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INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS
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RESEARCH WORKSHOP ON CONDENSED MATTER PHYSICS
(21 June - 3 September 1993)

WORKING PARTY ON SMALL SEMICONDUCTOR STRUCTURES
(2 - 13 August 1993)

FREQUENCY DEPENDENT CURRENT PARTITION

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Frequency-Dependent Current-Partition in Multiprobe Conductors

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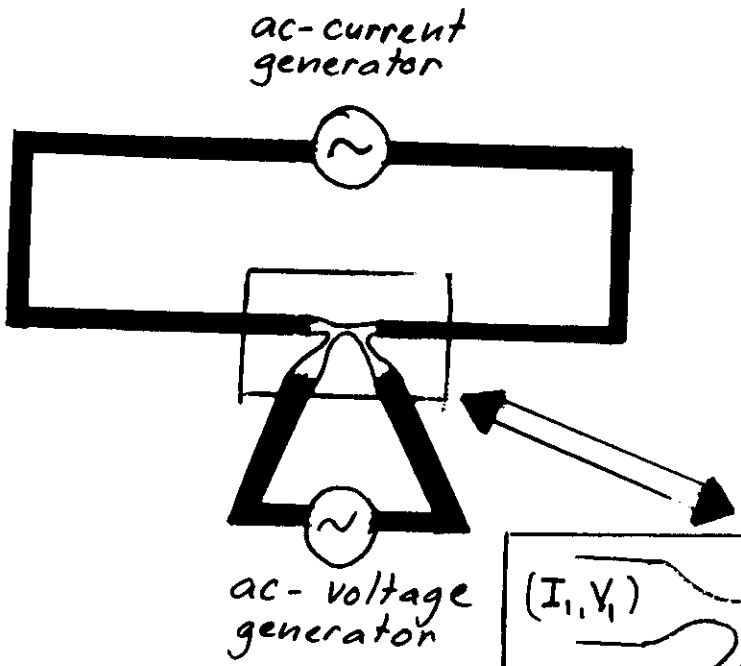
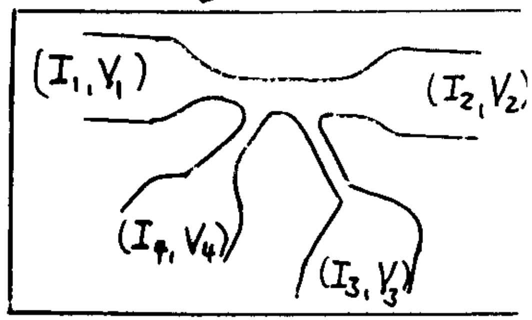
AC-Response of Mesoscopic Conductor Embeded in an External Circuit





$$\delta I_\alpha(\omega) = \sum_\beta g_{\alpha\beta}(\omega)\, \delta V_\beta(\omega)$$

Contrast with formal linear response:

$$H = H_0 + H_1$$

$$H_1 = - \int d^3r\, \rho(r,t)\, U_{ext}(r,t) + \int d^3r\, \vec{j}(r,t)\, \vec{A}_{ext}(r,t)$$

Current Conservation

$\partial \rho / \partial t + \operatorname{div} \vec{j} = 0$

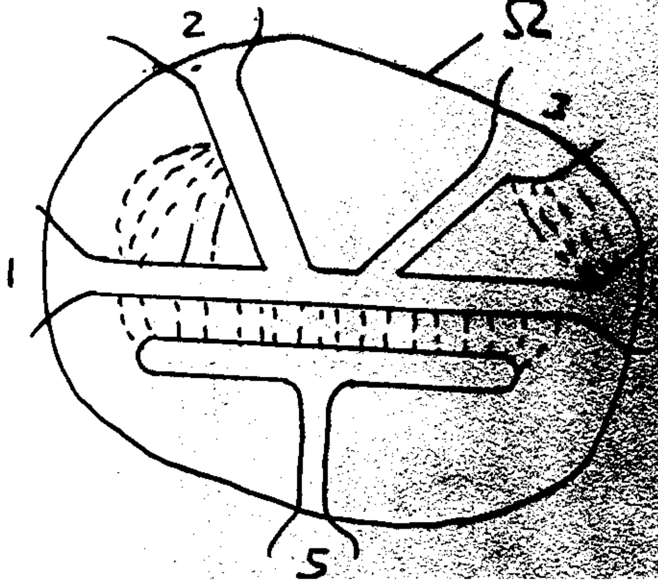
$\operatorname{div} \vec{E} = \frac{1}{\varepsilon} \rho \quad , \quad \vec{D} = \varepsilon \vec{E}$

$$\operatorname{div}\left(\frac{\partial \vec{D}}{\partial t} + \vec{j}\right) = 0$$

Assume that external circuit can be descrибed in quasi-stationary limit $\leftrightarrow$ $\partial \vec{D}/\partial t \approx 0$ in external circuit. $\Rightarrow$

$$\boxed{\sum_\alpha \delta I_\alpha(\omega) = 0}$$

$\int_\Omega d^3r \, \rho(\omega, r) =$



Program

$\delta\mu_\alpha(\omega)$

$\downarrow$

$\delta I_\alpha(\omega) = \sum_\beta g^e_{\alpha\beta}(\omega)\,(\delta\mu_\beta(\omega)/e)$

$$\left.\begin{array}{l} \delta n_\alpha(\omega, r) \\ \downarrow \\ \delta U^i_\alpha(\omega, r) \\ \delta n^i_\alpha(\omega, r) \end{array}\right\} \text{self-consistent internal potential}$$

$\downarrow$

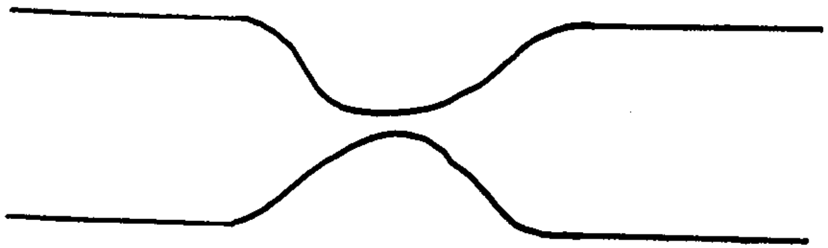
$\delta I_\alpha(\omega) = \sum_\beta g^i_{\alpha\beta}(\omega)\,(\delta\mu_\beta(\omega)/e)$

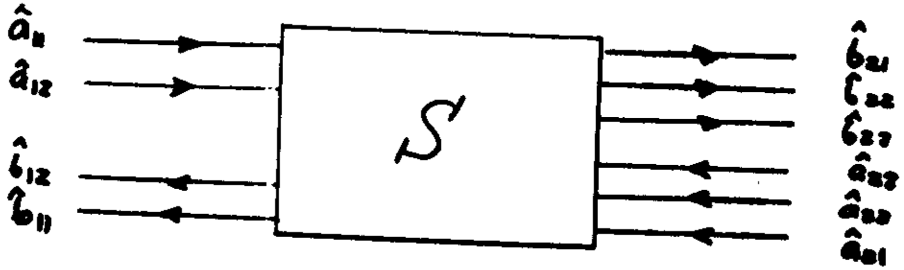
$\downarrow$

$\delta I_\alpha(\omega) = \sum_\beta \left(g^e_{\alpha\beta}(\omega) + g^i_{\alpha\beta}(\omega)\right)(\delta\mu_\beta(\omega)/e)$

Response to External Potentials

Current operator





$$\hat{\psi}(r,t) = \hat{\psi}_+(r,t) + \hat{\psi}_-(r,t) \rightarrow \hat{j}(r,t) \qquad ([illegible])$$

$$\hat{I}_\alpha(t) = \hat{I}_{\alpha+}(t) - \hat{I}_{\alpha-}(t) =$$

$$= \frac{e}{h}\int dE\, dE' \left(\hat{a}_\alpha^\dagger(E)\hat{a}_\alpha(E') - \hat{b}_\alpha^\dagger(E)\hat{b}_\alpha(E')\right)e^{i(E-E')t/\hbar}$$

"current amplitude": $\left(\frac{e}{h}\right)^{1/2} \hat{a}(E)\, dE$

Current Response to Oscillating External Potentials

$$\mu_\alpha(t) = \mu_0 + \delta\mu_\alpha \cos(\omega t)$$

$\rightarrow$

$$\hat{a}'_{\alpha m}(E,t) = \hat{a}_{\alpha m}(E)\, e^{-i(E_{\alpha m}-\mu_0)t/\hbar - i\varphi(t)}$$

$$\varphi(t) = \int dt'\, \delta\mu_\alpha \cos(\omega t') = (\delta\mu_\alpha/\hbar\omega)\sin(\omega t)$$

$\Longrightarrow$

$$\hat{a}'_{\alpha m}(E) = \hat{a}_{\alpha m}(E) + \hat{a}_{\alpha m}(E+\hbar\omega)(\delta\mu_\alpha/2\hbar\omega) - \hat{a}_{\alpha m}(E-\hbar\omega)(\delta\mu_\alpha/2\hbar\omega) + \ldots$$

$$\underline{\hat{b}}_\alpha(E) = \sum_\beta \underline{\underline{S}}_{\alpha\beta}(E)\, \underline{\hat{a}}_\beta(E)$$

$\Longrightarrow$

$$g^e_{\alpha\beta}(\omega) = \frac{e^2}{h}\int dE\, \mathrm{Tr}\left[\mathbf{1}_\alpha(E)\delta_{\alpha\beta} - S^\dagger_{\alpha\beta}(E)S_{\alpha\beta}(E+\hbar\omega)\right]\frac{f_\beta(E)-f_\beta(E+\hbar\omega)}{\hbar\omega}$$

Low-frequency expansion

$$g_{\alpha\beta}(\omega) = \frac{e^2}{h}\int dE\, Tr\left[1_\alpha(E)\delta_{\alpha\beta} - S^{\dagger}_{\alpha\beta}(E) S_{\alpha\beta}(E+\hbar\omega)\right] \frac{f_\beta(E) - f_\beta(E+\hbar\omega)}{\hbar\omega}$$

$$g_{\alpha\alpha}(0) = \frac{e^2}{h}\int dE\ (-df/dE)\ R_{\alpha\alpha}(E)$$

$$g_{\alpha\beta}(0) = -\frac{e^2}{h}\int dE\ (-df/dE)\ T_{\alpha\beta}(E)$$

$$\Sigma_\alpha\, g_{\alpha\beta}(0) = \Sigma_\beta\, g_{\alpha\beta}(0) = 0$$

First order in ω

$$\boxed{g_{\alpha\beta}(\omega) = g_{\alpha\beta}(0) - i\omega e^2 \int dE\, (-df/dE)(dN_{\alpha\beta}/dE)}$$

$$\frac{dN_{\alpha\beta}}{dE} = \frac{1}{4\pi i}\, Tr\left[S^{\dagger}_{\alpha\beta}(E)\frac{\partial S_{\alpha\beta}(E)}{\partial E} - \frac{\partial S^{\dagger}_{\alpha\beta}(E)}{\partial E} S_{\alpha\beta}(E)\right]$$

Emittance of α

$$\boxed{\Sigma_\beta\, g_{\alpha\beta}(\omega) = -i\omega e^2 \int dE\, (-df/dE)(dN_\alpha/dE) \neq 0}$$

$$dN_\alpha/dE = \Sigma_\beta\, (dN_{\alpha\beta}/dE)$$

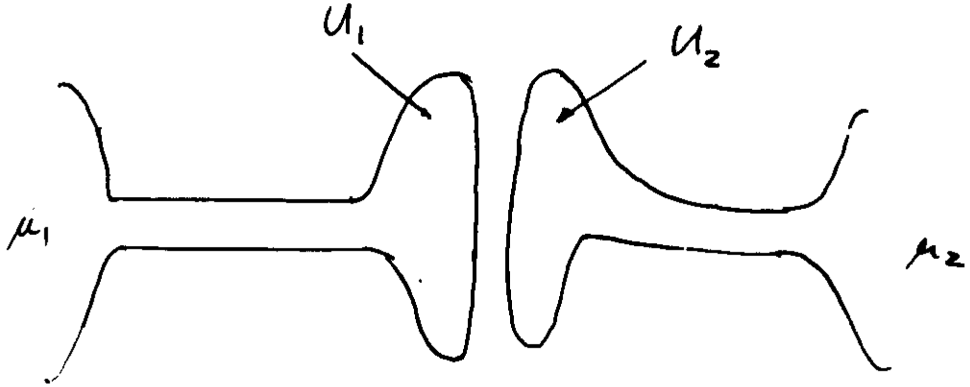
Injectance of β

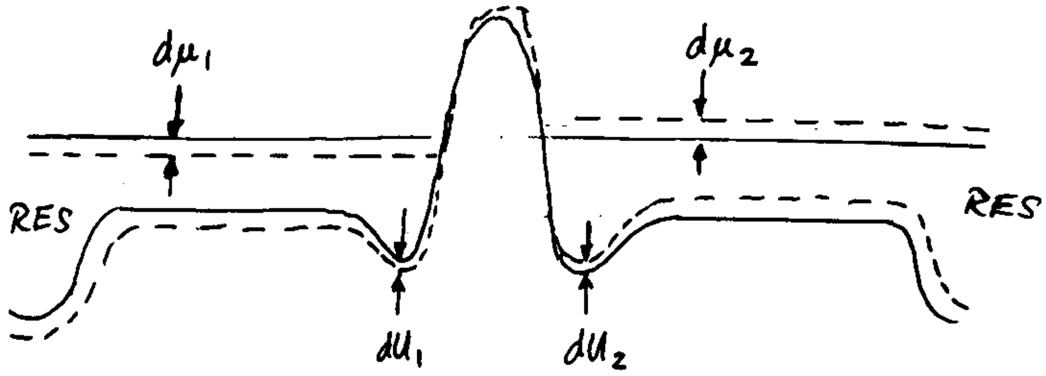
$$\boxed{\Sigma_\alpha\, g_{\alpha\beta}(\omega) = -i\omega e^2 \int dE\, (-df/dE)(d\tilde{N}_\beta/dE) \neq 0}$$

$$d\tilde{N}_\beta/dE = \Sigma_\alpha\, (dN_{\alpha\beta}/dE)$$

$$B = 0 \Rightarrow dN_\alpha/dE = d\tilde{N}_\beta/dE$$

Mesoscopic Capacitor





$$dQ_1 = e\,(dN_1/dE)(d\mu_1 - e\,dU_1) = C_e\,(dU_1 - dU_2)$$

$$dQ_2 = e\,(dN_2/dE)(d\mu_2 - e\,dU_2) = C_e\,(dU_2 - dU_1)$$

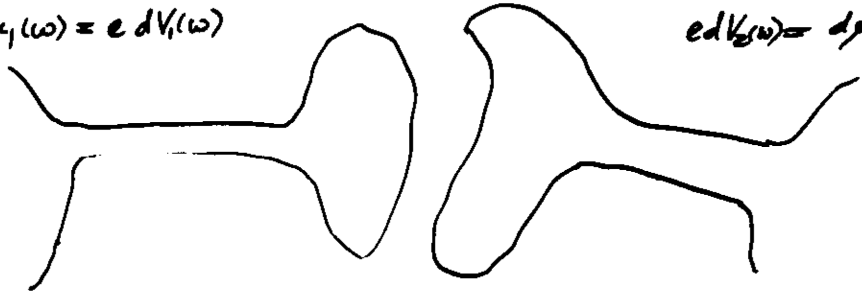
$$dQ = dQ_1 = -dQ_2 = C_\mu\,(d\mu_1 - d\mu_2)/e \Rightarrow$$

$$\boxed{\frac{1}{C_\mu} = \frac{d\mu}{e\,dQ} = \frac{1}{C_e} + \frac{1}{e^2\, dN_1/dE} + \frac{1}{e^2}\frac{1}{dN_2/dE}}$$

electro-chemical capacitance

Admittance of a Mesoscopic Capacitor

$d\mu_1(\omega) = e\, dV_1(\omega)$ $\qquad$ $e\, dV_2(\omega) = d\mu_2(\omega)$



$$dI_1(\omega) = g^e_{11}(\omega)\, dV_1(\omega) + g^i_{11}(\omega)\, dU_1(\omega) = -i\omega C_e (dU_1(\omega) - dU_2(\omega))$$
$$dI_2(\omega) = g^e_{22}(\omega)\, dV_2(\omega) + g^i_{22}(\omega)\, dU_2(\omega) = -i\omega C_e (dU_2(\omega) - dU_1(\omega))$$

Invariance against global potentialshift $\Rightarrow$

$$g^i_{11}(\omega) = -g^e_{11}(\omega) \;;\quad g^i_{22}(\omega) = -g^e_{22}(\omega)$$

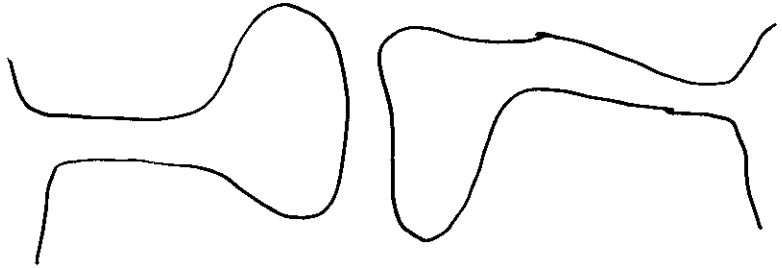
$$\Rightarrow \quad dI(\omega) = g^I(\omega)\,(dV_1(\omega) - dV_2(\omega))$$

$$\boxed{g^I(\omega) = -i\omega C_\mu + \omega^2 C_\mu^2 R_q + O(\omega^3)}$$

$R_q = R_1 + R_2$ $\qquad$ charge relaxation resistance

$$R_k = \frac{h}{2e^2} \frac{\int dE\, (-df_k/dE)\, Tr[S^\dagger_{\alpha k}(E)\, dS_{\alpha k}(E)/dE]^2}{\left(\int dE\, (-df_k/dE)\, Tr[S^\dagger_{\alpha k}(E)\, dS_{\alpha k}(E)/dE]\right)^2}$$

Noise Spectrum of a Capacitor



$$g^I(\omega) = -i\omega C_\mu + \omega^2 C_\mu^2 R_q + O(\omega^3)$$

$\langle(\Delta I(\omega))^2\rangle$ follows from FDT $\qquad$ MB PRB45, 3807(9 PRB46, 12485 (9

$$I_1 = g_{11}(\omega)\,(dV_1(\omega) - dU_1(\omega)) + \delta I_1(\omega) = -i\omega C_e (dU_1(\omega) - dU_2(\omega$$
$$I_2 = g_{22}(\omega)\,(dV_2(\omega) - dU_2(\omega)) + \delta I_2(\omega) = -i\omega C_e (dU_2(\omega) - dU_1(\omega$$

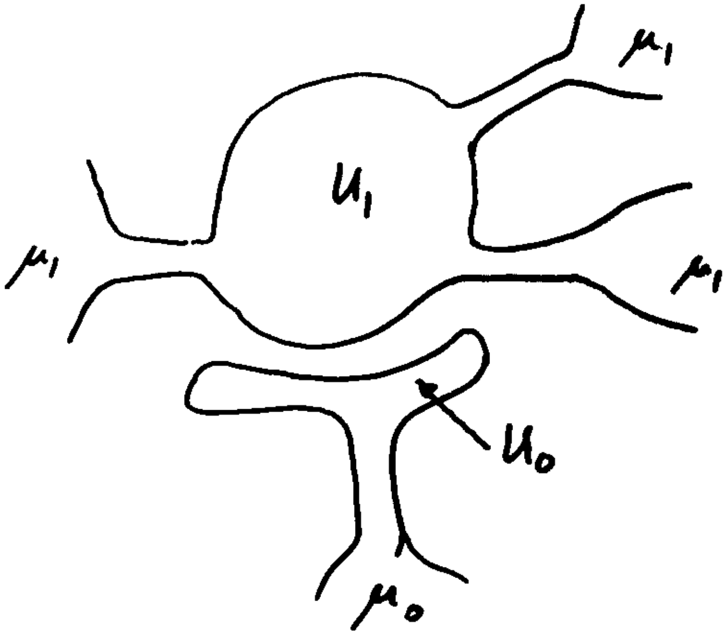
zero external impedance $\rightarrow dV_1 = dV_2 = 0 \Rightarrow$

$$\langle(\Delta I(\omega))^2\rangle = 2\,|g^I(\omega)|^2 \left(\frac{g'_{11}(\omega)}{|g_{11}(\omega)|^2} + \frac{g'_{22}(\omega)}{|g_{22}(\omega)|^2}\right) \varepsilon(\hbar\omega, kT)$$
$$= 2\,(g^I(\omega))'\, \varepsilon(\hbar\omega, kT)$$
$$= 2kT\,\omega^2 C_\mu^2 R_q + O(\omega^4)$$

infinite external impedance $\rightarrow dI_1 = dI_2 = 0 \Rightarrow$

$$\boxed{\langle(dV(\omega))^2\rangle = 2kT R_q + O(\omega^2)}$$

Electro-Chemical Capacitance



$$dQ_1 = e\left(\frac{dN_1}{dE}\right)(d\mu_1 - e\,dU_1) = C\,(dU_1 - dU_2)$$

$$dQ_2 = e\left(\frac{dN_0}{dE}\right)(d\mu_0 - e\,dU_0) = C\,(dU_2 - dU_1)$$

$$\frac{dN_1}{dE} = \sum_\alpha \frac{dN_\alpha}{dE} = \sum_\alpha \frac{d\tilde{N}_\alpha}{dE} = \sum_{\alpha\beta} \frac{dN_{\alpha\beta}}{dE}$$

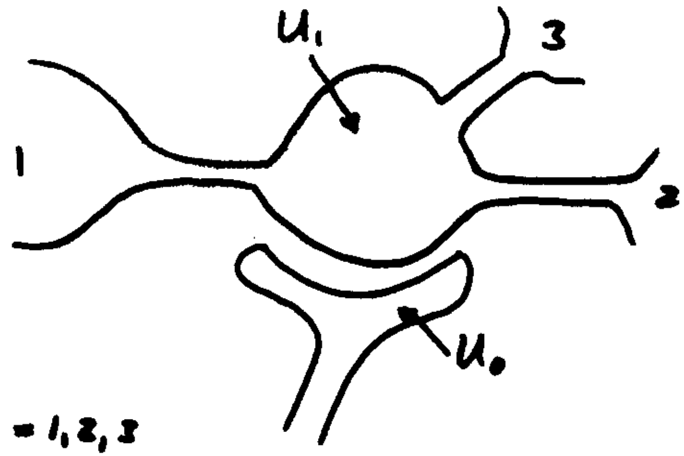
$\rightarrow$

$$dQ = dQ_1 = -dQ_2 = C_\mu\,(d\mu_1 - d\mu_2)/e$$

$\rightarrow$

$$\boxed{\frac{1}{C_\mu} = \frac{1}{C} + \frac{1}{e^2\frac{dN_0}{dE}} + \frac{1}{e^2\frac{dN_1}{dE}}}$$

Simple Model with two Internal Voltages



$\alpha = 1, 2, 3$

$$I_\alpha = \sum_\beta g^e_{\alpha\beta}\,dV_\beta + g^i_{\alpha 1}\,dU_1$$

$$I_0 = g^e_{00}\,dV_0 + g^i_{00}\,dU_0$$

$$\sum_\alpha I_\alpha = -i\omega C\,(dU_1 - dU_0)$$

$$I_0 = -i\omega C\,(dU_0 - dU_1)$$

Invariance against global potential shift

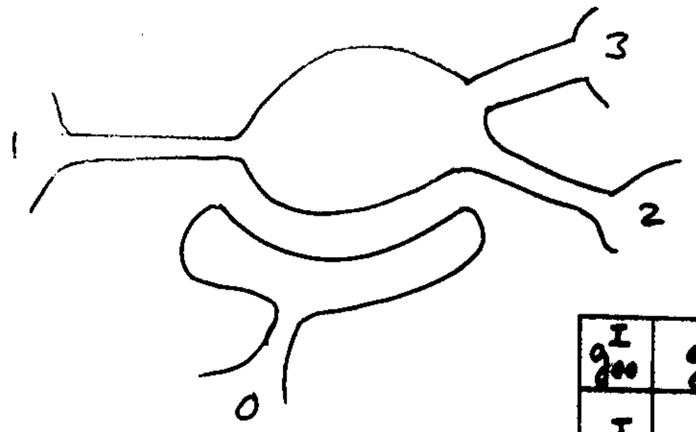
$$(dV_\alpha,\ dU_0,\ dU_1) \leftrightarrow (dV_\alpha - dU_1,\ dU_0 - dU_1,\ 0) \leftrightarrow$$

$$(dV_\alpha - dU_0,\ 0,\ dU_1 - dU_0)$$

$\Longrightarrow$

$$g^i_{00} = -g^e_{00}, \quad g^i_{\alpha 1} = -\sum_\beta g^e_{\alpha\beta}$$

Simple Model: Lowest Order Admittance Tensor



$$\frac{1}{C_\mu} = \frac{1}{C} + \frac{1}{e^2(dN_0/dE)} + \frac{1}{e^2(dN_1/dE)}$$

$$\frac{1}{C_0} = \frac{1}{C} + \frac{1}{e^2(dN_0/dE)}$$

g^I_{00}	g^I_{01}	g^I_{02}	g^I_{03}
g^I_{10}	g^I_{11}	g^I_{12}	g^I_{13}
g^I_{20}	g^I_{21}	g^I_{22}	g^I_{23}
g^I_{30}	g^I_{31}	g^I_{32}	g^I_{33}

$$g^I_{00}(\omega) = -i\omega C_\mu$$

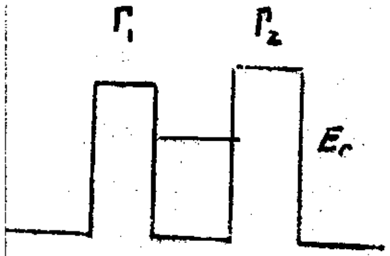
$$g^I_{0\alpha}(\omega) = i\omega C_\mu \frac{d\tilde{N}_\alpha/dE}{dN_1/dE}$$

$$g^I_{\alpha 0}(\omega) = i\omega C_\mu \frac{dN_\alpha/dE}{dN_1/dE}$$

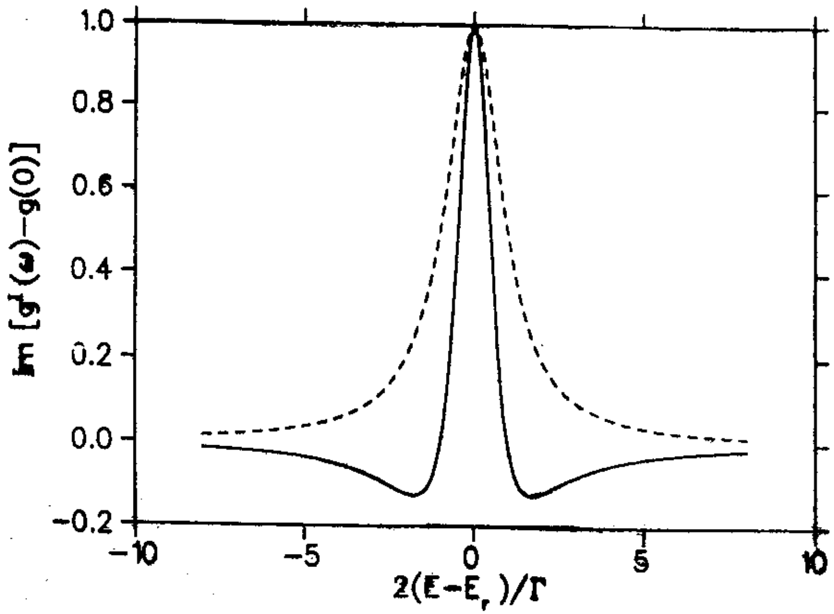
$$g^I_{\alpha\beta}(\omega) = g_{\alpha\beta}(0) - i\omega e^2\, dN_{\alpha\beta}/dE + i\omega e^2 \frac{C_\mu}{C_0} \frac{(dN_\alpha/dE)(d\tilde{N}_\beta/dE)}{(dN_1/dE)}$$

$$\frac{C_\mu - C_0}{C_0} = -\frac{C_\mu}{e^2(dN_1/dE)}$$

Two simple limits: | N [illegible]



$$S_{\alpha\beta}(E) = \left(\delta_{\alpha\beta} - \frac{i\,\Gamma_\alpha^{1/2}\Gamma_\beta^{1/2}}{E - E_r - eU + i\frac{\Gamma}{2}}\right) e^{i(\delta_\alpha + \delta_\beta)}$$

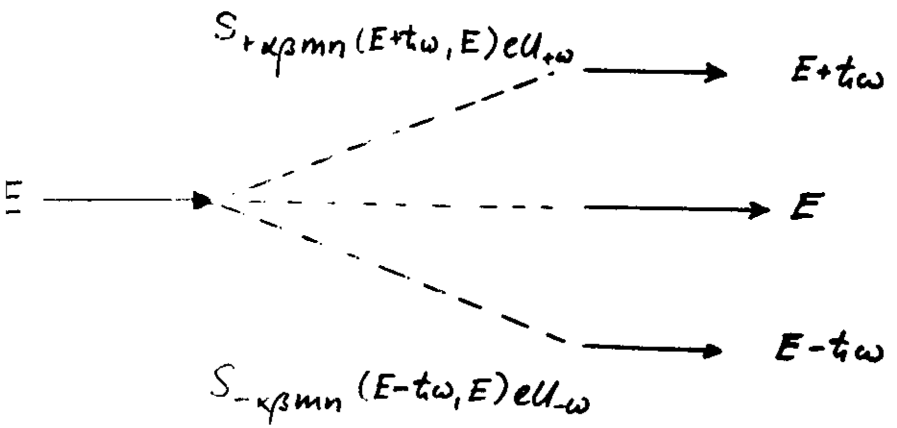


$$g^I(\omega) = g(0) + i\left(\frac{\omega e^2}{2\pi\Gamma}\right)\left(\frac{\Gamma_1\Gamma_2}{|\Delta|^2}\right)\frac{(\Gamma^2/2 - |\Delta|^2)}{|\Delta|^2}$$

$$|\Delta|^2 = (E - E_r)^2 + \Gamma^2/4$$

...rrent Response to an Arbitrary Internal Potential

$$(r,t) = v(r)\,(U_{+\omega} e^{-i\omega t} + U_{-\omega} e^{i\omega t})$$



$$_m(E) = \sum_{\beta n} \Big(S_{\alpha\beta mn}(E)\,\hat{a}_{\beta n}(E) + S_{-\alpha\beta mn}(E, E+\hbar\omega)\,eU_{\omega}\,\hat{a}_{\beta n}(E+\hbar\omega) + S_{+\alpha\beta mn}(E, E-\hbar\omega)\,eU_{\omega}\,\hat{a}_{\beta n}(E-\hbar\omega) + \cdots \Big)$$

$$^{i}(\omega) = -\frac{e^2}{h}\sum_\beta \int dE$$

$$[S^{\dagger}_{\alpha\beta}(E)\,S_{+\alpha\beta}(E+\hbar\omega, E)\,f(E) + S^{\dagger}_{\alpha\beta}(E, E+\hbar\omega)\,S_{\alpha\beta}(E+\hbar\omega)\,f(E+\hbar\omega)]$$

nitarity: $\sum_\beta [S^{\dagger}_{-\alpha\beta}(E, E+\hbar\omega)\,S_{\alpha\beta}(E+\hbar\omega) + S^{\dagger}_{\alpha\beta}(E)\,S_{+\alpha\beta}(E+\hbar\omega, E)] = 0$

$$^{i}(\omega) = -\frac{e^2}{h}\sum_\beta \int dE\; Tr\big[S^{\dagger}_{\alpha\beta}(E)\,S_{+\alpha\beta}(E+\hbar\omega, E)\big](f(E) - f(E+\hbar\omega))$$

Current Response to Slowly Changing Internal Potential

$$g^{i}_{\alpha}(\omega) = -\frac{e^2}{h}\sum_\beta \int dE\; Tr\big[S^{\dagger}_{\alpha\beta}(E)\,S_{+\alpha\beta}(E+\hbar\omega, E)\big](f(E) - f(E+\hbar\omega))$$

$\omega \to 0$?

$$S(E, U(r)) \to S(E, U(r,t))$$

$$U(r,t) = v(r)\,(U_{\omega} e^{-i\omega t} + U_{\omega} e^{i\omega t})$$

$$\lim_{\omega \to 0} S_{+\alpha\beta}(E+\hbar\omega, E) = \int d^3r\, \frac{\delta S_{\alpha\beta}(E)}{\delta eU(r)}\, v(r)$$

$$\boxed{g^{i}_{\alpha}(\omega) = i e^2 \omega \int d^3r\, \frac{dn(\alpha, r)}{dE}\, v(r)}$$

$$\boxed{\frac{dn(\alpha, r)}{dE} = -\frac{1}{4\pi i}\int dE \left(-\frac{df}{dE}\right)\sum_\beta Tr\left[S^{\dagger}_{\alpha\beta}(E)\,\frac{\delta S_{\alpha\beta}(E)}{\delta eU(r)} - \frac{\delta S^{\dagger}_{\alpha\beta}(E)}{\delta eU(r)}\,S_{\alpha\beta}(E)\right]}$$

integrated emissivity modulated by internal potential

Internal Potential from Charge Neutrality

Charge Neutrality Condition

$$\frac{dn(r,\beta)}{dE}\,\mu_\beta + \left(\frac{\delta n(r)}{\delta e U(r)}\right) e U_\beta(r) = 0$$

Screening by all locally available charges

$$\frac{\delta n(r)}{e\,\delta U(r)} = -\sum_\alpha \frac{dn(\alpha,r)}{dE} = -\sum_\alpha \frac{dn(r,\alpha)}{dE} = -\frac{dn_{tot}(r)}{dE}$$

$\Longrightarrow$

$$eU_\beta(r) = \frac{dE}{dn(r)}\,\frac{dn(r,\beta)}{dE}\,d\mu_\beta$$

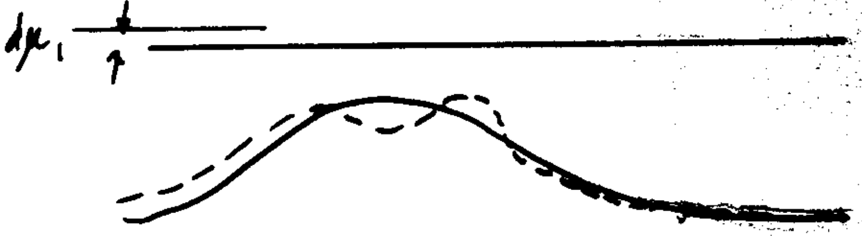
$\Longrightarrow$

$$g^i_\alpha(\omega)\, U_{\beta+\omega} = ie^2\omega \int d^3r\, \frac{dn(\alpha,r)}{dE}\, v_\beta(r)\, U_{\beta+\omega}$$

$$= ie\,\omega \int d^3r\, \frac{dn(\alpha,r)}{dE}\,\frac{dE}{dn(r)}\,\frac{dn(r,\beta)}{dE}\,\delta\mu_\beta(\omega)$$

$\Longrightarrow$

$$\boxed{g^i_{\alpha\beta}(\omega) = ie^2\omega \int d^3r\, \frac{dn(\alpha,r)}{dE}\,\frac{dE}{dn(r)}\,\frac{dn(r,\beta)}{dE}}$$



Admittance of a charge neutral conductor

- Charge neutrality

$$g^e_{\alpha\beta}(\omega) = g_{\alpha\beta}(0) - i\omega e^2\,(dN_{\alpha\beta}/dE)$$

$$g^i_{\alpha\beta}(\omega) = i\omega e^2 \int d^3r\, \frac{\delta n(\alpha,r)}{\delta n(r)}\,\frac{dn(r,\beta)}{dE}$$

$$g^I_{\alpha\beta}(\omega) = g^e_{\alpha\beta}(\omega) + g^i_{\alpha\beta}(\omega)$$

$$\boxed{g^I_{\alpha\beta}(\omega) = g_{\alpha\beta}(0) - i\omega e^2\left[\frac{dN_{\alpha\beta}}{dE} - \int d^3r\, \frac{\delta n(\alpha,r)}{\delta n(r)}\,\frac{dn(r,\beta)}{dE}\right]}$$

$$\delta n(r) = \sum_\alpha \delta n(\alpha,r) \Longrightarrow \sum_\alpha g^I_{\alpha\beta}(\omega) = 0$$

$$\sum_\beta g^I_{\alpha\beta}(\omega) = 0$$

- Compare with simple model: $(C \to 0)$

$$g^I_{\alpha\beta}(\omega) = g_{\alpha\beta}(0) - i\omega e^2\left[\frac{dN_{\alpha\beta}}{dE} - \frac{(dN_\alpha/dE)(d\tilde{N}_\beta/dE)}{(dN/dE)}\right]$$

Long range non-local screening

$$g^I_{\alpha\beta}(\omega) = g_{\alpha\beta}(0) - i\omega e^2\left[\frac{dN_{\alpha\beta}}{dE} - \int d^3r\, d^3r'\, \frac{dn(\alpha,r)}{dE}\, g(r,r')\, \frac{dn(r',\beta)}{dE}\right]$$

Summary

- Current Conservation
- Electro-chemical capacitance
- Response to external potentials
  Response to internal potentials
- Determine $U(r,t)$ self-consistently

References

Büttiker, Prêtre, Thomas, PRL 70, 4114 (1993)
Büttiker, Thomas, Prêtre, Phys. Lett. A (to appear)
Büttiker, Thomas, Prêtre, preprint on frequency dependent current partition (write to M. B.)