in horizontal plane

$E_y \sim \dot{\beta}_y \left(- n_y \dot{\beta}_z - n_x n_y \dot{\beta}_x \right)$

$\dot{\beta}_y$: acceleration in vertical plane; $n_y \dot{\beta}_z$: off-axis vertically; $n_x n_y \dot{\beta}_x$: off-axis horizontally & vertically

These relations can be used to examine the E-field and hence polarization components for different sources.

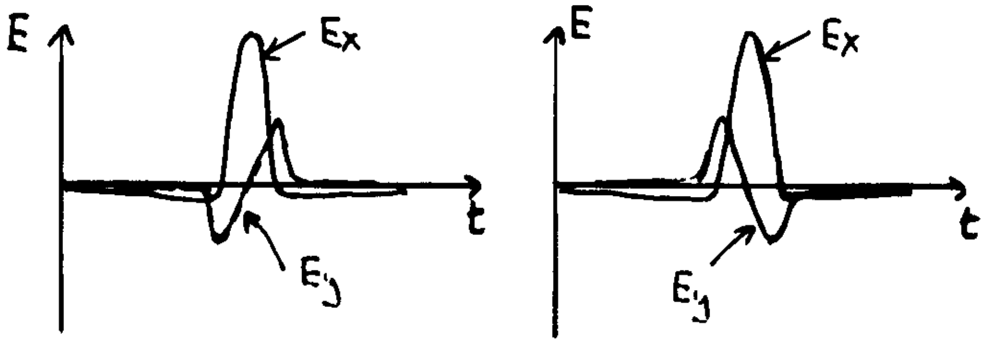
$\dot{\beta}_y = 0$



$\dot{\beta}_x$ symmetric (E_x)

$\dot{\beta}_z$ anti-symmetric (E_y)

on-axis ($n_y = 0$) $E_y = 0$; linearly polarized in horizontal plane



Off-axis: n_y +ve $\qquad$ n_y −ve

Phase relationship between E_x, E_y $\Rightarrow$ circularly polarized component, that changes sign above / below axis.

$$\underline{A} = \int \underline{E}(t)\, e^{i\omega t}\, dt$$

$$= \int \overset{\text{sym.}}{E_x} \cos(\omega t) + i\, \overset{\text{Asym.}}{E_y} \sin(\omega t)\; dt$$

$$\therefore \boxed{\begin{array}{ll} A_x & \text{Real} \\ A_y & \text{Imaginary} \end{array}} \quad (n_y > 0)$$

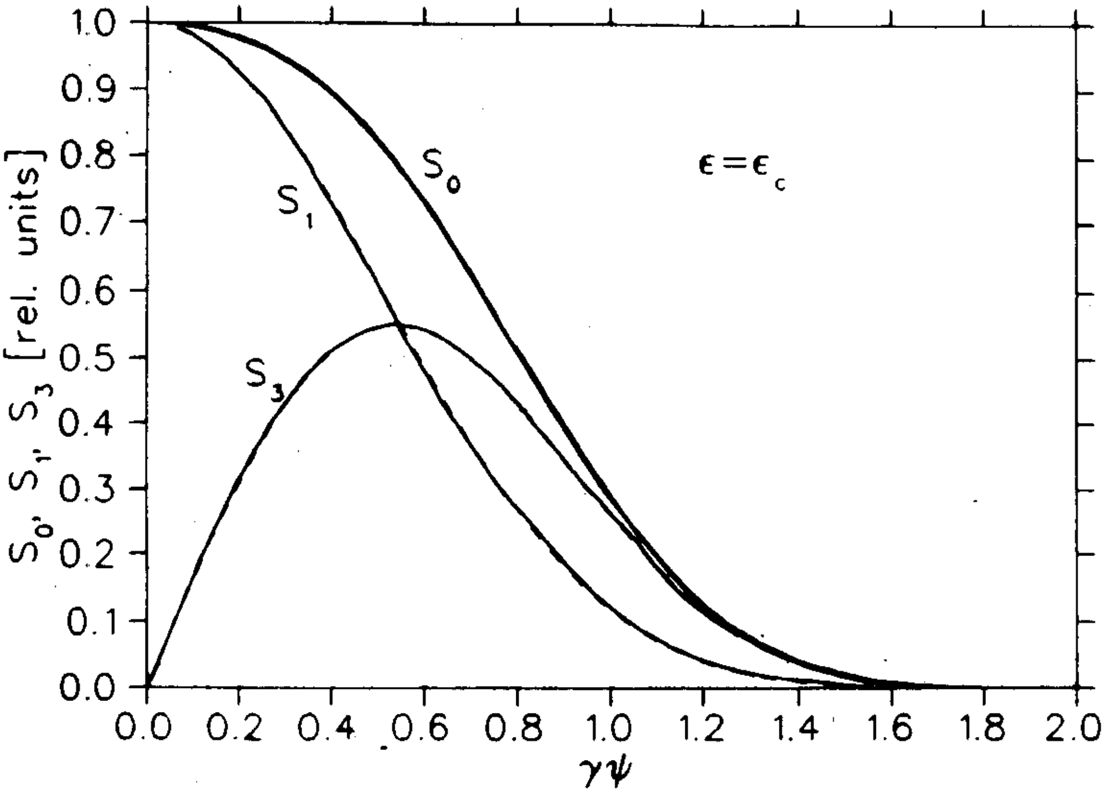
$$\underline{A} = \left(K_{2/3}(\eta),\; i \frac{X}{(1+X^2)^{1/2}} K_{1/3}(\eta),\; 0 \right)$$

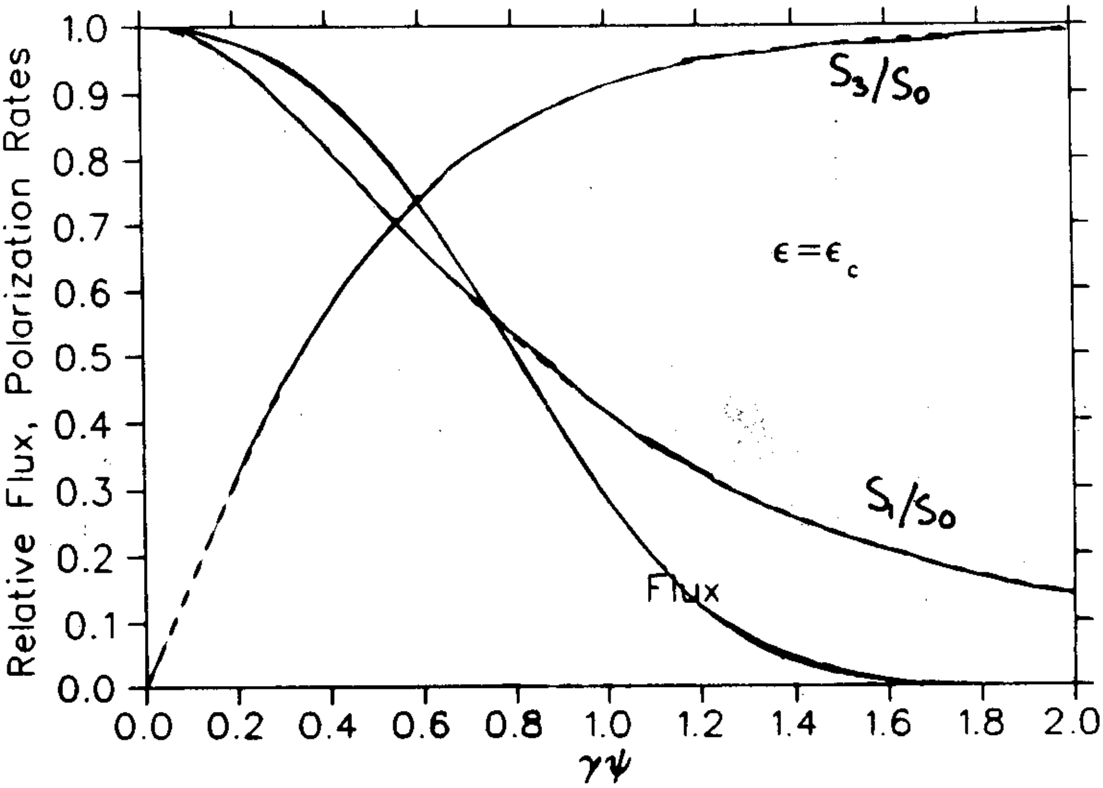
$$X = \gamma\theta \qquad \eta = \frac{\lambda_c}{2\lambda}\left(1 + X^2\right)^{3/2}$$

from which it can be shown that:

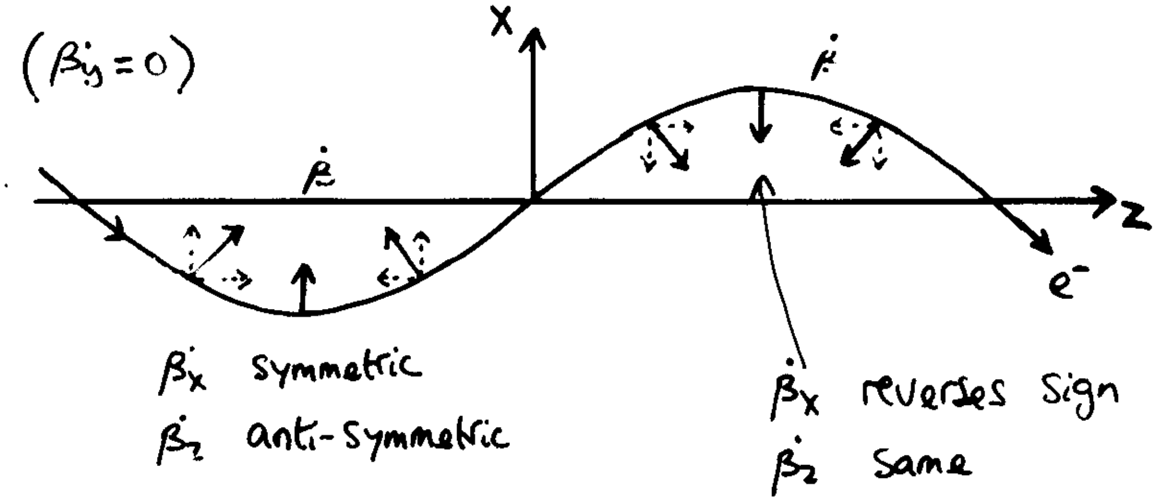
$$S_1 = I_x - I_y$$
$$S_2 = 0$$
$$S_3 = \pm 2\sqrt{I_x I_y}$$

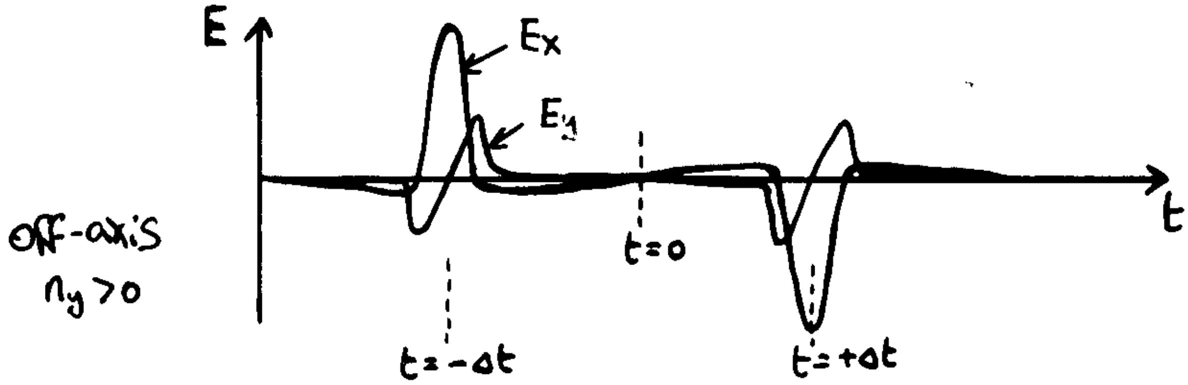
NB. valid only for a bending magnet; not a general result.





Plane periodic magnet





$$\underline{A} = \int E(t)\, e^{i\omega t}\, dt$$

$$= \underbrace{\int_1 E(t)\, e^{-i\omega\Delta t} e^{i\omega t}\, dt}_{\text{1st pole}} + \underbrace{\int_2 E(t)\, e^{i\omega\Delta t} e^{i\omega t}\, dt}_{\text{2nd pole}}$$

$$= e^{-i\phi}\left(\int_1 E_x \cos\omega t + i\int_1 E_y \sin\omega t\right) \qquad \phi = \omega\Delta t$$

$$+\, e^{i\phi}\left(\int_2 -E_x \cos\omega t + i\int_2 E_y \sin\omega t\right)$$

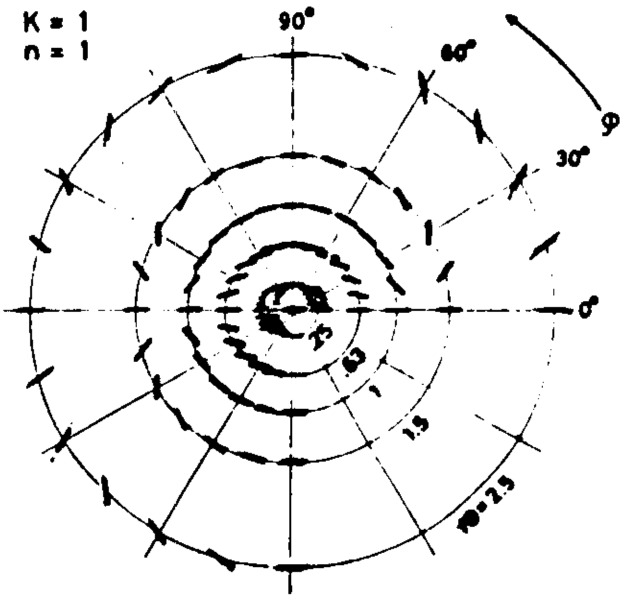
$$= -2i\sin\phi\left(\int E_x \cos\omega t\right) + 2i\cos\phi\left(\int E_y \sin\omega t\right)$$

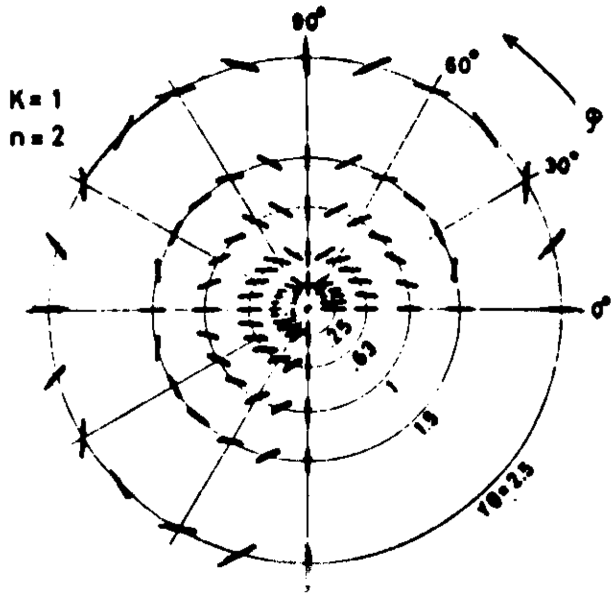
$$\therefore \boxed{\begin{array}{ll} A_x & \text{Imaginary} \\ A_y & \text{Imaginary} \end{array}} \quad (n>0)$$

from which it can be shown:

$$S_1 \neq 0$$
$$S_2 \neq 0$$
$$S_3 = 0$$

- complex variation of polarization direction with i, θ, ϕ :

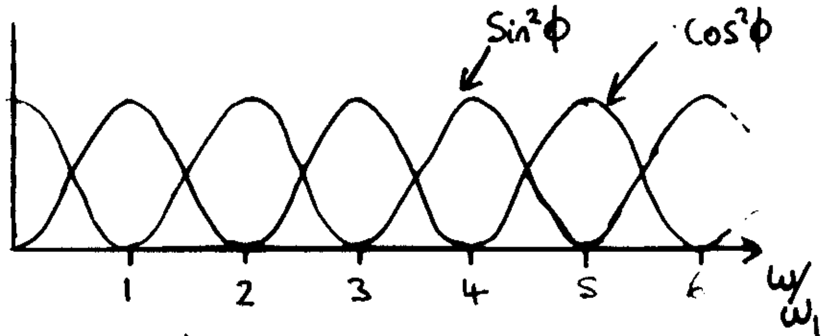




Wiggler

$A_x = a \sin\phi \qquad A_y = b\cos\phi \qquad \phi = \omega\Delta t = \pi\,\omega/\omega_1$

$I_x = a^2 \sin^2\phi \qquad I_y = b^2\cos^2\phi \quad$ etc



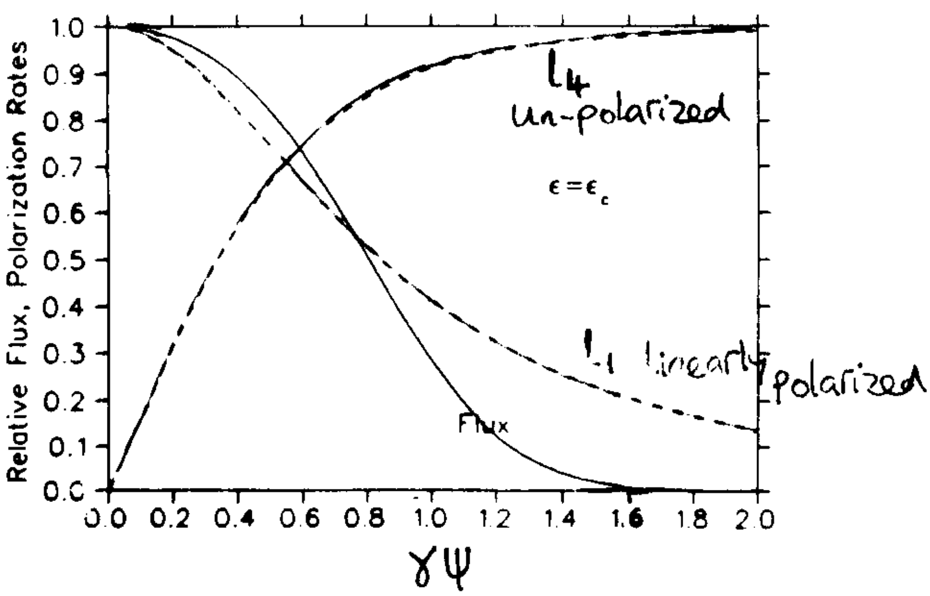
$\therefore$ averaging over frequency

$\langle \sin^2\phi\rangle = \langle\cos^2\phi\rangle = 1/2$

$\langle \sin\phi\cos\phi\rangle = 0$

it can then be shown $\langle S_2\rangle = 0$

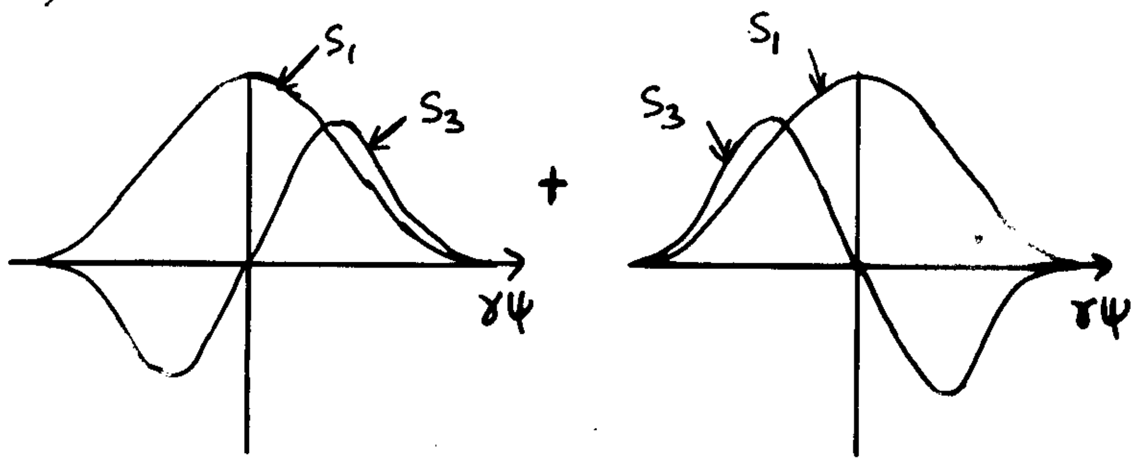
$\therefore$ all radiation not linearly polarized is <u>un-polarized</u>



This result can also be obtained directly by summing the <u>intensities</u> from 2 bending magnet sources ie) no interference

Since the intensities add, so also do the Stokes parameters.

ie)



($S_2 = 0$) ($S_2 = 0$)

Thus, $S_2 = S_3 = 0$

The cancellation of the right- and left-handed radiation (S3 positive and S3 negative) leads directly to an un-polarized component.

$$S_0 = \sqrt{S_1^2 + S_2^2 + S_3^2} + S_4 \quad \text{in general}$$

$$S_0 = \sqrt{S_1^2 + S_3^2} \qquad \underline{\text{Single pole}},\ \psi > 0$$

$$2S_0 = \sqrt{(2S_1)^2} + S_4 \qquad \underline{\text{two poles}},\ \psi > 0$$

5. Insertion Device Sources of Circularly Polarized Radiation

3 main types –

① Helical / Elliptical Devices (U/W)

$$B_x = B_{x0} \sin(kz)$$
$$B_y = B_{y0} \cos(kz)$$

hence $\dot{\beta}_y \sim \sin(kz)$

$\dot{\beta}_x \sim \cos(kz)$

hence $\left.\begin{array}{l} A_x = \text{Real} \\ A_y = \text{Imaginary} \end{array}\right\}$ as in bending magnet, off-axis

$\rightarrow \quad S_1 \neq 0, \quad S_2 = 0, \quad S_3 \neq 0 \quad$ (on-axis)

② Asymmetric Devices (W)

planar type with $B_x = 0$

but $|B_y|$ of positive and negative poles unequal

$\rightarrow$ Linear polarization on-axis

$S_1 \neq 0$ and $S_3 \neq 0$ off-axis (as in bend. magnet)

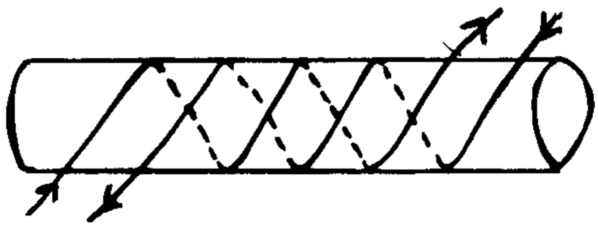
③ Interference Devices (U)

Interference of radiation from two separate orthogonal undulators can produce variable polarization, on-axis. ("crossed-undulator")

Helical / elliptical undulators

- types in which the performance is restricted by both horizontal and vertical magnet gaps

1. Bifilar helix

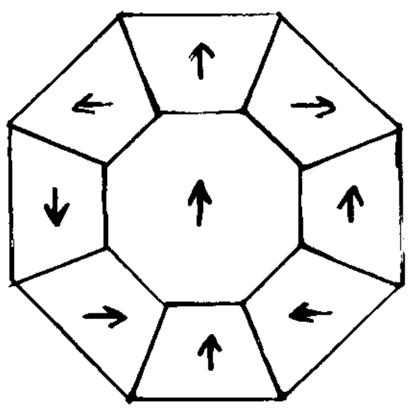


- used in first FEL experiment (Madey et. al., Stanford 1977)
- first use in storage ring : VEPP2M, Novosibirsk

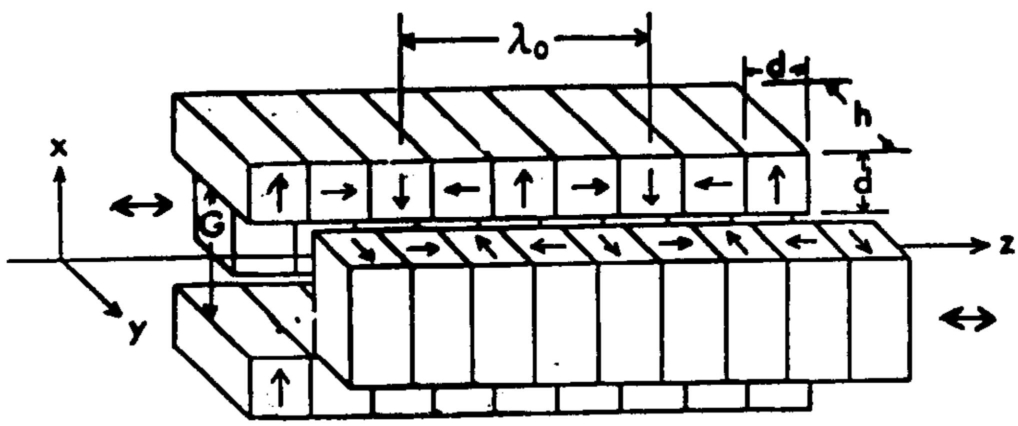
s/c helical undulator (1984), 8 periods of 2.4 cm, K = 1.0

2. Helical electromagnet

3. Helical permanent magnet (Halbach, 1981)

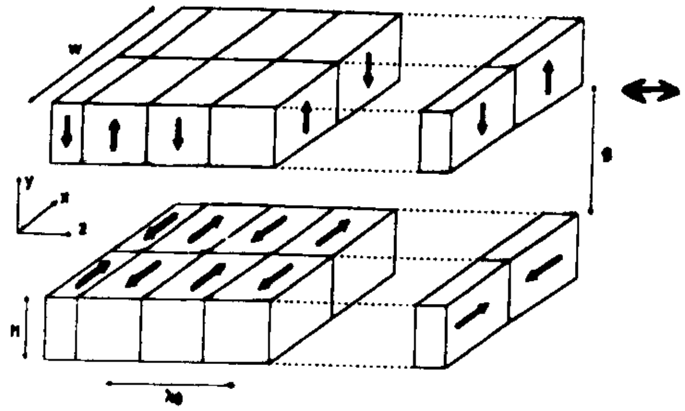


4. Variably polarized undulator (Onuki, 1986)

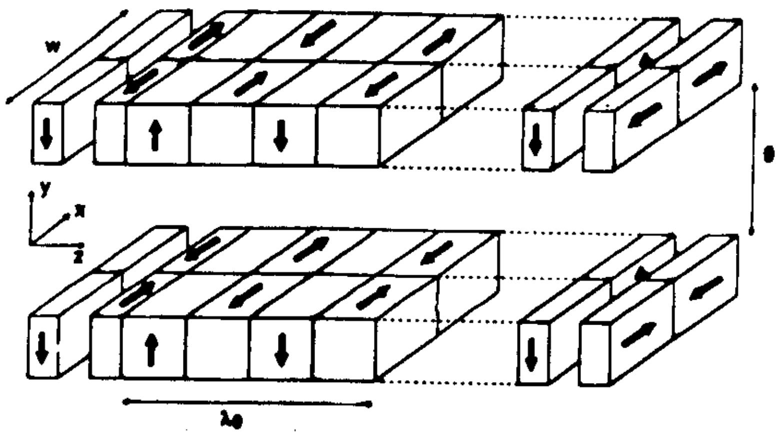


- "planar" types in which the performance is limited only by the horizontal magnet gap

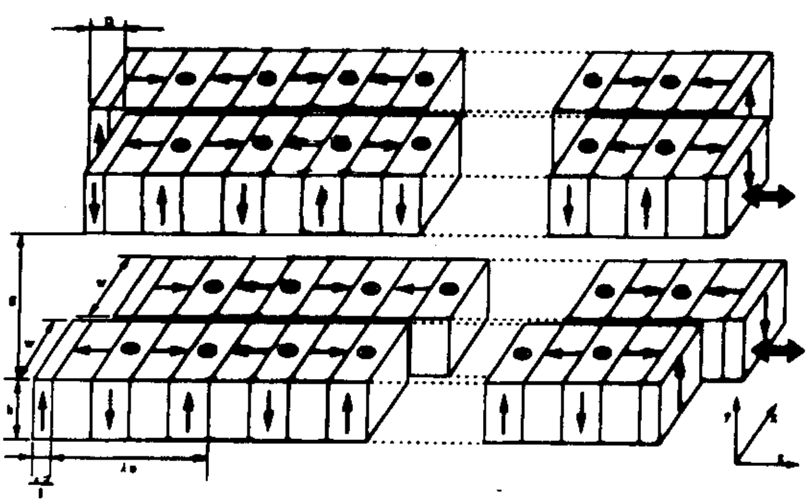
5. Planar variably polarized undulator (Elleaume, 1990)



6. Planar helical undulator (Diviacco and Walker, 1990)



7. Planar variably polarized undulator (Sasaki, 1992)



electron motion: x-y motion:

with $B_x = B_{x0} \sin\left(\frac{2\pi z}{\lambda_0}\right)$ $B_y = B_{y0} \cos\left(\frac{2\pi z}{\lambda_0}\right)$

get $\beta_x = \frac{K_y}{\gamma} \sin\left(\frac{2\pi z}{\lambda_0}\right)$ $\beta_y = \frac{K_x}{\gamma} \cos\left(\frac{2\pi z}{\lambda_0}\right)$

where $K_x = 93.4\, B_{x0} \lambda_0$ $K_y = 93.4\, B_{y0} \lambda_0$

z-motion: $\beta_x^2 + \beta_y^2 + \beta_z^2 = \beta^2$ (constant)

$$\therefore \beta_z \simeq \beta\left(1 - \frac{K_x^2}{4\gamma^2} - \frac{K_y^2}{4\gamma^2} + \frac{K_y^2 - K_x^2}{4\gamma^2} \cos\frac{4\pi z}{\lambda_0}\right)$$

$$\beta^* = \beta\left(1 - \frac{K_x^2}{4\gamma^2} - \frac{K_y^2}{4\gamma^2}\right)$$ average velocity along z

interference condition: $\lambda_0\left(\frac{1}{\beta^*} - \cos\theta\right) = i\lambda$

$$\boxed{\lambda = \frac{\lambda_0}{2\gamma^2} \frac{1}{i}\left(1 + \frac{K_x^2}{2} + \frac{K_y^2}{2} + \gamma^2\theta^2\right)}$$

i/ $K_x = K_y$

pure helical motion.

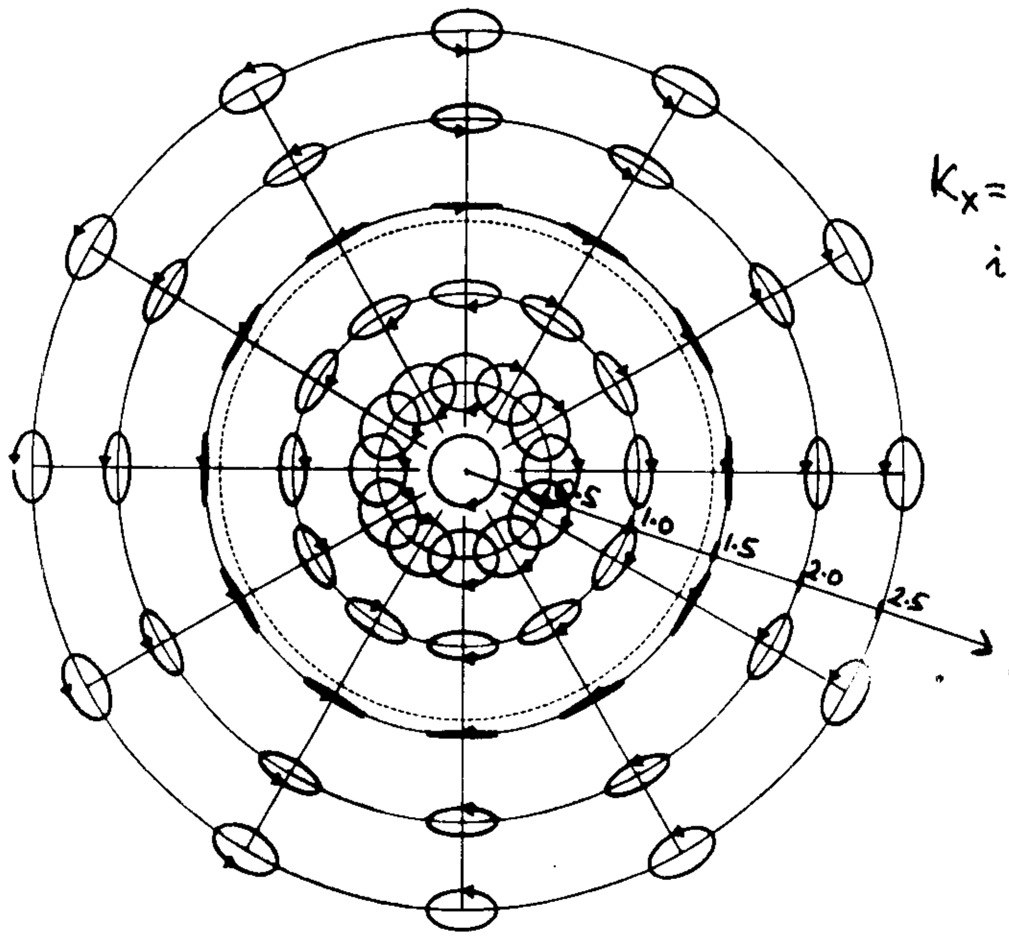
β_z = constant $\therefore$ single harmonic (on-axis)

$$\lambda = \frac{\lambda_0}{2\gamma^2} \frac{1}{i}\left(1 + K^2 + \gamma^2\theta^2\right) \qquad K = K_x = K_y$$

ii/ $K_x \neq K_y$

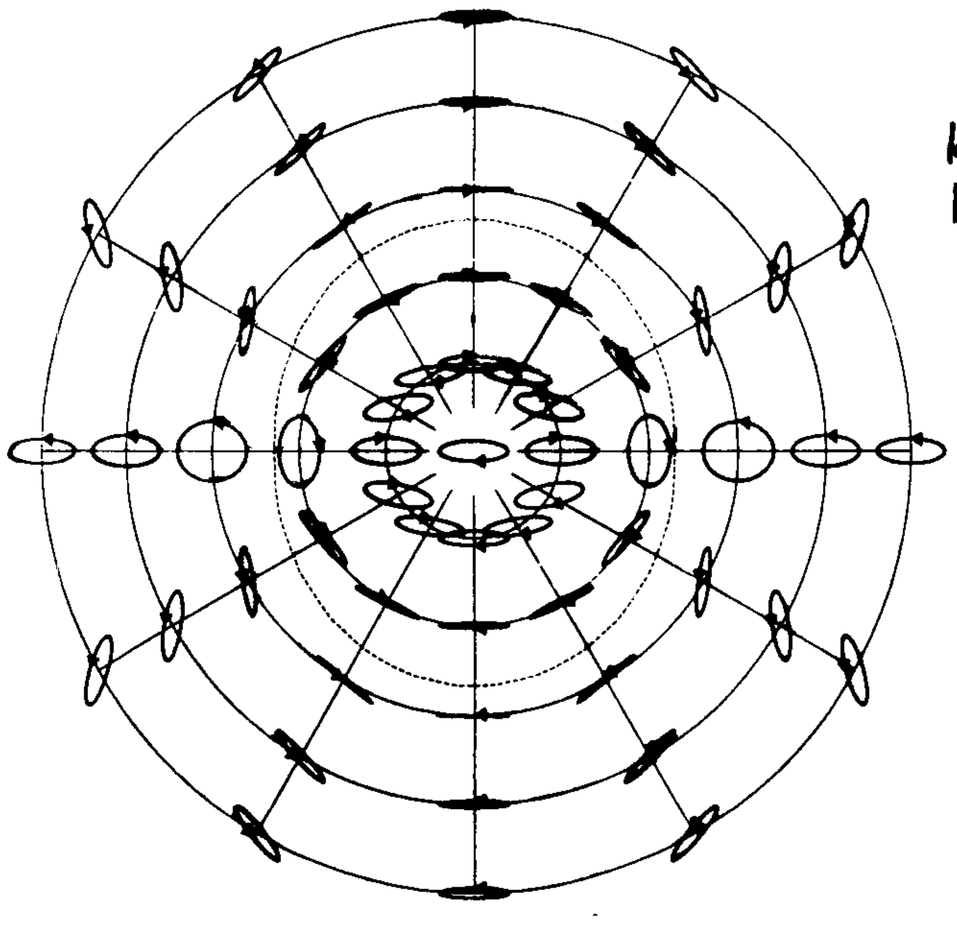
elliptical motion

$\beta_z \neq$ constant $\therefore$ all odd harmonics on-axis



$K_x = K_y = 1.0$

$i = 1$



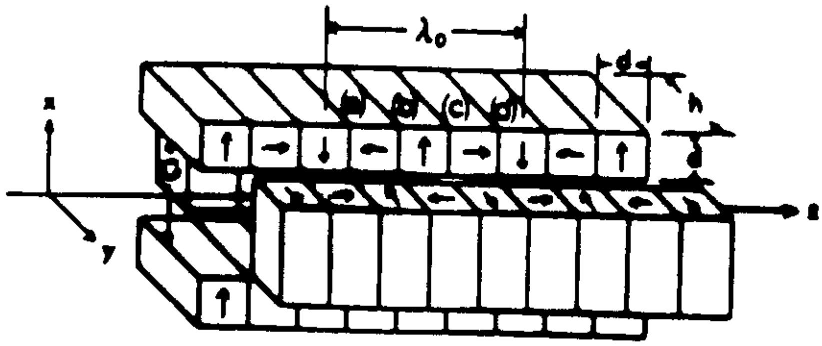
$K_x = 0.25$

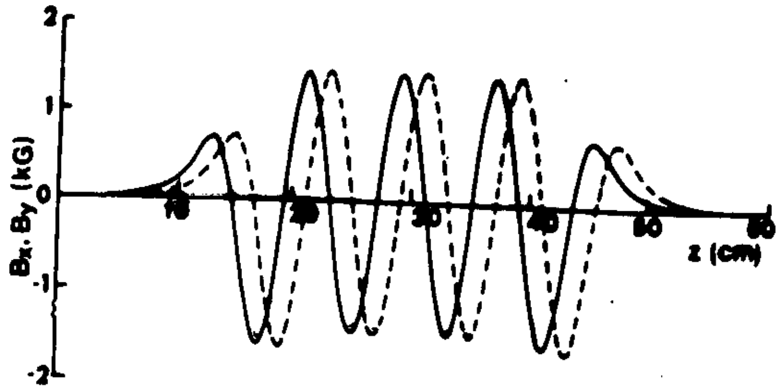
$K_y = 1.0$

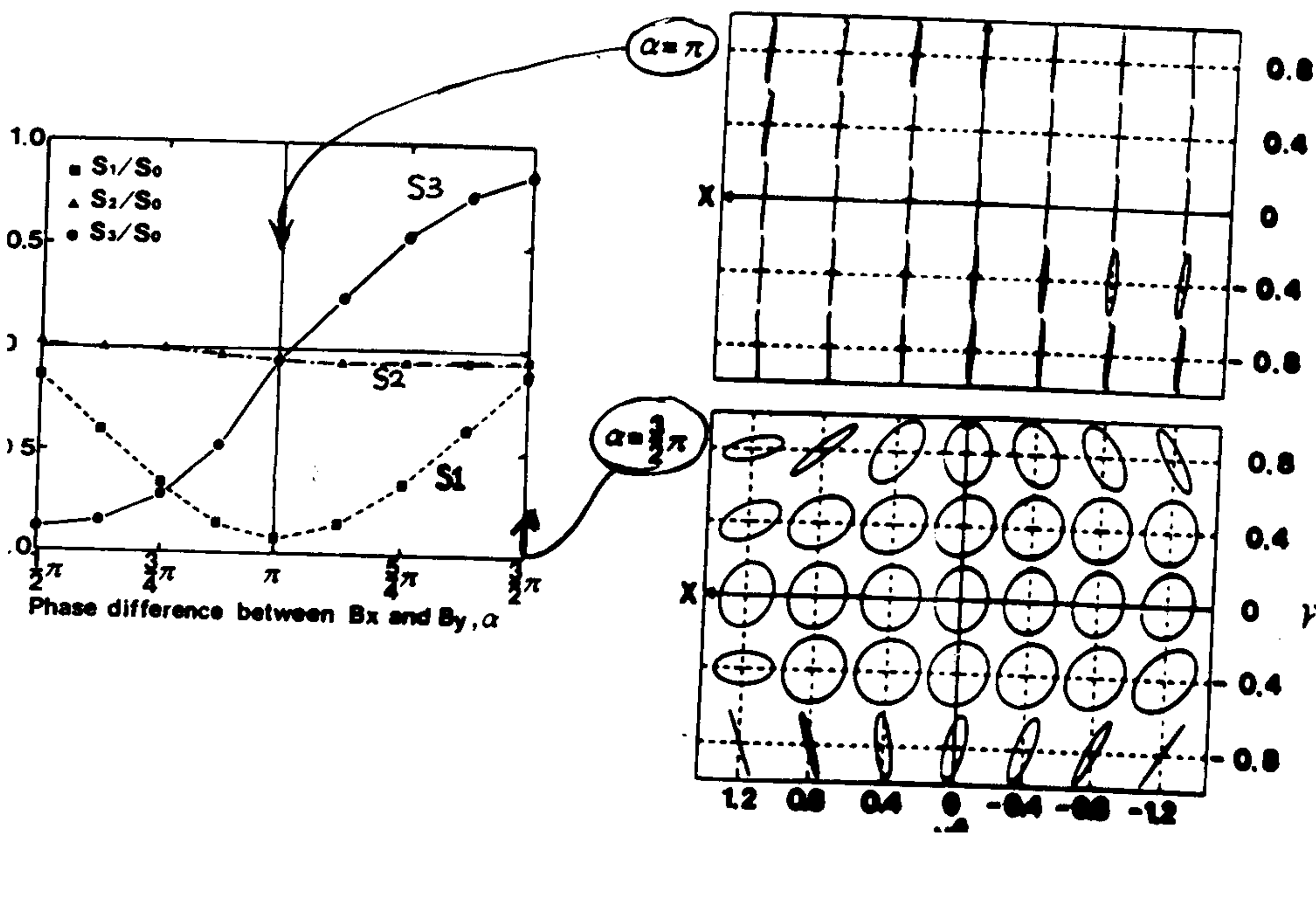
$i = 1$

Example - Variably Polarized Undulator developed at Electrotechnical Lab., Japan (TERAS).

(K. Yagi et. al., Rev. Sci. Instr. 63 (1992) 396.)





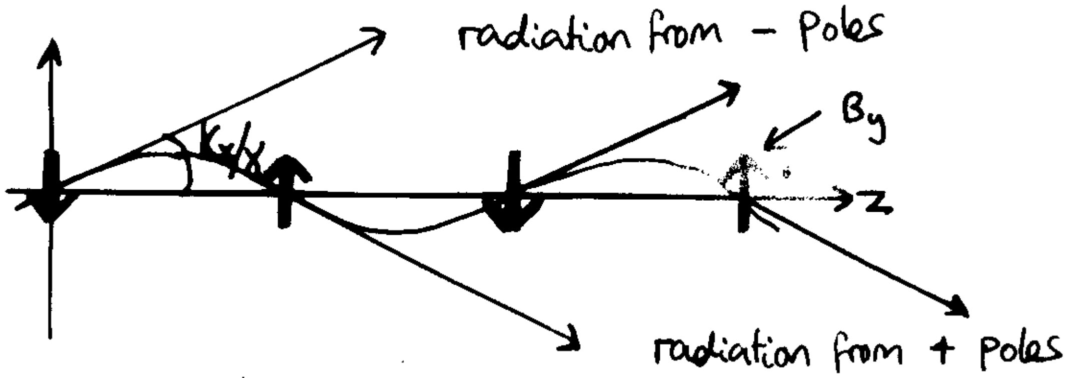


Elliptical wiggler

- extension of the elliptical undulator scheme with strong vertical field and weak horizontal field : $K_y \geq 10$, $K_x \sim 1$

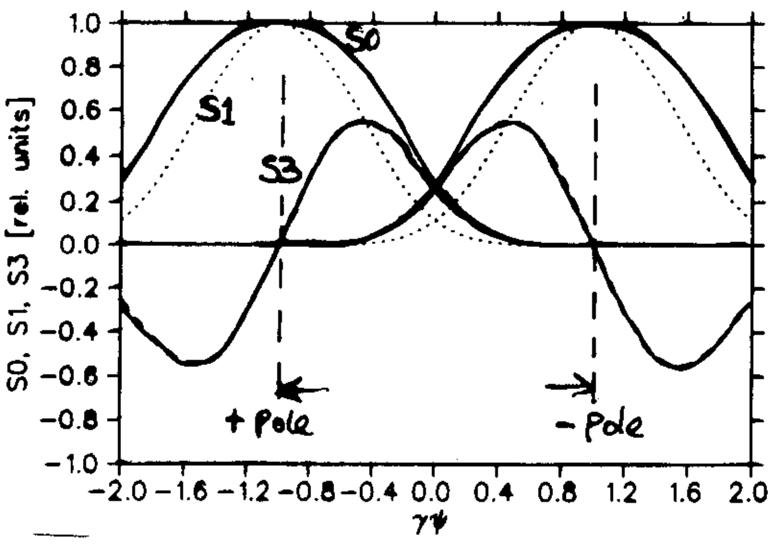
$$B_x = -B_{x0} \sin\left(\frac{2\pi}{\lambda_0} z\right) \qquad B_y = -B_{y0} \cos\left(\frac{2\pi}{\lambda_0} z\right)$$

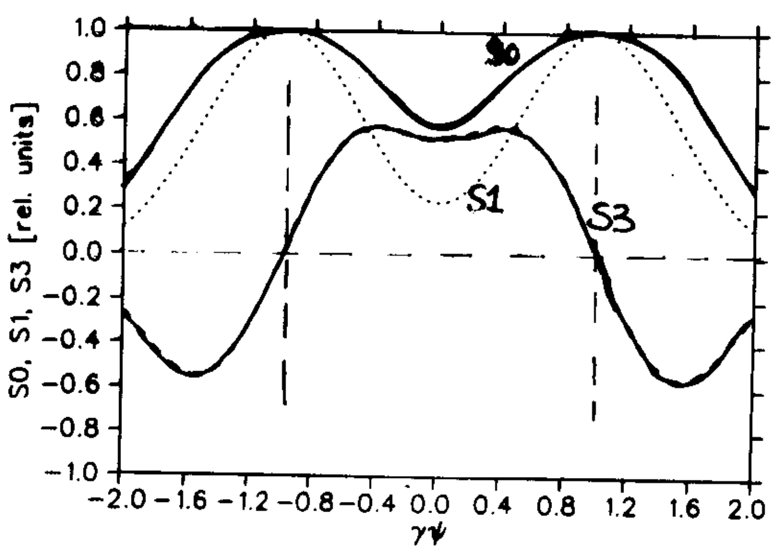
$$y' = \frac{K_x}{\gamma} \cos\left(\frac{2\pi}{\lambda_0} z\right)$$



$\epsilon = \epsilon_c$

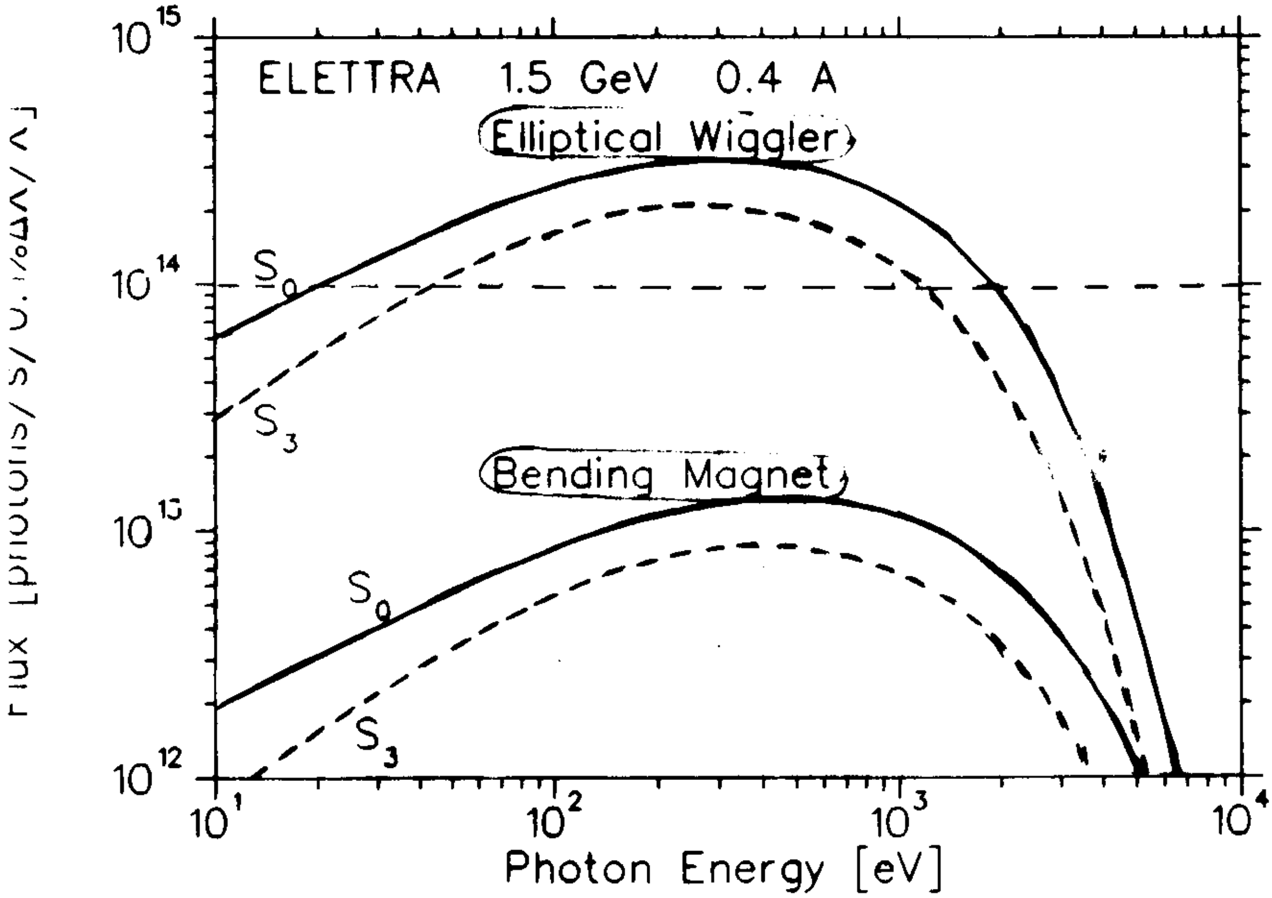
$K_x = 1$





Elliptical wiggler produces a broad spectrum with the flux enhanced by the number of poles.

Example.



$\lambda_0 = 0.23 \quad N = 12$

$B_{y0} = 0.6\,T \quad K_y = 12.9 \quad \epsilon_c = 0.9\,keV$

$B_{x0} = 0.047\,T \quad K_x = 1.0$

- Elliptical wigglers are operating in the Photon Factory and TRISTAN Accumulator Ring

<u>Example - Elliptical Wiggler developed at KEK, Japan (Photon Factory and Tristan Accumulator Ring)</u>

(H. Kitamura and S. Yamamoto, Rev. Sci. Instr. 63 (1992) 1104.)

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First Production of Intense Circularly Polarized Hard X Rays from a Novel Multipole Wiggler in an Accumulation Ring

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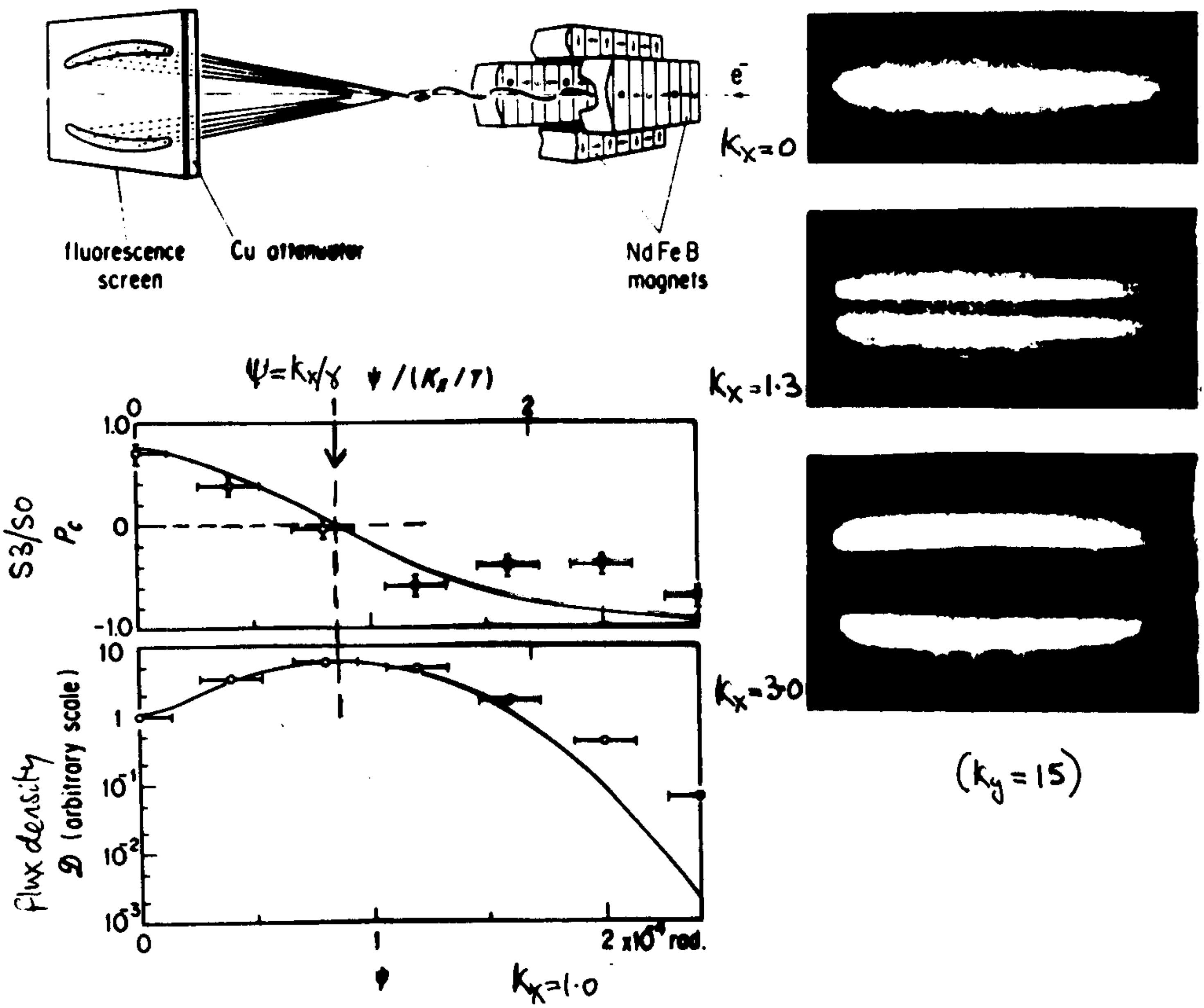
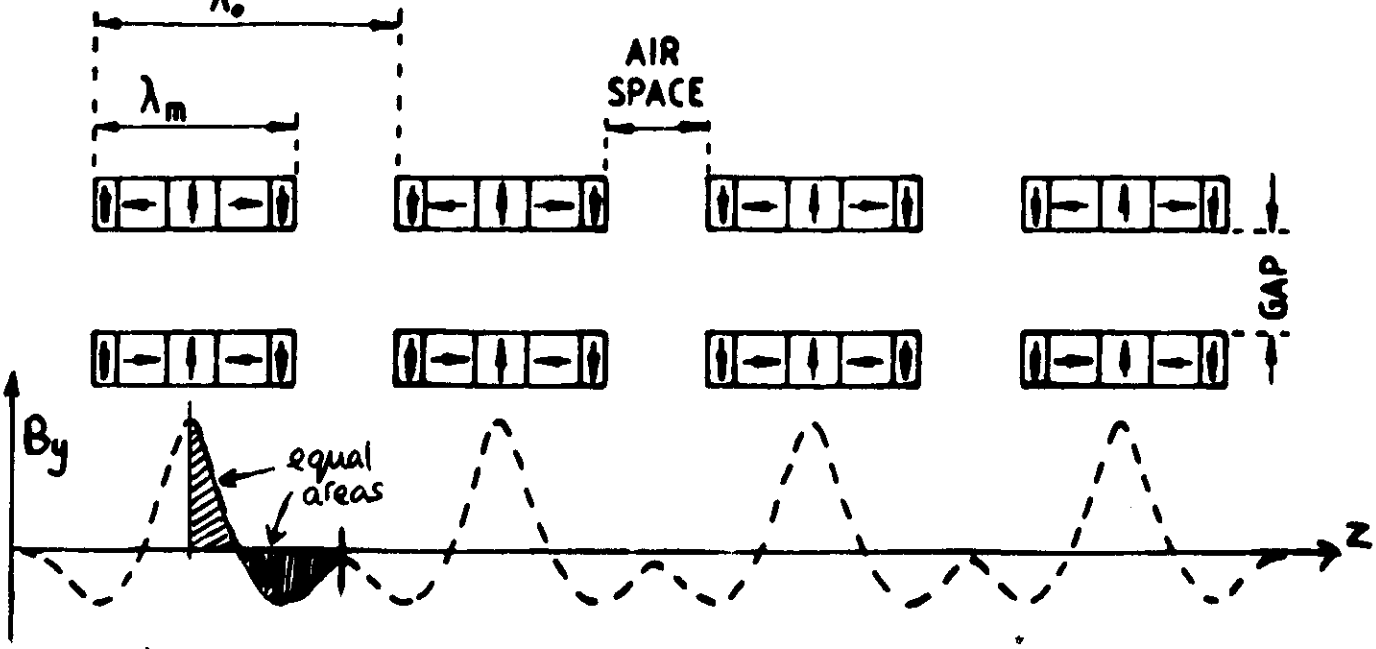
(Received 21 February 1989)

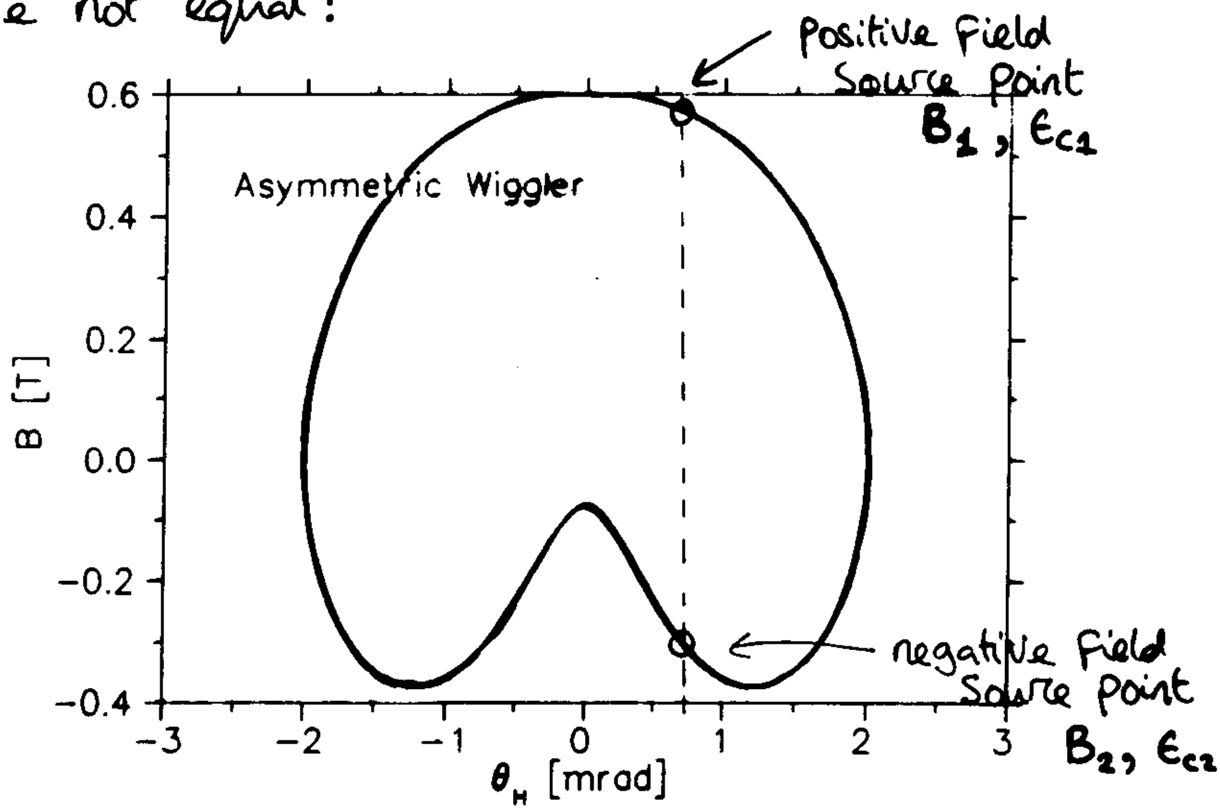




- although the field integrals are equal for positive and negative poles, the field amplitudes are <u>not</u>

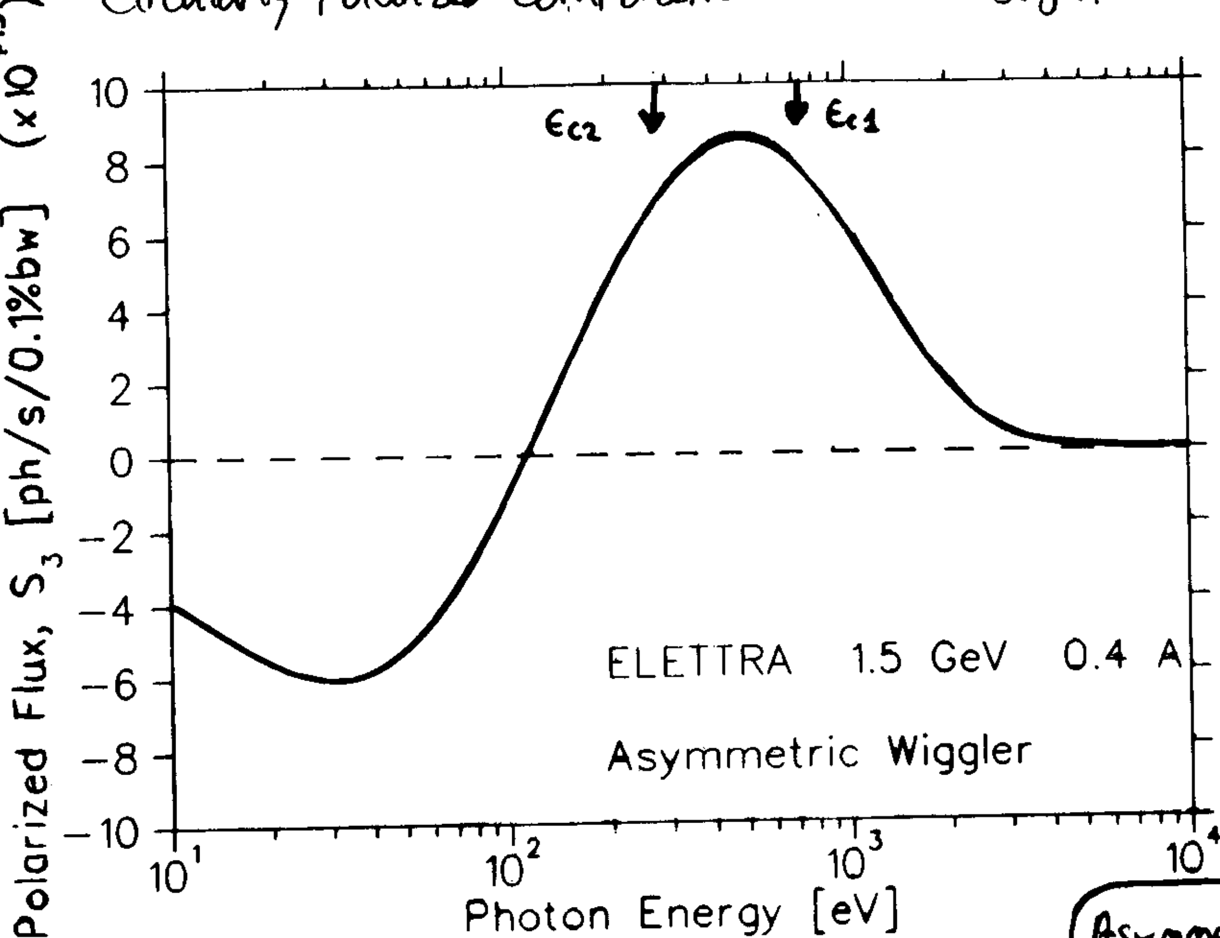
In a normal wiggler, radiation emitted at a given horizontal angle θ_H has two source points (per period) with <u>equal</u> positive and negative field amplitudes.

In an asymmetric wiggler the field values for a given θ_H are not equal:



The radiation intensity is the sum of the intensities of the two source points, with different critical energies $\epsilon_{c1} > \epsilon_{c2}$.

- near ϵ_{c1} : contribution of B_2 source point negligible, so like a series of +ve field bending magnets ∴ circularly polarized radiation off-axis vertically
- below ϵ_{c1} : contributions of both source points similar, giving a cancellation of the circularly polarized component as in a normal wiggler
- below ϵ_{c2} : for a given acceptance angle vertically, the B_1 source point contributes more than B_2, so the circularly polarized component reverses sign.



Asymmetric Wigglers operating in DORIS (Hamburg) SUPERACO (Orsay)

Example.

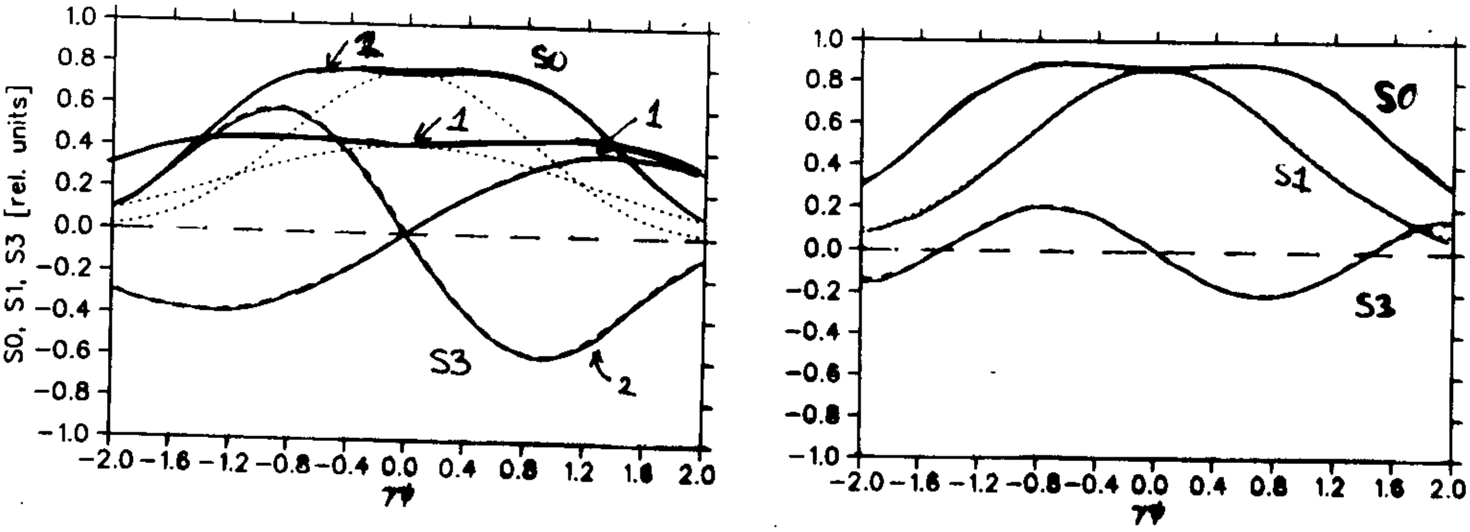
$\lambda_0 = 150\,\text{mm}$, $\lambda_m = 100\,\text{mm}$, $N = 20$

horizontal acceptance $\pm 1\,\text{mrad}$

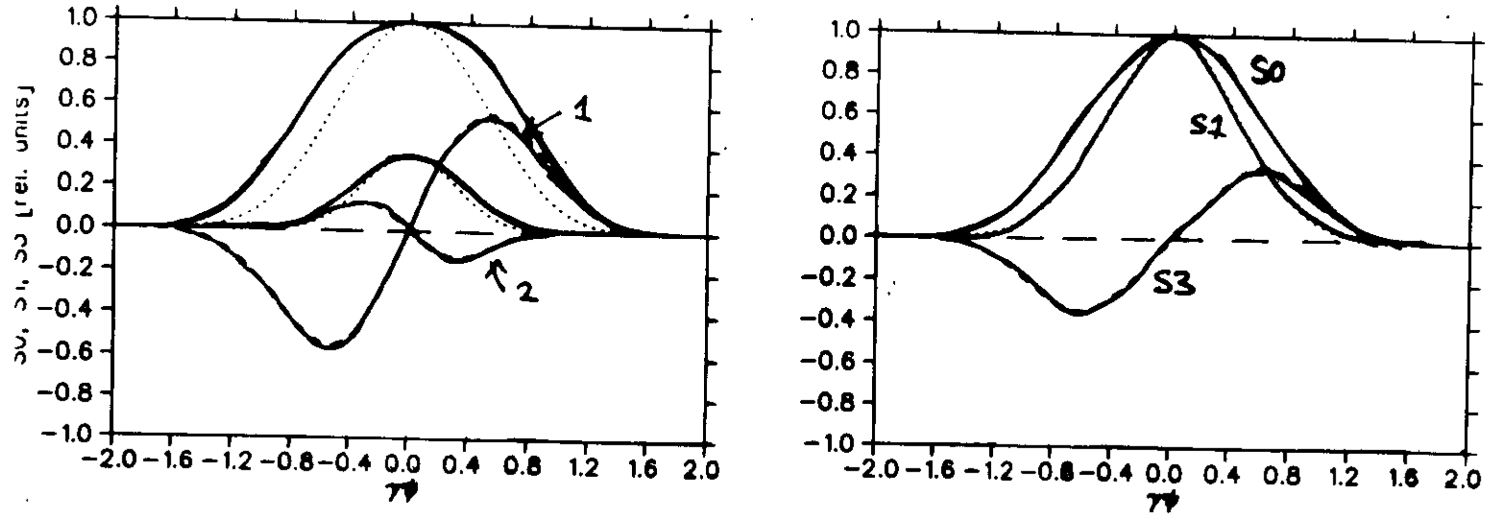
vertical acceptance $0 \to 0.7\,\text{mrad}$ (2/8)

EXAMPLE : $\epsilon_{c1}/\epsilon_{c2} = 3$

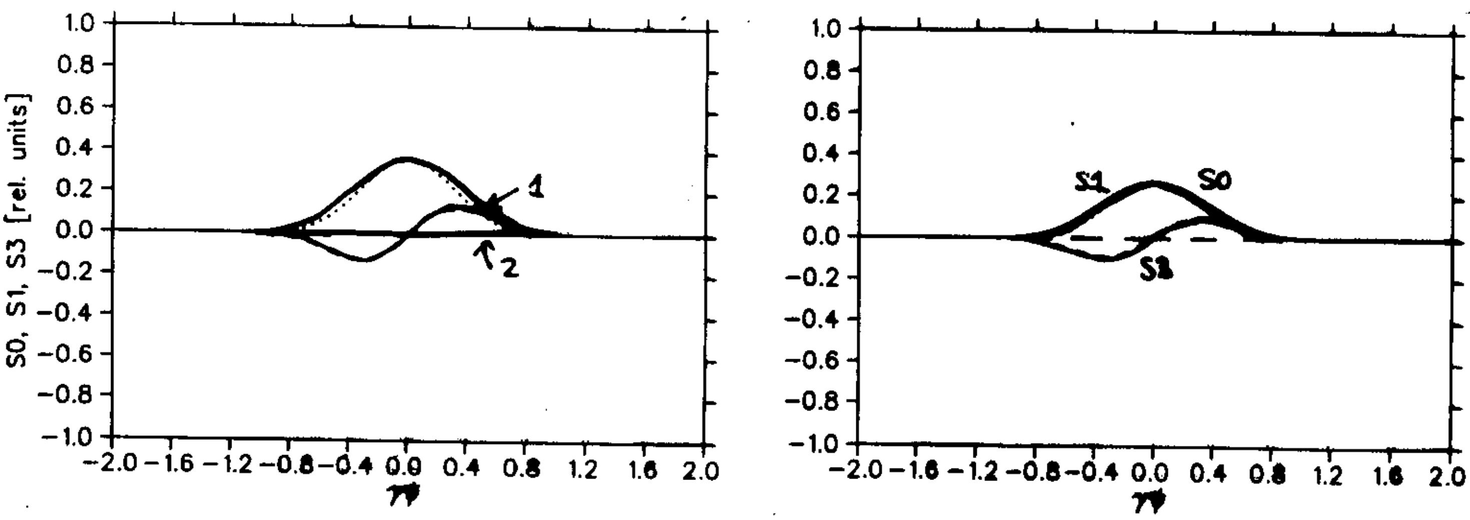
$\epsilon/\epsilon_{c1} = 0.1$



$\epsilon/\epsilon_{c1} = 1$

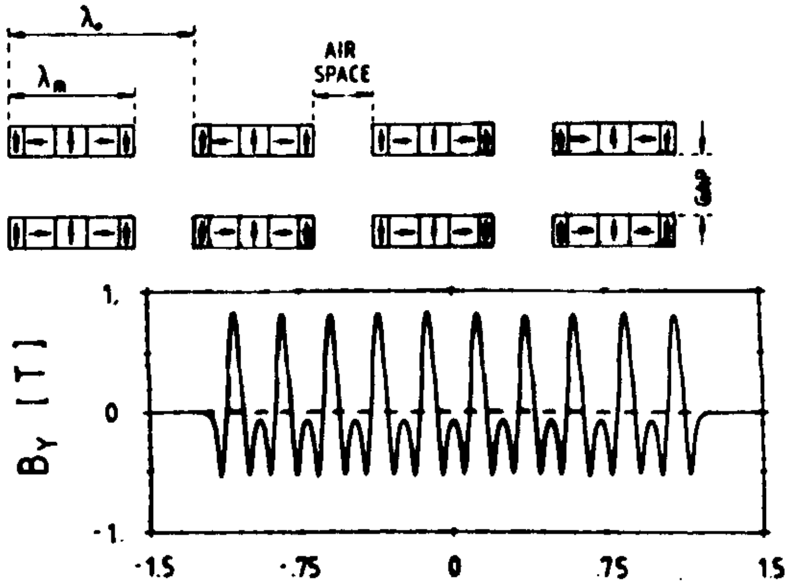


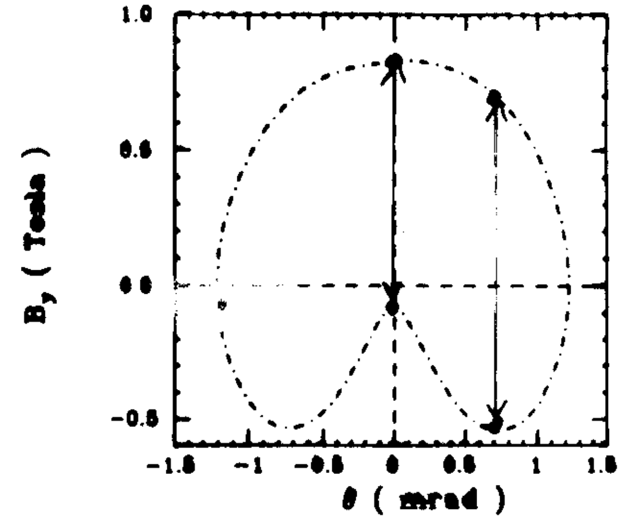
$\epsilon/\epsilon_{c1} = 3$

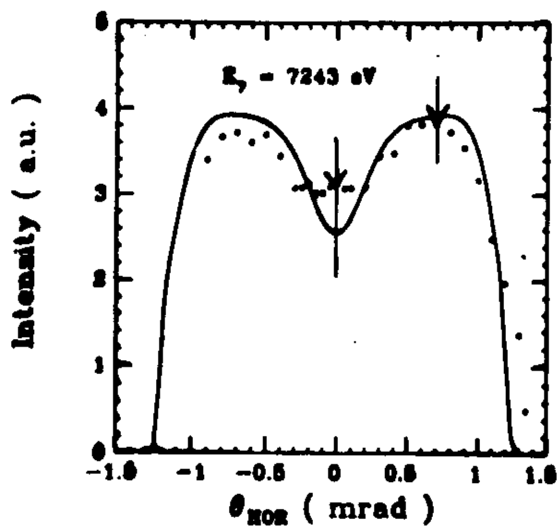


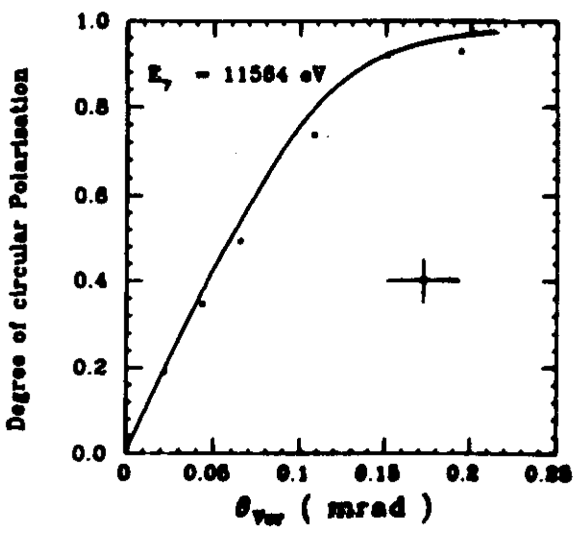
Example - Asymmetric Wiggler developed at HASYLAB, Germany (DORIS III).

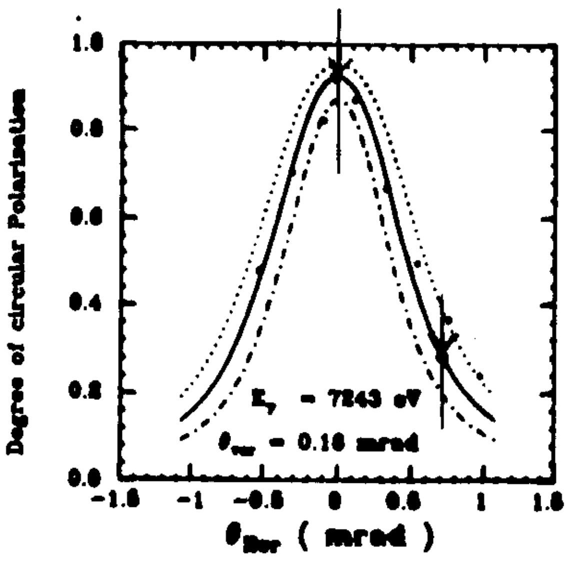
(J. Pflüger and G. Heintze, Nucl. Instr. Meth. A289 (1990) 300.)



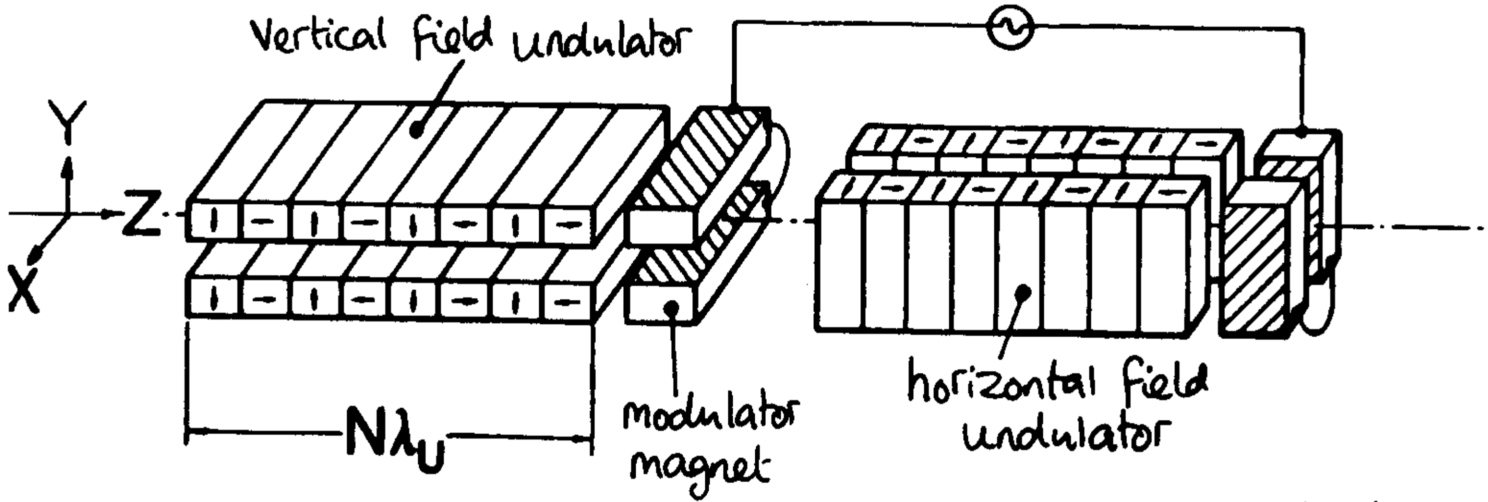








Crossed (interference) undulators (Nikitin et al., 1978; Kim, 1984)



$$\underline{A} = \int \underline{E}\, e^{i\omega t}$$

modulator phase

$$= \int \underline{E}_1\, e^{i\omega t} + \int \underline{E}_2\, e^{i(\omega t + \underline{\omega N t_0} + \psi)}$$

$$\Delta = 2\pi \frac{\Delta\omega}{\omega_1}$$

$$\underline{E}_1 = E(t)\,\hat{x} \qquad \underline{E}_2 = E(t)\,\hat{y}$$

$$\therefore \underline{A} = \left(\int E(t)\, e^{i\omega t}\right)\left[\hat{x} + e^{i(N\Delta + \psi)}\,\hat{y}\right]$$

hence,

$$S_1 = 0.0$$

$$S_2/S_0 = \cos(N\Delta + \psi)$$

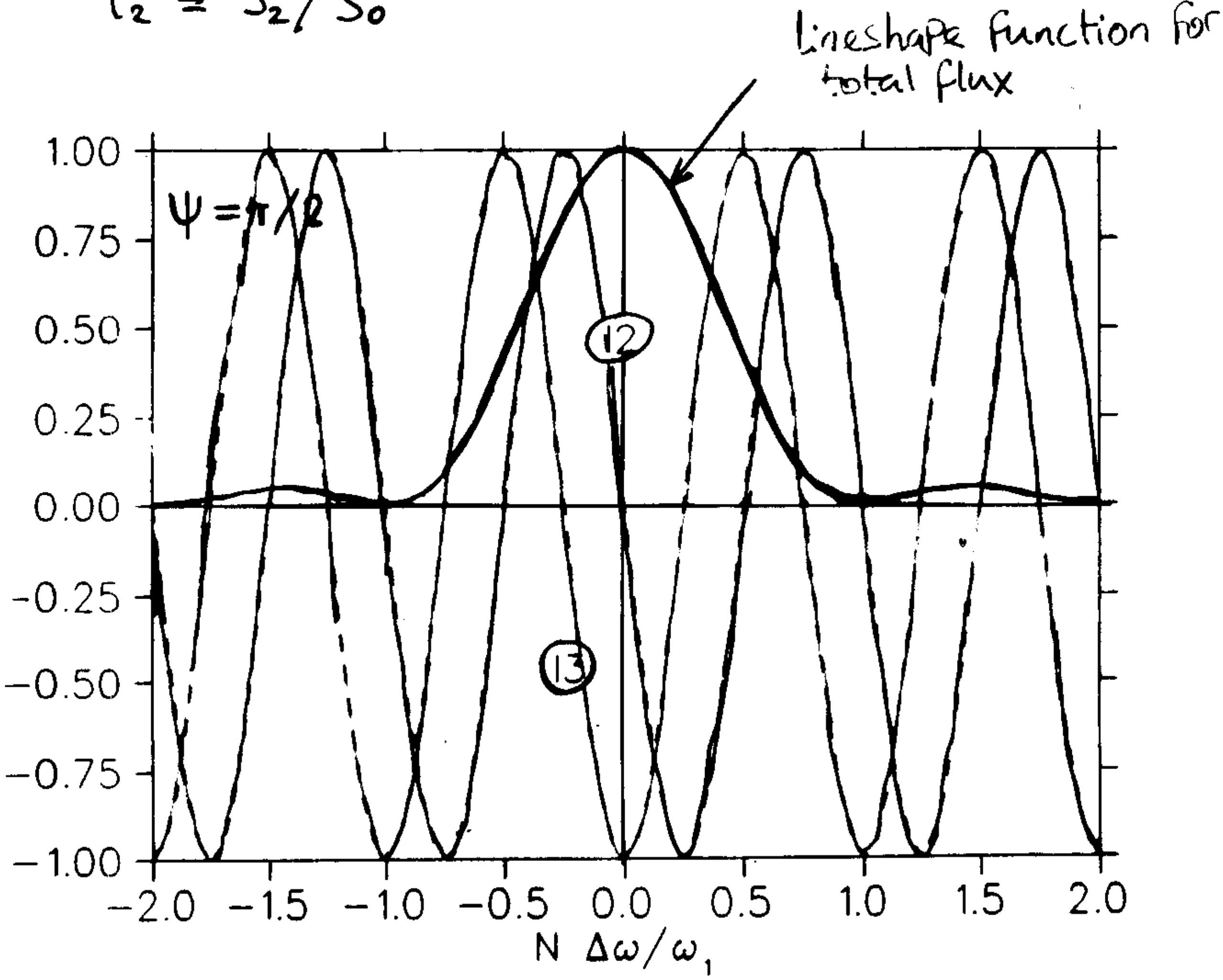
$$S_3/S_0 = \sin(N\Delta + \psi)$$

Thus, if $\Delta = 0$: (on-axis wavelength, no detuning)

ψ	S_2/S_0	S_3/S_0	Polarization
0	1	0	linear +45°
90°	0	1	Circular Right
180°	−1	0	linear −45°
270°	0	−1	Circular Left
360°	1	0	linear +45°

$l_3 = S_3 / S_0$

$l_2 = S_2 / S_0$



rapid variation of polarization parameters (l_2, l_3) within the lineshape function.

$\therefore$ to obtain significant polarization:

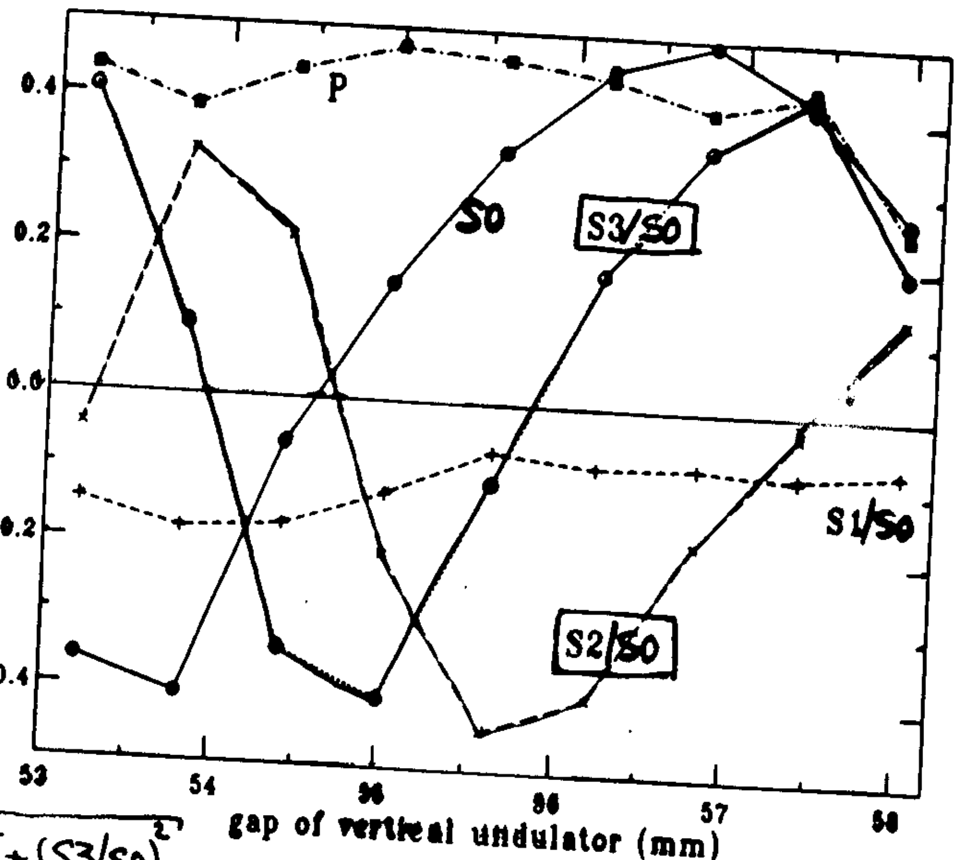
1. $\left(\frac{\Delta\lambda}{\lambda}\right)_{mono.} \lesssim \frac{1}{2N}$

2. angular divergence of electron beam and acceptance angle must be small, $\theta \lesssim \frac{1}{\gamma}\sqrt{\frac{1+k^2/2}{2iN}}$

[but there is a minimum circular polarization rate of ≈ 0.3 available]

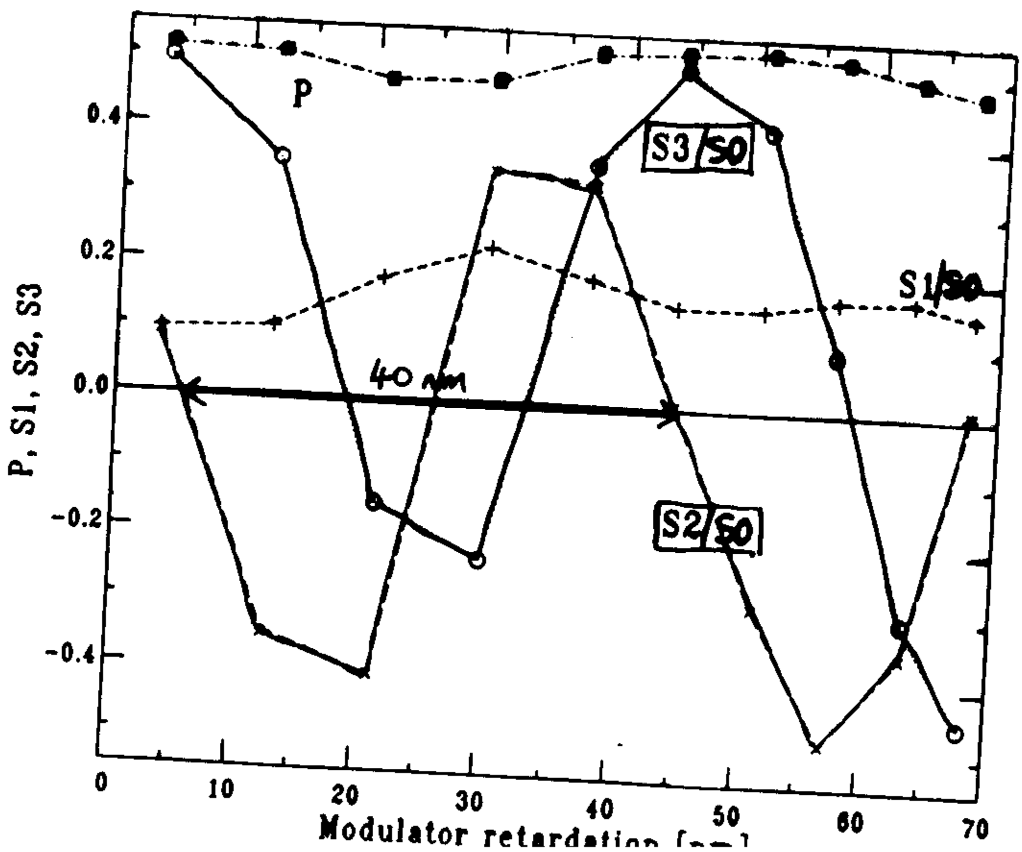
Example - Crossed Undulator developed at BESSY, Berlin

(J. Bahrdt et. al., Rev. Sci. Instr. 63 (1992) 63; SR News Mar./Apr. 1992)



$(S1/S0)^2 + (S2/S0)^2 + (S3/S0)^2$

:40nm

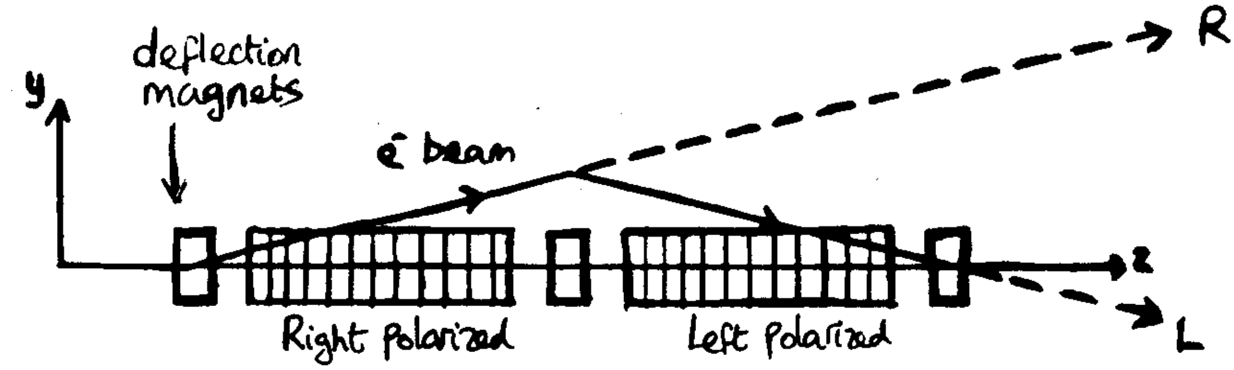


Polarization switching

Rate of switching
- ① 2 beam chopper - fast ~ kHz
- ② electromagnet 10-100 Hz
- ③ mechanical - slow ~ 1 Hz

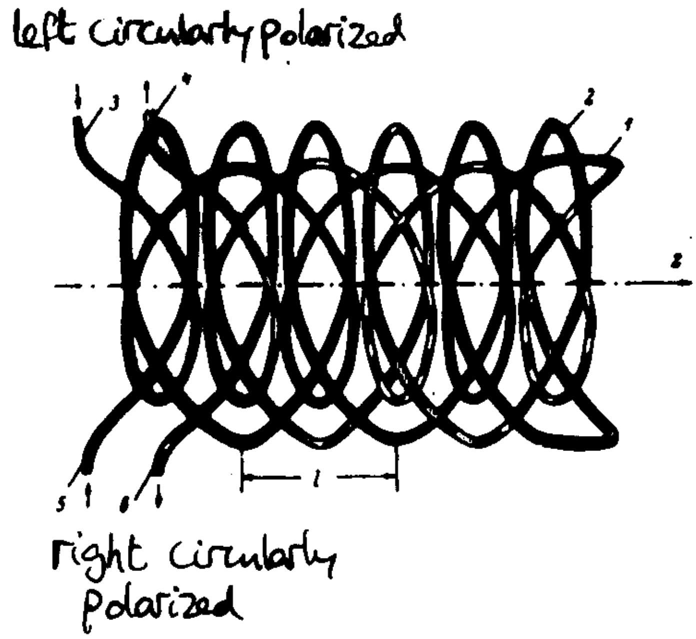
① 2 beams, above/below orbit plane + special monochromator (e.g. "Double-headed Dragon", NSLS)

- bending magnet
- asymmetric wiggler
- elliptical undulator/wiggler + d.c. e^- beam chicane (ESRF):

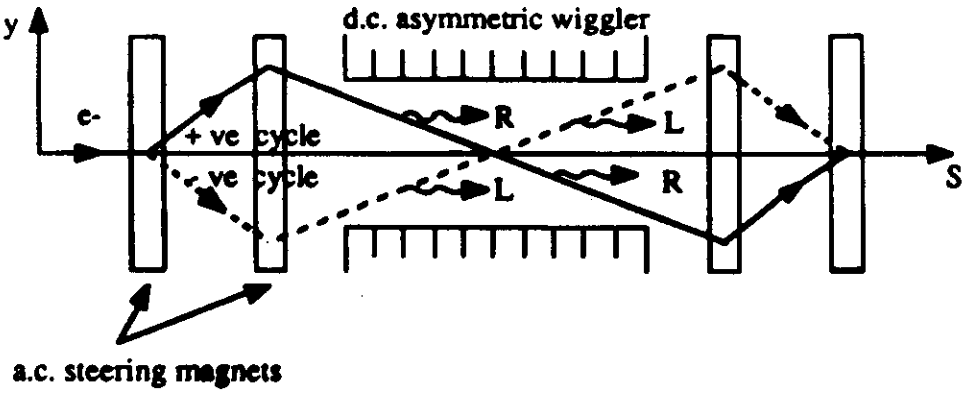


② Electromagnets :

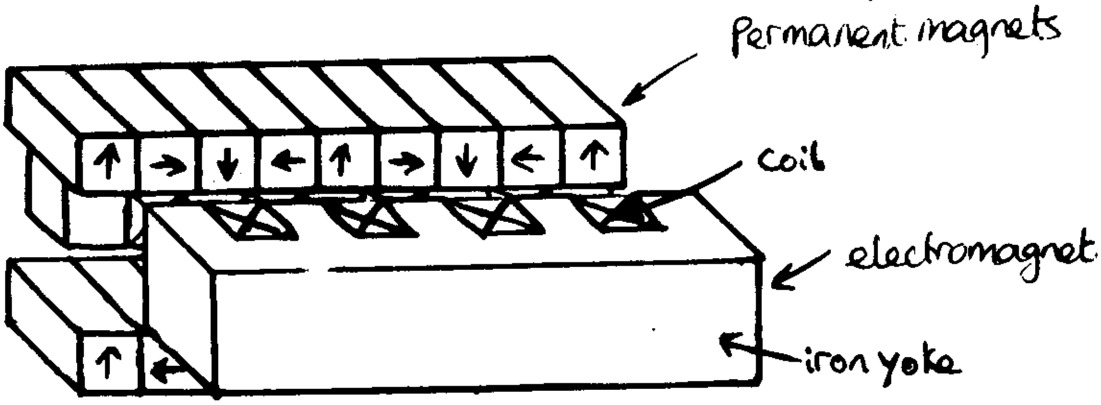
- "universal helical undulator" :



- modulated asymmetric wiggler :



- a.c. elliptical wiggler :



- plans at ALS, ELETTRA, NSLS, APS

- crossed-undulators + a.c. modulator (**BESSY**, plan ≤ 10 Hz)

(3) Mechanical translation of magnet arrays :

- elliptical undulator (**ETL**, tested ~ 3 Hz)
- planar elliptical undulators (Elleaume, Sasaki types)
  - planned for SSRL