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RESEARCH WORKSHOP ON CONDENSED MATTER PHYSICS
(21 June - 3 September 1993)

WORKING PARTY ON SMALL SEMICONDUCTOR STRUCTURES
(2 - 13 August 1993)

MAGNETOTUNNELLING AND CHAOS - I

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These are preliminary lecture notes, intended only for distribution to participants

Magnetotunnelling and Chaos

in

Double-barrier Heterostructures

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Lecture 1

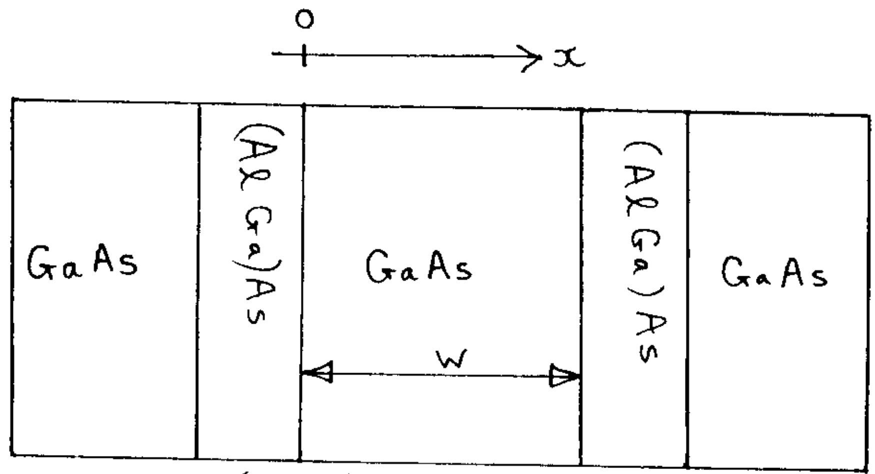
1) Background to resonant tunnelling theory : Coherent and sequential models.

2) Magnetotunnelling in resonant- tunnelling diodes containing a wide quantum well. Enhancement of the transition rates by wavefunction squeezing. Semiclassical interpretation.

3) Origin of sub-threshold resonances in the I(V) characteristics of laterally-gated and δ-doped resonant-tunnelling diodes.

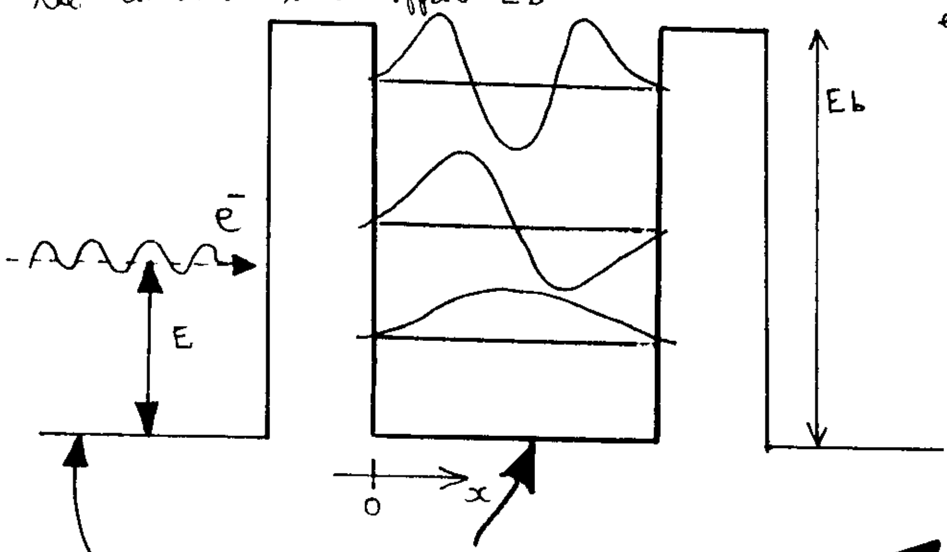
4) Probing the wavefunctions of quantum confined states by resonant magnetotunnelling. Application to arbitrary 0D - 2D transitions.

(1) Suppose we have a semiconductor heterostructure (below) comprising two monatomically-flat (AlGa)As layers sandwiched within 3 GaAs layers.

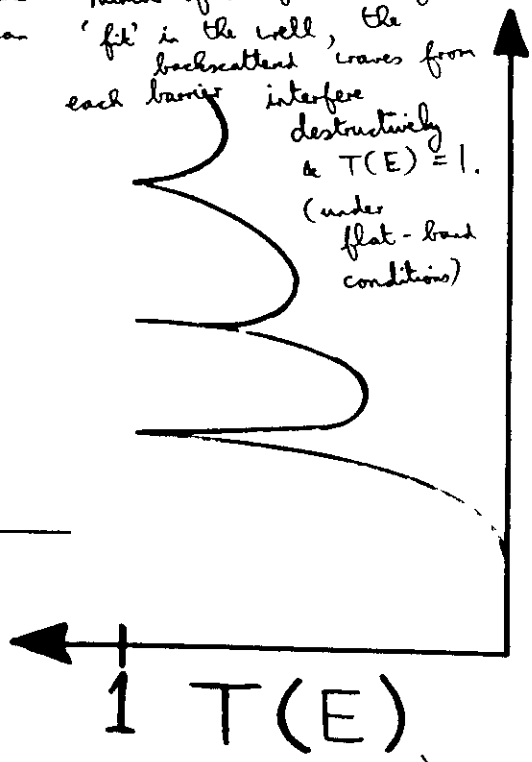


(4) If we now consider the two barriers acting together, the amplitude of the reflected wave (of an electron incident from the left) is determined by interference between the waves backscattered from each barrier. The reflection and transmission coefficients $T(E)$ vary rapidly with the incident kinetic energy E. However, whenever an integer number of half-wavelengths 'fit' in the well, the backscattered waves from each barrier interfere destructively & $T(E) = 1$. (under flat-band conditions)

(2) Because the (AlGa)As layer has a larger fundamental band gap, the conduction band edge is positioned at a higher energy as shown below. The (AlGa)As layers therefore act as potential barriers to electrons incident on them with kinetic energy less than the conduction band offset E_b



CONDUCTION BAND EDGE

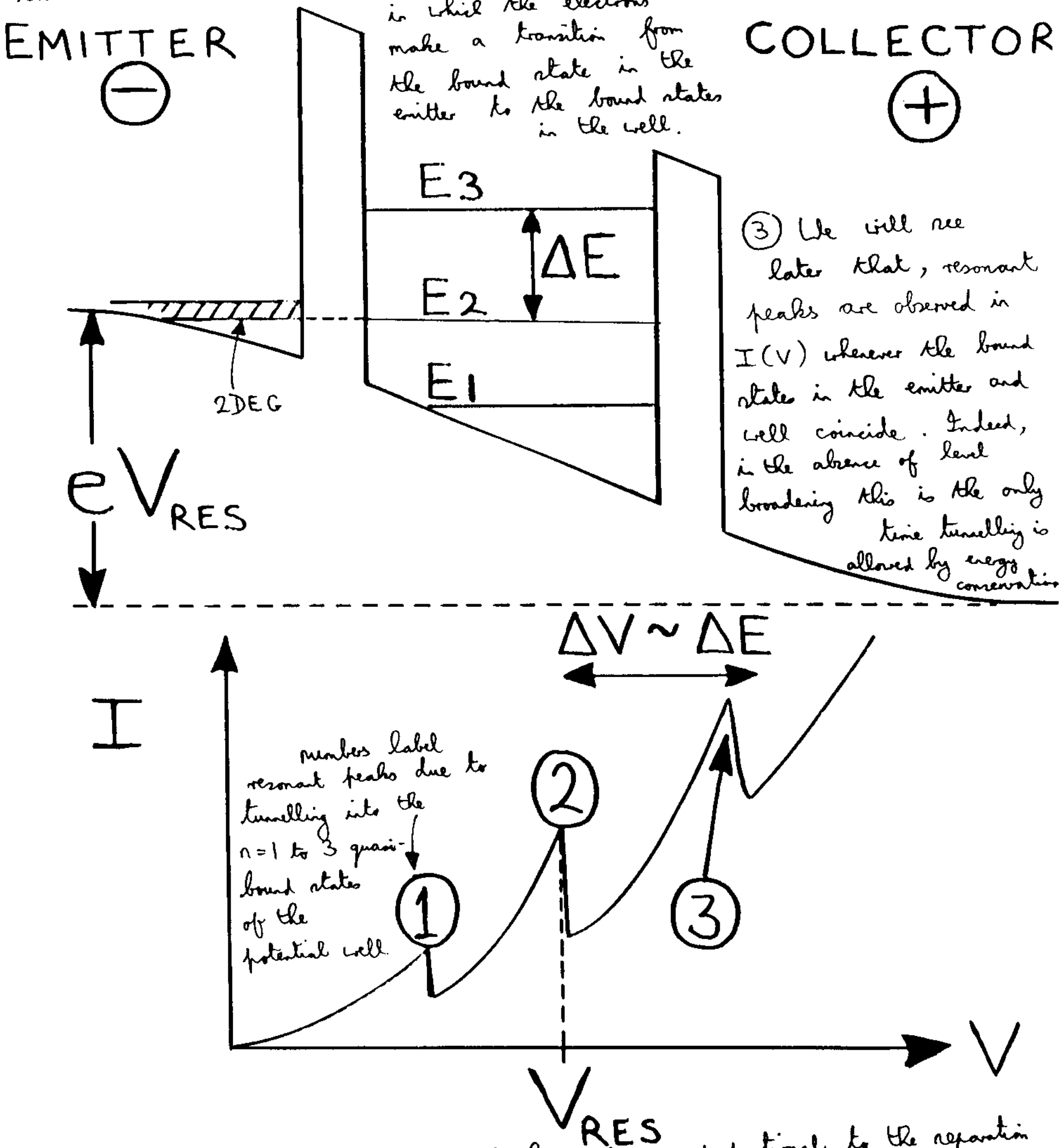


(3) The incident electrons are mainly reflected by each barrier but have a small probability of being transmitted ⇒ the double-barrier heterostructure is ∴ analogous to the optical Fabry-Perot interferometer.

(5) The requirement that $\frac{n\lambda}{2} = W$ where λ is the electron wavelength, W the well width also determines the allowed energy levels of the isolated well. Consequently $T(E) = 1$ (resonant tunnelling) whenever the incident kinetic energy equals one of the energy levels within the well.

(1) We [illegible] applying a bias to the device. This shifts the energy levels in the emitter relative to those in the well.

(2) If the device has a lightly n-doped emitter, a two-dimensional electron gas (2DEG) is formed in the emitter contact, adjacent to the LH barrier. In this case, because there are no incident plane-wave states occupied in the emitter, it is more helpful to view resonant tunnelling as a sequential process in which the electrons make a transition from the bound state in the emitter to the bound states in the well.



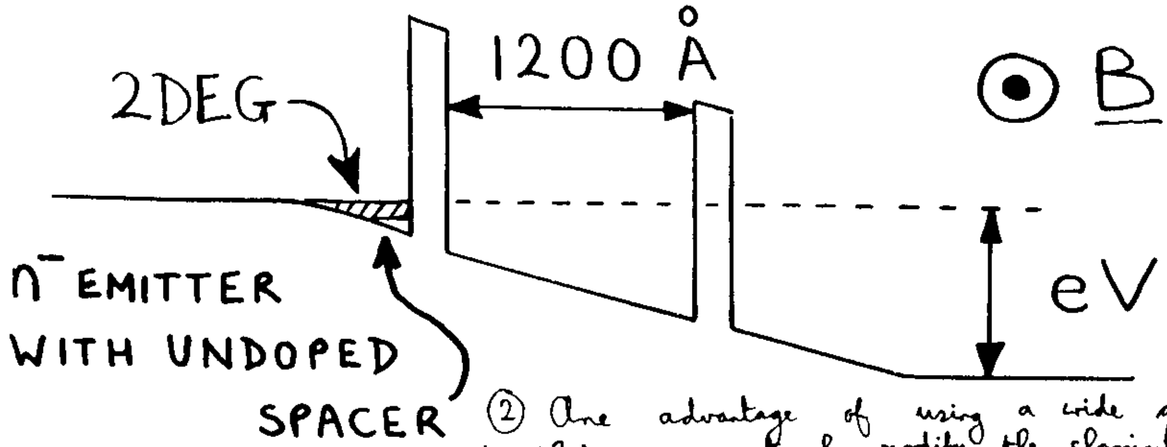
(3) We will see later that, resonant peaks are observed in I(V) whenever the bound state in the emitter and well coincide. Indeed, in the absence of level broadening this is the only time tunnelling is allowed by energy conservation

(4) The separation of the resonant peaks, ΔV, is proportional to the separation of the energy levels ΔE. Consequently, resonant tunnelling is a spectroscopic tool which enables us to probe the energy level pattern of the well.

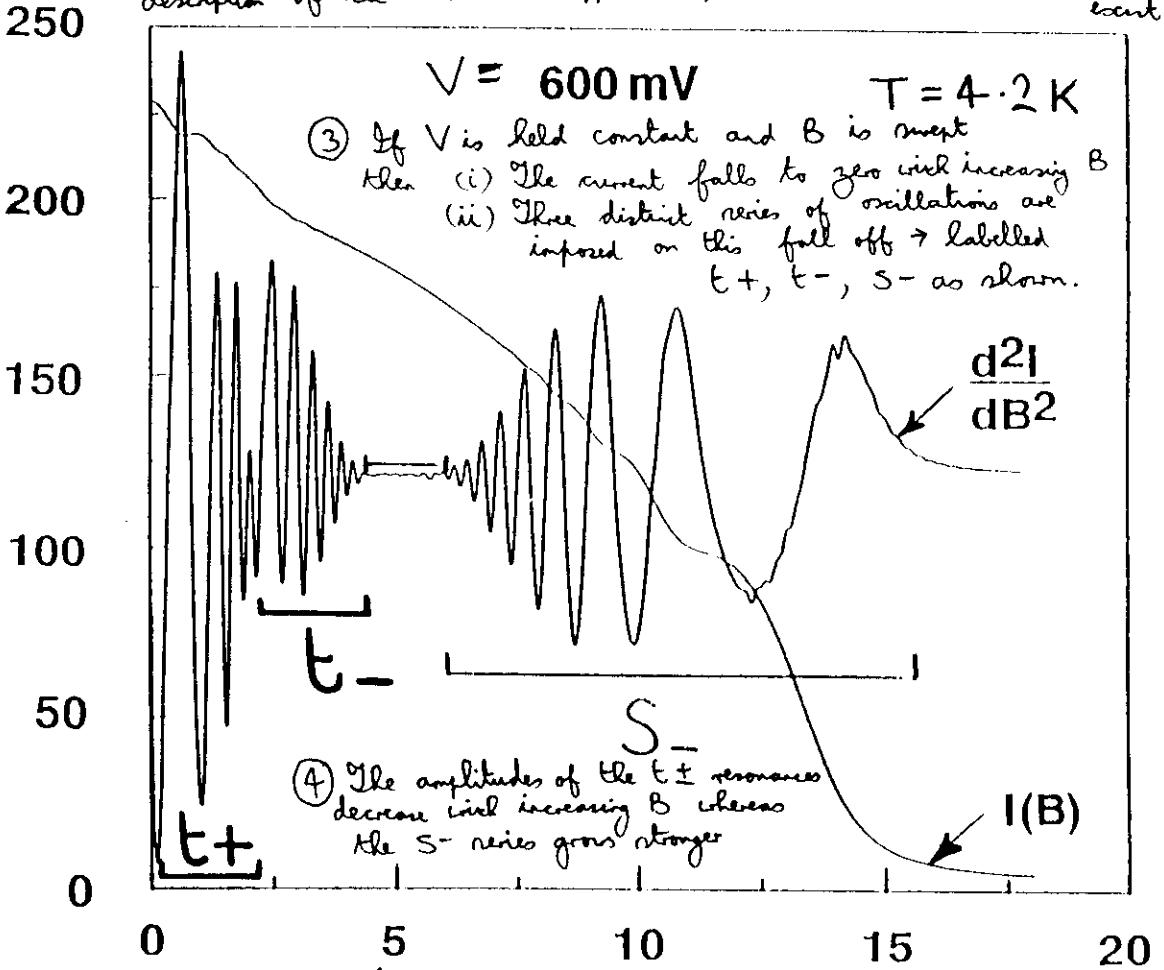
resonant tunnelling diode with a wide (~1000 Å) potential well

I(B) DATA FOR 1200 Å WELL

(Leadbeater et al. J. Phys.: Condens. Matter 1, 4865, (1989) & Alves et al, Solid State Electron 32, 1627, (1989))



② One advantage of using a wide quantum well is that we can strongly modify the classical motion in the well, and the corresponding quantum eigenstates, by applying a strong magnetic field in the plane of the barriers (because the classical Lorentz force acts over a large distance). Also a semiclassical description of the motion is applicable, because up to 100 bound states exist.



③ If V is held constant and B is swept then (i) The current falls to zero with increasing B
(ii) Three distinct series of oscillations are imposed on this fall off → labelled t+, t−, S− as shown.

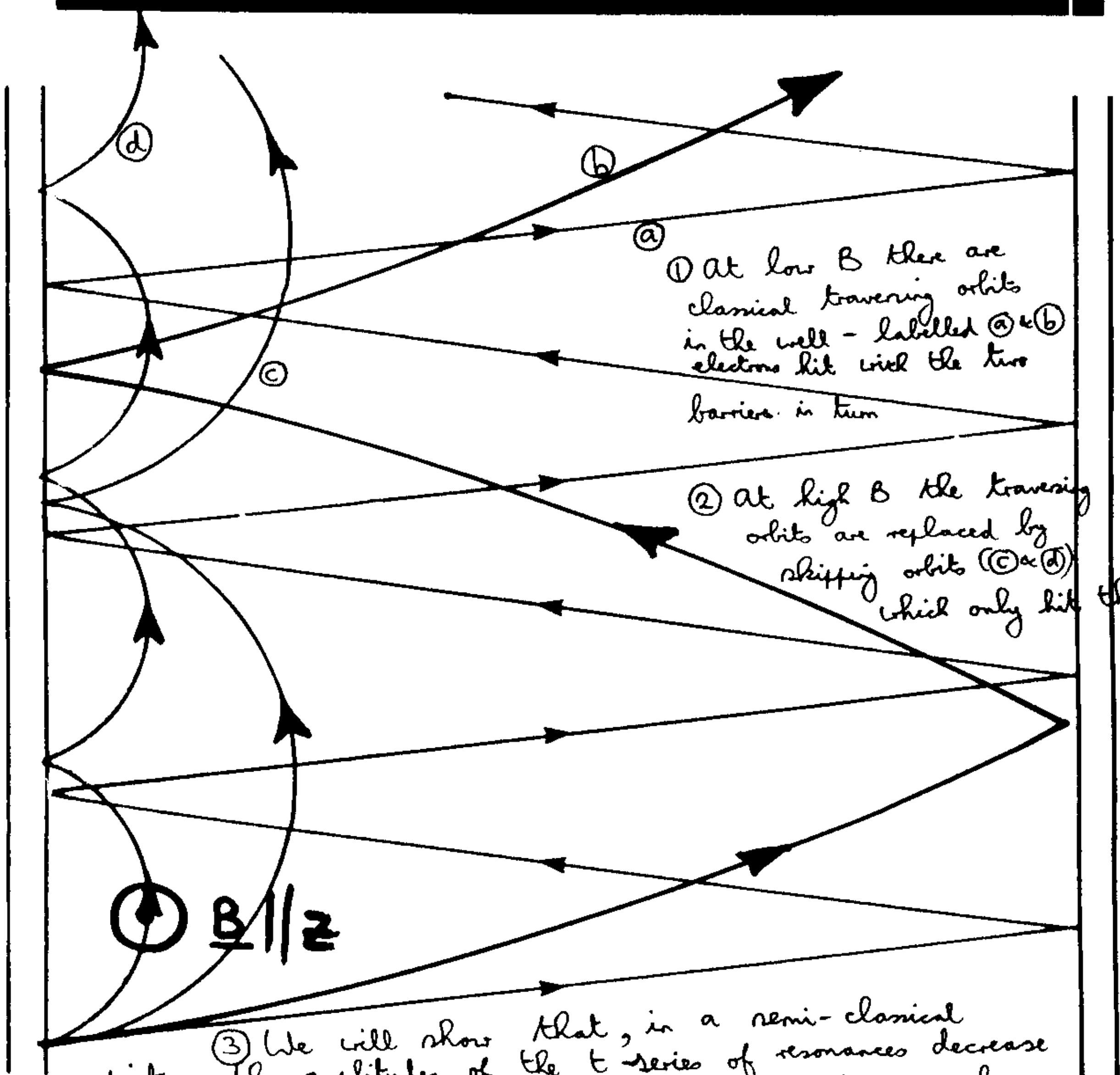
④ The amplitudes of the t± resonances decrease with increasing B whereas the S− series grows stronger

⑤ Can we explain the origin of the three series, their periodicities and the field dependence of the oscillatory amplitudes?

The key to understanding the oscillatory structure is to look at the

ATTEMPT RATES OF (classical) TRAVERSING AND SKIPPING WELL STATES

& see how these change with increasing magnetic field.



① at low B there are classical traversing orbits in the well - labelled (a)&(b) electrons hit with the two barriers in turn

② at high B the traversing orbits are replaced by skipping orbits ((c)&(d)) which only hit the LH (emitter) barrier in a classical picture.

③ We will show that, in a semi-classical picture, the amplitudes of the t-series of resonances decrease with increasing B, whereas the s-series grows stronger because:

(i) **ATTEMPT RATES OF TRAVERSING STATES ↓ WITH INCREASING B.**

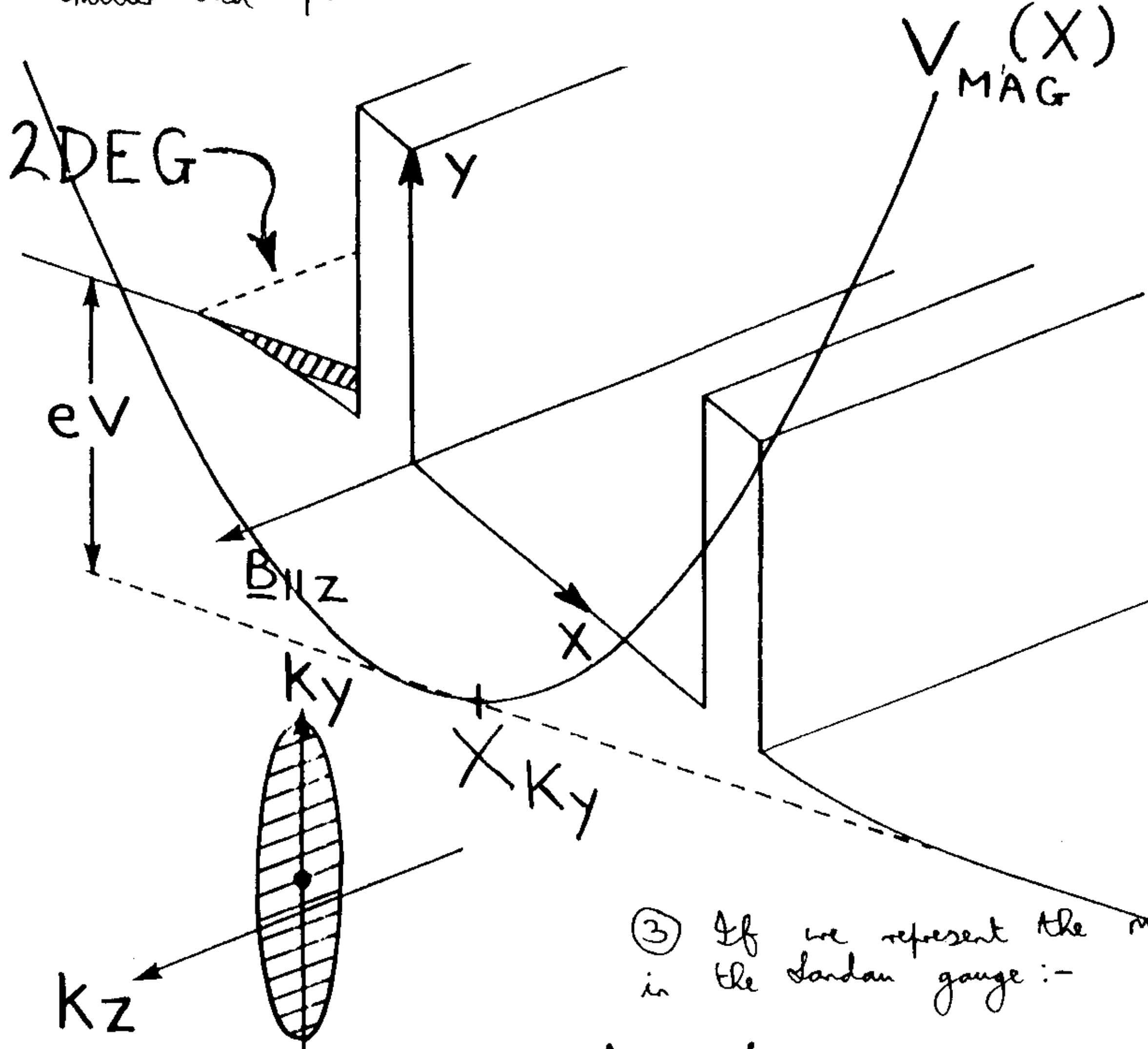
and

(ii) ATTEMPT RATES OF SKIPPING STATES INCREASE WITH B

CONDUCTION BAND UNDER BIAS

① Of course, the resonant structure must be quantum mechanical in origin so we need a full quantum model for the tunnel current to explain the origin of the three distinct series of structure and the variation of the oscillatory amplitudes with B. The beauty of a semiclassical picture is that we can then interpret the quantum mechanical results in a very simple physical way.

② As a first step in the quantum mechanical calculation let's consider the effect of an in-plane ($\underline{B} \parallel \underline{z}$) $\underline{B}$-field on the states in the emitter and potential well.



③ If we represent the magnetic field in the Landau gauge :-

$$\underline{A} = (0, BX, 0)$$

its effect is described through a 1D parabolic potential as shown above. Since this potential is a function of x alone. The electrons RETAIN free motion along the barrier interfaces & have plane wave eigenfunctions in these directions.

④ In the 2DEG the electrons therefore populate a Fermi disk in $\underline{k}$-space. However the origin is shifted along k_Y by an amount proportional to B. (We'll see why later).

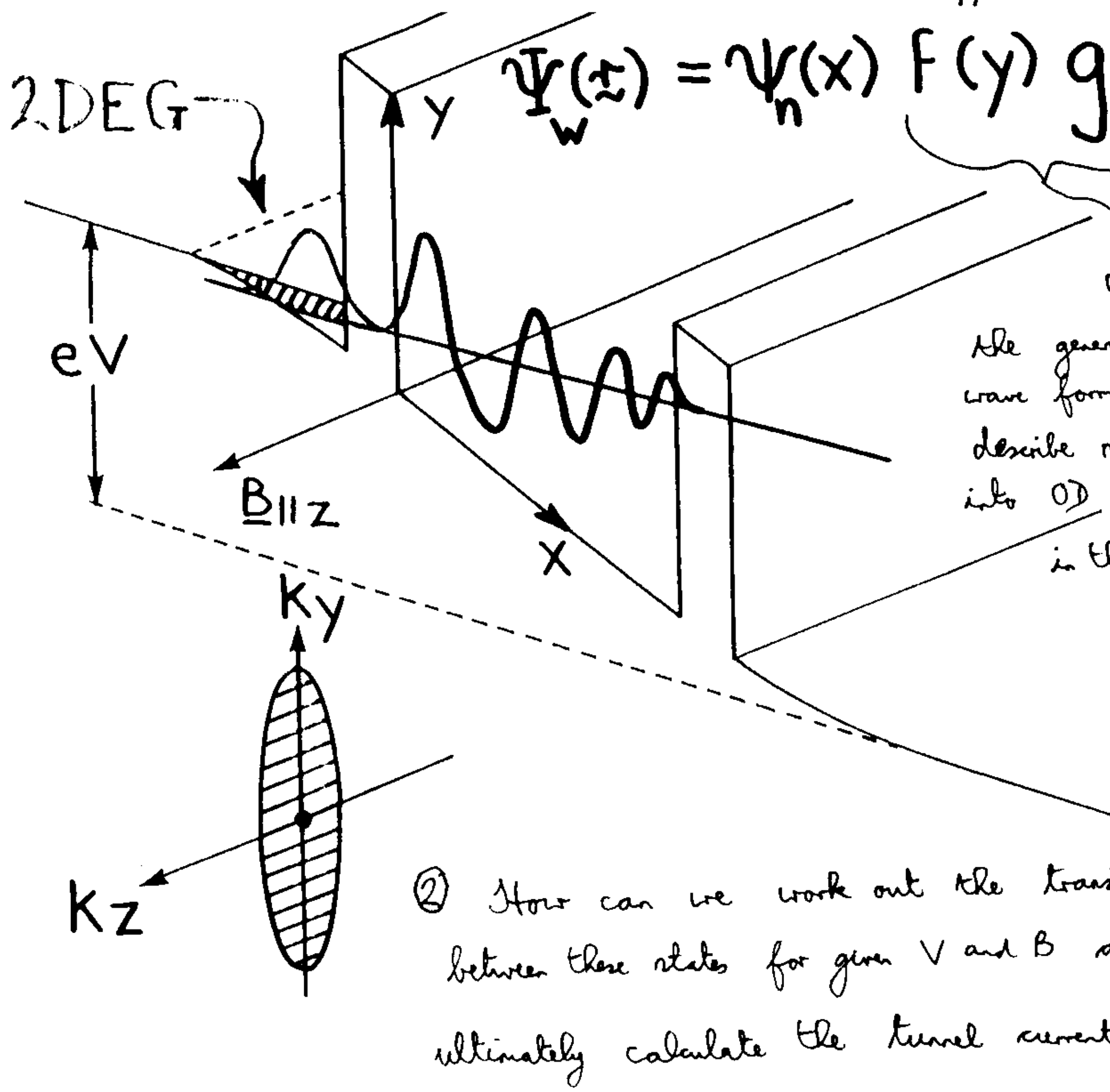
CONDUCTION BAND UNDER BIAS

① Motion normal to the interfaces is bound though in both the emitter and the quantum well. Suppose that we can represent each state by a separable wavefunction comprising lateral plane waves multiplied by a bound wavefunction which varies with x and satisfies a 1D Schrödinger equation in which the effective 1D potential is the conduction band profile in the appropriate region (emitter or well) supplemented by the 1D parabolic magnetic potential

$$\Psi_{2DEG}(\underline{r}) = \psi(x)\, \overbrace{e^{ik_y y}\, e^{ik_z z}}^{\text{PLANE WAVES AT PRESENT!}}$$

$$\Psi_w(\underline{r}) = \psi_n(x)\, \underbrace{f(y)\, g(z)}$$

we'll use the general non-plane wave form later, to describe magnetotunnelling into 0D donor impurities in the well.



② How can we work out the transition rates between these states for given V and B and thus ultimately calculate the tunnel current?

(1) Bardeen Transfer [illegible] formalism [illegible] rates between the bound states. The result is analogous to the Fermi Golden rule & essentially shows that the transition rate increases with the overlap of the initial & final wavefunction

2DEG → QUANTUM WELL TRANSITION RATE

(2) Using a WKB wavefunction for the n^{th} bound state of the potential well and representing the 2D emitter bound state by the Fang-Howard wavefunction (Fang & Howard, PRL, 16, 797, 1966) the Bardeen formalism enables us to show that the transition rate W for a 2D electron in the emitter with $\underline{k} = (k_y, k_z)$ into the n^{th} bound state of the potential well is:

$$W_n(k_y, k_z) \propto F_n \, T_n \underbrace{\left| \int_y f^*(y) e^{i k_y y} dy \right|^2 \left| \int_z g^*(z) e^{i k_z z} dz \right|^2} \times \underbrace{\delta(E_W - E_{2DEG})}_{\text{ENERGY CONSERVATION}}$$

(3) Key term: transition rate is proportional to semiclassical attempt rate F_n of electrons in well i.e. number of hits per second on the emitter barrier

T_n: Transmission coefficient of emitter barrier only.

WAVEVECTOR CONSERVATION

IF $\quad f(y) = e^{i k_y y}$

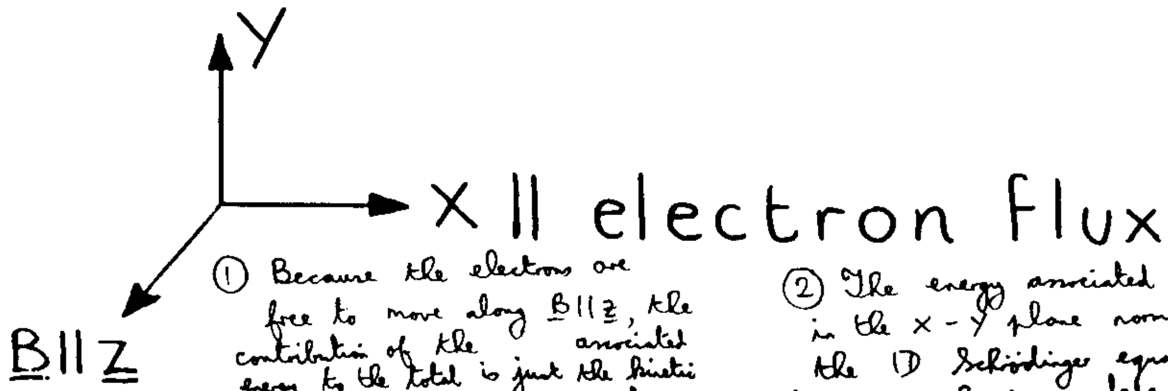
AND $\quad g(z) = e^{i k_z z}$

(4) Note that tunnelling transitions MUST satisfy both energy and wavevector conservation

(5) To identify the allowed transitions we must calculate the total energy of the emitter and well bound states as function of k_y & k_z

(6) We can then calculate the rates of those transitions, and so calculate the current & explain the imposed resonant structure.

ENERGY $E(k_y, k_z)$ IN THE 2DEG AND QUANTUM WELL



① Because the electrons are free to move along $\underline{B} \parallel \underline{z}$, the contribution of the associated energy to the total is just the kinetic energy

② The energy associated with motion in the x - y plane normal to $\underline{B}$ satisfies the 1D Schrödinger equation below. Note that the energy depends on k_y

$$E(k_y, k_z) = \frac{\hbar^2 k_z^2}{2m^*} + E(k_y)$$

because the effective 1D potential also depends on k_y.

$$-\frac{\hbar^2}{2m^*}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E(k_y)\,\psi(x)$$

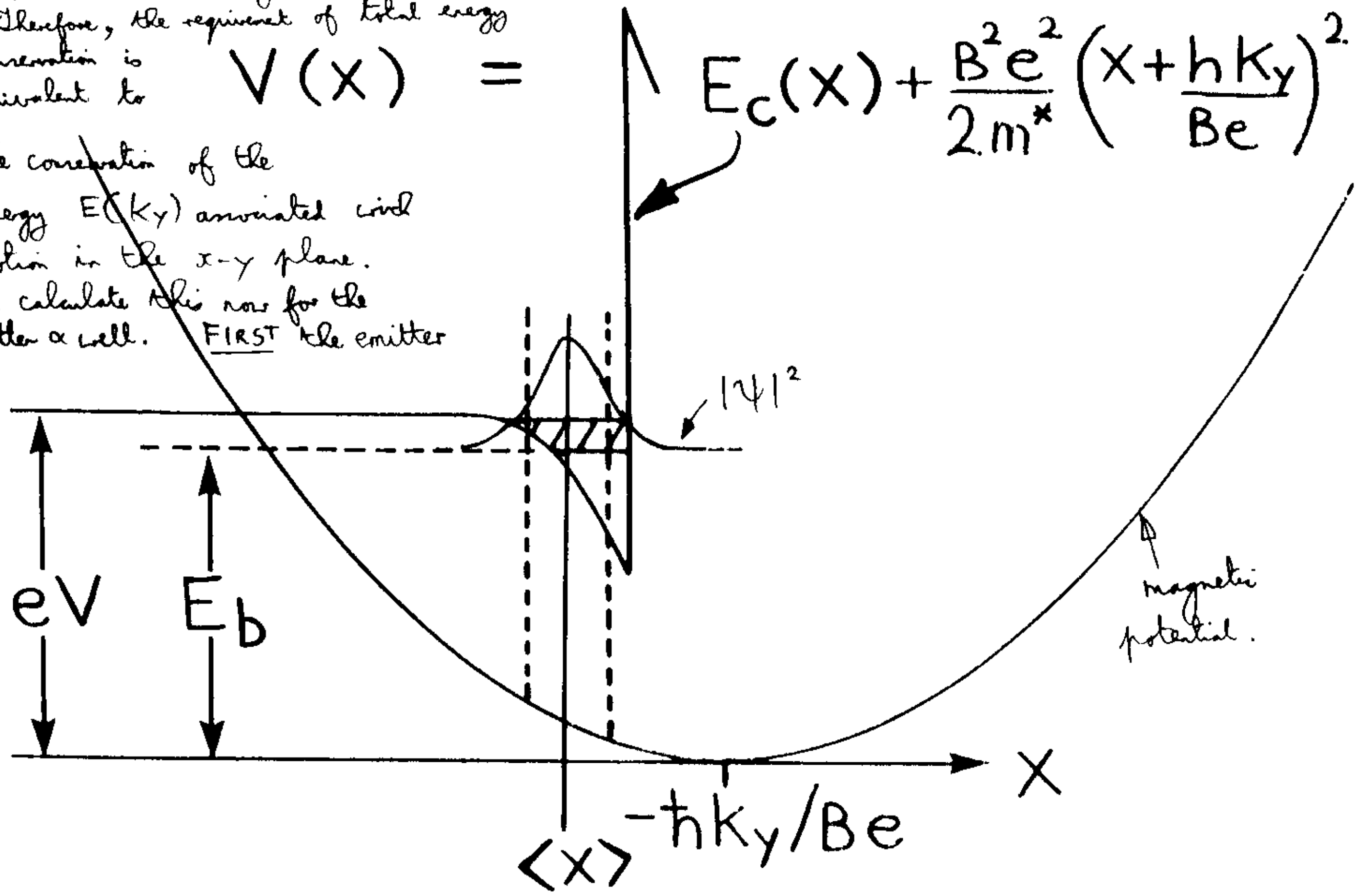
$$V(x) = E_c(x) + \frac{B^2 e^2}{2m^*}\left(x + \frac{\hbar k_y}{Be}\right)^2$$

③ The effective 1D potential comprises the conduction band profile (very different for the emitter and well) supplemented by the parabolic magnetic potential. The origin of the magnetic potential (where it is smallest) is $X_{k_y} = -\hbar k_y / Be$. For given k_y, the magnetic potential is therefore identical for electrons in both the emitter and the well.

ENERGY LEVELS $E(k_y)$ IN THE 2DEG

① If k_z is conserved, then the kinetic energy $\hbar^2 k_z^2/2m^*$ must also be conserved by default.
Therefore, the requirement of total energy conservation is equivalent to the conservation of the energy $E(k_y)$ associated with motion in the x-y plane.
We calculate this now for the emitter & well. FIRST the emitter

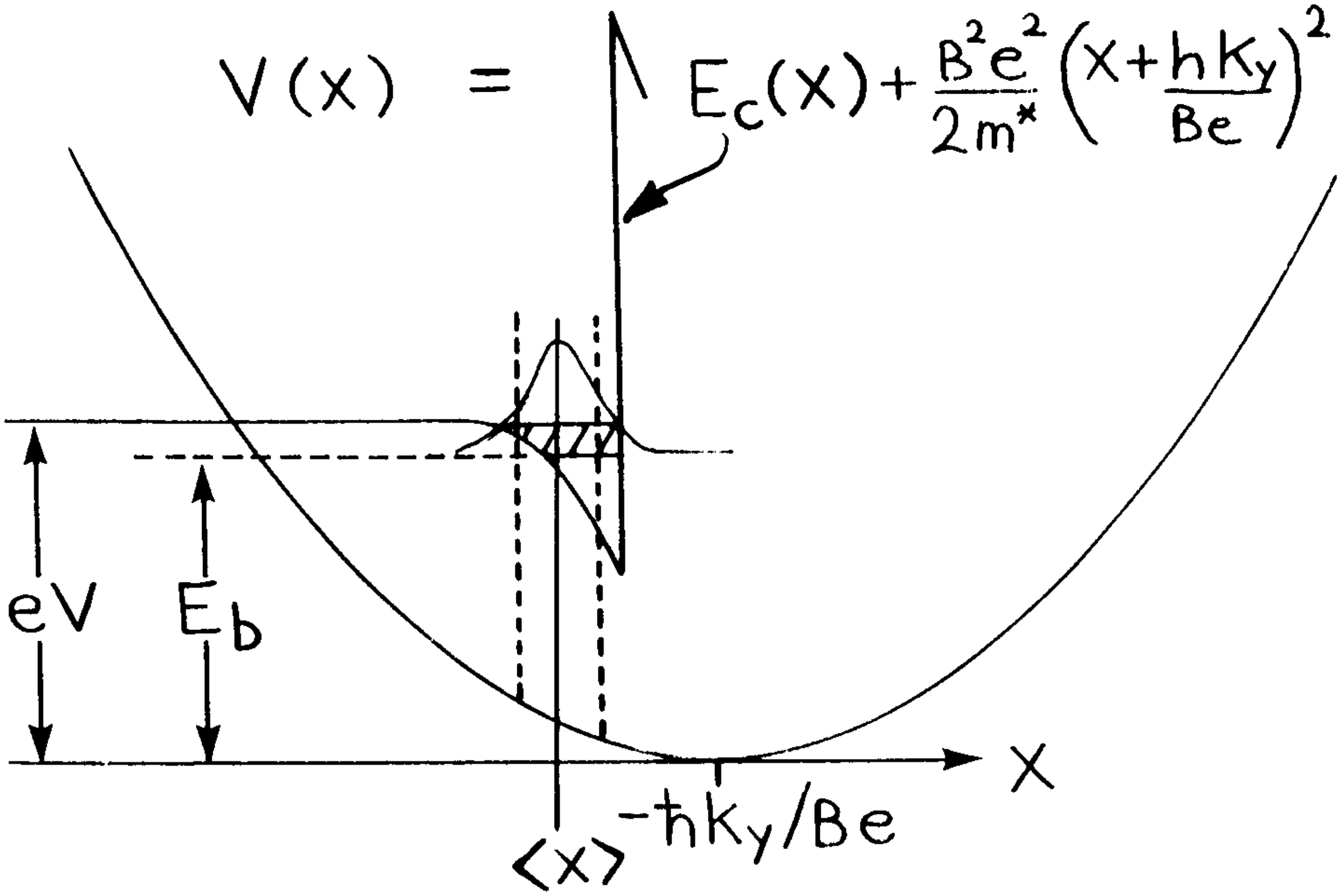
$$V(X) = E_c(X) + \frac{B^2e^2}{2m^*}\left(X + \frac{\hbar k_y}{Be}\right)^2$$



$$E(k_y) = E_b + \frac{B^2e^2}{2m^*}\left(\langle x \rangle + \frac{\hbar k_y}{Be}\right)^2$$

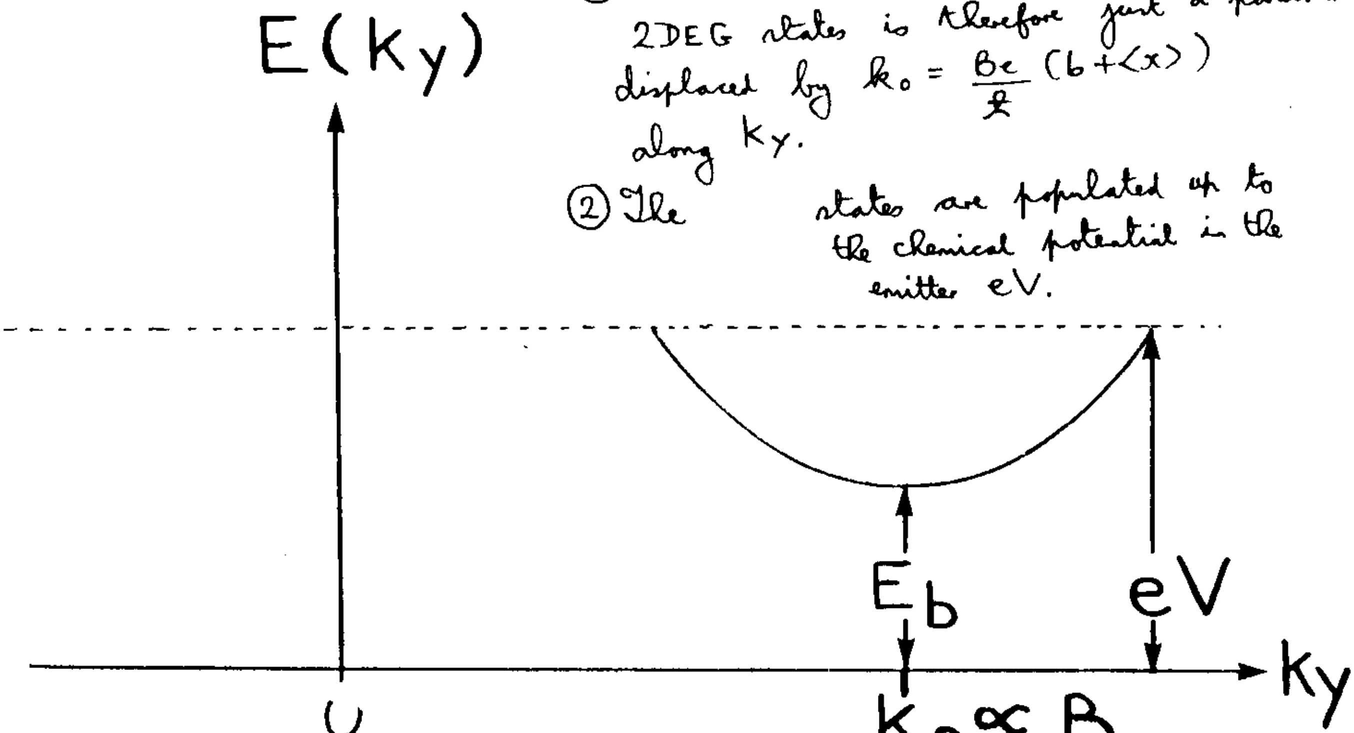
② Suppose that the magnetic field is sufficiently small that the magnetic potential varies slowly over the range of x where the 2D charge is concentrated (within vertical broken lines above). Then the magnetic field does not change that charge distribution to 1st order. Instead it simply adds a bodily shift to the bound state energy E_b, equal (approximately) to value of the magnetic potential at the mean position $\langle x \rangle$ of the 2DEG, giving the energy $E(k_y)$ associated with motion in the x-y plane above.

ENERGY LEVELS $E(k_y)$ IN THE 2DEG



① The dispersion curve $E(k_y)$ for the 2DEG states is therefore just a parabola displaced by $k_0 = \frac{Be}{\hbar}(b + \langle x \rangle)$ along k_y.

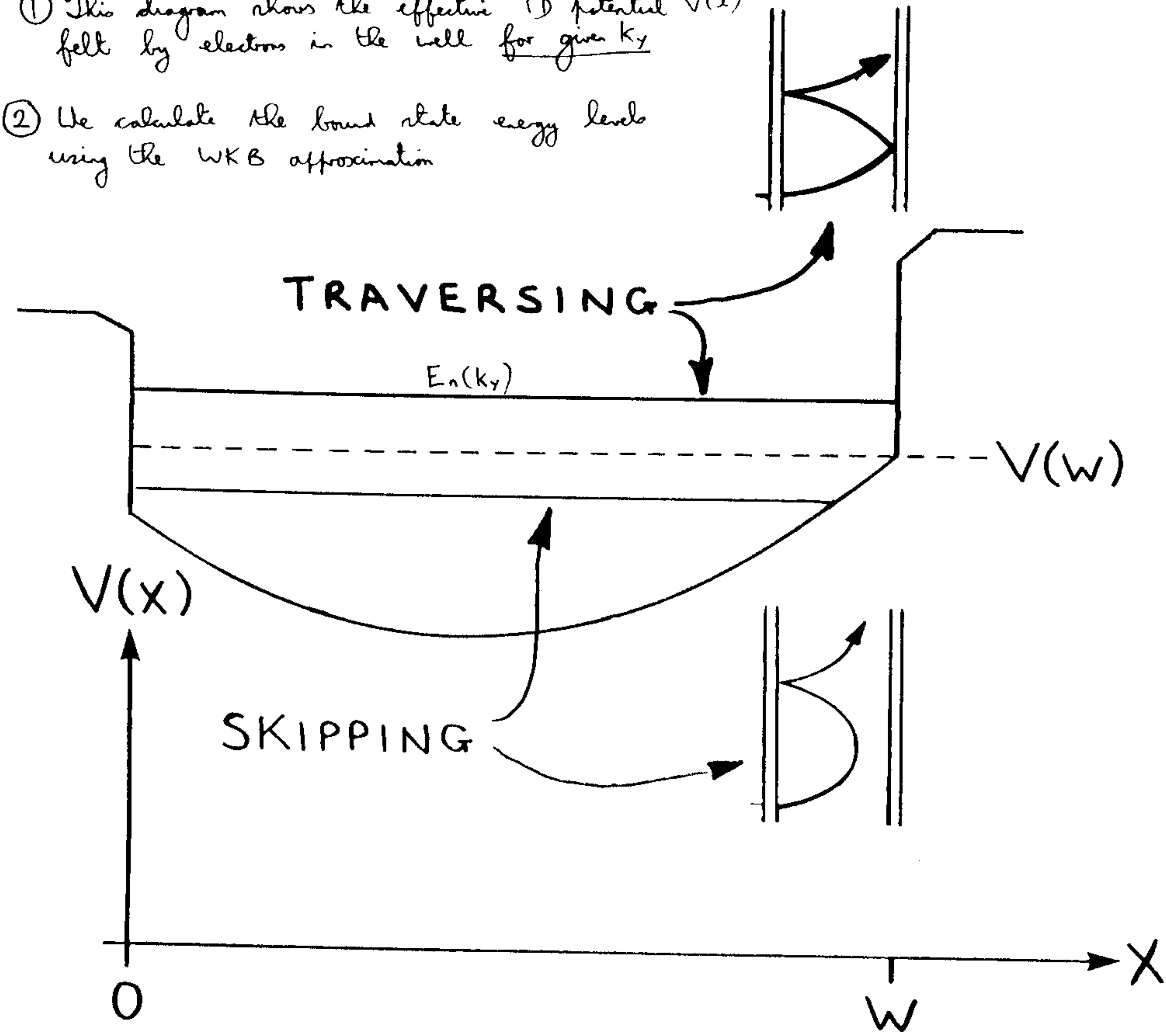
② The states are populated up to the chemical potential in the emitter eV.



ENERGY LEVELS $E_n(k_y)$ IN THE WELL

① This diagram shows the effective 1D potential $V(x)$ felt by electrons in the well for given k_y

② We calculate the bound state energy levels using the WKB approximation

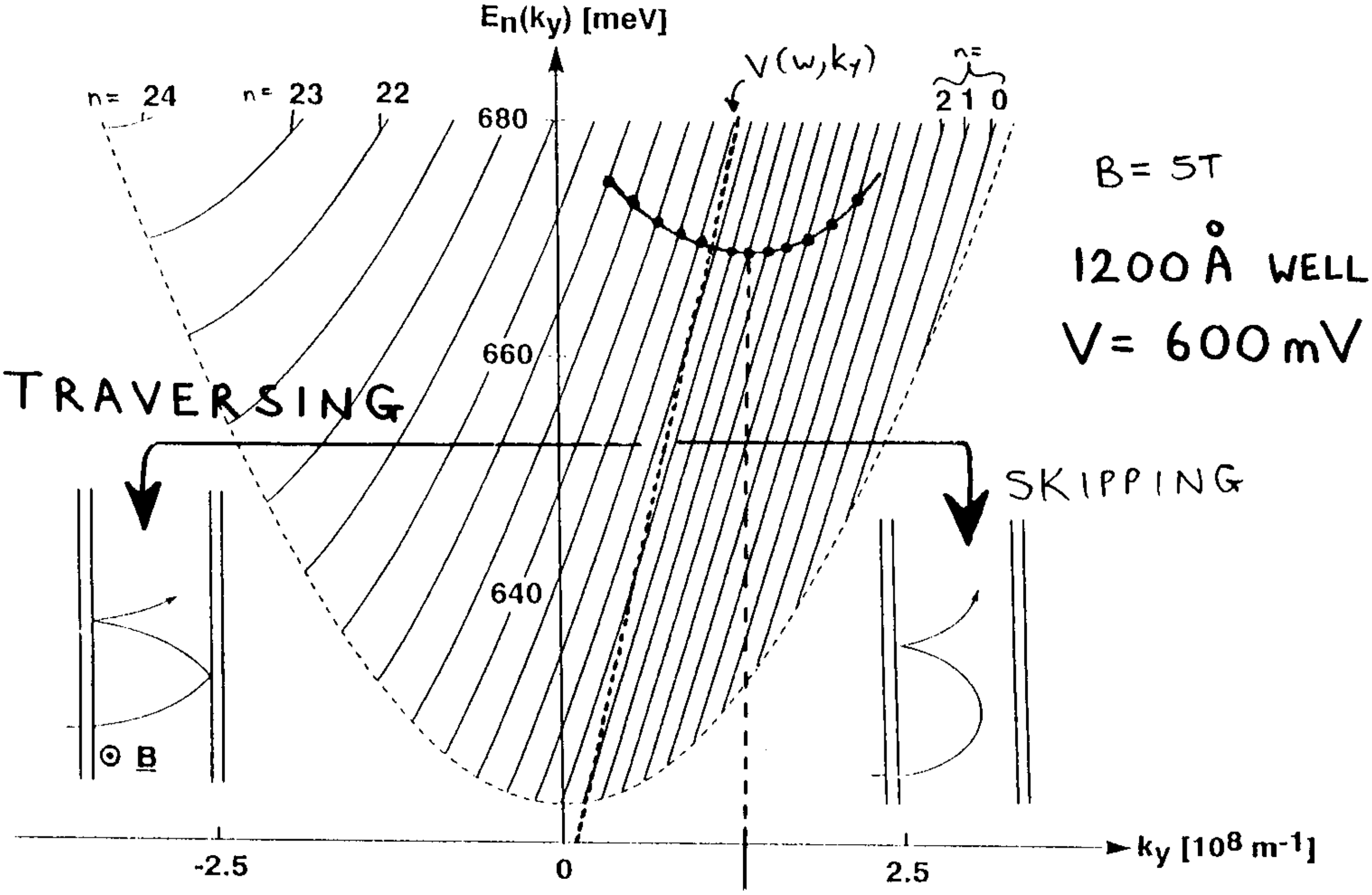


③ We can classify the energy levels $E_n(k_y)$ according to whether they lie above or below the potential energy at the RHS of the well $V(W)$. (i) If $E_n(k_y) < V(W)$ (soft turning point)

the electrons never reach the RH (collector) barrier in a semiclassical picture & skip along the emitter barrier as shown.

(ii) If $E_n(k_y) > V(W)$ (hard turning point) the electrons do hit the collector barrier and follow orbits which traverse the entire well, as above.

ENERGY LEVELS $E_n(K_y)$ FOR QUANTUM WELL AND 2DEG



$$K_0 \propto B$$

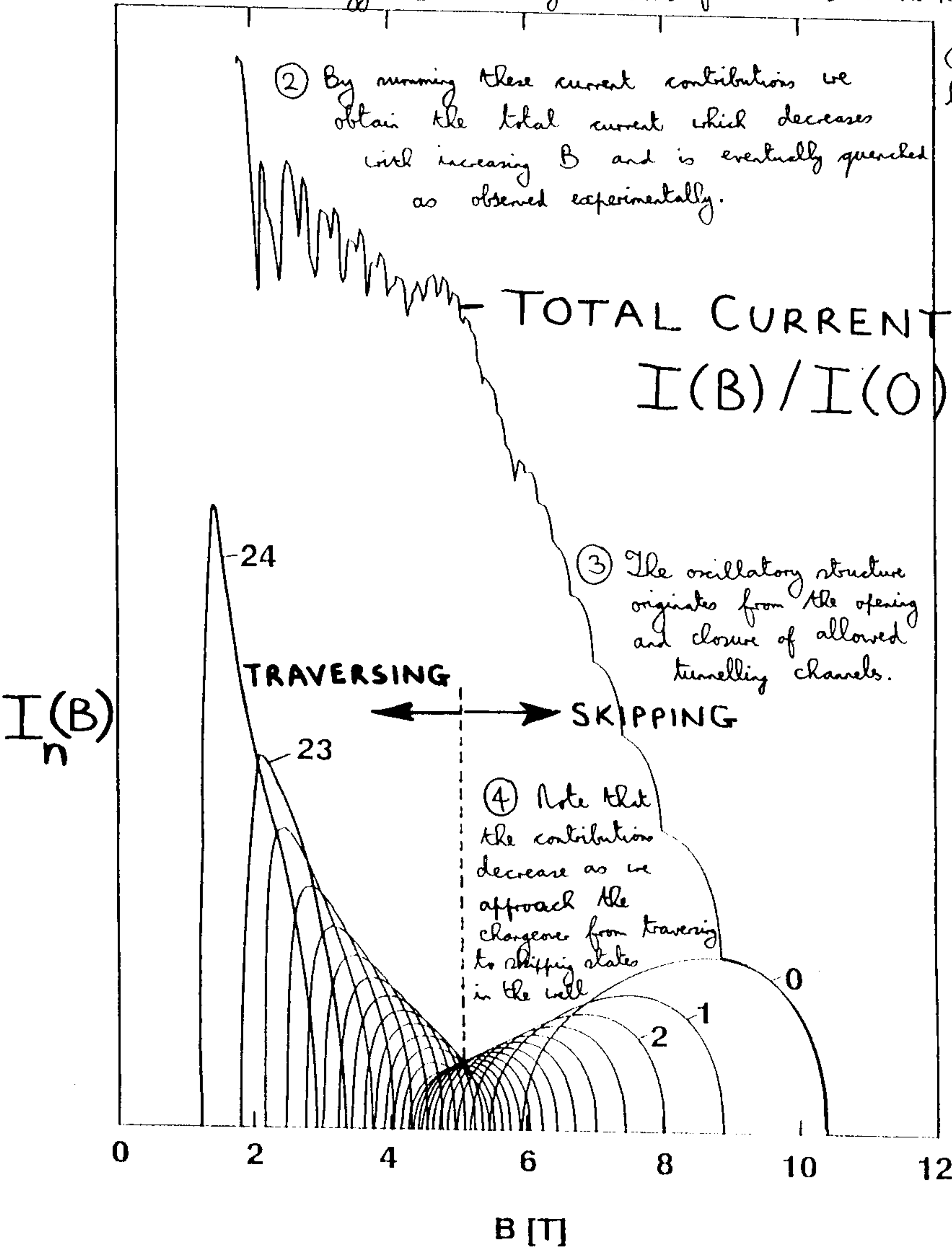
① Here we show the n = 0 to 24 energy levels of the well calculated as
functions of k_y. The dashed (almost linear) curve shows V(W) as a function
of k_y & marks the boundary between traversing and skipping orbits. The parabola
showing the occupied states in the emitter is also displayed. The intersects
between the 2DEG parabola & the $E_n(k_y)$ curves (black dots) indicate k_y values
for which the energy levels in the emitter & well coincide. Each dot represents an allowed tunnelling channel which makes a contribution $I_n(B)$ to the total current (left).

$$I(B) = \sum_n I_n(B)$$

②
With increasing B, the 2DEG parabola shifts
further along the k_y axis so that tunnelling channels are opened
(closed) as intersects with the $E_n(k_y)$ curves are gained (lost) from the RH (LH) edges of the parabola.
This gives the oscillations in I(B)

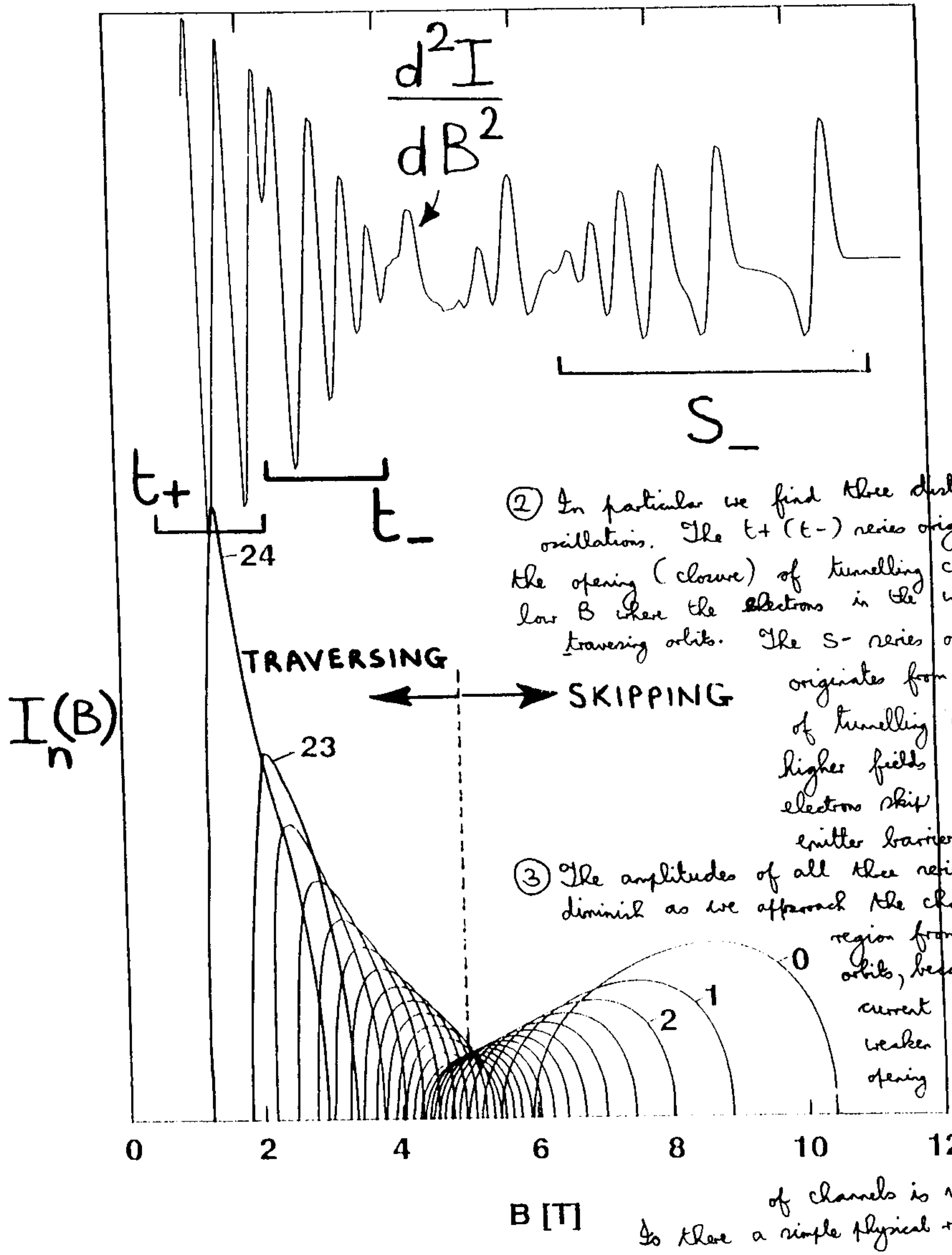
CURRENT CONTRIBUTIONS $I_n(B)$

① We can calculate each current contribution by using the transition rates given earlier. The lower curves here show the magnetic field dependence of the current contribution due to energy- & k- conserving transitions from the 2DEG to the potential well. (n = 0 to 24 bound states)



CURRENT CONTRIBUTIONS $I_n(B)$

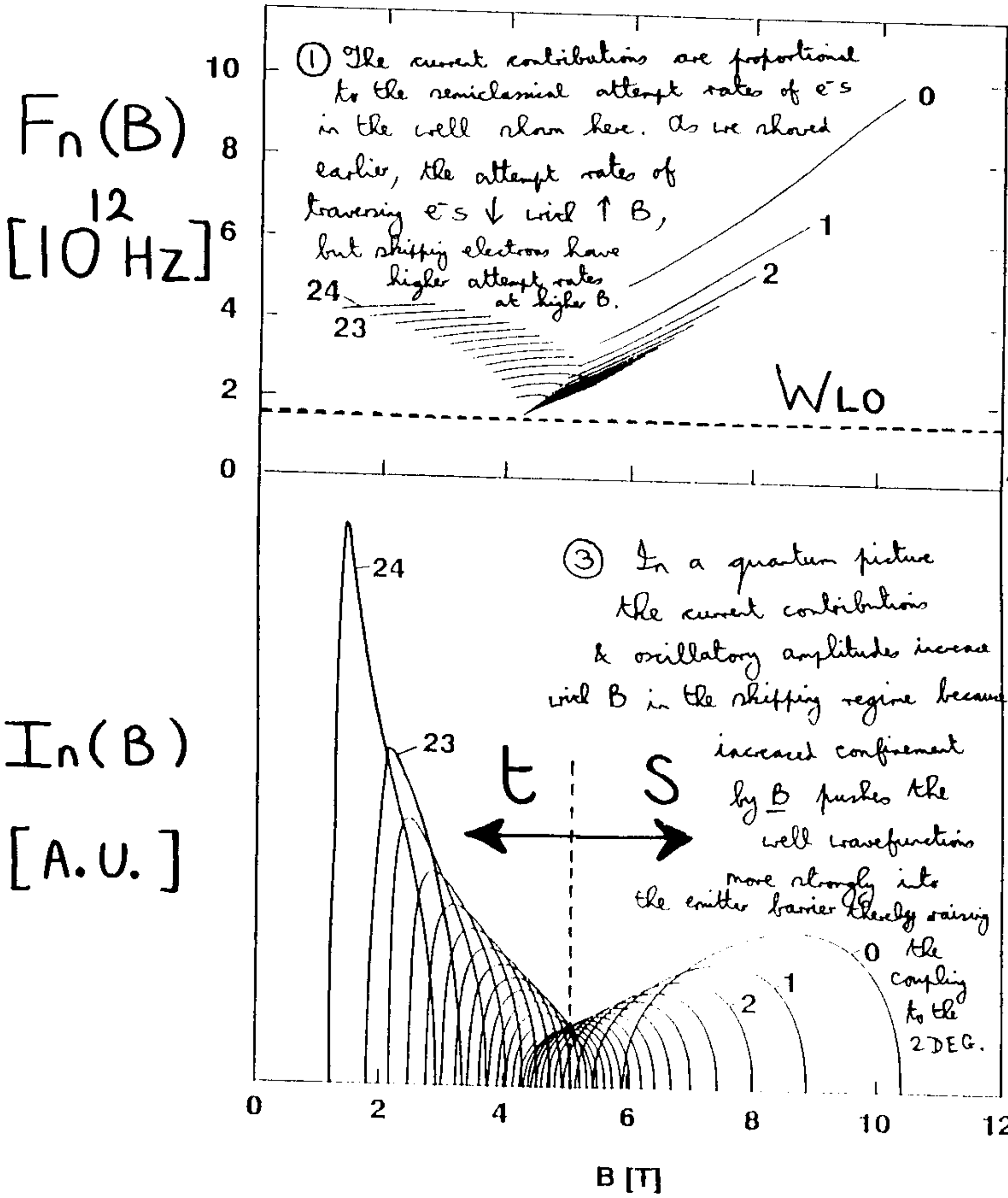
① By taking a second derivative plot of the $I(B)$ curve we reproduce the experimental data.



② In particular we find three distinct series of oscillations. The t_+ (t_-) series originates from the opening (closure) of tunnelling channels at low B where the electrons in the well execute traversing orbits. The S_- series of oscillations originates from the closure of tunnelling channels at higher fields where the electrons skip along the emitter barrier interface.

③ The amplitudes of all three series diminish as we approach the changeover region from traversing orbits, because the current contributions weaken & the opening and closure of channels is very rapid. Is there a simple physical reason for this??

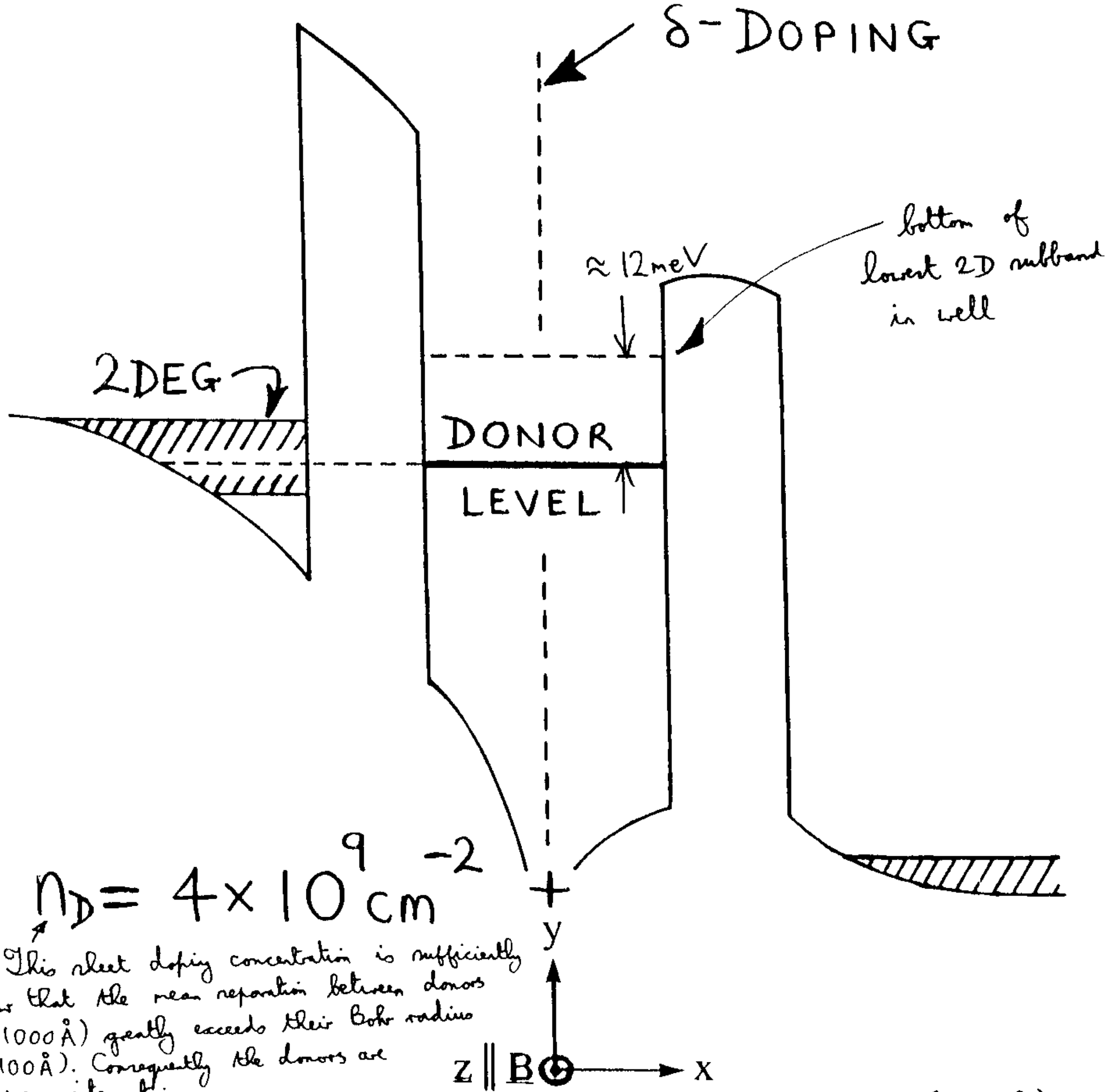
CURRENT CONTRIBUTIONS $I_n(B)$ $\propto$ ATTEMPT RATES $F_n(B)$



② Also, correspondence principle shows energy level spacing $\Delta E \propto F_n$. As F_n drops, so does the level spacing so the energy levels are poorly resolved. In fact, in the changeover region $F_n \simeq 2 \times W_{LO}$, where W_{LO} is the LO phonon emission rate so the level broadening is comparable with the separation.
Thus, the $I_n(B)$ values & oscillatory amplitudes in d^2I/dB^2 decrease with $F_n(B)$ for two distinct reasons:
(1) The current contributions fall
(2) The energy levels become closely-spaced and are poorly resolved

- LOW ATTEMPT RATES $\Longrightarrow$
  1) SMALL CONTRIBUTIONS $I_n \propto F_n$
  2) $\Delta E_n \propto F_n$ SO OSCILLATIONS ALSO <u>POORLY RESOLVED</u>

CONDUCTION BAND PROFILE FOR δ-DOPED SAMPLE UNDER BIAS



(3) This sheet doping concentration is sufficiently low that the mean separation between donors (≈ 1000 Å) greatly exceeds their Bohr radius (≈ 100 Å). Consequently the donors are non-interacting.

(1) We now consider samples with a much narrower well (100 Å) and which contain a delta-doped layer (Si dopant) at the centre of the well.

(2) Under zero applied bias, the donors are ionized. However the attractive potential of the ionized donors creates a 0D bound state ≈ 12 meV below the lowest 2D continuum states (broken horizontal line in well above)

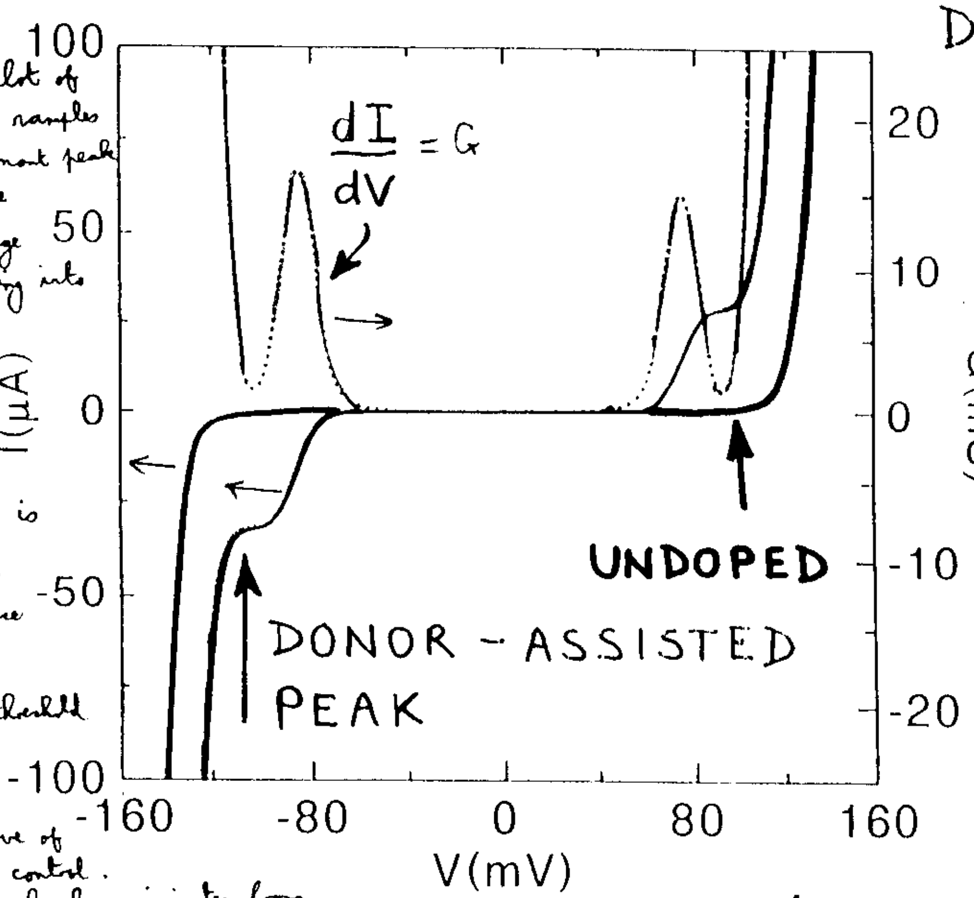
DONOR-ASSISTED PEAK IN I(V) OF δ-DOPED SAMPLE

DATA FROM J.W. SAKAI

① The I(V) plot of the δ-doped samples reveals a resonant peak well below the threshold voltage for tunnelling into the 2D continuum states in the well.

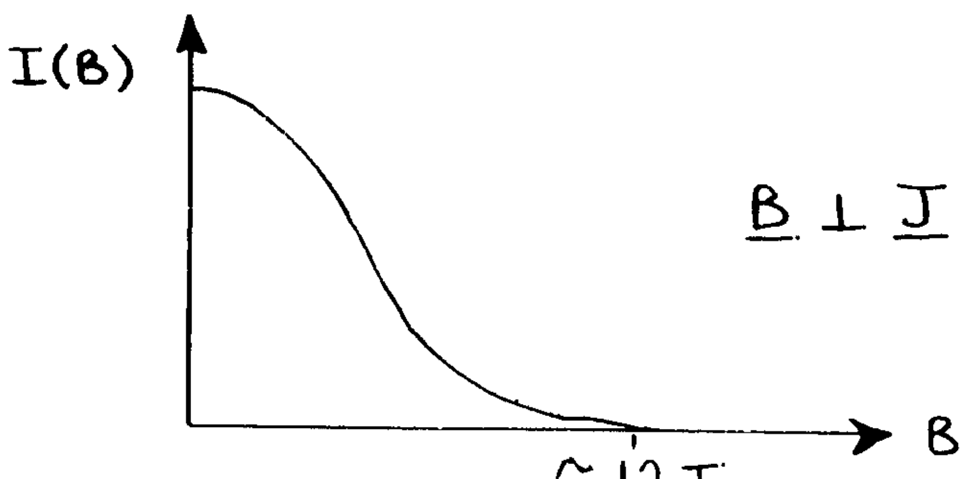
② The peak is emphasised in the conductance plot. G(V)

③ No sub-threshold peak is observed in the I(V) curve of an undoped control.



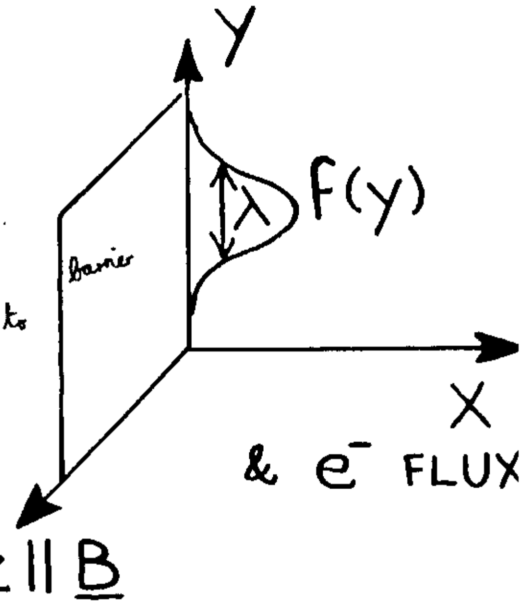
This peak ∴ clearly originates from tunnelling from 2D states in the 2DEC to the 0D bound states of the donor impurities

DONOR-ASSISTED PEAK REDUCED (& QUENCHED) BY IN-PLANE B-FIELD

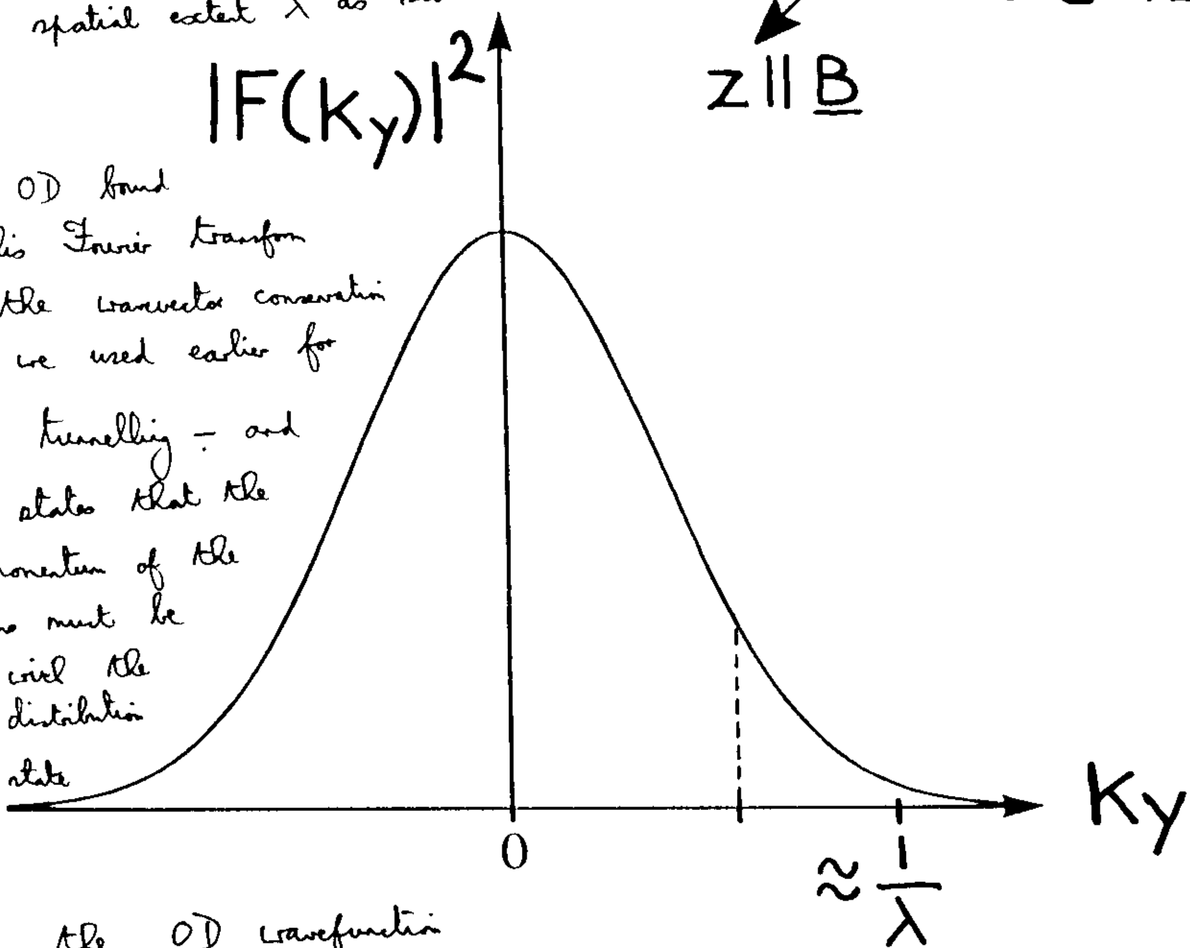


A MODEL FOR 2D → 0D TUNNELLING

$$I \propto \left| \int_y f^*(y) e^{i k_y y} dy \right|^2$$



① As we saw earlier, the transition rate for an electron with wavevector component k_y in the 2DEG into the 0D bound donor state is proportional to the Fourier component of the lateral bound donor wavefunction $f(y)$, which we take to be of approximate spatial extent λ as shown left.



② For the 0D bound state this Fourier transform replaces the wavevector conservation requirement we used earlier for 2D - 2D tunnelling - and essentially states that the lateral momentum of the 2D electrons must be consistent with the momentum distribution of the 0D state

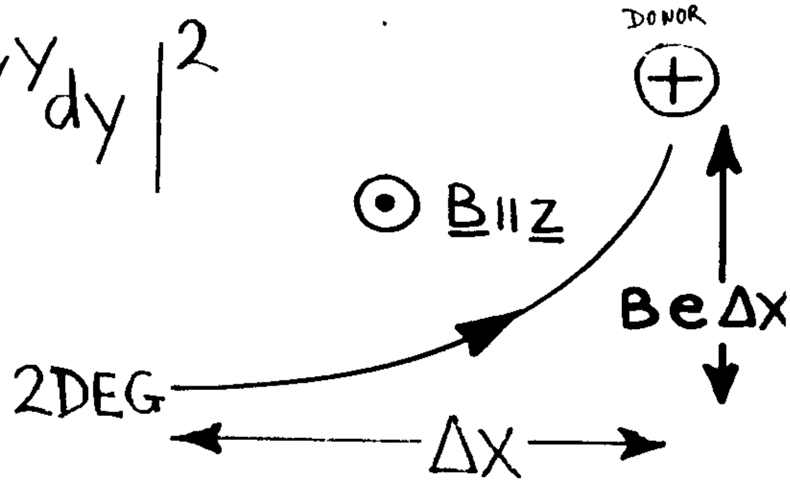
③ Because the 0D wavefunction is of finite spatial extent, its Fourier transform also only has significant magnitude within a region of width $\approx 2/\lambda$, centred on $k_y = 0$ as shown above.

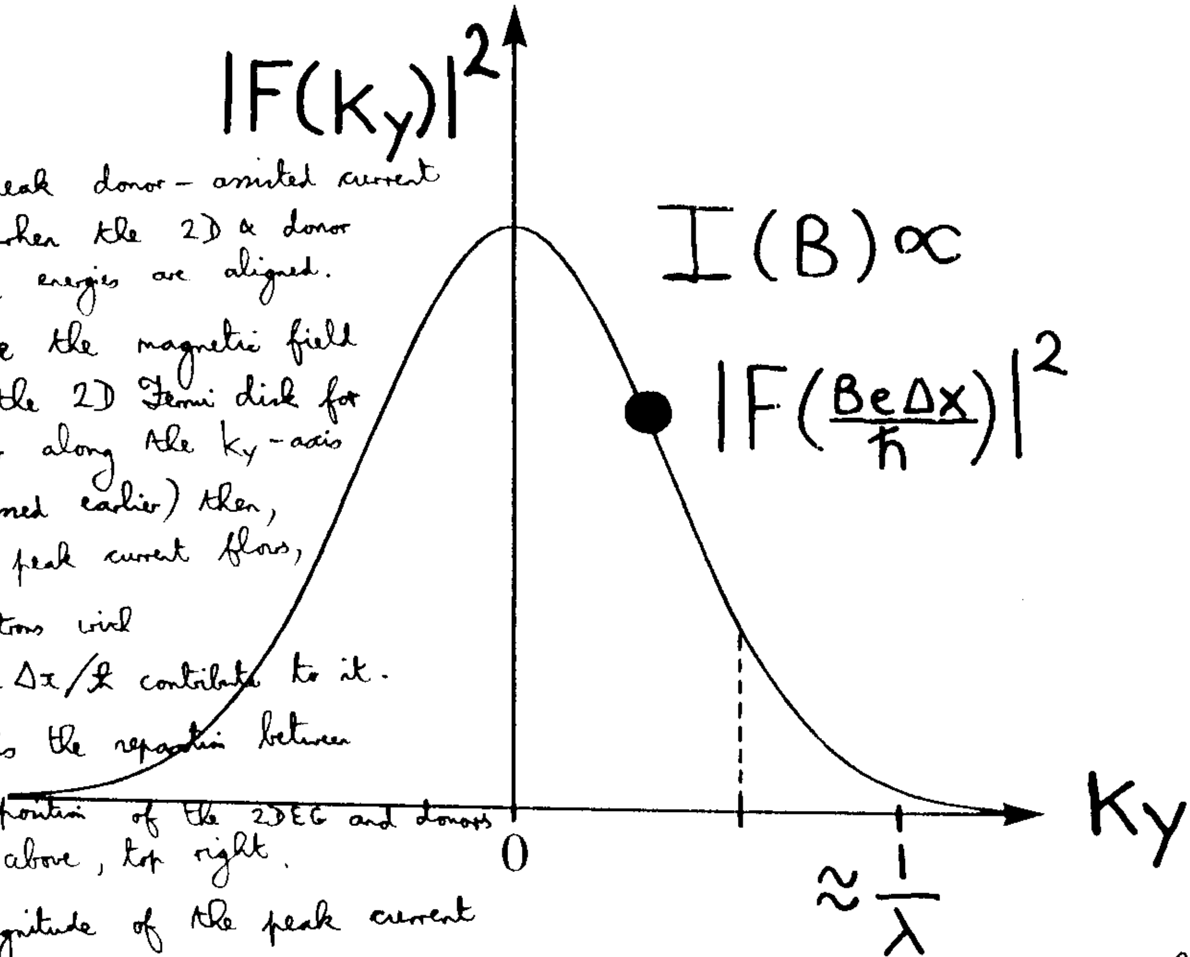
A MODEL FOR 2D → 0D TUNNELLING

$$I \propto \left| \int_Y F^*(y)\, e^{i K_y Y}\, dy \right|^2$$

$$\hbar K_y = B e \Delta x$$

(WHEN 2D & DONOR LEVELS ALIGNED)





(1) The peak donor-assisted current flows when the 2D & donor bound state energies are aligned.

(2) Because the magnetic field shifts the 2D Fermi disk for the emitter along the k_y-axis (as discussed earlier) then, when the peak current flows, ONLY electrons with $k_y = Be\Delta x/\hbar$ contribute to it. Here Δx is the separation between the mean position of the 2DEG and donors as shown above, top right.

(3) The magnitude of the peak current $I_{PEAK}(B) \propto \left|F\left(\frac{Be\Delta x}{\hbar}\right)\right|^2$ and therefore decreases with increasing B, as k_y moves towards the tails of the Fourier transform.

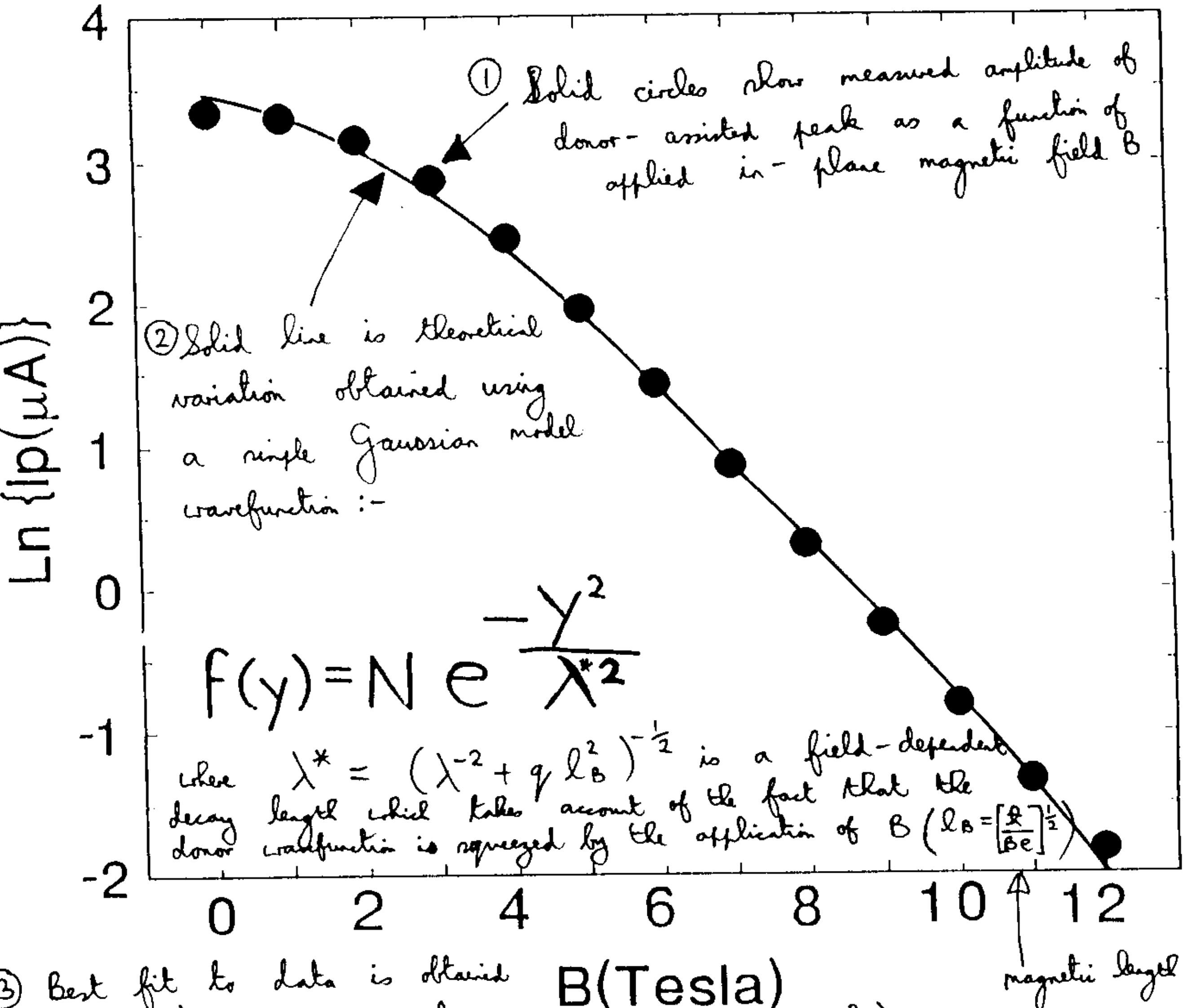
(4) The experimental $I(B)$ plot therefore effectively maps out the Fourier transform of the lateral donor wavefunction – from which the wavefunction itself can, in principle, be reconstructed.

(5) From the field $B \approx 12$T at which the donor-assisted peak is quenched we:

ESTIMATE $\lambda \approx 94$ Å

c.f. BOHR RADIUS IN GaAs ≈ 100 Å

MAGNETO-CURRENT DIRECTLY MEASURES DONOR WAVEFUNCTION ?



③ Best fit to data is obtained by taking $\lambda = 94 \pm 10$ Å (c.f. Bohr radius of ≈ 100 Å)
and $q = 0{\cdot}8 \pm 0{\cdot}2$

SUMMARY OF MAGNETOTUNNELLING WITH $\underline{B} \perp \underline{J}$

- OSCILLATORY AMPLITUDES IN $\frac{d^2 I}{dB^2}$ FOR WIDE WELLS $\propto F$ OR $|\psi|^2$

- AN IN-PLANE $\underline{B}$-FIELD CAN, IN PRINCIPLE MEASURE 0D WAVEFUNCTIONS IN 2D → 0D TUNNELLING