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SMR/697-14

RESEARCH WORKSHOP ON CONDENSED MATTER PHYSICS
(21 June - 3 September 1993)

WORKING PARTY ON SMALL SEMICONDUCTOR STRUCTURES
(2 - 13 August 1993)

QUANTUM STATISTICAL EFFECTS IN ELECTRON AND PHOTON TRANSPORT IN SMALL WAVE GUIDES

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Quantum Statistical Fluctuations in Small Conductors and Photon Wave Guides

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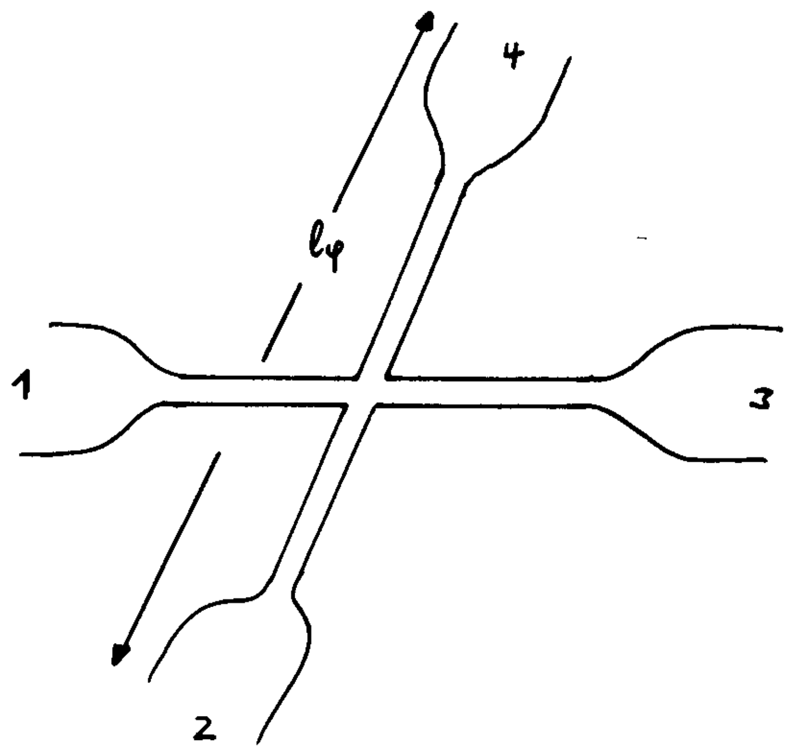
Discussions
R. Landauer, Y. Imry, Th. Martin

Electron-Current and Photon Intensity Fluctuations in Small Wave Guides



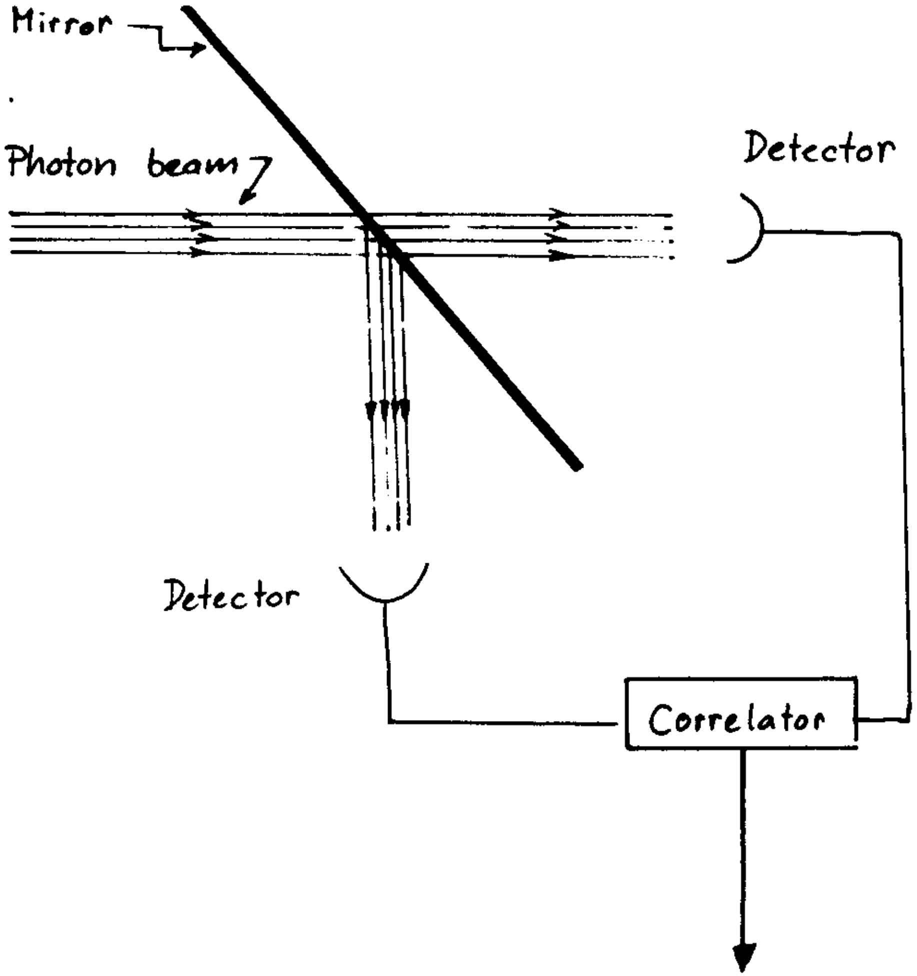
$\alpha = 1, 2, 3, 4$

$$\Delta I_\alpha(t) = I_\alpha(t) - \langle I_\alpha \rangle$$

$$\langle \Delta I_\alpha(t+\tau)\, \Delta I_\beta(t) \rangle = ?$$

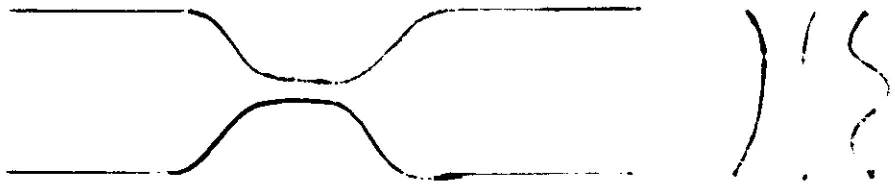
Hanbury Brown and Twiss

Proc. Roy. Soc. Ser. A 242, 300 (1957)



Conductance and Transmission

Landauer, Imry, Büttiker



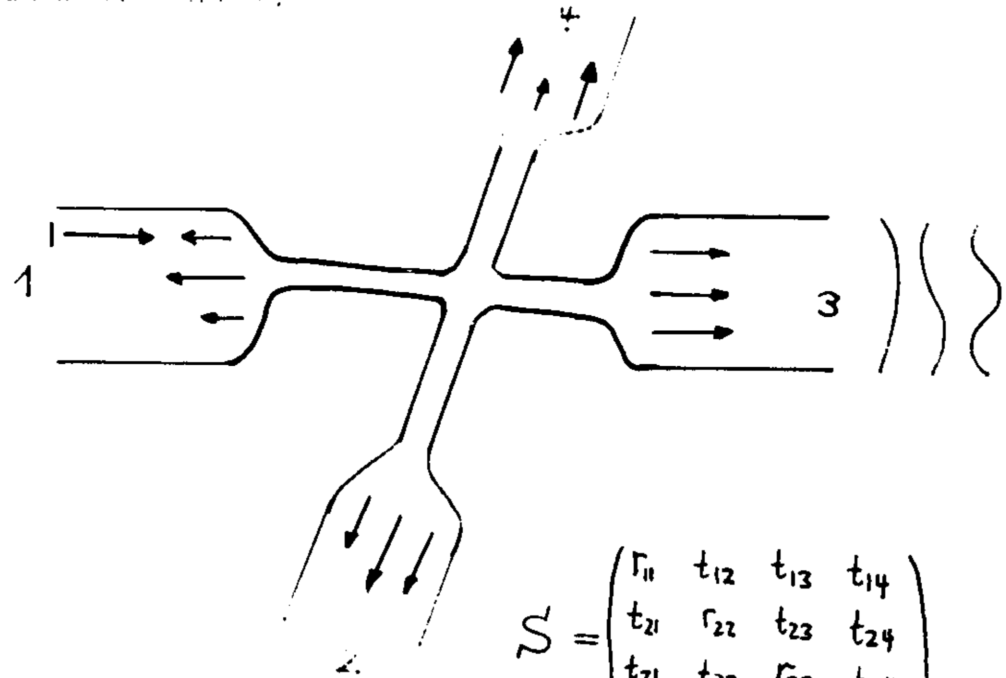
scattering state

$$S = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix}$$

$$\boxed{G = \frac{e^2}{h}\, Tr(t^\dagger t) = \frac{e^2}{h} \sum_{nm} T_{nm}}$$

Conductance and Transmission

Büttiker (1986)



$$S = \begin{pmatrix} r_{11} & t_{12} & t_{13} & t_{14} \\ t_{21} & r_{22} & t_{23} & t_{24} \\ t_{31} & t_{32} & r_{33} & t_{34} \\ t_{41} & t_{42} & t_{43} & r_{44} \end{pmatrix}$$

$$\boxed{I_\alpha = \frac{e}{h}\int dE \left[(M_\alpha - R_{\alpha\alpha}) f_\alpha - \sum_\beta T_{\alpha\beta}\, f_\beta \right]}$$

$$R_{\alpha\alpha} = \sum_{nm} R_{\alpha\alpha, nm} = Tr\,(r^{+}_{\alpha\alpha}\, r_{\alpha\alpha})$$

$$T_{\alpha\beta} = \sum_{nm} T_{\alpha\beta, nm} = Tr\,(t^{+}_{\alpha\beta}\, t_{\alpha\beta})$$

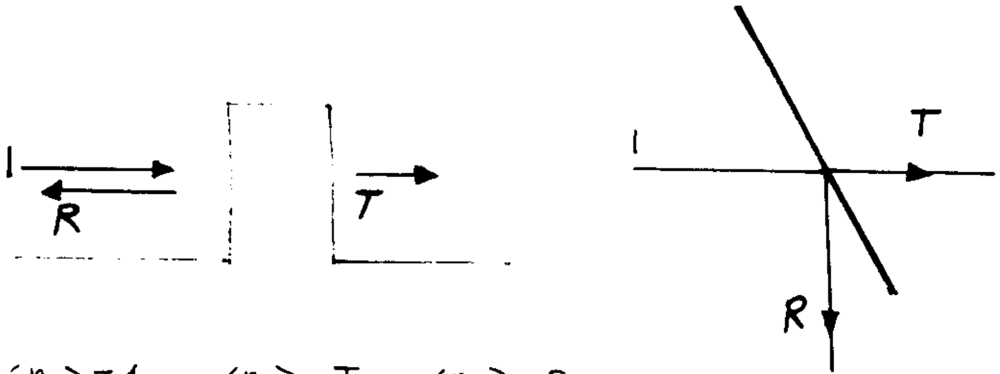
$$\frac{e}{h}\int dE \quad \longleftrightarrow \quad \int dE\, \nu(E)$$

Shot Noise

$$\boxed{\langle (\Delta I)^2 \rangle_\nu = 2e\,\Delta\nu\,\langle I \rangle}$$

$$\left\langle \left(\int_t^{t+\tau} \Delta I(t')\, dt' \right)^2 \right\rangle = e^2 \langle (N_\tau - \langle N_\tau \rangle)^2 \rangle = e^2 \langle N_\tau \rangle$$

Single Particle Scattering



$\langle n_I \rangle = 1$, $\langle n_T \rangle = T$, $\langle n_R \rangle = R$

$\langle n_T n_R \rangle = 0$

$\Delta n_T = n_T - \langle n_T \rangle$

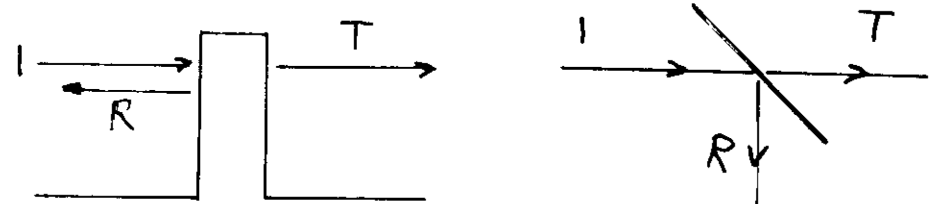
$\Delta n_R = n_R - \langle n_R \rangle$

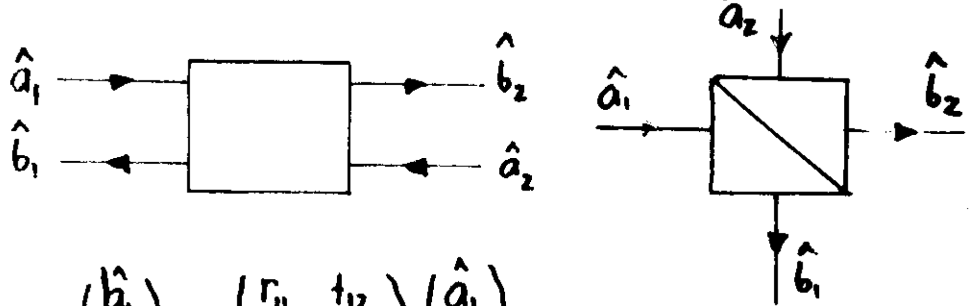
$\langle \Delta n_T \Delta n_R \rangle = \langle n_T n_R \rangle - \langle n_T \rangle \langle n_R \rangle = -RT$

$\langle (\Delta n_T)^2 \rangle = -\langle \Delta n_T \Delta n_R \rangle = RT$

$\langle (\Delta n_R)^2 \rangle = -\langle \Delta n_T \Delta n_R \rangle = RT$

Scattering and Second Quantization





$$\begin{pmatrix} \hat b_1 \\ \hat b_2 \end{pmatrix} = \begin{pmatrix} r_{11} & t_{12} \\ t_{21} & r_{22} \end{pmatrix} \begin{pmatrix} \hat a_1 \\ \hat a_2 \end{pmatrix}$$

$$|1\rangle = \hat a_1^\dagger |0\rangle$$

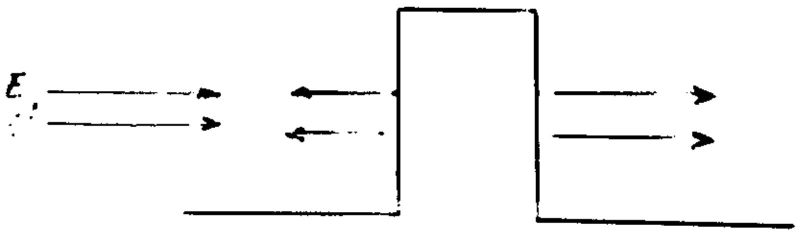
$$\langle n_R n_T \rangle = \langle 1| b_1^\dagger b_1 b_2^\dagger b_2 |1\rangle = 0$$

$$|n\rangle = \frac{1}{\sqrt{n!}} (a_1^\dagger)^n |0\rangle$$

$$\langle n_R n_T \rangle = \langle n| b_1^\dagger b_1 b_2^\dagger b_2 |n\rangle = RT\, n(n-1)$$

$$\langle (\Delta n_T)^2 \rangle = \langle (\Delta n_R)^2 \rangle = -\langle \Delta n_T \Delta n_R \rangle = nRT$$

Two Particle versus One Particle Effects



$$\langle \Delta n_R \, \Delta n_T \rangle \propto$$

$$dE\, dE' \langle n(E), m(E') | b_1^+(E)\, b_1(E')\, b_2^+(E')\, b_2(E) | n(E), m(E') \rangle$$

$$= \mp \, dE\, dE' \langle n(E), m(E') | b_1^+(E)\, b_2^+(E')\, b_1(E')\, b_2(E) | n(E), m(E') \rangle$$

↑ outcome of experiment depends on symmetry of wave function

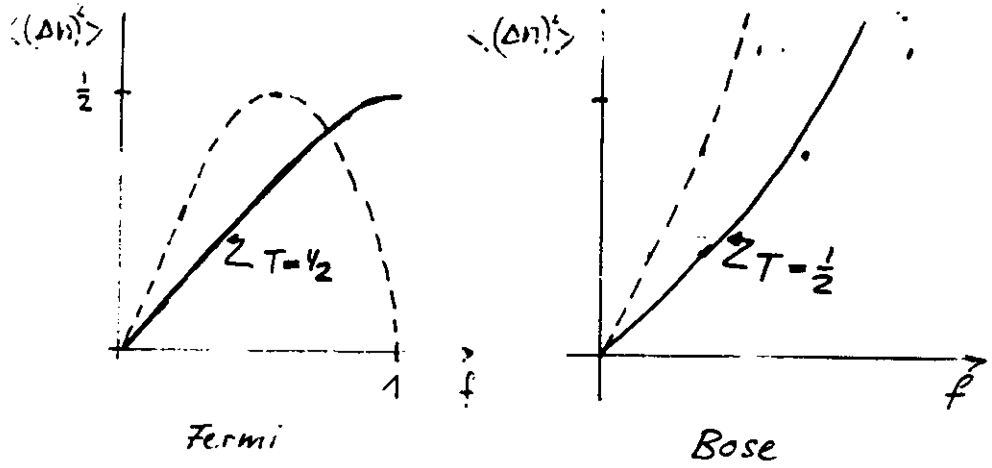
$\langle \Delta n_R \; \Delta n_T \rangle =$		$\mp R(E) T(E) f^2(E)$
$\langle (\Delta n_T)^2 \rangle =$	$T(E)\, f(E)(1 \mp f(E))$	$\pm R(E) T(E) f^2(E)$
$\langle (\Delta n_R)^2 \rangle =$	$R(E)\, f(E)(1 \mp f(E))$	$\pm R(E) T(E) f^2(E)$
	$f(E)(1 \mp f(E)) = k_B T \left(-\frac{df}{dE}\right)$	
	single particle	two particle

$$\langle (\Delta n_I)^2 \rangle = \langle (\Delta n_T + \Delta n_R)^2 \rangle$$

Effect of Scattering on Occupation Noise

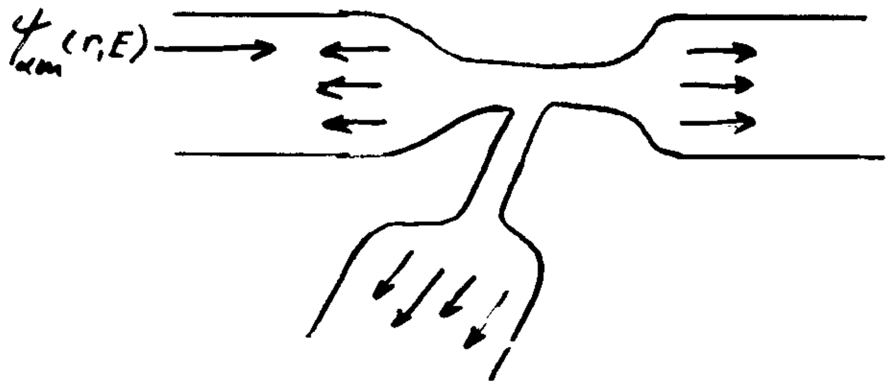
$$\langle (\Delta n_I)^2 \rangle = 2 f (1 \mp f) \qquad - - - - -$$

$$\langle (\Delta n_T)^2 \rangle = 2 T f (1 \mp T f)$$



Calculation of Fluctuations with the help of Scattering States

Fluctuations



$$\psi(r,t) = \sum_{\alpha m} \int \frac{dE_{\alpha m}}{(h v_{\alpha m})^{1/2}}\, a_{\alpha m}(E_{\alpha m})\, \psi_{\alpha m}(r,E)\, e^{-i\omega_{\alpha m} t}$$

$$\hbar\omega_{\alpha m} = E_{\alpha m} - \mu_\alpha$$

$$\boxed{\hat{\psi}(r,t) = \sum_{\alpha m} \int \frac{dE_{\alpha m}}{(h v_{\alpha m})^{1/2}}\, \hat{a}_{\alpha m}(E_{\alpha m})\, \psi_{\alpha m}(r,E)\, e^{-i\omega_{\alpha m} t}}$$

$$[\hat{a}^{\dagger}_{\alpha m}(E), \hat{a}_{\beta n}(E')]_{\pm} = \delta_{\alpha\beta}\, \delta_{mn}\, \delta(E-E')$$

Current Fluctuations

$$\Delta \hat{I}_\alpha(t) = \hat{I}_\alpha(t) - \langle \hat{I}_\alpha \rangle$$

$$\tfrac{1}{2}\langle \Delta\hat{I}_\alpha(\omega)\,\Delta\hat{I}_\beta(\omega') + \Delta\hat{I}_\beta(\omega')\,\Delta\hat{I}_\alpha(\omega)\rangle$$

$$= 2\pi \,\langle \Delta I_\alpha\, \Delta I_\beta \rangle_\omega\, \delta(\omega+\omega')$$

Low frequency limit:

$$\lim_{\omega\to 0} \hat{I}_\alpha(\omega) = \frac{e}{h}\int dE\, \left(\hat{a}_\alpha^{+}\hat{a}_\alpha - \hat{b}_\alpha^{+}\hat{b}_\alpha\right)$$

$$\boxed{\hat{b}_\alpha = \sum_\beta S_{\alpha\beta}\, \hat{a}_\beta}$$

$$\lim_{\omega\to 0} \hat{I}_\alpha(\omega) = \frac{e}{h}\int dE \sum_{\gamma\delta} \hat{a}_\gamma^{+} A_{\gamma\delta}(\alpha)\, \hat{a}_\delta$$

$$\boxed{A_{\gamma\delta}(\alpha) = \delta_{\alpha\gamma}\,\delta_{\alpha\delta} - S_{\alpha\gamma}^{+} S_{\alpha\delta}}$$

$$\boxed{\langle \Delta I_\alpha\, \Delta I_\beta\rangle_{\omega\to 0} = 2\frac{e^2}{h}\int dE \sum_{\gamma\delta} \mathrm{Tr}\left(A_{\gamma\delta}(\alpha) A_{\delta\gamma}(\beta)\right) f_\gamma (1\mp f_\delta)}$$

Equilibrium and Transport Fluctuations

$$\langle \Delta I_\alpha\, \Delta I_\beta\rangle = \langle \Delta I_\alpha\, \Delta I_\beta\rangle_{eq} + \langle \Delta I_\alpha\, \Delta I_\beta\rangle_{tr}$$

"equilibrium fluctuations" $(\alpha\neq\beta)$

$$\langle \Delta I_\alpha\, \Delta I_\beta\rangle_{eq} = -2\Delta\nu \frac{e^2}{h}\int dE \left[T_{\beta\alpha} f_\alpha(1\mp f_\alpha) + T_{\alpha\beta} f_\beta (1\mp f_\beta)\right]$$

- negative independent of statistics

"transport fluctuations" $(\alpha\neq\beta)$

$$\boxed{\langle \Delta I_\alpha\, \Delta I_\beta\rangle_{tr} = \mp 2\Delta\nu \frac{e^2}{h}\int dE \sum_{\gamma\delta} f_\gamma f_\delta\, \mathrm{Tr}\left(S_{\alpha\gamma}^{+} S_{\alpha\delta} S_{\beta\delta}^{+} S_{\beta\gamma}\right)}$$

- negative for Fermions
- positive for Bosons

Comment on current conservation

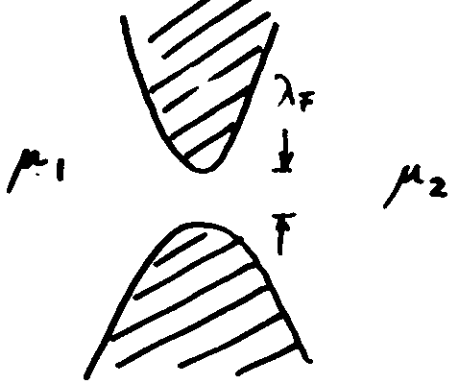
$$\sum_\alpha I_\alpha = 0 \quad \xrightarrow[\omega\to 0]{} \quad \sum_\alpha \Delta I_\alpha = 0 \quad \Longrightarrow$$

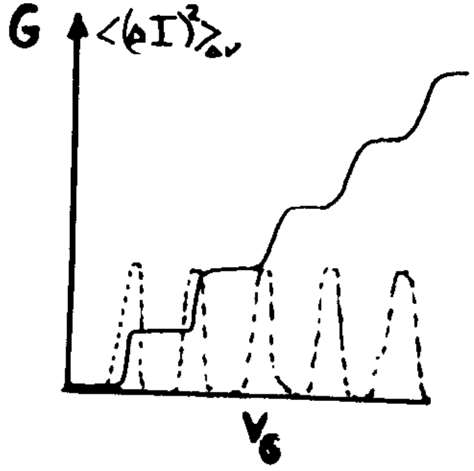
$$0 = \langle (\sum_\alpha \Delta I_\alpha)^2\rangle = \underbrace{\sum_\alpha \langle (\Delta I_\alpha)^2\rangle}_{>0} + \underbrace{\sum_{\alpha\neq\beta}\langle \Delta I_\alpha\, \Delta I_\beta\rangle}_{<0}$$

Shot Noise in a Quantum Point Contact

$kT=0$, $\langle I \rangle = \frac{e^2}{h} Tr(t^{\dagger}t)(V_1-V_2)$

Lesovik (1989)





Experiment: Li et al (1990)

$$S = \begin{pmatrix} r_{11} & t_{12} \\ t_{21} & r_{22} \end{pmatrix}$$

$$\boxed{\langle(\Delta I)^2\rangle_{\Delta\nu} = 2e\Delta\nu \frac{e^2}{h} |V_1-V_2| \; Tr(r_{11}^{\dagger} r_{11} t_{21}^{\dagger} t_{21})}$$

Büttiker (1990)

T_n eigenvalue of $t_{21}^{\dagger} t_{21}$

$1-T_n$ eigenvalue of $r_{11}^{\dagger} r_{11}$

$\Longrightarrow$

$$\langle(\Delta I)^2\rangle_{\Delta\nu} = 2e\Delta\nu \frac{e^2}{h} |V_1-V_2| \sum_n T_n (1-T_n)$$

If for all n, $T_n < 1$, then full shot noise

$$\langle(\Delta I)^2\rangle_{\nu} = 2e\Delta\nu \langle I \rangle$$

(Khlus, 1987)

Lesovik (1989)

Shot Noise in a Quantum Point Contact

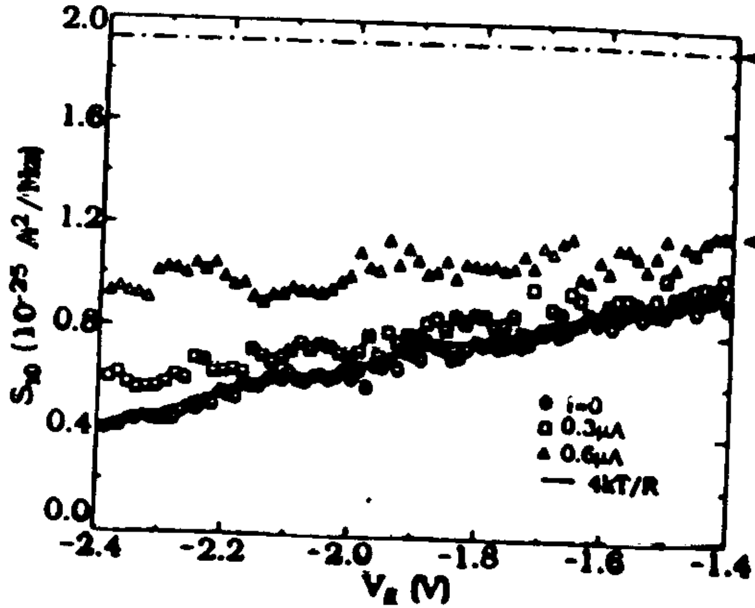
Experiment: Li et al (1990)

Correct theory:

$$\boxed{\langle(\Delta I)^2\rangle_{\nu} = 2e\Delta\nu \frac{e^2}{h} |V_1-V_2| \sum_n T_n(1-T_n)}$$

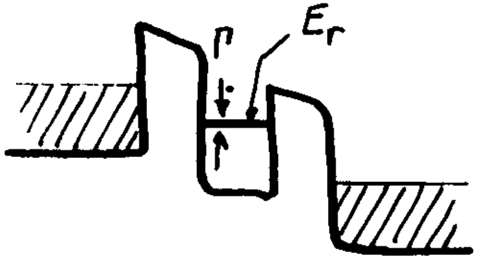
Uncorrelated electrons:

$$\langle(\Delta I)^2\rangle_{\nu} = 2e\Delta\nu \frac{e^2}{h} |V_1-V_2| \sum_n T_n = 2e\Delta\nu I$$



Shot Noise in a Resonant Double Barrier

Experiment: Li et al (1990)

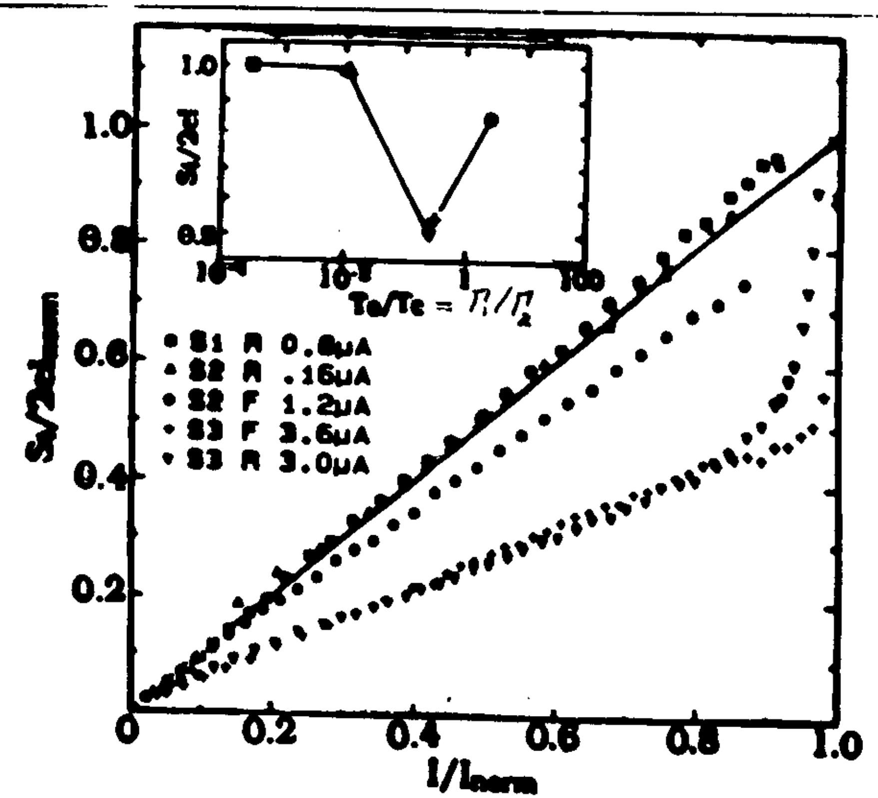


$$T = \frac{\Gamma_1 \Gamma_2}{(E-E_r)^2 + \Gamma^2/4}$$

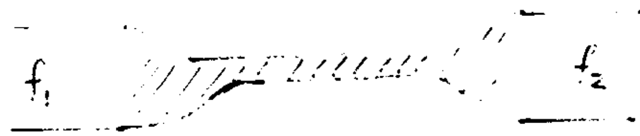
$$\Gamma = \Gamma_1 + \Gamma_2$$

$$T_{res} = \frac{4\Gamma_1\Gamma_2}{\Gamma^2}$$

$$\boxed{\langle (\Delta I)^2 \rangle_\nu = 2e\Delta\nu \, \frac{e}{h} \int dE \; T(1-T) = 2e\Delta\nu \, I \left(1 - \tfrac{1}{2} T_{res}\right)}$$



Two Port Wave Guide: $kT \neq 0$

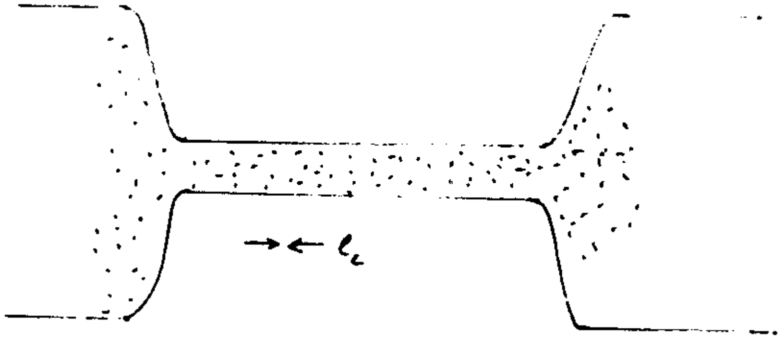


$$\langle (\Delta I)^2 \rangle = 2\Delta\nu \frac{e^2}{h} \sum_n \int dE \; \underbrace{\left[T_n f_1 (1 \mp f_1) + T_n f_2 (1 \mp f_2) \right.}_{\text{equilibrium like-noise}} \pm \underbrace{\left. T_n (1-T_n)(f_1 - f_2)^2 \right]}_{\text{transport noise}}$$

- For Fermi systems the noise in the presence of transport typically exceeds the equilibrium noise
- For Bose systems the noise in the presence of transport is typically smaller than the equilibrium noise

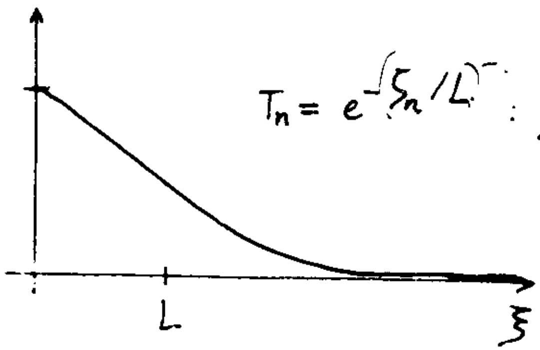
Shot Noise in Metallic Diffusive Conductors

Beenakker and Büttiker (199..



$l_e \ll l_\varphi \ll \xi = N l_e$

$\langle (\Delta I)^2 \rangle_\nu = \frac{2}{3} e \Delta\nu \langle I \rangle$

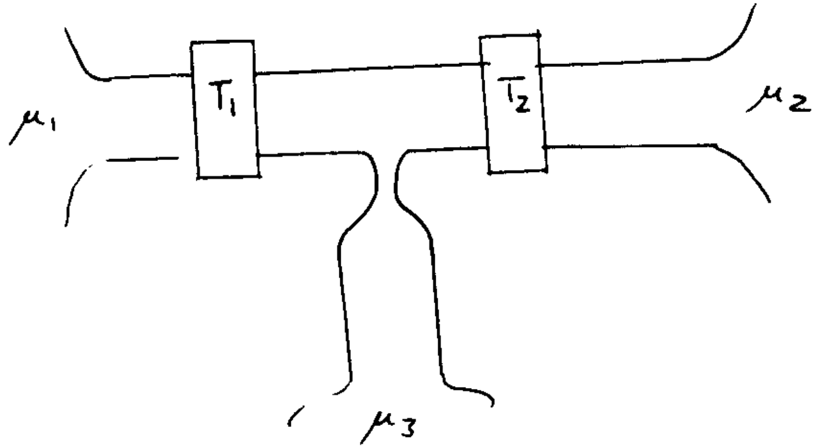


$T_n = e^{-(\xi_n / L)}$: ξ_n uniform

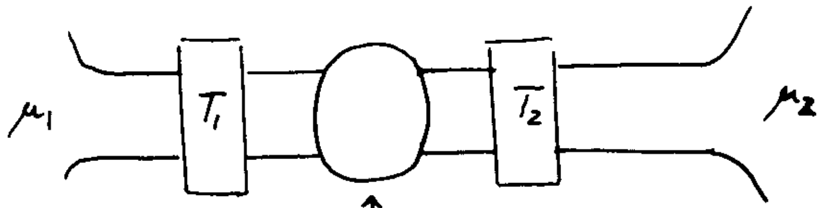
de Jong and Beenakker

Shot Noise of Series Resistors

Beenakker and Büttiker



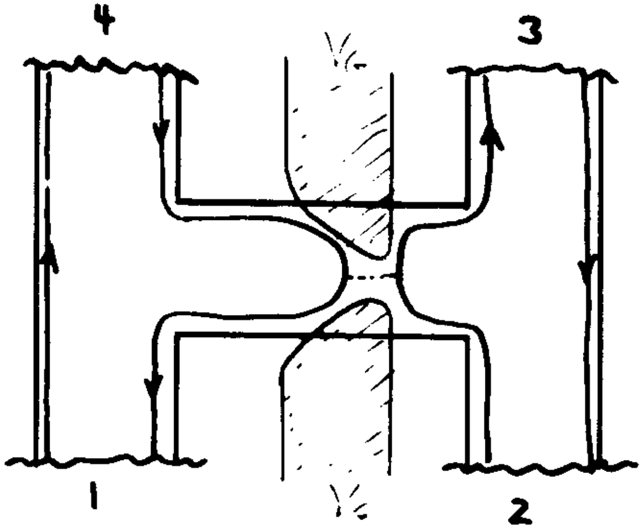
$\Delta I_3 = 0 \implies \delta\mu_3 = -\frac{\delta I_3}{T_{31} + T_{32}}$



For completely incoherent transmission and very opaque barriers

$$\boxed{\langle (\Delta I_1)^2 \rangle = 2 e \Delta\nu I \, \frac{R_1^2 + R_2^2}{(R_1 + R_2)^2}}$$

Example of Conductor in which Incident and Reflected "Beams" are separated



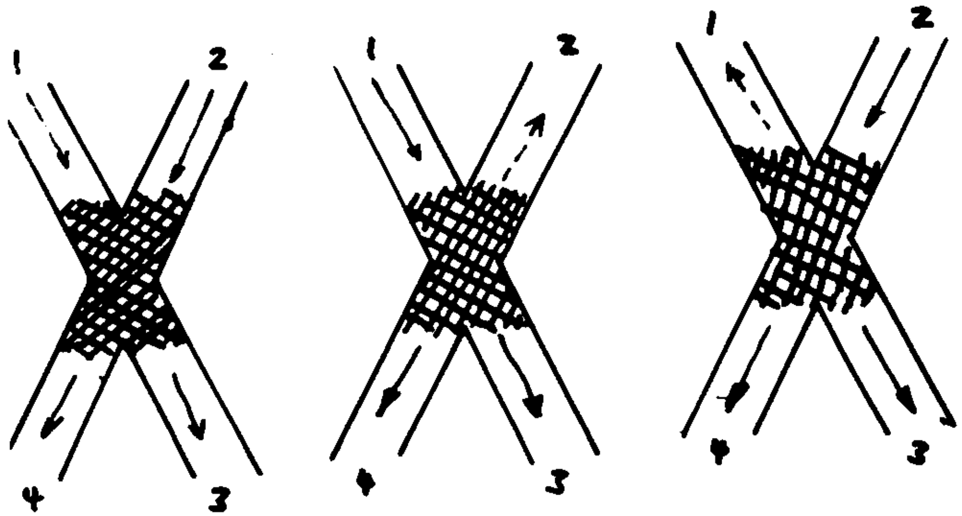
$kT=0$: $\mu_1 = \mu_4 \equiv \mu$, $\mu_3 = \mu_2 \equiv \mu_0$

$\langle(\Delta I_1)^2\rangle = \langle(\Delta I_3)^2\rangle = -\langle \Delta I_1 \Delta I_3\rangle = 2\Delta\nu \frac{e^2}{h} TR(\mu-\mu_0)$

$kT \neq 0$: $f_1 = f_4 \equiv f$, $f_3 = f_2 \equiv f_0$

$$\boxed{\langle \Delta I_1 \Delta I_3 \rangle = \mp 2\Delta\nu \frac{e^2}{h} \int dE\, RT(f-f_0)^2}$$

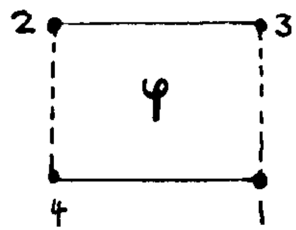
Cross-Correlations in the Presence of Two Sou



A: $f_1 = f_2 = f > f_3 = f_4 = f_0$ B: $f_1 = f > f_2 = f_3 = f_4 = f_0$ C: $f_2 = f > f_1 = f_3 = f_4$

$$\langle \Delta I_3 \Delta I_4 \rangle_A = \underline{\langle \Delta I_3 \Delta I_4\rangle_B} + \underline{\langle \Delta I_3 \Delta I_4 \rangle_C}$$

$$\mp 2\Delta\nu\,(e^2/h)\int dE\,(f-f_0)^2\Big[\underbrace{\underbrace{Tr(S_{31}^{\dagger} S_{32} S_{42}^{\dagger} S_{41})}_{A e^{i\varphi}} + \underbrace{Tr(S_{32}^{\dagger} S_{31} S_{41}^{\dagger} S_{4}}_{+A e^{-i\varphi}}}_{2A\cos\varphi}$$



Summary

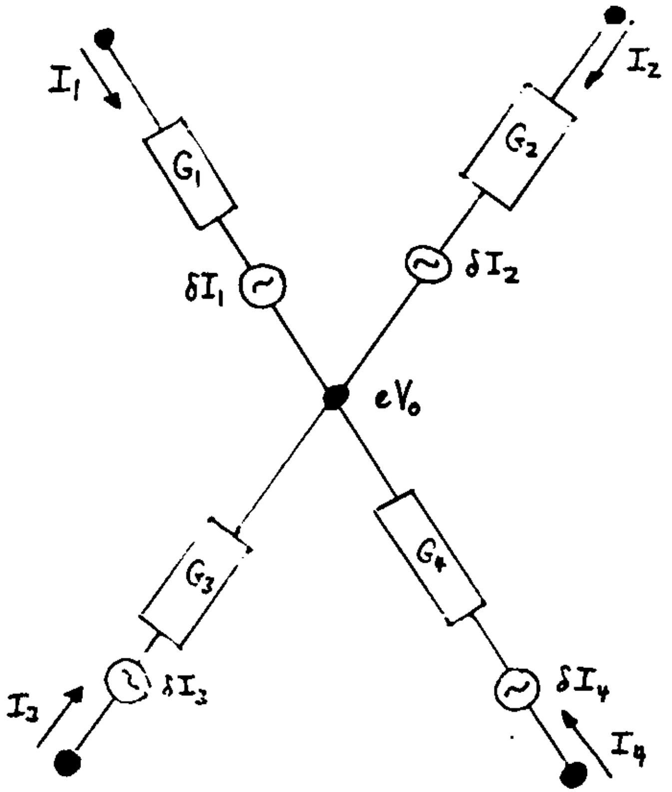
- Fluctuation theory of transport in mesoscopic wires and wave guides
- (Zero-frequency) noise can be expressed in terms of scattering matrices
- Two particle contributions to shot noise / open quantum channels / exchange effects

M. B. PRB 46, 12485 (1992) (Gives more than 70 Refs)

Summary

- Fluctuation theory of transport in mesoscopic wires and wave guides
- Two particle contributions to shot noise / exchange effects
- Admittance matrix related to fluctuation spectra

Cross - Correlations: Circuit Model



$$\langle \Delta I_3 \, \Delta I_4 \rangle_C = \langle \Delta I_3 \, \Delta I_4 \rangle_A + \langle \Delta I_3 \, \Delta I_4 \rangle_B$$